2. Review of Digital Signal Processing

2.1. (Review of LTI Systems)

Consider a time-varying discrete-time system described by

$$y(n) = 2nx(n-1) + 2n^2 x(n-2) - x(n-3)$$

Determine if this system is Linear. Explain why (or why not).

Solution:

Assume $x_1(n) \to y_1(n)$ and $x_2(n) \to y_2(n)$

Let $x(n) = a_1 x_1(n) + a_2 x_2(n)$ where a_1, a_2 are scalars

$$y(n) = 2n(a_1 x_1(n-1) + a_2 x_2(n-1)) + 2n^2(a_1 x_1(n-2) + a_2 x_2(n-2)) - (a_1 x_1(n-3) + a_2 x_2(n-3))$$

$$y(n) = (2na_1 x_1(n-1) + 2n^2 a_1 x_1(n-2) - a_1 x_1(n-3)) + (2na_2 x_2(n-1) + 2n^2 a_2 x_2(n-2) - a_2 x_2(n-3))$$

$$y(n) = a_1 y_1(n) + a_2 y_2(n)$$

The system is linear as it satisfies the superposition and homogeneity properties.

2.2. (Review of LTI Systems)

For each of the analog systems described by the following input-output relationship: Determine if the system (i) has memory, (ii) is invertible, (iii) is Bounded-Input Bounded-Output (BIBO) stable, (iv) is Time-Invariant and (v) is Linear. Explain why (or why not).

Solution:

	Memory	Invertible	BIBO Stable	Time Invariant	Linear
$y(t) = \int_{-\infty}^{+\infty} 3x(\tau - \alpha)d\tau$	Yes	Yes	Yes, BIBO stable	Yes	Yes
$y(t) = \log_e(2x(t))$	No	Yes	Yes, BIBO Stable	Yes	No
$y(t) = 3x(2t-5)$	Yes	Yes	Yes, BIBO Stable	Yes	Yes
$y(t) = \cos(x(t-5))$	Yes	No	Yes, BIBO Stable	Yes	No
$y(t) = \cos(2t)x(t-1))$	Yes	Yes	Yes, BIBO Stable	No	No

(We added Causality determination here below

$$y(t) = \int_{-\infty}^{+\infty} 3x(\tau - \alpha)d\tau$$

(i) has memory
(ii) is invertible

(iii) $y(t_0) = \int_{-\infty}^{+\infty} 3x(\tau - \alpha)d\tau$ => Not Causal, depends on all inputs, has memory

(iv) $|y(t)| = |\int_{-\infty}^{+\infty} 3x(\tau - \alpha)d\tau\ | = 3\int_{-\infty}^{+\infty} |x(\tau - \alpha)|\, d\tau < \infty$ for all time => BIBO stable

(v) $y_d(t) = y(t-t_0)$ => Time-Invariant

(vi) Superposition applies => Linear

$y(t) = \log_e(2x(t))$

(i) $y(0) = \ln[2x(0)]$ => has no memory

(ii) $x(t) = (1/2)\, e^{y(t)}$ => Invertible

(iii) $y(t_0) = \ln[2x(t_0)]$ => Causal, depends on current inputs only, no memory

(iv) $|y(t)| = |\ln[2x(t)]| < |x(t)|$ for all time. => BIBO stable

(v) $y_d(t) = \ln[2x(t-t_0)]$

$y(t-t_0) = \ln[2x(t-t_0)] = y_d(t)$ => Time-Invariant

(vi) $y(t) = \ln[x_1(t_0) + x_2(t_0)] \neq \ln[x_1(t_0)] + \ln[x_2(t_0)]$ => Not Linear

$y(t) = 3x(2t-5)$

(i) $y(0) = 3x(-5)]$ => has memory

(ii) $x(2t-5) = (1/3)y(t)$ => Invertible

(iii) $y(t_0) = 3x(2t_0 - 5)$ => Causal, depends on past inputs only

(iv) $|y(t)| = |3x(2t - 5)| < |3||x(2t-5)| < G <$ infinity, for all time. => BIBO stable

(v) $y_d(t) = 3x(2(t-t_0) - 5)$

$y(t-t_0) = 3x(2(t-t_0) - 5) = y_d(t)$ => Time-Invariant

(vi) $y(t) = 3[x_1(2t – 5) + x_2(2t – 5)] = 3[x_1(2t – 5)] + 3[x_2(2t – 5)]$ => Linear

$y(t) = \cos(x(t-5))$

(i) $y(0) = \cos[x(-5)]$ => has memory

(ii) $\cos(-\theta) = \cos(\theta)$ => not invertible

(iii) $y(t_0) = \cos[x(t_0-5)]$ => Causal

(iv) $|y(t)| < 1$ for all time => BIBO stable

(v) $y_d(t) = \cos[x(t- t_0-5)]$

$y(t-t_0) = \cos[x(t- t_0-5)] = y_d(t)$ => Time-Invariant

(vi) $\cos(\theta_1 + \theta_2) \neq \cos(\theta_1) + \cos(\theta_2)$ => Not Linear

$y(t) = \cos(2t)x(t-1))$

(i) $y(0) = \cos(0)\, x(-1)$ => has memory

(ii) $\cos(-\theta) = \cos(\theta)$ => not invertible

(iii) $y(t_0) = \cos(2t_0)\, x(t_0-1)$ => Causal

(iv) $|y(t_0)| < |\cos(2t_0)\, x(t_0-1)| < |x(t_0-1)|$ for all time => BIBO stable

(v) $y_d(t) = \cos(2t)\, x(t-t_0-1)$

$y(t-t_0) = \cos(2(t-t_0))\, x(t-t_0-1) \neq y_d(t)$ => Not Time-Invariant

(vi) $\cos(\theta_1 + \theta_2) \neq \cos(\theta_1) + \cos(\theta_2)$ => Not Linear

2.3. (Review of DSP)

The samples of a digital signal are spaced $0.2 \text{ x } 10^{-5}$ seconds apart. What is the maximum possible frequency content of the original analog signal in order to avoid aliasing?

Solution:

Sampling frequency=Fs=1/T=500 KHz
Maximum possible frequency of the original analog signal in order to avoid aliasing
=Fs/2
=250 KHz

2.4. (Review of DSP)

(a) What are three advantages of digital signal processing over analog signal processing?

Solution:

- Digital implementation allows the realization of certain characteristics not possible with analog implementation, such as exact linear phase, code-division multiplexing and multi-rate signal processing.
- Digital signals can be cascaded without input/output loading problems unlike analog signals.
- Digital signals can be stored almost indefinitely without any loss of information on various storage media such as magnetic tapes and disks and optical drives. On the other hand stored analog signals deteriorate rapidly as time progresses and cannot be recovered in their original forms.

(b) What are three dis-advantages of digital signal processing over analog signal processing?

Solution:

- Increased system complexity in the digital processing of analog signals because of need for additional pre- and post processing devices such as A/D and D/A convertors and their associated filters and complex digital circuitry.
- For digital signal processing only a limited range of frequencies are available for processing. This property limits its application particularly in the digital processing of analog signals.
- Digital systems are constructed using active devices that consume electrical power. On the other hand variety of analog processing algorithms are implemented using passive circuits employing inductors, capacitors, etc. that do not consume power. Moreover active devices are less reliable that passive components.

2.5. (Review of DSP)

(a)The discrete-time signal $x(n) = \cos(3\pi n/8), \quad -\infty \le n \le +\infty$ was obtained by sampling an analog signal $x(t) = \cos(2\pi ft), \quad -\infty \le t \le +\infty$ at a sampling rate of $f_s = 8$ KHz.
What are any two possible values of the analog frequency, f that could have resulted in the sequence $x(n)$?

Solution:

x(n)=cos(2πfn/fs)=cos(3πn/8)
For fs=8000 Hz; f=1500 Hz
Also cos((2π-3π/8)n)=cos(3πn/8)
The analog second frequency which got aliased to 1500Hz is
F=6500 Hz

(b) A certain continuous-time seismic signal is 3 minutes long. The spectrum of the signal ranges from dc to 400Hz. This analog signal is to be sampled and converted to a digital signal for digital signal processing on a DSP processor system. (i) What is the theoretical minimum number of samples that must be taken? (ii) Assume that each sample is to be represented by an 8-bit binary number. What is the minimum storage in bits required to handle this signal?

Solution:

(i) The minimum sampling frequency is 800 Hz
Hence the total number of samples= fs*length of the signal
=800*3
=2400

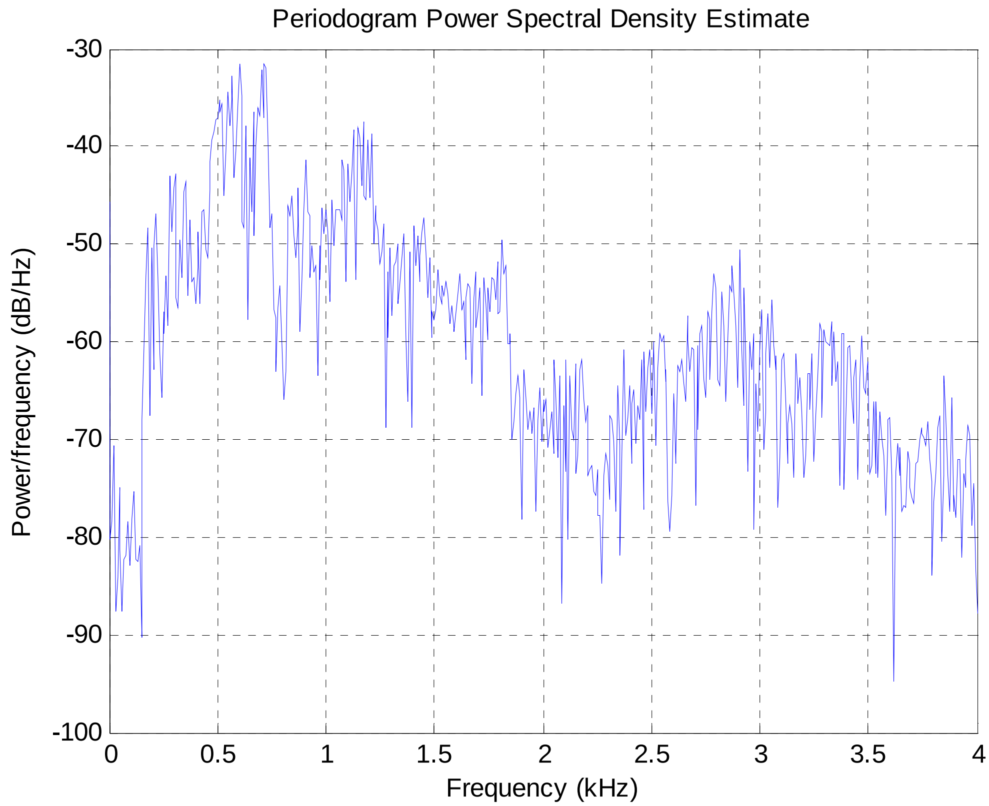
(ii) The minimum storage in bits is = 2400*8= 19200

2.6. (Review of Stochastic Signal Processing)

Using MATLAB, plot the PSD of a portion of the speech signal in Figure 1.2.

Solution:

```
[y,fs,N]=wavread('goaway.wav');
plot(y);
Hs = spectrum.periodogram('Hamming');
psd(Hs,y,'Fs',fs,'NFFT',1024,'SpectrumType','onesided');
```



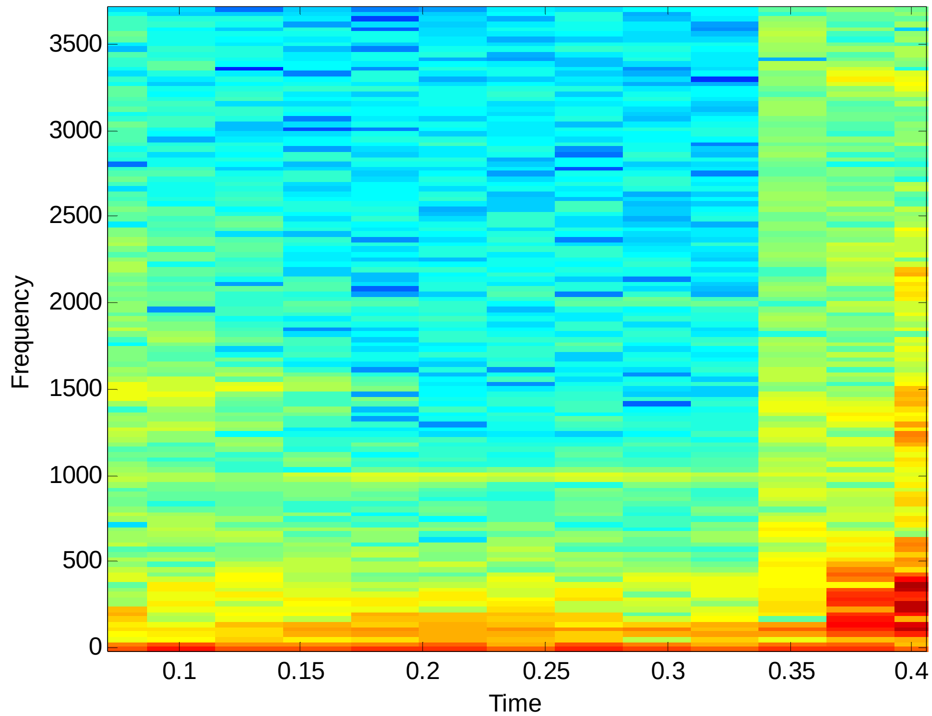
2.7. (Review of Stochastic Signal Processing)

Use MATLAB to explore effect of window length on the spectrogram of speech signals.

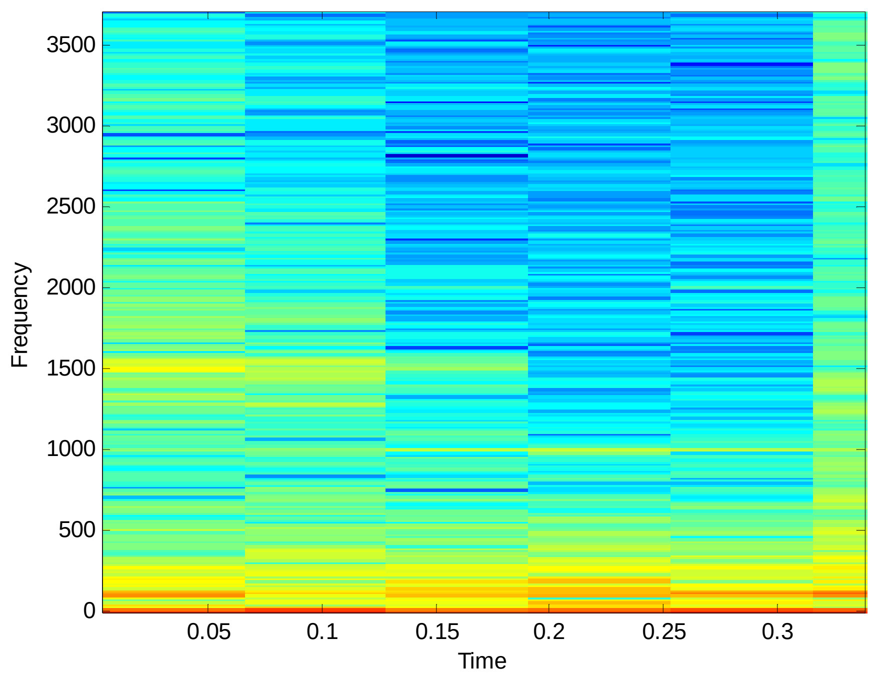
<u>Solution:</u>

```
load y %input speech
n = 1:length(y);
plot(n,y);
xlabel('Time index n');ylabel('Amplitude');
pause
nfft = input('Type in the window length = ');
ovlap = input('Type in the desired overlap = ');
figure;
specgram(y,nfft,7418,hamming(nfft),ovlap)
```

Window length=256 Overlap=50



Windows length=512 Overlap=50

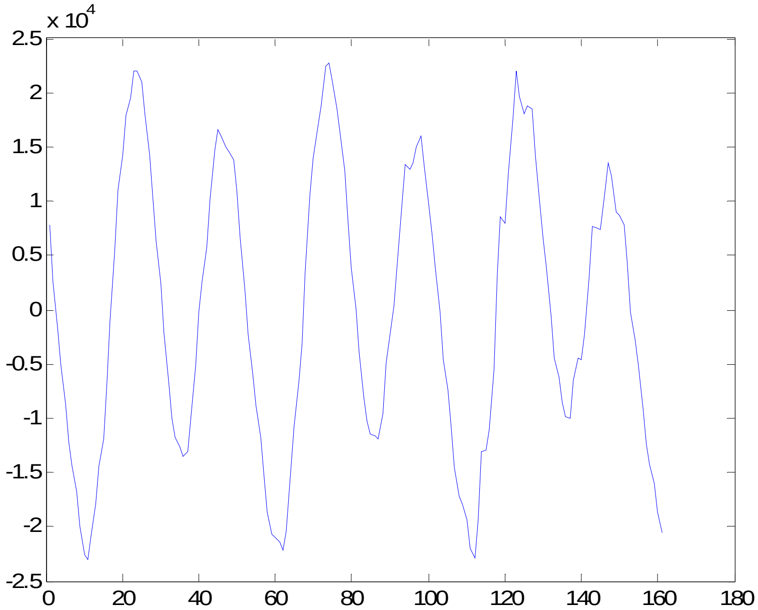


2.8. (Pitch Period Estimation Methods)

Using the sample speech signal in Figure 1.2, compute the pitch using the methods of both AMDF and Autocorrelation. Try the generalized autocorrelation for k=1. Compare the result with the AMDF method. Write a MATLAB program to do this example.

Solution:

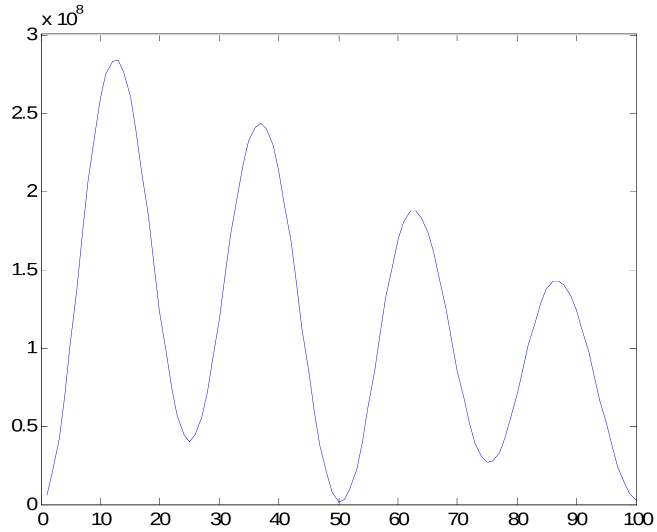
Take a portion of speech signal of a length of 20ms as shown below and the pitch is computed for that speech frame.



The MATLAB code is as shown below-

```
load y;
x1=y(5600:5760);N=length(x1);k=2;
x=double(x1);
for tou=1:100
    sum = double(0);
    for i=1:N
        if(i+tou>N)
          break;
        else
        sum=sum+(abs(x(i)-x(i+tou)))^2;
        end
    end
    E(tou)=1/N*(sum)^1/k;
end
[value,j]=max(E);
fprintf('The pitch=%f',8000/j);
```

The plot of E is as shown. For k=2, it is autocorrelation method and for k=1 it is AMDF method. The corresponding index of maximum E value will give us the pitch.



The pitch value of the speech frame is - 615 Hz.