Chapter 1

Functions, Graphs, and Limits

1.1 Functions

1. $f(x) = 3x + 5,$
$f(0) = 3(0) + 5 = 5,$
$f(-1) = 3(-1) + 5 = 2,$
$f(2) = 3(2) + 5 = 11.$

2. $f(x) = -7x + 1$
$f(0) = -7(0) + 1 = 1$
$f(1) = -7(1) + 1 = -6$
$f(-2) = -7(-2) + 1 = 15$

3. $f(x) = 3x^2 + 5x - 2,$
$f(0) = 3(0)^2 + 5(0) - 2 = -2,$
$f(-2) = 3(-2)^2 + 5(-2) - 2 = 0,$
$f(1) = 3(1)^2 + 5(1) - 2 = 6.$

4. $h(t) = (2t+1)^3$ $\quad h(-1) = (-2+1)^3 = -1$
$h(0) = (0+1)^3 = 1$ $\quad h(1) = (2+1)^3 = 27$

5. $g(x) = x + \frac{1}{x},$
$g(-1) = -1 + \frac{1}{-1} = -2,$
$g(1) = 1 + \frac{1}{1} = 2,$
$g(2) = 2 + \frac{1}{2} = \frac{5}{2}.$

6. $f(x) = \frac{x}{x^2+1}$
$f(0) = \frac{0}{0+1} = 0$
$f(-1) = \frac{-1}{(-1)^2+1} = -\frac{1}{2}$
$f(2) = \frac{2}{2^2+1} = \frac{2}{5}$

7. $h(t) = \sqrt{t^2 + 2t + 4},$
$h(2) = \sqrt{2^2 + 2(2) + 4} = 2\sqrt{3},$
$h(0) = \sqrt{0^2 + 2(0) + 4} = 2,$
$h(-4) = \sqrt{(-4)^2 + 2(-4) + 4} = 2\sqrt{3}.$

8. $g(u) = (u+1)^{3/2}$
$g(0) = (0+1)^{3/2} = 1$
$g(-1) = (-1+1)^{3/2} = 0$
$g(8) = (8+1)^{3/2} = \left(\sqrt{9}\right)^3 = 27$

9. $f(t) = (2t-1)^{-3/2} = \frac{1}{\left(\sqrt{2t-1}\right)^3},$
$f(1) = \frac{1}{\left[\sqrt{2(1)-1}\right]^3} = 1,$
$f(5) = \frac{1}{\left[\sqrt{2(5)-1}\right]^3} = \frac{1}{\left[\sqrt{9}\right]^3} = \frac{1}{27},$
$f(13) = \frac{1}{\left[\sqrt{2(13)-1}\right]^3} = \frac{1}{\left[\sqrt{25}\right]^3} = \frac{1}{125}.$

10. $f(t) = \frac{1}{\sqrt{3-2t}}$
$f(1) = \frac{1}{\sqrt{3-2(1)}} = 1$
$f(-3) = \frac{1}{\sqrt{3-2(-3)}} = \frac{1}{3}$
$f(0) = \frac{1}{\sqrt{3-2(0)}} = \frac{1}{\sqrt{3}}$

11. $f(x) = x - |x - 2|$,
$f(1) = 1 - |1 - 2| = 1 - |-1| = 1 - 1 = 0$,
$f(2) = 2 - |2 - 2| = 2 - |0| = 2$,
$f(3) = 3 - |3 - 2| = 3 - |1| = 3 - 2 = 2$.

12. $g(x) = 4 + |x|$
$g(-2) = 4 + |-2| = 6$
$g(0) = 4 + |0| = 4$
$g(2) = 4 + |2| = 6$

13. $h(x) = \begin{cases} -2x + 4 & \text{if} \quad x \le 1 \\ x^2 + 1 & \text{if} \quad x > 1 \end{cases}$

$h(3) = (3)^2 + 1 = 10$
$h(1) = -2(1) + 4 = 2$
$h(0) = -2(0) + 4 = 4$
$h(-3) = -2(-3) + 4 = 10$

14. $f(t) = \begin{cases} 3 & \text{if} \quad t < -5 \\ t + 1 & \text{if} \quad -5 \le t \le 5 \\ \sqrt{t} & \text{if} \quad t > 5 \end{cases}$

$f(-6) = 3$
$f(-5) = -5 + 1 = -4$
$f(16) = \sqrt{16} = 4$

15. $g(x) = \dfrac{x}{1 + x^2}$
Since $1 + x^2 \ne 0$ for any real number, the domain is the set of all real numbers.

16. Since $x^2 - 1 = 0$ for $x = \pm 1$, $f(x)$ is defined only for $x \ne \pm 1$ and the domain does not consist of the real numbers.

17. $f(t) = \sqrt{1 - t}$
Since negative numbers do not have real square roots, the domain is all real numbers such that $1 - t \ge 0$, or $t \le 1$. Therefore, the domain is not the set of all real numbers.

18. The square root function only makes sense for non-negative numbers. Since $t^2 + 1 \ge 0$ for all real numbers t the domain of $h(t) = \sqrt{t^2 + 1}$ consists of all real numbers.

19. $g(x) = \dfrac{x^2 + 5}{x + 2}$
Since the denominator cannot be 0, the domain consists of all real numbers such that $x \ne -2$.

20. $f(x) = x^3 - 3x^2 + 2x + 5$
The domain consists of all real numbers.

21. $f(x) = \sqrt{2x + 6}$
Since negative numbers do not have real square roots, the domain is all real numbers such that $2x + 6 \ge 0$, or $x \ge -3$.

22. $f(t) = \dfrac{t + 1}{t^2 - t - 2}$
$t^2 - t - 2 = (t - 2)(t + 1) \ne 0$
if $t \ne -1$ and $t \ne 2$.

23. $f(t) = \dfrac{t + 2}{\sqrt{9 - t^2}}$
Since negative numbers do not have real square roots and denominators cannot be zero, the domain is the set of all real numbers such that $9 - t^2 > 0$, namely $-3 < t < 3$.

24. $h(s) = \sqrt{s^2 - 4}$ is defined only if $s^2 - 4 \ge 0$ or equivalently $(s - 2)(s + 2) \ge 0$. This occurs when the factors $(s - 2)$ and $(s + 2)$ are zero or have the same sign. This happens when $s \ge 2$ or $s \le -2$ and these values of s form the domain of h.

25. $f(u) = 3u^2 + 2u - 6$ and $g(x) = x + 2$, so

$$\begin{aligned} f(g(x)) &= f(x + 2) \\ &= 3(x + 2)^2 + 2(x + 2) - 6 \\ &= 3x^2 + 14x + 10. \end{aligned}$$

26. $f(u) = u^2 + 4$

$f(x-1) = (x-1)^2 + 4 = x^2 - 2x + 5$

27. $f(u) = (u-1)^3 + 2u^2$ and $g(x = x + 1$, so

$$f(g(x)) = f(x+1) = [(x+1)-1]^3 + 2(x+1)^2 = x^3 + 2x^2 + 4x + 2.$$

28. $f(u) = (2u+10)^2$

$$f(x-5) = [2(x-5)+10]^2 = (2x-10+10)^2 = 4x^2$$

29. $f(u) = \frac{1}{u^2}$ and $g(x) = x - 1$, so

$$f(g(x)) = f(x-1) = \frac{1}{(x-1)^2}.$$

30. $f(u) = \frac{1}{u}$

$$f(x^2 + x - 2) = \frac{1}{x^2 + x - 2}$$

31. $f(u) = \sqrt{u+1}$ and $g(x) = x^2 - 1$, so

$$f(g(x)) = f(x^2 - 1) = \sqrt{(x^2-1)+1} = \sqrt{x^2} = |x|.$$

32. $f(u) = u^2$, $f\left(\frac{1}{x-1}\right) = \frac{1}{(x-1)^2}$

33. $f(x) = 4 - 5x$

$$\frac{f(x+h)-f(x)}{h} = \frac{4-5(x+h)-(4-5x)}{h}$$

$$\frac{4-5x-5h-4+5x}{h} = \frac{-5h}{h} = -5$$

34. For $f(x) = 2x + 3$,

$$\frac{f(x+h)-f(x)}{h} = \frac{(2(x+h)+3)-(2x+3)}{h} = \frac{2x+2h+3-2x-3}{h} = \frac{2h}{h} = 2$$

35. $f(x) = 4x - x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{4(x+h)-(x+h)^2-(4x-x^2)}{h} = \frac{4x+4h-(x^2+2xh+h^2)-4x+x^2}{h} = \frac{4x+4h-x^2-2xh-h^2-4x+x^2}{h} = \frac{4h-2xh-h^2}{h} = \frac{h(4-2x-h)}{h} = 4-2x-h$$

36. $f(x) = x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-x^2}{h} = \frac{x^2+2xh+h^2-x^2}{h} = \frac{h(2x+h)}{h} = 2x+h$$