Chapter 2

Problem 1

Apply energy equation entry to (1) to (2)

$$\frac{p_1}{\rho g}+\frac{V_1}{2g}=\frac{p_0}{\rho g}+\frac{V_2^2}{2g}=\frac{p_{entry}}{\rho g}+\frac{V_{entry}^2}{2g}+4$$

$$\frac{p_{entry}}{\rho g}=2m \quad V_{entry}=0 \quad \frac{p_0}{\rho g}=0$$

$$\therefore \qquad \frac{V_2^2}{2g}=2+4$$

$$V_2=\sqrt{12g}=\underline{10.85\ m/s}$$

To find p_1, first find V_1

$$V_1A_1=V_2A_2$$

$$V_1=V_2\frac{A_2}{A}=10.85\times\frac{300^2}{200^2}$$

$$V_1=24.41\ m/s$$

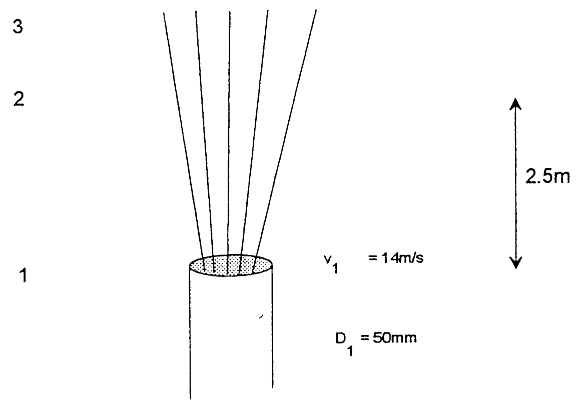
hence
$$\frac{p_1}{\rho g}+\frac{24.41^2}{2g}=6$$

$$\frac{p_1}{\rho g}=6-30.37$$

$$p_1=-239\ kN/m^2 \quad !$$

The solution is physically impossible as p_1 cannot be less than negative atmospheric pressure. Either frictional losses would reduce the velocities and/or cavitation would occur.

Problem 2



Assume atmospheric pressure at all heights and apply the energy equation.

$$\frac{V_1^2}{2g} + 0 = \frac{V_2^2}{2g} + 2.5 = Z_3$$

$$\therefore \quad \frac{V_2^2}{2g} = \frac{14^2}{2g} - 2.5$$

$$V_2 = 12.12 \ m/s$$

Apply continuity

$$V_1 A_1 = V_2 A_2$$

$$A_2 = \frac{V_1}{V_2} A_1$$

$$\text{or } D_2^2 = \frac{V_1}{V_2} D_1^2 = \frac{14}{12.12} \times 0.05^2 = 0.00289$$

$$\underline{D_2 = 53.7m}$$

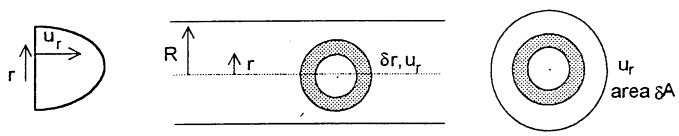
$$Z_3 = \frac{V_1^2}{2g} = \underline{9.99 \ m}$$

Problem 3

Given $u_r = K\left(R^2 - r^2\right)$

and $\alpha = \dfrac{1}{\overline{V}^3 A}\int u_r^3\, dA$

then



$$A = \pi R^2 \qquad \delta A = 2\pi r\,\delta r$$

$$\therefore \quad \int u^3 dA = 2\pi K^3 \int_0^R r\left(R^2 - r^2\right)^3 dr$$

$$= 2\pi K^3 \int r\left(R^4 + r^4 - 2R^2 r^2\right)\left(R^2 - r^2\right) dr$$

$$= 2\pi K^3 \int r\left(R^6 + R^2 r^4 - 2R^4 r^2 - R^4 r^2 - r^6 + 2R^2 r^4\right) dr$$

$$= 2\pi K^3 \int_0^R \left(r R^6 + 3R^2 r^5 - 3R^4 r^3 - r^7\right) dr$$

$$= 2\pi K^3 \left[\frac{r^2 R^6}{2} + \frac{3R^2 r^6}{6} - \frac{3R^4 R^4}{4} - \frac{r^8}{8}\right]_0^R$$

$$= 2\pi K^3 \left[\frac{R^8}{2} + \frac{R^8}{2} - \frac{3R^8}{4} - \frac{R^8}{8}\right]$$

$$= 2\pi K^3 \frac{R^8}{8}$$

$$= \frac{\pi K^3 R^8}{4}$$

$$\overline{V} = \frac{Q}{A} = \frac{\int_0^R u_r 2\pi r\, dr}{\pi R^2}$$

$$= \frac{2}{R^2}\int_0^R K\left(R^2 - r^2\right) r\, dr$$

$$= \frac{2K}{R^2}\int_0^R R^2 r - r^3\, dr$$

$$= \frac{2K}{R^2}\left[\frac{R^2r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{2K}{R^2}\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$\overline{V} = \frac{KR^2}{2}$$

$$\therefore \quad \overline{V}^3 A = \frac{K^3R^6}{8}\pi R^2 = \frac{\pi R^8 K^3}{8}$$

$$\therefore \quad \alpha = \frac{\pi K^3 R^8}{4} . \frac{8}{\pi R^8} K^3$$

$$\underline{\underline{\alpha = 2}}$$

$$\beta = \frac{1}{\overline{V}^2} A \int u^2 dA$$

$$\int u^2 dA = 2\pi K \int_0^R r\left(R^2 - r^2\right)^2 dr$$

$$= 2\pi K^2 \int_0^R \left(rR^4 + r^5 - 2R^2r^3\right) dr$$

$$= 2\pi K^2\left[\frac{r^2R^4}{2} + \frac{r^6}{6} - \frac{2R^2}{4}r^4\right]_0^R$$

$$= 2\pi K^2\left[\frac{R^6}{2} + \frac{R^6}{6} - \frac{R^6}{2}\right]$$

$$= \frac{\pi K^2}{3} R^6$$

$$\overline{V}^2 A = \frac{K^2}{4} R^4 . \pi R^2 = \frac{K^2 \pi}{4} R^6$$

$$\therefore \beta = \frac{\pi K^2}{3} R^6 . \frac{4}{K^2 \pi R^6}$$

$$\underline{\underline{= \frac{4}{3}}}$$

Problem 4

Momentum forces

$$F_{mx} = \rho Q\left(V_2 \cos 45 - V_1\right) = +4.36 kN$$

$$F_{my} = \rho Q\left(-V_2 \sin 45 - 0\right) = -5.64 kN$$

Pressure Forces

$$F_{px} = p_1 A_1 - p_2 A_2 \cos 45 = 176.9 kN$$

$$F_{py} = 0 + p_2 A_2 \sin 45 = +19.5 kN$$

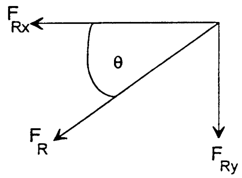
Reaction Forces

$\left(F_m = \Sigma F_p + F_R\right)$ $\quad F_{Rx} = -F_{px} + F_{mx} = -176.9 + 4.36 = -172.54$

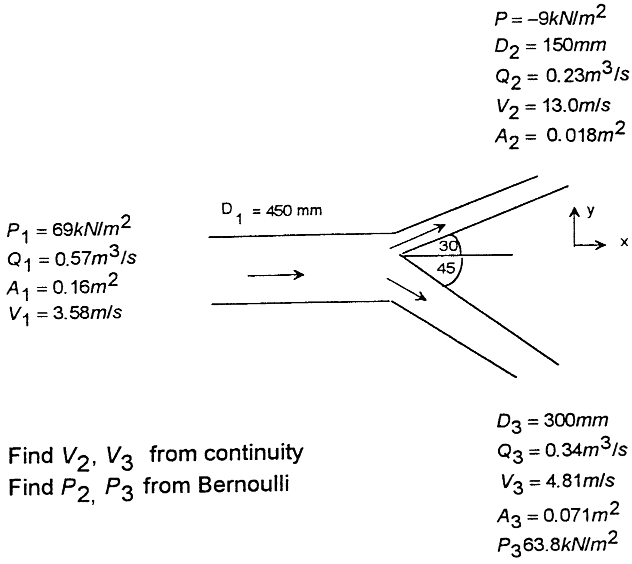
$$F_{Ry} = -F_{py} + F_{my} = -19.5 - 5.64 = -25.14$$

$$F_R = \sqrt{172.5^2 + 25.14^2} = 174.4 kN$$

$$\theta = \tan^{-1}\frac{25.14}{172.54} = 8.3°$$



Problem 5



Find V_2, V_3 from continuity
Find P_2, P_3 from Bernoulli

Momentum

x direction $\quad Fmx = \rho Q_2 V_2 \cos 30 + \rho Q_3 V_3 \cos 45 - \rho Q_1 V_1$

$= \rho(2.589 + 1.156 - 2.04)$

$=$ 1.705 kN

y direction $\quad Fmy = \rho Q_2 V_2 \sin 30 - \rho Q_3 V_3 \sin 45 - 0$

$= \rho(1.495 - 1.156)$

$=$ 0.34 kN

Pressure

x direction $\quad Fpx = P_1 A_1 - P_2 A_2 \cos 30 - P_3 A_3 \cos 45$

$=$ 11.04 -0.14-3.203

$=$ + 7.98 kN

y direction $\quad Fpy = -P_2 A_2 \sin 30 + P_3 A_3 \sin 45$

$=$ -(-0.081)+ 3.203

$=$ +3.28 kN

Problem 5 (continued)

Total force equations $\left(F_p + F_R = F_m\right)$

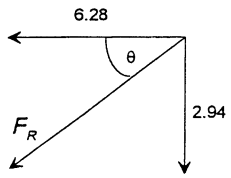
x direction F_{Rx} = 1.705 - 7.98

= -6.28 kN

y direction F_{Ry} = 0.34 - 3.28

= -2.94 kN

Resultant Force



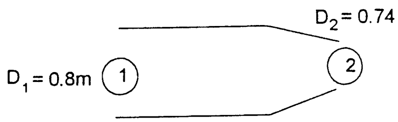
$$F_R = \sqrt{6.28^2 + 2.94^2}$$

$$F_R = 6.93\ kN$$

$$\theta = \tan^{-1}\left(\frac{2.94}{6.28}\right)$$

$$\theta = 25.1$$

Problem 6



$$\frac{\Delta p}{\rho g} = 30 \text{ mm water}$$

$$\therefore \ \Delta p = 249.3 N/m^2$$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{V_2^2 - V_1^2}{2g} = \left(\frac{p_1 - p_2}{\rho g}\right)$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{\Delta p}{\rho} = \frac{294.3}{1.3} = 226.38$$

also $V_1 A_1 = V_2 A_2$

$$\therefore \quad V_1 = \frac{V_2 A_2}{A_1} = \frac{V_2 \times 0.74^2}{0.8^2} = 0.8556 V_2$$

$$\therefore \quad \frac{V_2^2 - 0.8556^2 V_2^2}{2} = 226.38$$

$$0.134 V_2^2 = 226.38$$

$$V_2 = 41.1 \ m/s$$

Mass flow rate = ρQ

$$Q = V_2 A_2 = \frac{41.1 \times 0.74^2 \times \pi}{4} = 17.68 m^3/s$$

$$\rho Q = 1.3 \times 17.68 = 23.0 \ kg/s$$

Problem 7

$$Q = C_d \frac{\pi D_1^2}{4}\left(\frac{1}{\sqrt{m^4 - 1}}\right)\sqrt{2gh^*}$$

$$h^* = R_p\left(\frac{\rho g}{\rho} - 1\right) = 0.015\left(\frac{13.6}{1} - 1\right) = 0.189m$$

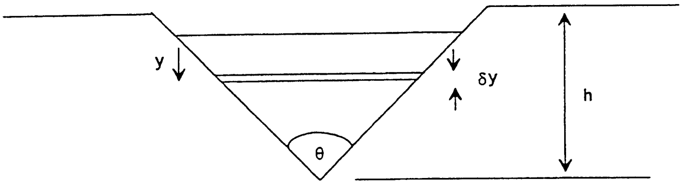
$$m = \frac{D_1}{D_2} = \frac{300}{200} = 1.5$$

$$\therefore \qquad Q = 0.98\frac{\pi \times 0.3^2}{4}\left(\frac{1}{\sqrt{1.5^4 - 1}}\right)\sqrt{2g \times 0.189}$$

$$= \qquad 0.0662\ m^3 s$$

$$= \qquad 66.2\ l/s$$

Problem 8



For an elemental strip at depth y

velocity u $= \sqrt{2gy}$

area $= 2(h-y)\tan\left(\frac{\theta}{2}\right)\delta y$

$\delta Q = \sqrt{2gy}\,2(h-y)\tan\frac{\theta}{2}$

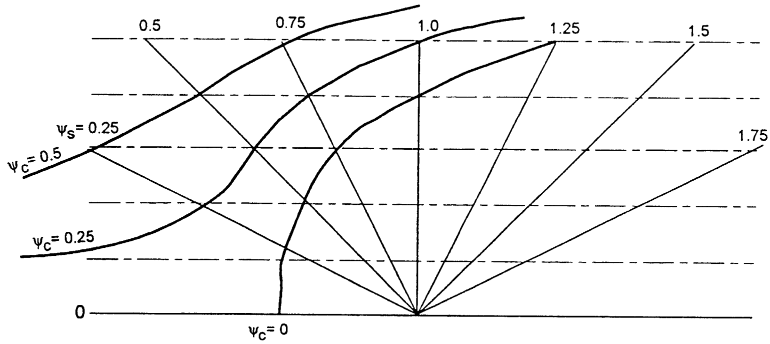
$$\therefore Q = 2\sqrt{2g}\tan\frac{\theta}{2}\int_{y=0}^{y=h}(h-y)y^{\frac{1}{2}}dy$$

$$= 2\sqrt{2g}\tan\frac{\theta}{2}\int_{y=0}^{y=h}\left(hy^{\frac{1}{2}} - y^{\frac{3}{2}}\right)dy$$

$$= 2\sqrt{2g}\tan\frac{\theta}{2}\left[\frac{2}{3}hy^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}}\right]_{y=0}^{y=h}$$

$$Q_{ideal} = \frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}h^{\frac{5}{2}}$$

Problem 9



Velocity → 0 at stagnation point where radial velocity V_r is equal and opposite to velocity of linear flow i.e. V_r = 5 m/s.

i.e. $V_r r\theta = \dfrac{Q\theta}{2\pi} = \psi$

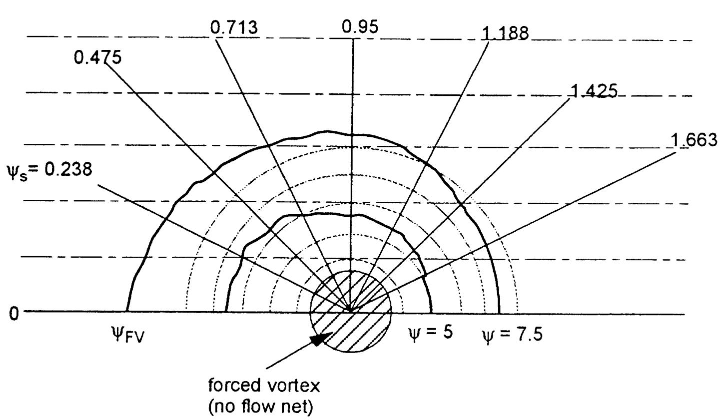
Therefore

$$r = \frac{Q}{2\pi V_r} \quad \text{for } \theta = 0$$

Therefore

$$r = \frac{4}{2\pi \times 5} \quad = 0.127 \text{ m} \quad = 127 \text{ mm}$$

Problem 10



For forced vortex $V = \omega r \quad = 100 \times 0.15 \quad = 15 \text{ m/s}$

For source $V_r = \dfrac{3.8}{\pi} \times 0.30 \quad = 4.03 \text{ m/s}$

Resultant velocity $= \sqrt{15^2 + 4.03^2} \quad = 15.53 \text{ m/s}$

$$\frac{dH}{dr} = \frac{2\omega^2 r}{g} \quad \text{therefore } H = \frac{\omega^2 r^2}{g} \quad = \frac{100^2 \times 0.15^2}{9.81} = 22.94 \text{ m}$$

At 0.15 m radius the free vortex begins, so

$$V = \frac{K}{r} \qquad 15 = \frac{K}{0.15} \qquad \text{therefore} \qquad K = 2.25$$

At 250 mm radius $V = \dfrac{2.25}{0.25} \quad = 9 \text{ m/s}$

$$V_{source} = V_r = \frac{3.8}{2\pi \times 0.25} \qquad = 2.434 \text{ m/s}$$

Resultant velocity $= \sqrt{9^2 + 2.424^2} \quad = 9.32 \text{ m/s}$

$$H = 22.94 \quad = \frac{p}{\rho g} + \frac{V^2}{2g} \quad = \frac{p}{\rho g} + \frac{9.32^2}{2 \times 9.81}$$

therefore $p = 181.6 \text{ kN/m}^2$