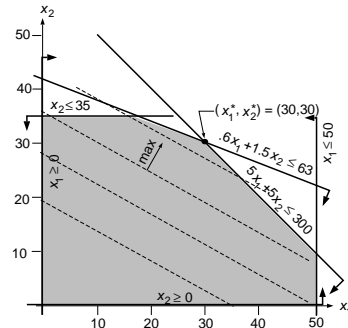


Chapter 2 Solutions ^{1 2}

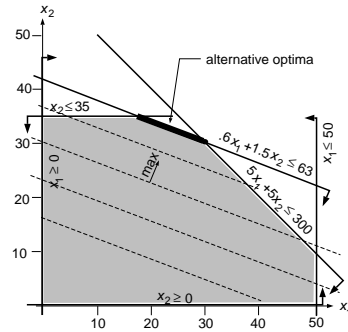
2-1. (a) $\max 200x_1 + 350x_2$ (max total profit), s.t. $5x_1 + 5x_2 \leq 300$ (legs), $0.6x_1 + 1.5x_2 \leq 63$ (assembly hours), $x_1 \leq 50$ (wood tops), $x_2 \leq 35$ (glass tops), $x_1 \geq 0$, $x_2 \geq 0$

(b) x_1^* =basic=30, x_2^* =deluxe=30

(c)



(d)



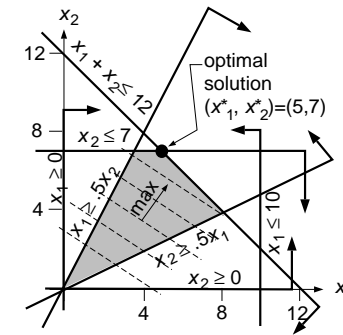
All optimal from $\mathbf{x} = (30, 30)$ to $\mathbf{x} = (17.5, 35)$.

2-2. (a) $\max .11x_1 + .17x_2$ (max total return), s.t. $x_1 + x_2 \leq 12$ (\$12 million investment), $x_1 \leq 10$ (max \$10 million domestic), $x_2 \leq 7$ (max \$7 million foreign), $x_1 \geq .5x_2$ (domestic at least half foreign), $x_2 \geq .5x_1$ (foreign at least half domestic), $x_1 \geq 0$, $x_2 \geq 0$ **(b)** x_1^* =domestic=\$5 million, x_2^* =foreign=\$7 million

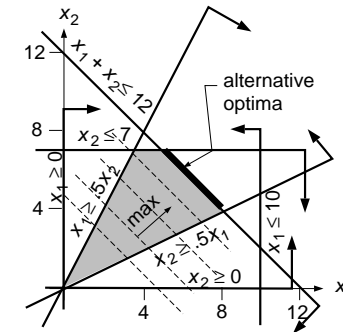
¹Supplement to the 2nd edition of *Optimization in Operations Research*, by Ronald L. Rardin, Pearson Higher Education, Hoboken NJ, ©2017.

²As of September 24, 2015

(c)



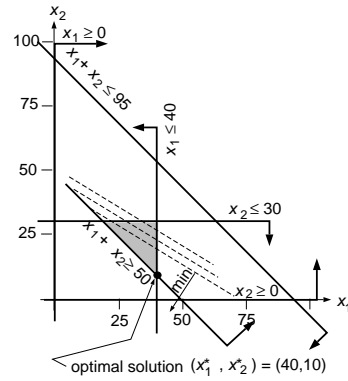
(d)



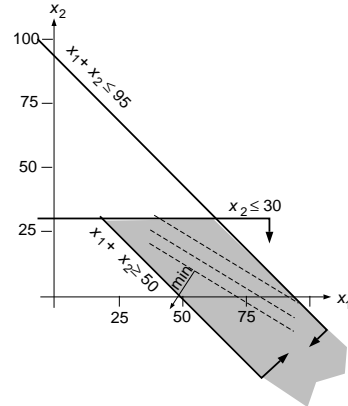
All optimal from $\mathbf{x} = (5, 7)$ to $\mathbf{x} = (8, 4)$.

2-3. (a) $\min 3x_1 + 5x_2$ (min total cost), s.t. $x_1 + x_2 \geq 50$ (at least 50 thousand acres), $x_1 \leq 40$ (at most 40 thousand from Squawking Eagle), $x_2 \leq 30$ (at most 30 thousand from Crooked Creek), $x_1 \geq 0$, $x_2 \geq 0$ **(b)** x_1^* =Squawking Eagle=40 thousand, x_2^* =Crooked Creek=10 thousand

(c)

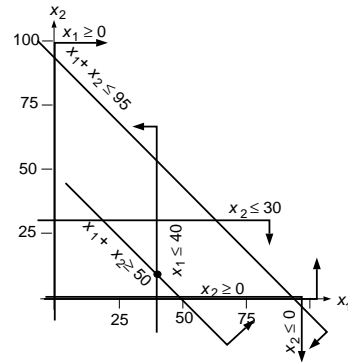


(d)



Improves forever in direction $\Delta x_1 = 1$,
 $\Delta x_2 = -1$.

(e)

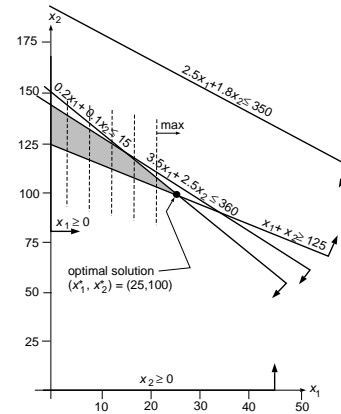


$x_2 = 0$ leaves no feasible.

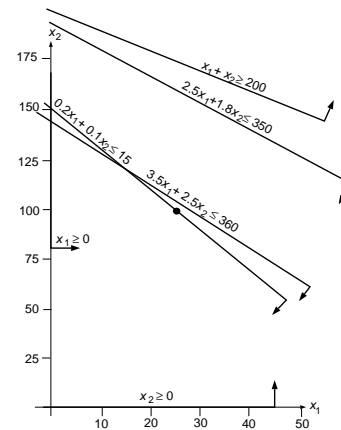
2-4. (a) $\max x_1$ (max beef content), s.t.

$x_1 + x_2 \geq 125$ (weight at least 125),
 $2.5x_1 + 1.8x_2 \leq 350$ (calories at most 350),
 $0.2x_1 + 0.1x_2 \leq 15$ (fat at most 15),
 $3.5x_1 + 2.5x_2 \leq 360$ (sodium at most 360),
 $x_1 \geq 0, x_2 \geq 0$ (b) $x_1^* = \text{beef} = 25\text{g}$,
 $x_2^* = \text{chicken} = 100\text{g}$

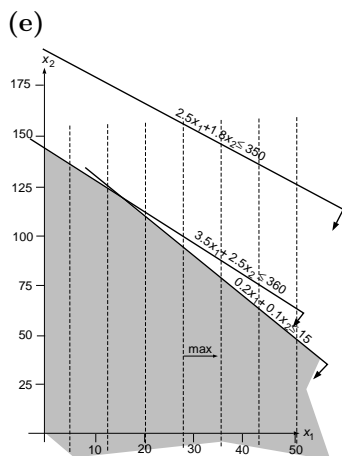
(c)



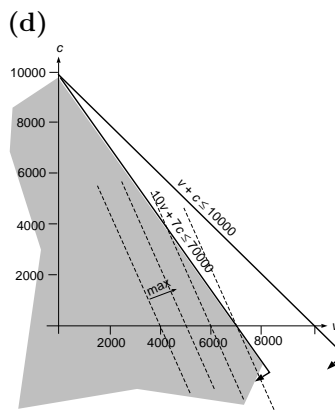
(d)



$x_1 + x_2 \geq 200$ leaves no feasible.

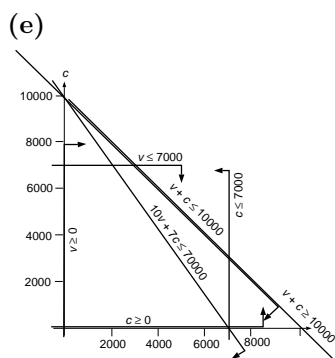


Improve forever in direction $\Delta x_1 = 1$,
 $\Delta x_2 = -2$.

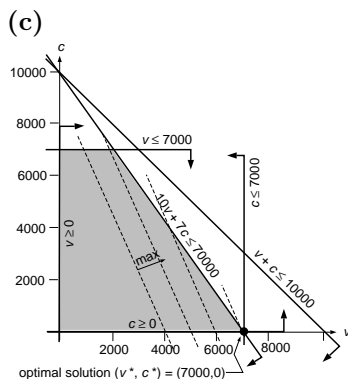


Improves forever in direction $\Delta v = 10$,
 $\Delta c = -7$.

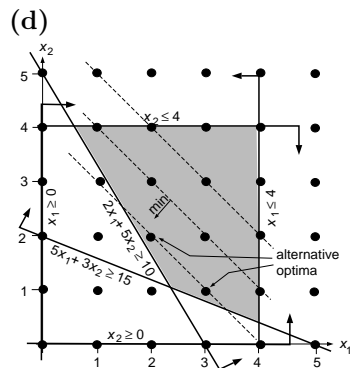
2-5. (a) max $450v + 200c$ (max total profit),
 s.t. $10v + 7c \leq 70000$ (water at most 70000
 units), $v + c \leq 10000$ (total acreage 10000),
 $v \leq 7000$ (at most 70% vegetables), $c \leq 7000$
 (at most 70% cotton), $v \geq 0$, $c \geq 0$ **(b)**
 $v^* = 7000$, $c^* = 0$



No solution with $v + c = 10000$.

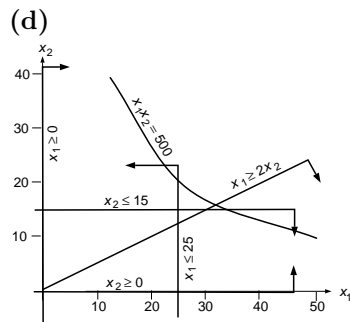
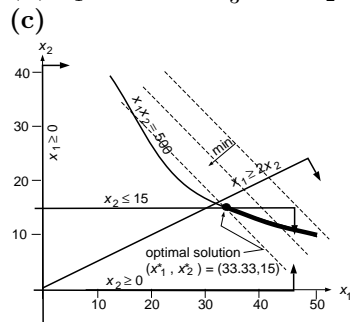


2-6. (a) min $x_1 + x_2$ (min used stock), s.t.
 $5x_1 + 3x_2 \geq 15$ (cut at least 15 long rolls),
 $2x_1 + 5x_2 \geq 10$ (cut at least 10 short rolls),
 $x_1 \leq 4$ (at most 4 times on pattern 1), $x_2 \leq 4$
 (at most 4 times on pattern 2), $x_1, x_2 \geq 0$ and
 integer. **(b)** Partial cuts make no physical
 sense because all unused material is scrap. **(c)**
 Either $x_1^* = x_2^* = 2$, or $x_1^* = 3$, $x_2^* = 1$



(e) Both (2, 2) and (3, 1) are feasible and lie on the best contour of the objective.

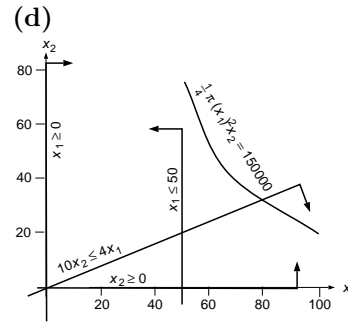
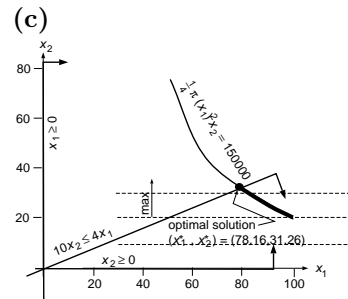
2-7. (a) min $16x_1 + 16x_2$ (min total wall area), s.t. $x_1x_2 = 500$ (500 sqft pool), $x_1 \geq 2x_2$ (length at least twice width), $x_2 \leq 15$ (width at most 15 ft), $x_1 \geq 0, x_2 \geq 0$
(b) x_1^* =length= $33\frac{1}{3}$ feet, x_2^* =width=15 feet



$x_1 \leq 25$ leaves no feasible.

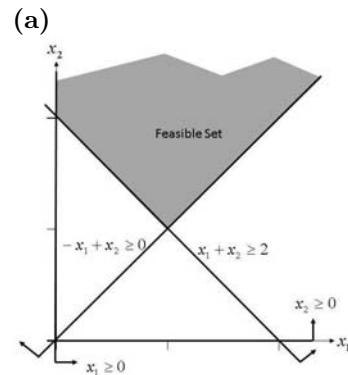
2-8. (a) max x_2 (max number of floors), s.t. $\pi/4(x_1)^2x_2 = 150000$ (150000 sqft floor space), $10x_2 \leq 4x_1$ (height at most 4 times diameter), $x_1 \geq 0, x_2 \geq 0$ **(b)** x_1^* = diameter

= 78.16 feet, x_2^* = floors = 31.26



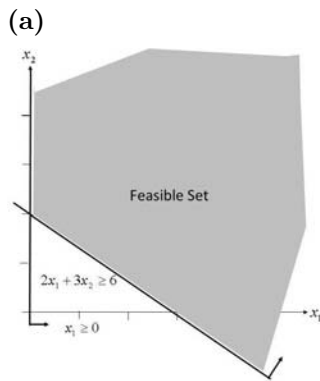
$x_1 \leq 50$ leaves no feasible.

2-9.



(b) min x_2 **(c)** min $x_1 + x_2$ **(d)** max x_2 **(e)** $x_2 \leq 1/2$

2-10.



(b) $\min x_1 + x_2$ (c) $\min x_1$ (d) $\max x_1$ (e) $x_1 + x_2 \leq 1$

2-11. (a) $\min \sum_{i=3}^4 i \sum_{j=1}^2 y_{i,j}$

(b) $\max \sum_{i=1}^4 i y_{i,3}$

(c) $\max \sum_{i=1}^p \alpha_i y_{i,4}$

(d) $\min \sum_{i=1}^t \delta_i y_i$

(e) $\sum_{j=1}^4 y_{i,j} = s_i, i = 1, \dots, 3$

(f) $\sum_{j=1}^4 a_{j,i} y_j = c_i, i = 1, \dots, 3$

2-12. (a) $\sum_{i=1}^{17} x_{i,j,t} \leq 200, j = 1, \dots, 5; t = \dots, 7; 35$ constraints

(b) $\sum_{j=1}^5 \sum_{t=1}^7 x_{5,j,t} \leq 4000; 1$ constraint

(c) $\sum_{j=1}^5 x_{i,j,t} \geq 100, i = 1, \dots, 17; t = 1, \dots, 7; 119$ constraints

2-13. model; param m; param n; param p; set products := 1 .. m; set lines := 1 .. n; set weeks := 1 .. p; var x{i in products, j in lines, t in weeks} >= 0; subject to

part (a)

linecap {j in lines, t in weeks}: sum {i in products} x[i,j,t] <= 200;

part (b)

prod5lim: sum {j in lines, t in weeks} x[5,j,t] <= 4000;

part (c)

minprodn{i in products, t in weeks}: sum {j in lines} x[i,j,t] >= 100;

#

data; param m := 17; param n := 5; param p := 7;

2-14. (a)

$\sum_{j=1}^9 x_{i,j,t} \leq p_i, i = 1, \dots, 47; t = 1, \dots, 10; 470$ constraints

(b) $0.25 \sum_{i=1}^{47} \sum_{j=1}^9 x_{i,j,t} \leq \sum_{i=1}^{47} x_{i,4,t}; t = 1, \dots, 5; 5$ constraints

(c) $x_{i,1,t} \geq x_{i,j,t} i = 1, \dots, 47; j = 1, \dots, 9; t = 1, \dots, 10; 4230$ constraints

2-15. model; param m; param n; param q; set plots := 1 .. m; set crops := 1 .. n; set years := 1 .. q; param p {i in plots}; var x{i in plots, j in crops, t in years} >= 0; subject to

part (a)

acrelims {i in plots, t in years}: sum {j in crops} x[i,j,t] <= p[i];

part (b)

crop4min {t in years: t <= 5}: $0.25 * \sum \{i \text{ in plots, } j \text{ in crops}\} x[i,j,t] <= \sum \{i \text{ in plots}\} x[i,4,t];$

part (c)

beam1st {i in plots, j in crops, t in years}: $x[i,1,t] >= x[i,j,t];$

#

data; param m := 47; param n := 9;

param q := 10;

2-16. (a) $f(y_1, y_2, y_3) \triangleq (y_1)^2 y_2 / y_3,$

$g_1(y_1, y_2, y_3) \triangleq y_1 + y_2 + y_3, b_1 = 13,$

$g_2(y_1, y_2, y_3) \triangleq 2y_1 - y_2 + 9y_3, b_2 = 0,$

$g_3(y_1, y_2, y_3) \triangleq y_1, b_3 = 0, g_4(y_1, y_2, y_3) \triangleq y_3, b_4 = 0$

(b) $f(y_1, y_2, y_3) \triangleq 13y_1 + 22y_2 + 10y_2y_3 + 100,$

$g_1(y_1, y_2, y_3) \triangleq y_1 - y_2 + 9y_3, b_1 = -5,$

$g_2(y_1, y_2, y_3) \triangleq 8y_2 - 4y_3, b_2 = 0, g_3(y_1, y_2, y_3)$

$\triangleq y_1, b_3 = 0, g_4(y_1, y_2, y_3) \triangleq y_2, b_4 = 0,$

$g_5(y_1, y_2, y_3) \triangleq y_3, b_5 = 0,$

2-17. (a) Linear because LHS is a weighted

sum of the decision variables. (b) Linear

because both LHS and RHS are weighted

sums of the decision variables. (c) Nonlinear

because LHS has reciprocal $1/x_9$. (d) Linear

because LHS is a weighted sum of the decision

variables. (e) Nonlinear because LHS has

$(x_j)^2$ terms. (f) Nonlinear because LHS has

$\log(x_1)$ term, and RHS has a product of

variables. **(g)** Nonlinear because LHS has max operator. **(h)** Linear because LHS is a weighted sum of the decision variables.

2-18. **(a)** LP because the objective and all constraints are linear. **(b)** NLP because of the nonlinear objective function with reciprocal of w_2 . **(c)** NLP because of the nonlinear first constraint. **(d)** LP because the objective and all constraints are linear.

2-19. **(a)** Continuous because fractions make sense. **(b)** Discrete because they either closed or not. **(c)** Discrete because a specific process must be used. **(d)** Continuous because fractions can probably be ignored.

2-20. **(a)** $\sum_{j=1}^8 x_j = 3$ **(b)**
 $x_1 + x_2 + x_4 + x_5 \geq 2$ **(c)** $x_3 + x_8 \leq 1$ **(d)**
 $x_4 \geq x_1$

2-21. **(a)** max $85x_1 + 70x_2 + 62x_3 + 93x_4$
(max total score), s.t.

$700x_1 + 400x_2 + 300x_3 + 600x_4 \leq 1000$ (\$1 million available), $x_j = 0$ or 1, $j = 1, \dots, 4$

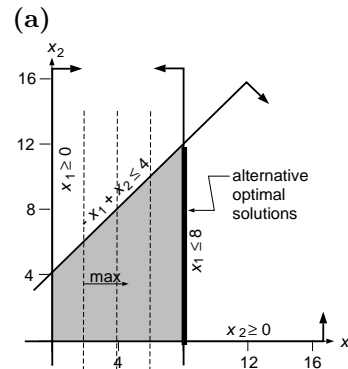
(b) Fund 2 and 4, i.e. $x_1^* = x_3^* = 0$,
 $x_2^* = x_4^* = 1$

2-22. **(a)** min $43y_1 + 175y_2 + 60y_3 + 35y_4$
(min total land cost), s.t. $y_2 + y_4 \geq 1$ (service NW),
 $y_1 + y_2 + y_4 \geq 1$ (service SW),
 $y_2 + y_3 \geq 1$ (service capital), $y_1 + y_4 \geq 1$
(service NE), $y_1 + y_2 + y_3 \geq 1$ (service SE),
 $y_j = 0$ or 1, $j = 1, \dots, 4$ **(b)** Build 3 and 4,
i.e. $y_1^* = y_2^* = 0$, $y_3^* = y_4^* = 1$

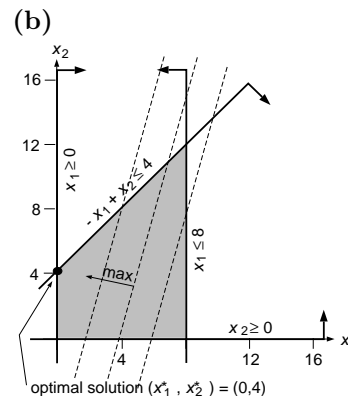
2-23. **(a)** ILP because the objective and all constraints are linear, but variables are discrete. **(b)** NLP because the objective is nonlinear and all variables are continuous. **(c)** INLP because the objective is nonlinear and variables are discrete. **(d)** LP because the objective and all constraints are linear, and all variables are continuous. **(e)** INLP because the one constraint is nonlinear, and z_3 are discrete. **(f)** ILP because the objective and all constraints are linear, but variables z_1 and z_3 are discrete. **(g)** LP because the objective and all constraints are linear, and all variables are continuous. **(h)** INLP because the objective is nonlinear and z_3 is discrete.

2-24. **(a)** Model (d) because LP's are generally more tractable than ILP's. **(b)** Model (d) because LP's are generally more tractable than NLP's. **(c)** Model (d) because LP's are generally more tractable than INLP's. **(d)** Model (f) because ILP's are generally more tractable than INLP's. **(e)** Model (g) because LP's are generally more tractable than ILP's.

2-25.

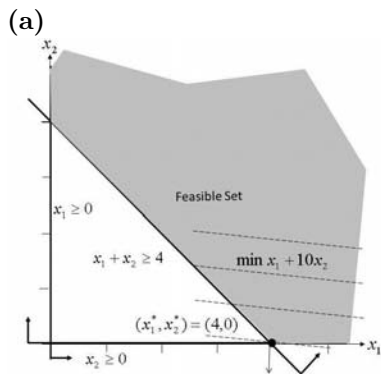


Alternative optima from $x_1^* = 8$, $x_2^* = 0$ to $x_1^* = 8$, $x_2^* = 12$

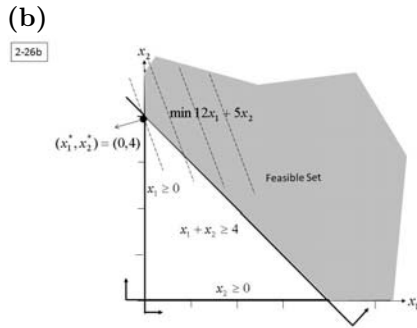


Unique optimum $x_1^* = 0$, $x_2^* = 4$ **(c)** Helping one can hurt the other.

2-26.



Unique optimum $x_1^* = 4$, $x_2^* = 0$



Unique optimum $x_1^* = 0$, $x_2^* = 4$ (c) Helping one can hurt the other.

2-27. (a) min
 $.092x_4 + .112x_5 + .141x_6 + .420x_9 + .719x_{12}$
 (min total cost),
 s.t. $x_4 + x_5 + x_6 + x_9 + x_{12} = 16000$ (16000m line),
 $.279x_4 + .160x_5 + .120x_6 + .065x_9 + .039x_{12} \leq 1600$ (at most 1600 Ohms resistance),
 $.00175x_4 + .00130x_5 + .00161x_6 + .00095x_9 + .00048x_{12} \leq 8.5$ (at most 8.5 dBell attenuation),
 $x_4, x_5, x_6, x_9, x_{12} \geq 0$

(b) Nonzeros: $x_5^* = 1000$, $x_{12}^* = 15000$

2-28. (a) Pump rates are the decisions to be made.

(b) $u_j \triangleq$ the capacity of pump j , $c_j \triangleq$ the pumping cost of pump j

(c) min $\sum_{j=1}^{10} c_j x_j$

(d) $x_1 + x_4 + x_7 \leq 3000$ (well 1),

$x_2 + x_5 + x_8 \leq 2500$ (well 2),

$x_3 + x_6 + x_9 + x_{10} \leq 7000$ (well 3)

(e) $x_j \leq u_j$, $j = 1, \dots, 10$

(f) $\sum_{j=1}^{10} x_j \geq 10000$

(g) $x_j \geq 0$, $j = 1, \dots, 10$

(h) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(i) $x_1^* = x_2^* = x_3^* = 1100$, $x_4^* = x_6^* = 1500$,
 $x_5^* = 1400$, $x_7^* = 400$; $x_8^* = x_{10}^* = 0$, $x_9^* = 1900$

2-29. (a) The decisions to be made are which projects to undertake.

(b) $p_j \triangleq$ the profit for project j , $m_j \triangleq$ the man-days required on project j , and $t_j \triangleq$ the CPU time required on project j .

(c) max $\sum_{j=1}^8 p_j x_j$

(d) $7 \leq \left(\sum_{j=1}^8 m_j x_j \right) / 240 \leq 10$

(e) $\sum_{j=1}^8 t_j x_j \leq 1000$ (computer time),

$\sum_{j=1}^8 x_j \geq 3$ (select at least 3);

$x_3 + x_4 + x_5 + x_8 \geq 1$ (include at least 1 of director's favorites)

(f) $x_j = 0$ or 1, $j = 1, \dots, 8$

(g) A single objective ILP because the one objective and all constraints are linear, but variables are discrete.

(h) $x_1^* = x_3^* = x_6^* = x_7^* = 1$, others = 0

2-30. (a) We must decide what quantities to move from surplus sites to fulfill each need.

(b) $s_i \triangleq$ the supply available at i , $r_j \triangleq$ the quantity needed at j , $d_{i,j} \triangleq$ the distance from i to j .

(c) min $\sum_{i=1}^4 \sum_{j=1}^7 d_{i,j} x_{i,j}$

(d) $\sum_{j=1}^7 x_{i,j} = s_i$, $i = 1, \dots, 4$

(e) $\sum_{i=1}^4 x_{i,j} = r_j$, $j = 1, \dots, 7$

(f) $x_{i,j} \geq 0$, $i = 1, \dots, 4$, $j = 1, \dots, 7$

(g) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(h) Nonzeros: $x_{1,1}^* = 81$, $x_{1,2}^* = 93$,
 $x_{1,3}^* = 166$, $x_{1,5}^* = 90$, $x_{1,6}^* = 85$, $x_{1,7}^* = 145$,
 $x_{2,2}^* = 301$, $x_{3,1}^* = 166$, $x_{3,4}^* = 105$, $x_{4,3}^* = 99$

2-31. (a) The values to be chosen are the

coefficients in the estimating relationship.

(b) $\min \sum_{j=1}^n (c_j - k/(1 + e^{a+bf_j}))^2$ (min total squared error)

(c) Single objective NLP because the objective is quadratic, there are no constraints, and all variables are continuous.

2-32. (a) The decisions to be made are where to assign each teacher.

(b) $\min \sum_{i=1}^{22} \sum_{j=1}^{22} c_{i,j} x_{i,j}$ (min total cost),
 $\max \sum_{i=1}^{22} \sum_{j=1}^{22} t_{i,j} x_{i,j}$ (max total teacher preference), $\max \sum_{i=1}^{22} \sum_{j=1}^{22} s_{i,j} x_{i,j}$ (max total supervisor preference), $\max \sum_{i=1}^{22} \sum_{j=1}^{22} p_{i,j} x_{i,j}$ (max total principal preference)

(c) $\sum_{j=1}^{22} x_{i,j} = 1, i = 1, \dots, 22$ (each teacher i)

(d) $\sum_{i=1}^{22} x_{i,j} = 1, j = 1, \dots, 22$ (each school j)

(e) $x_{i,j} = 0$ or $1, i, j = 1, \dots, 22$

(f) A multiobjective ILP because the 4 objectives and all constraints are linear, but variables are discrete.

2-33. (a) Each task must go to Assistant 0 or Assistant 1.

(b) $\max 100(1 - x_1) + 80x_1 + 85(1 - x_2) + 70x_2 + 40(1 - x_3) + 90x_3 + 45(1 - x_4) + 85x_4 + 70(1 - x_5) + 80x_5 + 82(1 - x_6) + 65x_6$

(c) $\sum_{j=1}^6 x_j = 3$

(d) $x_5 = x_6$

(e) $x_j = 0$ or $1, j = 1, \dots, 6$

(f) A single objective ILP because the one objective and all constraints are linear, but variables are discrete.

(g) $x_2^* = x_3^* = x_4^* = 1$, others $= 0$

2-34. (a) Batch sizes are the decisions to be made.

(b) $\min x_j/d_j, j = 1, \dots, 4$ (each burger j)

(c) $\sum_{j=1}^4 t_j d_j / x_j \leq 60$

(d) $0 \leq x_j \leq u_j, j = 1, \dots, 4$

(e) Multiobjective NLP because the first constraint is nonlinear and all variables are continuous.

2-35. (a) The issue is how many cars to move from where to where.

(b) Relatively large values can be rounded if fractional without much loss, and continuous is more tractable.

(c) $c_{i,j} \triangleq$ the cost of moving a car from i to j ,
 $p_j \triangleq$ the number of cars presently at j , $n_j \triangleq$ the number of cars required at j

(d) $\min \sum_{i=1}^5 \sum_{j=1, j \neq i}^5 c_{i,j} x_{i,j}$

(e) $\sum_{i=1, i \neq k}^5 x_{i,k} - \sum_{j=1, j \neq k}^5 x_{k,j} = n_k - p_k$,
 $k = 1, \dots, 5$ (each region k)

(f) $x_{i,j} \geq 0, i, j = 1, \dots, 5, i \neq j$

(g) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(h) Nonzero values: $x_{4,2}^* = 115, x_{4,3}^* = 165$,
 $x_{5,1}^* = 85, x_{5,3}^* = 225$

2-36. (a) We must decide how much of what fuel to burn at each plant.

(b) $\min \sum_{f=1}^4 \sum_{p=1}^{23} c_{f,p} x_{f,p}$

(c) $\min \sum_{f=1}^4 s_f \sum_{p=1}^{23} x_{f,p}$

(d) $\sum_{f=1}^4 e_f x_{f,p} \geq r_p, p = 1, \dots, 23$ (each plant p); 23 constraints

(e) $x_{f,p} \geq 0, f = 1, \dots, 4, p = 1, \dots, 23$; 92 constraints

(f) A multiobjective LP because the 2 objectives and all constraints are linear, and all variables are continuous.

2-37. (a) The available options are to buy whole logs or green lumber.

(b) Relatively large magnitudes can be rounded without much loss, and continuous is more tractable.

(c) \min

$70x_{10} + 200x_{15} + 620x_{20} + 1.55y_1 + 1.30y_2$

(d) $100(.09)x_{10} + 240(.09)x_{15} + 400(.09)x_{20} + .10y_1 + .08y_2 \geq 2350$

(e) $x_{10} + x_{15} + x_{20} \leq 1500$ (sawing capacity),
 $100x_{10} + 240x_{15} + 400x_{20} + y_1 + y_2 \leq 26500$ (drying capacity)

(f) $x_{10} \leq 50$ (size 10 log availability),

$x_{15} \leq 25$ (size 15 log availability), $x_{20} \leq 10$ (size 20 log availability), $y_1 \leq 5000$ (grade 1 green lumber availability)

(g) $x_{10}, x_{15}, x_{20}, y_1, y_2 \geq 0$

(h) A single objective LP because the one

objective and all constraints are linear, and all variables are continuous.

(i) $x_{10}^* = 50$, $x_{15}^* = 25$, $x_{20}^* = 5$, $y_1^* = 5000$, $y_2^* = 8500$

2-38. (a) Decisions to be made are when to schedule each film.

(b) $\min \sum_{j=1}^{m-1} \sum_{j'=j+1}^m a_{j,j'} \sum_{t=1}^n x_{j,t} x_{j',t}$
 (c) $\sum_{t=1}^n x_{j,t} = 1$, $j = 1, \dots, m$ (each film j)
 (d) $\sum_{j=1}^m x_{j,t} \leq 4$, $t = 1, \dots, n$ (each time t)
 (e) $x_{j,t} = 0$ or 1 , $j = 1, \dots, m$; $t = 1, \dots, n$
 (f) A single objective INLP because the one objective is nonlinear, and variables are discrete. (g) model; param m ; param n ; set films := 1 .. m ; set slots := 1 .. n ; var $x\{j \text{ in films}, t \text{ in slots}\}$ binary; param $a\{j \text{ in films}, jp \text{ in films}\}$; minimize totconflict: $\sum\{j \text{ in films}, jp \text{ in films}: j < m \text{ and } jp > j\} a[j,jp] * \sum\{t \text{ in slots}\} x[j,t] * x[jp,t]$; subject to allin $\{j \text{ in films}\}$: $\sum\{t \text{ in slots}\} x[j,t] = 1$; max4 $\{t \text{ in slots}\}$: $\sum\{j \text{ in films}\} x[j,t] \leq 4$;

2-39. (a) We need to decide both which offices to open and how to service customers from them.

(b) Offices must either be opened or not.

(c) $f_i \triangleq$ fixed cost of site i , $c_{i,j} \triangleq$ unit cost of audits at j from i , $r_j \triangleq$ required number of audits in state j

(d) $\min \sum_{i=1}^5 \sum_{j=1}^5 c_{i,j} r_j x_{i,j} + \sum_{i=1}^5 f_i y_i$

(e) $\sum_{i=1}^5 x_{i,j} = 1$, $j = 1, \dots, 5$ (each location j)

(f) $x_{i,j} \leq y_i$, $i, j = 1, \dots, 5$ (each site i , location j combination)

(g) $x_{i,j} \geq 0$, $i, j = 1, \dots, 5$, $y_i = 0$ or 1 , $i = 1, \dots, 5$

(h) A single objective ILP because the one objective and all constraints are linear, but the y_i variables are discrete.

(i) Nonzeros:

$x_{2,2}^* = x_{2,4}^* = x_{3,1}^* = x_{3,3}^* = x_{5,5}^* = 1$,

$y_2^* = y_3^* = y_5^* = 1$ (j) model; param m ; param n ; set sites := 1 .. m ; set

states := 1 .. n ; var $x\{i \text{ in sites}, j \text{ in states}\} \geq 0$; var $y\{i \text{ in sites}\}$ binary; param $c\{i \text{ in sites}, j \text{ in states}\}$; param $f\{i \text{ in sites}\}$ binary; param $r\{j \text{ in states}\}$; minimize totcost: $\sum\{i \text{ in sites}, j \text{ in states}\} c[i,j] * r[j] * x[i,j] + \sum\{i \text{ in sites}\} f[i] * y[i]$; $x[j,t] * x[jp,t]$; subject to doeach $\{j \text{ in states}\}$: $\sum\{i \text{ in sites}\} x[i,j] = 1$; switch $\{i \text{ in sites}, j \text{ in states}\}$: $x[i,j] \leq y[i]$; data; param $m := 5$; param $n := 5$; param $f := 1 \ 160 \ 2 \ 49 \ 3 \ 246 \ 4 \ 86 \ 4 \ 100$; param $r := 1 \ 200 \ 2 \ 100 \ 3 \ 300 \ 4 \ 100 \ 5 \ 200$; param c : $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 0.0 & 0.4 & 0.8 & 0.4 & 0.8 \\ 0.4 & 3 & 0.6 & 0.4 & 0.0 \\ 0.0 & 0.5 & 0.4 & 4 & 0.6 \\ 0.9 & 0.0 & 0.4 & 5 & 0.9 \end{matrix}$ $\begin{matrix} 0.7 & 0.0 & 0.8 & 0.4 \\ 0.4 & 0.7 & 0.4 & 0.0 \end{matrix}$;

2-40. (a) $\max \sum_{j=1}^8 r_j x_j$, subject to,

$\sum_{j=1}^8 x_j \leq 4$, $x_1 + x_2 + x_3 \geq 2$,

$x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$,

$x_2 + x_3 + x_4 + x_8 \geq 2$, $x_1 \dots x_8 = 0$ or 1 (b)

model; param n ; set games := 1 .. n ; #ratings param $r\{j \text{ in games}\}$; #home? param $h\{j \text{ in games}\}$; #state? param $s\{j \text{ in games}\}$; #cover? var $x\{j \text{ in games}\}$ binary; maximize totat: $\sum\{j \text{ in games}\} r[j] * x[j]$; subject to capacity: $\sum\{j \text{ in games}\} x[j] \leq 4$; home: $\sum\{j \text{ in games}\} h[j] * x[j] \geq 2$; away: $\sum\{j \text{ in games}\} (1-h[j]) * x[j] \geq 1$; state: $\sum\{j \text{ in games}\} s[j] * x[j] \geq 2$; data; param $n := 8$; param $r := 1 \ 3.0 \ 2 \ 3.7 \ 3 \ 2.6 \ 4 \ 1.8 \ 5 \ 1.5 \ 6 \ 1.3 \ 7 \ 1.6 \ 8 \ 2.0$; param $h := 1 \ 1 \ 2 \ 1 \ 3 \ 1 \ 4 \ 0 \ 5 \ 0 \ 6 \ 0 \ 7 \ 0 \ 8 \ 0$; param $s := 1 \ 0 \ 2 \ 1 \ 3 \ 1 \ 4 \ 1 \ 5 \ 0 \ 6 \ 0 \ 7 \ 0 \ 8 \ 1$; (c) The model is an ILP because all constraints and the objective are linear, but decision variables are binary.

2-41. (a) How to divide funds is the issue.

(b) $\max \sum_{j=1}^n v_j x_j$

(c) $\min \sum_{j=1}^n r_j x_j$

(d) $\sum_{j=1}^n x_j = 1$

(e) $x_j \geq \ell_j$, $j = 1, \dots, n$ (each category j)

(f) $x_j \leq u_j, j = 1, \dots, n$ (each category j)

(g) A multiobjective LP because the 2 objectives and all constraints are linear, and all variables are continuous.

2-42. (a) The issue is which module goes to which site.

(b) If $x_{i,j}x_{i',j'} = 1$ the i is at j and i' is at j' , so wire $d_{j,j'}$ will be required. Summing over all possible location pairs captures the wire requirements for i and i' .

(c) min

$$\sum_{i=1}^{m-1} \sum_{i'=i+1}^m a_{i,i'} \sum_{j=1}^n \sum_{j'=1}^n d_{j,j'} x_{i,j} x_{i',j'}$$

(d) $\sum_{j=1}^n x_{i,j} = 1, i = 1, \dots, m$ (each module i)

(e) $\sum_{i=1}^m x_{i,j} \leq 1, j = 1, \dots, n$ (each site j)

(f) $x_{i,j} = 0$ or $1, i = 1, \dots, m, j = 1, \dots, n$

(g) Single objective INLP because the one objective is nonlinear and variables are

discrete. (h) `model; param m; param n; set modules := 1 .. m; set sites := 1 .. n; var x{i in modules, j in sites} binary; param a{ i in modules, ip in modules }; param d{ j in sites, jp in sites }; minimize totdist: sum{ i in modules, ip in modules: i < m and ip > i } a[i,ip] sum{ j in sites, jp in sites : j < n and jp > j } d[j,jp]*x[i,j]*x[ip,jp]; subject to alli {i in modules }: sum{ j in sites } x[i,j] = 1; allj { j in sites }: sum { i in modules } x[i,j] <= 1;`

2-43. $\max 199x_1 + 229x_2 + 188x_3 + 205x_4 - 180y_1 - 224y_2 - 497y_3$, subject to,
 $23x_3 + 41x_4 \leq 2877y_1, 14x_1 + 29x_2 \leq 2333y_2,$
 $11x_3 + 27x_4 \leq 3011y_3,$
 $x_1 + x_2 + x_3 + x_4 \geq 205, y_1 + y_2 + y_3 \leq 2,$
 $x_1, \dots, x_4 \geq 0, y_1, \dots, y_3 = 0$ or 1

2-44. $\max 11x_{1,1} + 15x_{1,2} + 19x_{1,3} + 10x_{1,4} + 19x_{2,1} + 23x_{2,2} + 44x_{2,3} + 67x_{2,4} + 17x_{3,1} + 18x_{3,2} + 24x_{3,3} + 55x_{3,4}$, subject to, $15x_{1,1} + 24x_{2,1} + 17x_{3,1} \leq 7600, 19x_{1,2} + 26x_{2,2} + 13x_{3,2} \leq 8200, 23x_{1,3} + 18x_{2,3} + 16x_{3,3} \leq 6015,$
 $14x_{1,4} + 33x_{2,4} + 14x_{3,4} \leq 5000, 31x_{1,1} + 26x_{2,1} + 21x_{3,1} \leq 6600, 25x_{1,2} + 28x_{2,2} + 17x_{3,2} \leq 7900,$
 $39x_{1,3} + 22x_{2,3} + 20x_{3,2} \leq 5055, 29x_{1,4} +$

$31x_{2,4} + 18x_{3,4} \leq 7777, x_{1,1} + x_{2,1} + x_{3,1} \geq 200,$
 $x_{1,2} + x_{2,2} + x_{3,2} \geq 300, x_{1,3} + x_{2,3} + x_{3,3} \geq 250, x_{1,4} + x_{2,4} + x_{3,4} \geq 500, x_{j,t} \geq 0, j = 1, \dots, 3, t = 1, \dots, 4.$