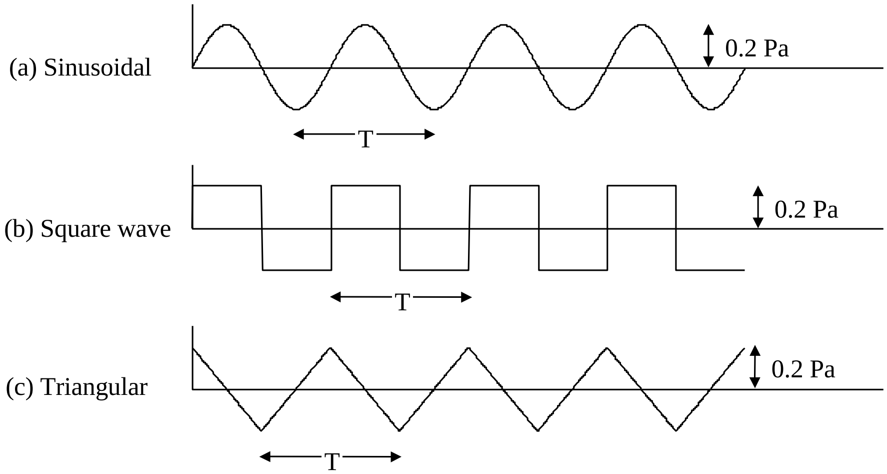
Solutions to Questions from Chapter 2:

1. Evaluate the mean square pressures and the corresponding sound pressure levels of the continuous sound pressure signals shown below:



The mean square pressure is $\overline{p^2} = \lim_{T\to\infty}\frac{1}{T}\int_0^T p(t)^2 dt,$

Write A for the amplitude

a) Sinusoidal

$$\overline{p^2} = \lim_{T\to\infty}\frac{1}{T}\int_0^T (A\sin\omega t)^2 dt = \frac{A^2}{2} = 0.02\ \mathrm{Pa}^2$$

L_p = 10 $\log_{10}$ (0.02/4×10^{-10}) = <u>77.0 dB</u> re 2×10^{-5} Pa

b) square wave

$$\overline{p^2} = \lim_{T\to\infty}\frac{1}{T}\left(\int_0^{T/2}(A)^2 dt + \int_{T/2}^{T}(-A)^2 dt\right) = A^2 = 0.04\ \mathrm{Pa}^2$$

L_p = 10 $\log_{10}$ (0.04/4×10^{-10}) = <u>80.0 dB</u> re 2×10^{-5} Pa

c) Triangular

$$\overline{p^2} = \frac{A^2}{T}\left(\int_0^{T/2}\left(\frac{4}{T}t-1\right)^2 dt + \int_{T/2}^{T}\left(3-\frac{4}{T}t\right)^2 dt\right)$$

$$= \frac{A^2}{T}\left(\int_0^{T/2}\left(\frac{4}{T}t-1\right)^2 dt + \int_0^{T/2}\left(\frac{4}{T}t'-1\right)^2 dt'\right)$$

$$= \frac{2A^2}{T}\left(\int_0^{T/2}\left(\frac{4}{T}t-1\right)^2 dt\right)$$

$$= \frac{A^2}{3} = 0.0133 \text{ Pa}^2$$

$L_p = 10 \log_{10} (0.0133/4\times10^{-10}) = \underline{75.2 \text{ dB}}$ re 2×10^{-5} Pa

2. What is the r.m.s. pressure amplitude of a sound signal with a sound pressure level of (i) 80 dB, (ii) 94 dB, (iii) 0 dB (re 2×10^{-5} Pa).

According to equation 2.7 for sound pressure level:

$$L_p = 20 \log_{10} \frac{p_{\mathrm{rms}}}{p_{\mathrm{ref}}}$$

where $p_{\mathrm{ref}} = 2 \times 10^{-5}$Pa. Hence $p_{\mathrm{rms}} = p_{\mathrm{ref}} \times 10^{(Lp/20)}$. This gives

80 dB: $p_{\mathrm{rms}} = 2\times10^{-5} \times 10^{(80/20)} = 0.2$ Pa

94 dB: $p_{\mathrm{rms}} = 2\times10^{-5} \times 10^{(94/20)} = 1$ Pa

0 dB: $p_{\mathrm{rms}} = 2\times10^{-5} \times 10^{(0/20)} = 2\times10^{-5}$ Pa

3. Three independent sound sources produce sound pressure levels at a receiver position of 80, 82 and 85 dB re 2×10^{-5} Pa respectively. Calculate the total sound pressure level when all three operate simultaneously. What would be the maximum possible sound pressure level if the three sources were coherent?

According to

$$L_p = 20 \log_{10} \frac{p_{\mathrm{rms}}}{p_{\mathrm{ref}}}$$

when $p_{\mathrm{ref}} = 2 \times 10^{-5}$Pa, $L_{p1} = 80$dB, $L_{p2} = 82$dB, $L_{p3} = 85$dB, these can be converted to mean-square pressures and added:

$10^{(80/10)} = 1.0 \times10^8$

$10^{(82/10)} = 1.5849\times10^8$

$10^{(85/10)} = \underline{3.1623\times10^8}$

sum $= 5.7472\times10^8$ $\qquad$ $10 \log_{10}(\text{sum}) = \underline{87.6 \text{ dB}}$

NB Pressures have not been converted to Pa but are given as multiples of 2×10^{-5} Pa. Multiplying by the reference value and then dividing again is unnecessary.

To find the maximum possible sound pressure level if the sources are coherent, add the pressure amplitudes instead of the mean-square values.

Add pressure amplitudes (assuming all in phase):

$2\times10^{-5}\times10^{(80/20)}$ = 0.20 Pa

$2\times10^{-5}\times10^{(82/20)}$ = 0.25 Pa

$2\times10^{-5}\times10^{(85/20)}$ = <u>0.35 Pa</u>

sum = 0.8 Pa $\quad$ $20\log_{10}(0.8/2\times10^{-5})$ = <u>92.0 dB</u>

Again it is actually unnecessary to multiply by the reference value and then divide again.

4. A sliding piston at the end of a semi-infinite tube moves with a maximum velocity of 0.1 ms^{-1} for a short duration. Given that the gas in the tube is air with density ρ_0 = 1.2 kg/m^3, ambient pressure $p_0 = 10^5$ N/m^2 and ratio of specific heats γ = 1.4, calculate the maximum pressure and density changes produced by the motion of the piston.

Maximum particle velocity equals the piston velocity: $u' = 0.1\mathrm{m/s}$

Assume that plane waves are generated, then from Euler's equation, $p' = \rho_0 c u'$

For an adiabatic compression,

$$pV^\gamma = \text{const.}$$

$$\frac{\mathrm{d}p}{p} + \gamma\frac{\mathrm{d}V}{V} = 0$$

From the mass conservation law,

$$\mathrm{d}m = \rho\mathrm{d}V + V\mathrm{d}\rho = 0$$

$$\frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}V}{V} = 0$$

Substituting this into the above equation,

$$\frac{\mathrm{d}p}{p} - \gamma\frac{\mathrm{d}\rho}{\rho} = 0$$

Thus,

$$\frac{p'}{\rho'} = \frac{dp}{d\rho}\bigg|_0 = \gamma \frac{p}{\rho}\bigg|_0 = \gamma \frac{p_0}{\rho_0} = c_0^2$$

With $p_0 = 10^5$ Pa, $\rho_0 = 1.2\ \mathrm{kg/m^3}$, $\gamma = 1.4$, the speed of sound in the air is $c_0 = 342$ m/s.

$$\therefore p' = \rho_0 c_0 u' = 41.0 \text{ Pa}, \qquad \rho' = \frac{p'}{c_0^2} = 3.5 \times 10^{-4}\ \mathrm{kg/m^3}$$

5. A harmonic plane wave having a pressure amplitude of 5×10^{-3} Pa travels in air. Calculate the amplitude of the acoustic particle velocity fluctuation and the value of the time-averaged acoustic intensity in the direction of propagation of the wave. If a wave with the same pressure amplitude travels in water (having an ambient density ρ_0 of 1000 $\mathrm{kgm^{-3}}$ and a sound speed c_0 of 1500 $\mathrm{ms^{-1}}$) what is the corresponding amplitude of the acoustic particle velocity and the value of the time-averaged acoustic intensity?

For a plane wave

$$p = \rho_0 c_0 u,$$

So

$$u = \frac{p}{\rho_0 c_0} = \frac{5 \times 10^{-3}}{1.2 \times 344} = 1.2 \times 10^{-5}\ \mathrm{m/s}$$

Intensity:

$$I = \frac{|p|^2}{2\rho_0 c_0} = \frac{(5 \times 10^{-3})^2}{2 \times 1.2 \times 344} = 6.1 \times 10^{-9}\ \mathrm{W/m^2}$$

For sound in water, the impedance $\rho_0 c_0$ will be different. Following the same procedure,

$$u = \frac{p}{\rho_0 c_0} = \frac{5 \times 10^{-3}}{1000 \times 1500} = 3.33 \times 10^{-9}\ \mathrm{m/s}$$

$$I = \frac{|p|^2}{2\rho_0 c_0} = \frac{(5 \times 10^{-3})^2}{2 \times 1000 \times 1500} = 8.33 \times 10^{-12}\ \mathrm{W/m^2}$$

6. Two travelling harmonic plane waves with different amplitudes propagate in a direction parallel to the x-axis, one in the positive x-direction and the other in the negative x-direction. The waves produce pressure fluctuations that are in phase at $x = 0$. Derive an expression for the modulus and phase of the net pressure fluctuation produced by the interference of the waves as a function of x and as a function of the ratio of the amplitudes of the positive and negative going waves. Illustrate your results graphically with a sketch of the dependence on x of the modulus and phase of the net pressure fluctuation for the case where the ratio of amplitudes is 2.

The two waves can be written as $p_1(x) = A\, e^{-\mathrm{i}kx}$ and $p_2(x) = B\, e^{\mathrm{i}kx}$, that is two plane waves travelling parallel to the x-axis in opposite directions. The two waves are in phase at $x = 0$, so we can assume $\varphi_A = \varphi_B = 0$, i.e. A and B are real. The total pressure is given by

$$p(x) = A\, e^{-\mathrm{i}kx} + B\, e^{\mathrm{i}kx}$$

This can be written as

$$p(x) = A(\cos kx - \mathrm{i}\sin kx) + B(\cos kx + \mathrm{i}\sin kx)$$

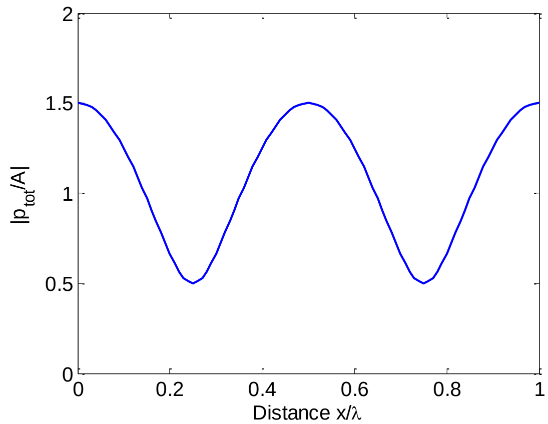
$$= (A + B)\cos kx + \mathrm{i}(B - A)\sin kx$$

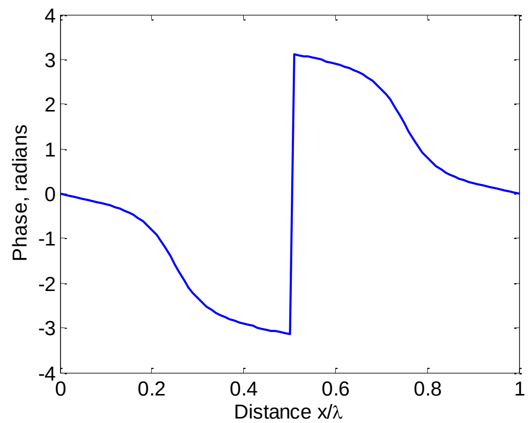
The modulus and phase are given by

$$|p(x)| = \sqrt{(A + B)^2\cos^2 kx + (B - A)^2\sin^2 kx}$$

$$\varphi(x) = \mathrm{atan}\left(\frac{(B - A)}{(A + B)}\tan kx\right)$$

The magnitude and phase are plotted below for $A = 1$ and $B = 0.5$.





7. Two uncorrelated spherically radiating acoustic sources (in air) are respectively 15 m and 5 m from a receiver point at which the sound pressure level is 80 dB re 2×10^{-5} Pa. Given that the sound power level of the more distant source is 110 dB re 10^{-12} W, calculate the sound power level of the nearer source.

For uncorrelated sources $p^2_{\mathrm{rms,tot}} = p^2_{\mathrm{rms,1}} + p^2_{\mathrm{rms,2}}$

Since $$L_{\mathrm{p,tot}} = 80 = 20 \cdot \log_{10}\frac{p_{\mathrm{rms,tot}}}{2\times10^{-5}}\ \mathrm{dB}$$

$$p_{\mathrm{rms,tot}} = 0.2\ \mathrm{Pa}$$

Also $L_{W,2} = 110 = 10 \cdot \log_{10}\frac{W_2}{1\times10^{-12}}$ dB. So $W_2 = 1\times10^{-12}\times10^{11} = 0.1\mathrm{W}$,

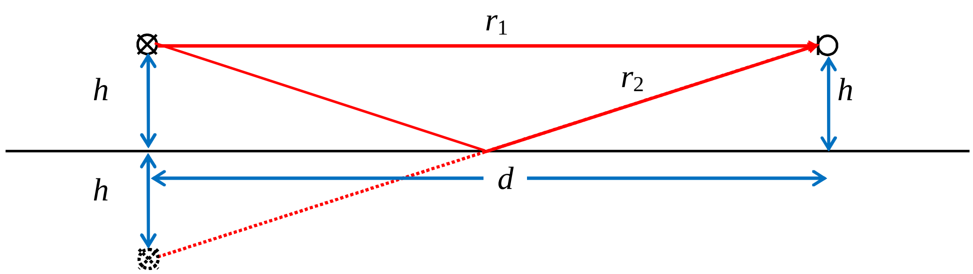
and the squared pressure due to the further source is

$$p^2_{\mathrm{rms,2}} = \frac{W\rho_0 c_0}{4\pi r_2^2} = \frac{0.1\times1.2\times343}{4\times\pi\times15^2} = 0.0146\ \mathrm{Pa}^2$$

Hence $p^2_{\mathrm{rms,1}} = 0.2^2 - 0.0146 = 0.0254\ \mathrm{Pa}^2$, and

$$L_{W,1} = 10\cdot\log_{10}\left(\frac{W_1}{W_{ref}}\right) = 10\cdot\log_{10}\left(\frac{4\pi r_1^2 p^2_{\mathrm{rms,1}}}{\rho_0 c_0}\frac{1}{W_{\mathrm{ref}}}\right) = 10\cdot\log_{10}\left(\frac{4\times\pi\times5^2\times0.0254}{1.2\times344}\frac{1}{1\times10^{-12}}\right) = 102.9\ \mathrm{dB}$$

8. A point monopole source radiating single frequency sound is placed 1 m above a rigid reflecting surface. The sound pressure is measured at a distance of 10 m from the source at the same height as the source above the plane. Calculate the complex ratio of this pressure to that produced at the same position in the absence of the reflecting plane and plot its magnitude as a function of frequency.



For a source of volume velocity Q, the sound pressure at a distance of $r_1 = d = 10$ m without the reflecting plane can be written as:

$$p_1 = \frac{\mathrm{i}\omega\rho_0 Q}{4\pi r_1} e^{-\mathrm{i}kr_1}$$

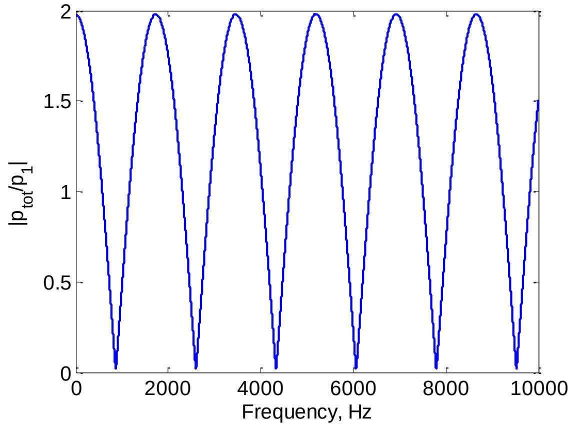
The sound pressure at the observation point including the reflecting plane can be written as the sum of the direct sound and the sound from an image source:

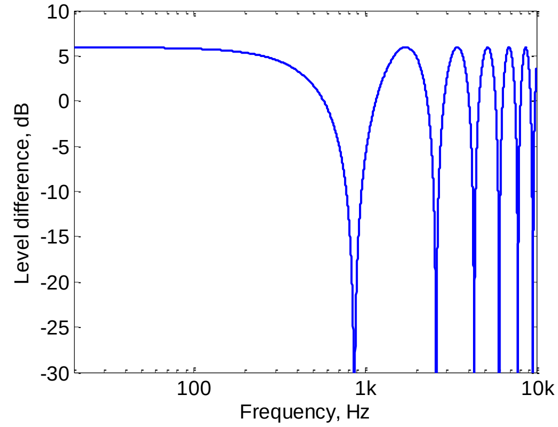
$$p_{\mathrm{tot}} = \frac{\mathrm{i}\omega\rho_0 Q}{4\pi r_1} e^{-\mathrm{i}kr_1} + \frac{\mathrm{i}\omega\rho_0 Q}{4\pi r_2} e^{-\mathrm{i}kr_2}$$

where $r_1 = 10$ m, $r_2 = \sqrt{10^2 + 2^2} = 10.2$ m

Thus $\quad R = \frac{p_{\mathrm{tot}}}{p_1} = 1 + \frac{r_1}{r_2} e^{-\mathrm{i}k(r_2 - r_1)}$

The results are shown below as pressure magnitude and in decibels.





9. (a) A rectangular air-filled enclosure has dimensions 5 m × 7 m × 3 m. All the surfaces are covered with plaster having an absorption coefficient of 0.07 at 500 Hz, except the floor (which measures 5 m × 7 m) which is covered with carpet having an absorption coefficient of 0.2 at 500 Hz. Calculate the average absorption coefficient at 500 Hz and hence estimate the reverberation time.

(b) A material having an unknown absorption coefficient is used to cover one of the walls (measuring 5 m×3 m) of the enclosure as a result of which the reverberation time in the enclosure is reduced to three-quarters of its original value. Estimate the absorption coefficient of the material.

(c) Explain why this estimate is likely to be unreliable.

(a) The average absorption coefficient

$$\bar{\alpha} = \frac{S_1\alpha_1 + S_2\alpha_2 + \cdots S_n\alpha_n}{S} = \frac{5\times7\times0.2+5\times7\times0.07+(3\times7+5\times3)\times2\times0.07}{2\times(5\times7+7\times3+5\times3)} = 0.102$$

So $$T_{60} = \frac{0.161\text{V}}{S\bar{\alpha}} = \frac{0.161\times5\times7\times3}{2\times(5\times7+7\times3+5\times3)\times0.102} = 1.17s$$

(b) Assume $T_{60_1} = \frac{0.161V}{S\bar{\alpha}_1}$ $T_{60_2} = \frac{0.161V}{S\bar{\alpha}_2}$

Then $\frac{T_{60_1}}{T_{60_2}} = \frac{\bar{\alpha}_2}{\bar{\alpha}_1} = \frac{4}{3}$; $\bar{\alpha}_2 = \frac{4}{3} \times 0.102 = 0.136$

Since $\bar{\alpha}_2 = \frac{\sum_{i=1}^{6} S_i \alpha_i}{S} = \frac{5\times7\times0.2+5\times7\times0.07+5\times3\times0.07+2\times7\times3\times0.07+5\times3\times\alpha_{\text{unknown}}}{2\times(5\times7+5\times3+7\times3)} = 0.136$

Then $\alpha_{\text{unknown}} = 0.392$

(c) The absorption is placed on one surface so the sound field will not be diffuse.

10. Derive an expression relating the sound power level of a spherically radiating source to the sound pressure level (in dB re 2×10^{-5} Pa) at a radial distance *r* from the source. Calculate the sound pressure level at 10 m from such a source of which the sound power is 5 W.

The total power output of a source can be written as

$$W = I_r\left(4\pi r^2\right) \tag{1}$$

The intensity I_r is given by

$$I_r = \frac{\left|p(r)\right|^2}{2\rho_0 c_0} \tag{2}$$

Substituting (2) into (1)

$$W = \frac{\left|p(r)\right|^2}{2\rho_0 c_0}\left(4\pi r^2\right) \tag{3}$$

Then

$$\frac{\left|p(r)\right|^2}{2} = \frac{W\rho_0 c_0}{4\pi r^2} \tag{4}$$

$$p_{rms}^2 = \frac{W\rho_0 c_0}{4\pi r^2} \tag{5}$$

Equation (5) can be expressed as

$$\frac{p_{rms}^2}{p_{ref}^2} = \frac{W}{W_{ref}} \frac{1}{4\pi r^2} \frac{\rho_0 c_0 W_{ref}}{p_{ref}^2} \tag{6}$$

Taking $10\log_{10}$ of both sides of Equation (6)

$$L_p = L_w - 10\log_{10}(4\pi r^2) + 10\log_{10}\left(\frac{\rho_0 c_0 W_{ref}}{p_{ref}^2}\right) \tag{7}$$

Equation (7) can be simplified as

$$L_p \approx L_w - 10\log_{10}(4\pi r^2)$$

For a source which has a sound power of 5 W, the sound pressure level at 10 m can be calculated as

$$L_p \approx 10\log_{10}\left(\frac{5}{10^{-12}}\right) - 10\log_{10}(4\pi \times 10^2) = 96 \text{ dB}$$