PART I. SOLUTIONS TO PROBLEM SETS

2.1 $v = \sqrt{2}\times 120\cos(\omega t + 30°) \Rightarrow V = 120\angle 30°$ V

$i = \sqrt{2}\times 10\cos(\omega t - 30°) \Rightarrow I = 10\angle -30°$ A

(a) $p(t) = |V||I|\,[\cos\phi + \cos(2\omega t + \angle V + \angle I)]$

$= 600 + 1200\cos 2\omega t$ W

$S = VI^* = 1200\angle 60° = P + jQ \Rightarrow P = 600\text{ W},\ Q = 1039\text{ VAr}$

(b) $Z = V/I = 12\angle 60° = 6 + j10.39 = R + jX \Rightarrow R = 6,\ X = 10.39$

2.2 (a) Using (2.3) we find $P_{max} = 1707 = |V||I|(\cos\phi + 1)$ and $P_{min} = -293 = |V||I|(\cos\phi - 1)$. Then, since $|V| = 100$, we get $|I| = 10$ and $\cos\phi = \pm 45°$. Pick $\phi = 45° \Rightarrow Z = 10\angle 45° = 7.07 + j7.07 = R + jX \Rightarrow R = 7.07,\ X = 7.07$

(b) $S = VI^* = Z|I|^2 = (7.07 + j7.07)\,10^2 \Rightarrow P = 707,\ Q = 707$

(c) For simplicity assume $i(t) = \sqrt{2}\,|I|\cos\omega t$. Then

$p_L(t) = v_L(t)\,i(t) = L\frac{di}{dt}\,i = -2\omega L|I|^2\cos\omega t\sin\omega t = -\omega L|I|^2\sin 2\omega t$

$P_{L\,max} = \omega L|I|^2 = 707 = Q$. Thus $P_{L\,max} = Q$. The same!

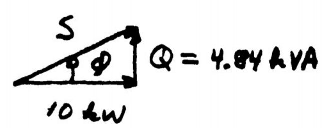
2.3 0.7 PF lagging $\Rightarrow \phi = 45.57°$, $Q = 5.10$ MVAr

0.9 PF lagging $\Rightarrow \phi = 25.84°$, $Q = 2.42$ MVAr. Capacitor must supply $5.10 - 2.42 = 2.68$ MVAr.

2.4 0.707 PF lagging $\Rightarrow S_{3\phi} = 200 + j200$ kVA. Cap supplies 50 kVAr. Resultant $S_{3\phi} = 200 + j150$ kVA $\Rightarrow$ PF $= 0.80$.

$|S| = \frac{|S_{3\phi}|}{3} = \frac{250\times 10^3}{3} = |V||I| = \frac{440}{\sqrt{3}}|I| \Rightarrow |I| = 328$ A

2.5 0.9 PF lagging $\Rightarrow \phi = 25.84°$



(a) $S = 10 + j4.84$ kVA

(b) $10\times 10^3 = 416\times|I|\times 0.9 \Rightarrow |I| = 26.71$ A

(c) Using (2.3), (or first principles) we get

$p(t) = 10\times 10^3 + 11.11\times 10^3\cos(2\omega t + 25.84°)$

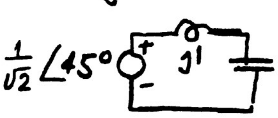
Note: the average value of $p(t)$ is 10 kW

2.6 Because system is balanced $V_{ab} = 208\angle 120°$, $V_{bc} = 208\angle 0°$. Using (2.17) or Fig 2.11, $V_{an} = 120\angle 90° \Rightarrow V_{bn} = 120\angle -30°$, $V_{cn} = 120\angle -150°$. Using per phase analysis, $I_a = 12\angle 105° \Rightarrow$ $I_b = 12\angle -15°$, $I_c = 12\angle -135°$.

2.7 $S = VI^* = V(YV)^* = Y^*|V|^2 = Y_C^* + Y_L^* + Y_R^*$
$= -j5 + j10 + 0.1 = 0.1 + j5$

2.8 (a) Using loop or nodal analysis we find, after much work,
$I_a = 0.9123\angle -90.351°$, $I_b = 0.9123\angle -209.65°$, $I_c = 0.9929\angle 30°$.
(b) Using per phase analysis $I_a = 1\angle 0° / j1.1 = 0.9091\angle -90°$,
then, $I_b = 0.9091\angle -210°$, $I_c = 0.9091\angle 30°$.

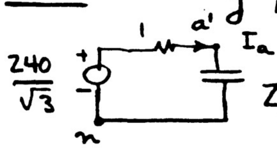
2.9 Proceeding by analogy with 3ϕ, we note
$E_{ab} = E_{an} - E_{bn} = E_{an}(1 - e^{-j\pi/2}) = \sqrt{2}\, E_{an}\, e^{j\pi/4}$.
Thus $E_{an} = \frac{1}{\sqrt{2}} E_{ab} e^{-j\pi/4}$, and $E_{an}, E_{bn}, E_{cn}, E_{dn}$ form a pos. seq. set of 4ϕ voltages. Doing per phase (phase a) analysis we have



$\Rightarrow I_a = \dfrac{\frac{1}{\sqrt{2}}\angle -45°}{j0.5} = \sqrt{2}\,\angle -135°$

Then $I_b = \sqrt{2}\angle -225°$, $I_c = \sqrt{2}\angle -315°$, $I_d = \sqrt{2}\angle -405°$

2.10 Using per phase circuit we find $I_a = 103.8\angle 41.5°$.



Then $|I_b| = 103.8$ A

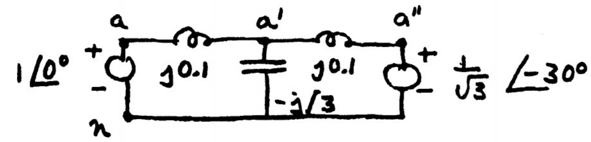
$Z_\lambda = \frac{1}{3}\frac{1}{j\omega C} = -j0.884$

$V_{a'n} = Z_\lambda I_a = 91.76\angle -48.5° \Rightarrow |V_{a'b'}| = \sqrt{3}\cdot 91.76 = 158.9$ V

$S_{load} = V_{a'n} I_a^* = 9524\angle -90°$
$S_{load}^{3\phi} = 3 S_{load} = 28574\angle -90°$ W

2.11 Assume pos. seq. operation. $V_{a''b''} = V_{a''n} - V_{b''n} = \sqrt{3}\, V_{a''n}\, e^{j\pi/6} \Rightarrow V_{a''n} = \frac{1}{\sqrt{3}} V_{a''b''}\, e^{-j\pi/6} = \frac{1}{\sqrt{3}} \angle -30°$

Per Phase Ckt



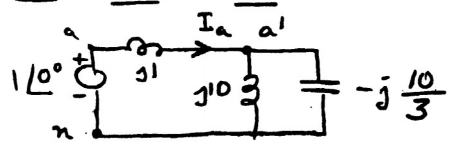
Using superposition & voltage divider law we get

$V_{a'n} = 0.899 \angle -10.89° \Rightarrow V_{b'n} = 0.899 \angle -130.89°$ and

$V_{c'n} = 0.899 \angle -250.89°$. Then $V_{a'b'} = 1.557 \angle 19.11°$

2.12 Assume pos. seq..

Per Phase Ckt



Combining parallel elements we have $Z_{\parallel} = -j5$. $I_a = 0.25 \angle 90°$

$V_{a'n} = -j5 \cdot j0.25 = 1.25 \angle 0°$

$V_{a'b'} = 2.165 \angle 30° \Rightarrow I_{cap} = .2165 \angle 120°$

$S^{3\phi}_{load} = 3\, V_{a'n}\, I_a^{*} = 0.3125 \angle -90°$

2.13

(a) $V_{bc} = 208 \angle -120°$, $V_{ca} = 208 \angle 120°$

$V_{an} = \frac{208}{\sqrt{3}} \angle -30° \Rightarrow I_a = 1.20 \angle -90°$

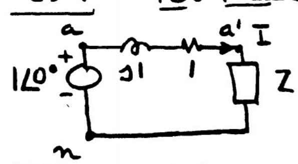
then $I_b = 1.20 \angle -210°$, $I_c = 1.20 \angle -330°$

(b) $V_{bc} = 208 \angle 120°$, $V_{ca} = 208 \angle -120°$

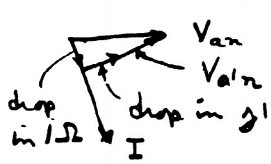
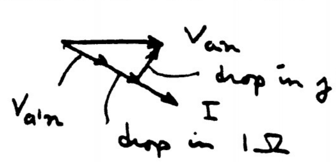
$V_{an} = \frac{208}{\sqrt{3}} \angle 30° \Rightarrow I_a = 1.20 \angle -30°$

$I_b = 1.20 \angle 90°$, $I_c = 1.20 \angle -150°$

2.14 Per Phase Ckt.



Problem reduces to picking Z so that $|V_{a'n}| > |V_{an}|$. It helps to draw some phasor diagrams.

I. $Z = j\omega L$	II $Z = R$	III $Z = -j\frac{1}{\omega C}$
		For example $Z = -j2$ $I = \frac{1}{\sqrt{2}} \angle 45°$ $V_{a'n} = \sqrt{2} \angle -45°$ $\lvert V_{a'n}\rvert > \lvert V_{an}\rvert$ this is O.K.
Clearly $\lvert V_{a'n}\rvert < \lvert V_{an}\rvert$	Clearly $\lvert V_{a'n}\rvert < \lvert V_{an}\rvert$	