Chapter 2

2.1 $$h = \frac{p}{\varrho g} = \frac{200 \times 10^3\ \mathrm{N \cdot m^{-2}}}{1590\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}}} = \mathbf{12.82\ m}$$

2.2 Pressure depends only on depth below free surface.

(a) $$p = \varrho g h = (820\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}})(3 - 0.15)\ \mathrm{m}$$
$$= 22\,930\ \mathrm{N \cdot m^{-2}} = \mathbf{22.93\ kPa}$$

(b) $$p = 820 \times 9.81\ \mathrm{N \cdot m^{-3}} \times (3 + 2)\ \mathrm{m} = \mathbf{40.2\ kPa}$$

(c) $$p = 820 \times 9.81\ \mathrm{N \cdot m^{-3}} \times \{3 + 2 - (1.2 \sin 30^\circ + 0.6)\}\ \mathrm{m}$$
$$= 820 \times 9.81 \times 3.8\ \mathrm{N \cdot m^{-2}} = \mathbf{30.57\ kPa}$$

(d) Load = Pressure × Area
$$= 820 \times 9.81 \times 3\ \mathrm{N \cdot m^{-2}} \times (3.5 \times 2.5)\ \mathrm{m^2} = \mathbf{211.2\ kN}$$

2.3 $$h_{\mathrm{air}} = \frac{p}{\varrho_{\mathrm{air}} g} = \frac{\varrho_{\mathrm{water}} g h_{\mathrm{water}}}{\varrho_{\mathrm{air}} g} = \frac{\varrho_{\mathrm{water}}}{\varrho_{\mathrm{air}}} h_{\mathrm{water}}$$
$$= \frac{1000\ \mathrm{kg \cdot m^{-3}} \times 287\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}} \times 288.15\ \mathrm{K}}{1.013 \times 10^5\ \mathrm{N \cdot m^{-2}}} 0.075\ \mathrm{m}$$
$$= \mathbf{61.2\ m}$$

2.4 pV = constant
$$\therefore \left(\frac{d}{4\ \mathrm{mm}}\right)^3 = \frac{101.3 \times 10^3\ \mathrm{Pa} + 1000\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}} \times 9\ \mathrm{m}}{101.3 \times 10^3\ \mathrm{Pa}}$$
whence d = **4.93 mm**

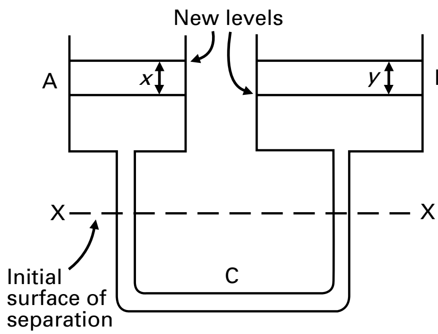
2.5 $$\Delta p = 820\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}} \times 2\ \mathrm{m} + (13.56 - 0.82)$$
$$\times 1000\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}} \times 0.225\ \mathrm{m} = \mathbf{44.2\ kPa}$$

$$\frac{\Delta p^*}{\varrho g} = \Delta h = \frac{0.225\ \text{m}(13.56 - 0.82)1000\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1}}{820\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1}}$$
$$= \mathbf{3.496\ m}$$

$$44\,200\ \text{N}\cdot\text{m}^{-2} = 820 \times 9.81 \times 2\ \text{N}\cdot\text{m}^{-2} + x(0.82 - 0.74)1000 \times 9.81\ \text{N}\cdot\text{m}^{-3}$$

whence $\boldsymbol{x} = \mathbf{35.83\ m}$

2.6



Movement of fluid in

$$\text{C} = 60\ \text{mm} \times 70\ \text{mm}^2$$
$$= (500\ \text{mm}^2)x$$
$$= (800\ \text{mm}^2)y$$
$$\therefore x = 8.4\ \text{mm};$$
$$y = 5.25\ \text{mm}$$

Measuring above XX: Initially $0.8h_{\text{A}} = 0.9h_{\text{B}}$

Later: $800 \times 9.81(\text{Old } h_{\text{A}} - 60 + 8.4)10^{-3}\ \text{Pa}$

$= p + 900 \times 9.81(\text{Old } h_{\text{B}} - 60 + 5.25)10^{-3}\ \text{Pa}$

$\therefore\ p = 9.81 \times 10^{-3}(-800 \times 51.6 + 900 \times 65.25)\text{Pa} = \mathbf{171.1\ Pa}$

2.7 From eqn 2.7 $p = p_0\left(1 - \dfrac{\lambda z}{T_0}\right)^{g/R\lambda}$

$$= 101.5\ \text{Pa}\left(1 - \frac{0.0065 \times 7500}{288.15}\right)^{9.81/287\times0.0065}$$
$$= \mathbf{38.3\ kPa}$$

2.8

$$\frac{p}{p_0} = \left(\frac{T_0 - \lambda z}{T_0}\right)^{g/R\lambda} = \left(\frac{T_{\text{top}}}{T_{\text{top}} + \lambda z}\right)^{g/R\lambda}$$

$$\therefore z = \frac{T_{\text{top}}}{\lambda}\left\{\left(\frac{p_0}{p}\right)^{R\lambda/g} - 1\right\}$$

$$= \frac{268.15}{0.0065}\ \text{m}\left\{\left(\frac{749}{566}\right)^{287\times0.0065/9.81} - 1\right\}$$

$$= \mathbf{2257\ m}$$

2.9 $F = (1.2 \times 1.8)\ \mathrm{m}^2 \times 1000\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}}$
$\times\ (x + 0.9 \sin 30°)\ \mathrm{m}$

(a) $2160 \times 9.81\ \mathrm{N \cdot m^{-1}} \times 0.45\ \mathrm{m} = \mathbf{9.54\ kN}$
(b) $2160 \times 9.81\ \mathrm{N \cdot m^{-1}} \times 0.95\ \mathrm{m} = \mathbf{20.13\ kN}$
(c) $2160 \times 9.81\ \mathrm{N \cdot m^{-1}} \times 30.45\ \mathrm{m} = \mathbf{645\ kN}$

Centre of pressure is at slant depth $\dfrac{(bd^3/12) + bd(2x + 0.9)^2}{bd(2x + 0.9)}$

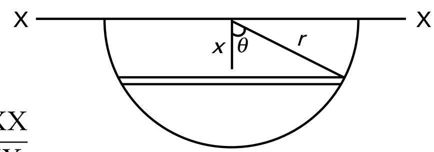
$$= \frac{d^2}{12(2x + 0.9)} + 2x + 0.9(\text{metres})$$

$$= \frac{(1.8\ \mathrm{m})^2}{12(2x + 0.9)} + 2x + 0.9\ \mathrm{m}$$

that is $\left\{ \dfrac{1.8^2}{12(2x + 0.9)} + 0.9 \right\}$ m from upper edge

= (a) **1.2 m;** (b) **1.042 m;** (c) **0.904 m** from upper edge

2.10 By symmetry, centre of pressure is on vertical centre-line



$$\text{Depth} = \frac{\text{2nd moment about XX}}{\text{1st moment about XX}}$$

$$= \frac{\int_0^r x^2 2(r^2 - x^2)^{1/2} \mathrm{d}x}{\int_0^r x 2(r^2 - x^2)^{1/2} \mathrm{d}x}$$

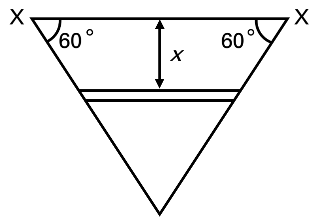
$$= \frac{\int_{\pi/2}^0 (r \cos\theta)^2 2r \sin\theta(-r \sin\theta \mathrm{d}\theta)}{\int_{\pi/2}^0 r \cos\theta 2r \sin\theta(-r \sin\theta \mathrm{d}\theta)}$$

$$= \frac{r \int_0^{\pi/2} \cos^2\theta \sin^2\theta \mathrm{d}\theta}{\int_0^{\pi/2} \sin^2\theta \cos\theta \mathrm{d}\theta}$$

$$= \frac{r \int_0^{\pi} \frac{1}{8} \sin^2 2\theta \mathrm{d}(2\theta)}{\left[\frac{1}{3} \sin^3\theta\right]_0^{\pi/2}}$$

$$= \frac{r/8\,[2\theta/2 - (1/4) \sin 4\theta]_0^{2\theta=\pi}}{1/3}$$

$$= \frac{3}{8} r \frac{\pi}{2} = \boldsymbol{\frac{3\pi d}{32}}$$

2.11 

Full depth $= (2.5\text{ m})\sin 60°$

Breadth of strip

$$= 2.5\text{ m}\left\{\frac{(2.5\text{ m})\sin 60° - x}{(2.5\text{ m})\sin 60°}\right\}$$

$$= 2.5\text{ m} - x\operatorname{cosec} 60°$$

$\therefore$ Second moment of area about XX

$$= \int_0^{(2.5\text{ m})\sin 60°} (2.5\text{ m} - x\operatorname{cosec} 60°)x^2\mathrm{d}x = \frac{2.5^4}{12}\sin^3 60°\text{ m}^4$$

First moment $= \text{Area} \times \dfrac{\text{Depth}}{3}$

$$= \frac{1}{2}2.5 \times 2.5\sin 60° \times \frac{2.5\sin 60°}{3}\text{ m}^3 = \frac{2.5^3}{6}\sin^2 60°\text{ m}^3$$

$\therefore$ Depth of C.P. $= \dfrac{2.5}{2}\sin 60°\text{ m} = \dfrac{\text{Depth}}{2}$

$\therefore$ Thrust is equally divided between XX and bottom.

Thrust = Area × Pressure at centroid

$$= \frac{1}{2}2.5^2\sin 60° \times 1000 \times 9.81 \times \frac{2.5\sin 60°}{3}\text{N} = 19\,160\text{ N}$$

$\therefore$ Load at bottom = **9580 N**; at each upper corner **4790 N**

2.12 Let shaft be at depth h below free surface. Then force on disc $= \pi R^2 \varrho g h$.

By parallel axes theorem, 2nd moment of area about free surface $= \pi R^4/4 + \pi R^2 h^2$.

1st moment of area about free surface $= \pi R^2 h$

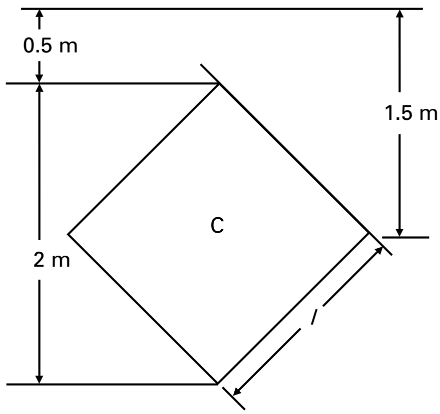
$\therefore$ Depth of C.P. $= \dfrac{R^2}{4h} + h$ below free surface, that is, $R^2/4h$ below shaft

$\therefore$ Turning moment on shaft

$$= \pi R^2 \varrho g h \times \frac{R^2}{4h} = \frac{\pi R^4 \varrho g}{4} \quad \text{[independent of } h\text{]}$$

$$= \frac{\pi(0.6\text{ m})^4 1000\text{ kg}\cdot\text{m}^{-3} \times 9.81\text{ N}\cdot\text{kg}^{-1}}{4} = \mathbf{999\ N\cdot m}$$

2.13



Force on plate

$= 1150\ \text{kg}\cdot\text{m}^{-3}$

$\times\ 9.81\ \text{N}\cdot\text{kg}^{-1}$

$\times\ 1.5\ \text{m}(\sqrt{2}\ \text{m})^2$

$= \mathbf{33.84\ kN}$

$$(Ak^2)_{\text{c},\ \perp\ \text{plate}} = \frac{Al^2}{6}$$

$$\therefore\ (Ak^2)_{\text{c, diagonal}} = \frac{Al^2}{12}$$

since diagonals are perpendicular

$\therefore$ Depth of C.P. below free surface

$$= \frac{(Al^2/12) + A\bar{y}^2}{A\bar{y}} = \bar{y} + \frac{l^2}{12\bar{y}} = \left\{1.5 + \frac{(\sqrt{2})^2}{12 \times 1.5}\right\}\text{m}$$

$= 1.611$ m, that is, **1.111 m from top of aperture**

$\therefore$ Total moment about hinge $= 33.84\ \text{kN} \times 1.111/\sqrt{2}\ \text{m}$

$= \mathbf{26.59\ kN\cdot m}$

2.14 Width of gates $= (3\ \text{m}) \sec 30^\circ = 3.464$ m

Thrust on 'deep' side of gate

$= (1000 \times 9.81 \times 4.5)(9 \times 3.464)\text{N} = 1.376\ \text{MN}$

Trust on 'shallow' side of gate

$= (1000 \times 9.81 \times 1.35)(2.7 \times 3.464)\text{N}$

$= \underline{0.124\ \text{MN}}$

Net thrust $= (1.376 - 0.124)\ \text{MN} = 1.252\ \text{MN}$

$\therefore$ Force between gates $= \dfrac{1.252\ \text{MN}}{2 \sin 30^\circ} = \mathbf{1.252\ MN}$

Resultant force F acts at height y given by

$F_1\dfrac{h_1}{3} - F_2\dfrac{h_2}{3} = Fy$, since F_1, F_2 act at $\dfrac{2}{3}h_1$, $\dfrac{2}{3}h_2$ below free surfaces

$$\therefore\ y = \frac{1.376 \times 9/3 - 0.124 \times 2.7/3}{1.252}\ \text{m} = 3.208\ \text{m}$$

Total hinge reaction R also acts at this height.

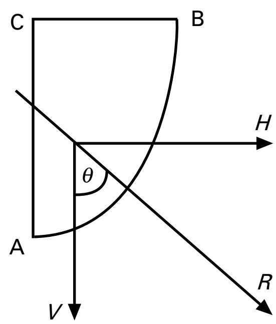
If top hinge is distance x above bottom hinge,

$R_{top}x = R(3.208 - 0.6)$ m

$\therefore\ x = \dfrac{R}{R_{top}} 2.608 \text{ m} = 3 \times 2.608 \text{ m} = 7.82 \text{ m},$

that is, **8.42 m above base**

2.15



Horizontal component H

$=$ Thrust on vertical projection AC divided by width

$= \dfrac{1}{2} 1000 \times 9.81 \times 27^2 \text{ N} \cdot \text{m}^{-1}$

$= 3.576 \text{ MN} \cdot \text{m}^{-1}$ acting at $\dfrac{2}{3} \times 27$ m

$=$ 18 m below BC

Vertical component $V =$ Weight of water ABC

$$\text{Area ABC} = \int_0^{27} x\text{d}y = \sqrt{18}\int_0^{27} y^{1/2}\text{d}y = \frac{2}{3}\sqrt{18}(27)^{3/2} \text{ m}^2$$

$$= 396.8 \text{ m}^2$$

$$\therefore \text{ Vertical component} = 1000 \times 9.81 \times 396.8 \text{ N} \cdot \text{m}^{-1}$$

$$= 3.893 \text{ MN} \cdot \text{m}^{-1}$$

It acts through centroid of ABC. Moments of area about AC:

$$396.8\bar{x} = \int_0^{27} x\text{d}y\frac{x}{2} = \int_0^{27} 9y\text{d}y = 9 \times \frac{27^2}{2} \text{ m}^3$$

whence $\bar{x} = 8.27$ m

$\theta = \arctan \dfrac{3.576}{3.893} = \mathbf{42.57°}$

Resultant $= \sqrt{(3.576^2 + 3.893^2)} \text{ MN} \cdot \text{m}^{-1} = \mathbf{5.29\ MN \cdot m^{-1}}$

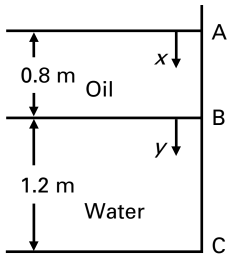
It intersects free surface at $(18 \tan\theta - 8.27)$ m from C

$=$ 8.27 m from C

that is $\{8.27 + \sqrt{(18 \times 27)}\}$ m

$= \mathbf{30.31\ m\ from\ B}$

2.16



Relevant forces are only those on vertical plane 0.5 m wide.

Total force on AB $= F_1$

$$= \int_0^{0.8\text{ m}} \varrho_{\text{oil}} g x b \mathrm{d}x = \left[\frac{1}{2}\varrho_{\text{oil}} g b x^2\right]_0^{0.8\text{m}}$$

$$= \frac{1}{2} 850 \text{ kg}\cdot\text{m}^{-3} \times 9.81 \text{ N}\cdot\text{kg}^{-1}$$

$$\times 0.5 \text{ m}(0.8\text{ m})^2 = 1334 \text{ N}$$

$$\text{Total force on BC} = \int_0^{1.2\text{ m}} (\varrho_{\text{oil}} g 0.8\text{ m} + \varrho_{\text{water}} g y) b \, \mathrm{d}y$$

$$= b\varrho_{\text{water}} g \int_0^{1.2\text{ m}} (0.85 \times 0.8\text{ m} + y)\mathrm{d}y$$

$$= b\varrho_{\text{water}} g \left[(0.68\text{ m})y + \frac{y^2}{2}\right]_0^{1.2\text{m}}$$

$$= 0.5 \times 1000 \times 9.81(0.816 + 0.72)\text{ N}$$

$$= 7535 \text{ N}$$

Total force $F = (7535 + 1334)\text{N} = \mathbf{8869\ N}$

Let total force act at height z above base of tank. Then moments about axis through C:

$$Fz = F_1\left(1.2 + \frac{0.8}{3}\right)\text{m} + \int_0^{1.2\text{m}} b\varrho_{\text{water}} g(0.68\text{ m} + y)(1.2\text{ m} - y)\mathrm{d}y$$

$$= 1334 \times 1.467 \text{ N}\cdot\text{m} + 0.5 \times 1000 \times 9.81 \text{ N}\cdot\text{m}^{-2}$$

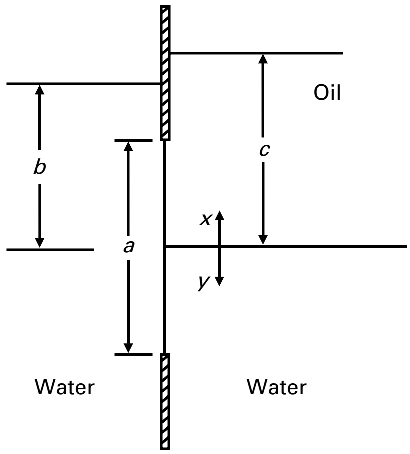
$$\times \left[(0.816\text{ m}^2)y + \frac{0.52\text{ m}}{2}y^2 - \frac{y^3}{3}\right]_0^{1.2\text{m}}$$

$$= 1957 \text{ N}\cdot\text{m} + 500 \times 9.81[0.9792 + 0.3744 - 0.576]\text{ N}\cdot\text{m}$$

$$= 5771 \text{ N}\cdot\text{m}$$

$$\therefore\ z = \frac{5771}{8869}\text{ m} = \mathbf{0.651\ m}$$

2.17



$\overrightarrow{\text{Force}} = \varrho g b a^2$

$$\overleftarrow{\text{Force}} = \sigma \varrho g \left(c - \frac{a}{4}\right)\frac{a^2}{2} + \left(\sigma \varrho g c + \varrho g \frac{a}{4}\right)\frac{a^2}{2}$$

$$= \sigma \varrho g c a^2 + \frac{\varrho g a^3}{8} \times (1 - \sigma)$$

For zero net force

$$\boldsymbol{b = \sigma c + \frac{a}{8}(1 - \sigma)}$$

Total moment $\curvearrowleft$ about centre-line for forces on left

$$= -\int_0^{a/2} \varrho g(b - x)ax\,\mathrm{d}x + \int_0^{a/2} \varrho g(b + y)ay\,\mathrm{d}y$$

$$= \varrho g a\left(-\frac{ba^2}{8} + \frac{a^3}{24} + \frac{ba^2}{8} + \frac{a^3}{24}\right) = \frac{1}{12}\varrho g a^4$$

$\therefore$ Net $\overrightarrow{\text{Force}}$ acts at $\frac{1}{12}\varrho g a^4 \div \varrho g b a^2 = a^2/12b$ below centre-line.

Total moment $\curvearrowright$ about centre-line for forces on right

$$= -\int_0^{a/2} \sigma \varrho g(c - x)ax\,\mathrm{d}x + \int_0^{a/2} (\sigma \varrho g c + \varrho g y)ay\,\mathrm{d}y$$

$$= \varrho g a\left(-\sigma c\frac{a^2}{8} + \sigma\frac{a^3}{24} + \sigma c\frac{a^2}{8} + \frac{a^3}{24}\right) = \frac{1}{24}\varrho g a^4(1 + \sigma)$$

$\therefore$ Net $\overleftarrow{\text{force}}$ acts at $\frac{1}{24}\varrho g a^4(1 + \sigma) \div \varrho g b a^2$

$= a^2(1 + \sigma)/24b$ below centre-line.

$\therefore$ Axis of couple is $\frac{1}{2}\left\{\frac{a^2}{12b} + \frac{a^2(1 + \sigma)}{24b}\right\}$

$= \boldsymbol{\frac{a^2}{48b}(3 + \sigma)}$ **below centre-line**

2.18 Pressure at centroid $= (15\,000 + 900 \times 9.81 \times 1)$ Pa $= 23\,829$ Pa

$\therefore$ Total force $= 23\,829$ Pa $\times\ 0.24$ m$^2 = 5719$ N

This acts on vertical centre-line

$\therefore$ Force on lock $= \dfrac{1}{2} \times 5719\ \text{N} = \mathbf{2859\ N}$

Air pressure is equivalent to $\dfrac{15\,000}{900 \times 9.81}\ \text{m} = 1.699\ \text{m}$ of oil

$\therefore$ Equivalent free (atmospheric) surface is at 2.699 m above centre-line

$\therefore$ Depth of C.P. below centre-line $= (Ak^2)_c / A\bar{y}$

$$= \frac{(0.4\ \text{m})^2}{12} \Big/ 2.699\ \text{m} = 0.00494\ \text{m}$$

Moments about horizontal axis through upper hinge:

$5719(0.125 + 0.00494)\ \text{N}\cdot\text{m} = 2859 \times 0.125\ \text{N}\cdot\text{m} + F_\text{L}(0.25\ \text{m})$

$\therefore$ Force on lower hinge $= F_\text{L} = \mathbf{1543\ N}$

and force on upper hinge $= (2859 - 1543)\ \text{N} = \mathbf{1317\ N}$

2.19 2.7 kg of iron occupy $\dfrac{2.7\ \text{kg}}{7500\ \text{kg}\cdot\text{m}^{-3}} = 0.00036\ \text{m}^3$

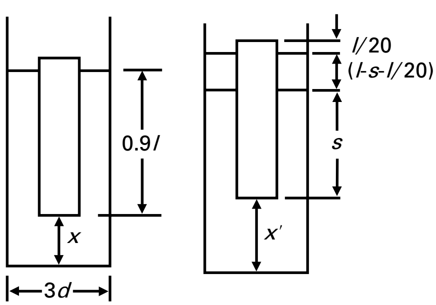
$\therefore$ Buoyancy force $= 0.00036\ \text{m}^3 \times 1000\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1}$

$= 0.36 \times 9.81\ \text{N}$

$\therefore$ Spring balance reads $(2.7 - 0.36)$ kgf $=$ **2.34 kgf**

Parcel balance reads $(5 + 0.36)$ kgf $=$ **5.36 kgf**

2.20



Archimedes for case II:

$$0.9l = 1 \times s + 0.8\left(\frac{19}{20}l - s\right)$$

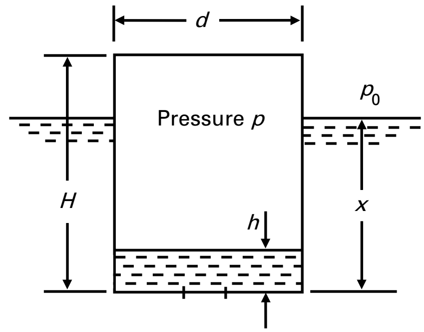
whence $s = 0.7l$

Volume of water is constant

$$\therefore \frac{\pi}{4}(3d)^2 x + 0.9l \times \left\{\frac{\pi}{4}(3d)^2 - \frac{\pi}{4}d^2\right\} = \frac{\pi}{4}(3d)^2 x' + 0.7l\left\{\frac{\pi}{4}(3d)^2 - \frac{\pi}{4}d^2\right\}$$

$\therefore 9x + 0.9l\{9 - 1\} = 9x' + 0.7l\{9 - 1\} \quad \therefore \boldsymbol{x' - x = 0.1778l}$

2.21



Archimedes $\frac{\pi}{4}d^2(x-h)\varrho g$

$= 27 \times 9.81\ \text{N}$

At base of cylinder, pressure

$= p_0 + \varrho g x = p + \varrho g h$

$\therefore p - p_0 = \varrho g(x-h)$

$$= \frac{27 \times 9.81\ \text{N}}{(\pi/4)(0.3\ \text{m})^2}$$

$= \mathbf{3747\ Pa}$

For isothermal compression $pV = \text{constant}$

$$\therefore\ p(H-h) = p_0 H$$

$$\therefore\ h = \frac{p-p_0}{p}H = \frac{3747}{105\,047} \times 450\ \text{mm} = \mathbf{16.05\ mm}$$

$$x - h = \frac{27 \times 9.81\ \text{N}}{(\pi/4)(0.3\ \text{m})^2 1000\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1}} = 0.382\ \text{m}$$

$\therefore \boldsymbol{x} = \mathbf{398\ mm}$

2.22 From eqn 2.7, p at 6000 m is $p_0\left(1 - \frac{\lambda z}{T_0}\right)^{g/R\lambda}$

$$= 101\ \text{kPa}\left(1 - \frac{0.0065 \times 6000}{288.15}\right)^{9.81/287\times0.0065} = 47.01\ \text{kPa}$$

$$\therefore\ \varrho \text{ at 6000 m is } \frac{47\,010}{287(288.15 - 0.0065 \times 6000)}\ \text{kg}\cdot\text{m}^{-3}$$

$$= 0.6574\ \text{kg}\cdot\text{m}^{-3}$$

which must be same as effective density of balloon.

$\therefore$ Total mass of balloon $= 0.6574 \times \frac{\pi}{6}0.8^3\ \text{kg} = 0.17625\ \text{kg}$

$\therefore$ Mass of helium $= (176.25 - 160)\text{g} = \mathbf{16.25\ g}$

2.23 $\text{BM} = Ak^2/V = \frac{\pi}{64}d^4 \Big/ \left(\frac{\pi}{4}d^2 \times 0.6l\right) = d^2/9.6l$

B is at $0.3l$ above base.

$\therefore$ When M and G coincide, BM $= 0.2l$

$\therefore d^2 = 0.2 \times 9.6l^2 \quad \therefore d/l = \sqrt{1.92} = \mathbf{1.386}$

2.24 Weight of pontoon

$$= (6 \times 3 \times 0.9)\ \text{m}^3 \times 1000\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1} = 158.9\ \text{kN}$$

$$\text{BM} = Ak^2/V = [(6 \times 3^3/12)/(6 \times 3 \times 0.9)]\ \text{m} = 0.833\ \text{m}$$

$$\text{GM} = \left(0.833 + \frac{0.9}{2} - 0.7\right)\ \text{m} = 0.583\ \text{m}$$

$$7600\ \text{N}\cdot\text{m} = W(\text{GM})\sin\theta$$

$$\therefore \sin\theta = \frac{7600}{158.9 \times 10^3 \times 0.583} \quad \therefore \theta = \mathbf{4.70°}$$

2.25 If relative density $= \sigma$, depth of immersion $h = 150\sigma$ mm
$\therefore$ Height of B $= 75\sigma$ mm

$$\text{BM} = Ak^2/V = \frac{\pi}{64}d^4 \Big/ \frac{\pi}{4}d^2h = \frac{d^2}{16h} = \frac{75^2}{16 \times 150\sigma}\ \text{mm}$$

$$= \frac{75}{32\sigma}\ \text{mm}$$

For stability BM > BG $\quad \therefore \dfrac{75}{32\sigma} > \dfrac{150}{2} - 75\sigma$

that is $\dfrac{1}{32\sigma} > 1 - \sigma$

$$\therefore 32\sigma^2 - 32\sigma + 1 > 0 \quad \therefore \sigma > \frac{16 + \sqrt{256 - 32}}{32} = 0.9677$$

$$\text{or } \sigma < \frac{16 - \sqrt{256 - 32}}{32} = 0.0323$$

this is unrealistic since cylinder is solid

$\therefore 0.9677 < \sigma < 1.0$

Mass of equal volume of water

$$= \frac{\pi}{4}(0.075)^2 \times 0.15\ \text{m}^3 \times 1000\ \text{kg}\cdot\text{m}^{-3} = 0.663\ \text{kg}$$

$\therefore$ Mass of cylinder is **between 0.641 kg and 0.663 kg**

2.26 $\text{Torque} = \dfrac{\text{Power}}{\omega} = \dfrac{3.34 \times 10^6}{1.4 \times 2\pi}\ \text{N}\cdot\text{m} = W(\text{GM})\sin\theta$

$$= 80 \times 10^6(\text{G}_1\text{M}_1)\sin 0.53°$$

whence $\text{G}_1\text{M}_1 = 0.513$ m

$\therefore \text{B}_1\text{M}_1 = (0.513 + 1.6 - 0.3)\ \text{m} = 1.813\ \text{m}$

$$B_1M_1 \times V_1 = Ak^2 = B_2M_2 \times V_2$$

$$\therefore B_2M_2 = 1.813 \text{ m} \times \frac{80 \times 10^6}{80 \times 10^6 - 400 \times 10^3 \times 9.81} = 1.907 \text{ m}$$

$$\frac{3.34 \times 10^6}{1.4 \times 2\pi} = 76.076 \times 10^6 (G_2M_2) \sin 0.75°$$

whence $G_2M_2 = 0.3813$ m

$$\therefore B_2G_2 = (1.907 - 0.381) \text{ m} = 1.525 \text{ m}$$

$$B_2G_1 = (1.6 - 0.3 + 0.075) \text{ m} = 1.375 \text{ m}$$

$$\therefore G_1G_2 = (1.525 - 1.375) \text{ m} = \mathbf{0.150 \text{ m}}$$

2.27 Volume of water displaced $= \dfrac{355}{1025}\text{m}^3 = 0.3463 \text{ m}^3 = \dfrac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi r^3\sqrt{3}$$

$\therefore r^3 = 0.1910 \text{ m}^3 \quad \therefore r = 0.576$ m

$$BM = Ak^2/V = \frac{\pi}{4}r^4 \Big/ \frac{1}{3}\pi r^2 h = \frac{3}{4}\frac{r^2}{h} = \frac{r}{4}\sqrt{3} = 0.2493 \text{ m}$$

B is at $\dfrac{3}{4}h = 0.748$ m above vertex, that is $(0.6\sqrt{3} - 0.748)$ m

$= 0.2912$ m below top

$\therefore$ M is $(0.2912 - 0.2493)$ m $= 0.0419$ m below top – this is limiting position of G.

Let beacon be x metres above top. Then moments about axis in top:

$$300(0.6\sqrt{3} - 0.75) - 55x = 355 \times 0.0419,$$

whence $\mathbf{x = 1.308}$ **m**

2.28 Depth of immersion $= 0.85 \times 0.8 \text{ m} = 0.68$ m

$\therefore$ B is 0.34 m above base

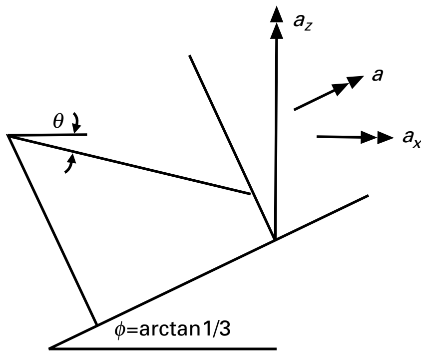
$$BM = Ak^2/V = \frac{\pi}{64}(1 \text{ m})^4 \Big/ \frac{\pi}{4}(1 \text{ m})^2 \times 0.68 \text{ m} = 0.0919 \text{ m}$$

$$\therefore GM = (0.0919 + 0.34 - 0.4) \text{ m} = 0.0319 \text{ m}$$

$$t = 2\pi\sqrt{\frac{k^2}{g(GM)}} = 2\pi\sqrt{\frac{l^2/12 + r^2/4}{g(GM)}} = 2\pi\sqrt{\frac{0.8^2/12 + 0.5^2/4}{9.81 \times 0.0319}} \text{ s}$$

$= \mathbf{3.822 \text{ s}}$

2.29 $0.405\ \mathrm{m}^3 = \left\{0.9^3 - 0.9\dfrac{0.9}{2}0.9\tan(\phi+\theta)\right\}\mathrm{m}^3$



whence $\tan(\phi+\theta) = \dfrac{8}{9}$

$$\therefore \tan\theta = \frac{8/9 - \tan\phi}{1 + \frac{8}{9}\tan\phi} = \frac{3}{7}$$

$$\tan\theta = \frac{3}{7} = \frac{a_x}{a_z + g}$$

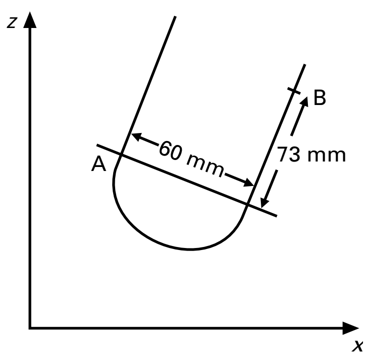
$$= \frac{a\cos\phi}{a\sin\phi + g}$$

whence $a = g\sqrt{10}/6$

Total mass $= (340 + 0.405 \times 850)$ kg $= 684.25$ kg

$\therefore\ F = 684.25 \times 9.81\sqrt{10}/6\ \mathrm{N} = \mathbf{3538\ N}$

2.30



$a_x = 2\cos 20°\ \mathrm{m \cdot s^{-2}}$;

$a_z = -2\sin 20°\ \mathrm{m \cdot s^{-2}}$

If A is origin, B is at

$\{(60\cos 20° + 73\sin 20°)$ mm,

$(73\cos 20° - 60\sin 20°)$ mm$\}$

that is (81.35 mm, 48.08 mm)

Pressure at B $= -\varrho a_x x - \varrho(g + a_z)z +$ constant

If p at A is taken as zero, constant $= 0$

$$\begin{aligned}\therefore p_{\mathrm{B}} &= -790\ \mathrm{kg \cdot m^{-3}} \times \{2\cos 20° \times 0.08135 \\ &\quad + (9.81 - 2\sin 20°)0.04808\}\ \mathrm{m^2 \cdot s^{-2}} \\ &= -790(0.1529 + 0.4388)\ \mathrm{Pa} = \mathbf{-467\ Pa}\end{aligned}$$