Chapter 2:
Electrical Conduction
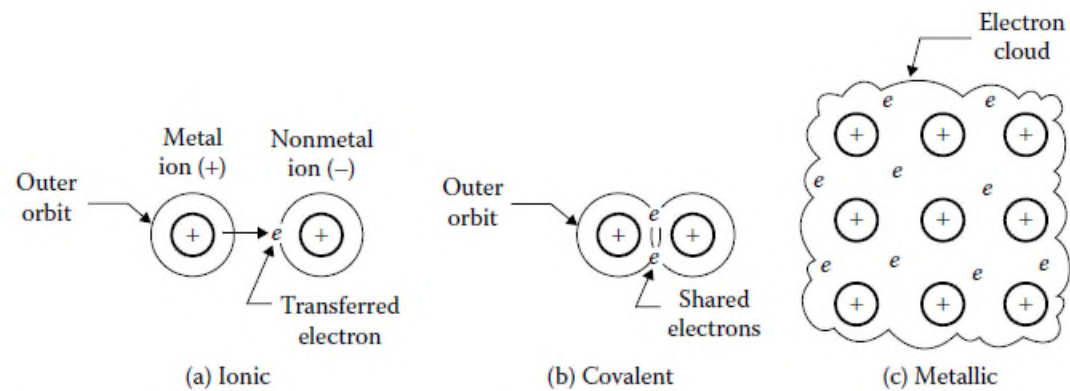# in Metals and Alloys

FIGURE 2.1 Illustration of (a) ionic bonds, such as those in sodium chloride and many other ceramics; (b) covalent bonds, such as those in silicon, many ceramics, and polymers; and (c) metallic bonds. (From Groover, M. P. 2007. *Fundamentals of Modern Manufacturing: Materials, Processes, and Systems.* New York: Wiley. With permission.)
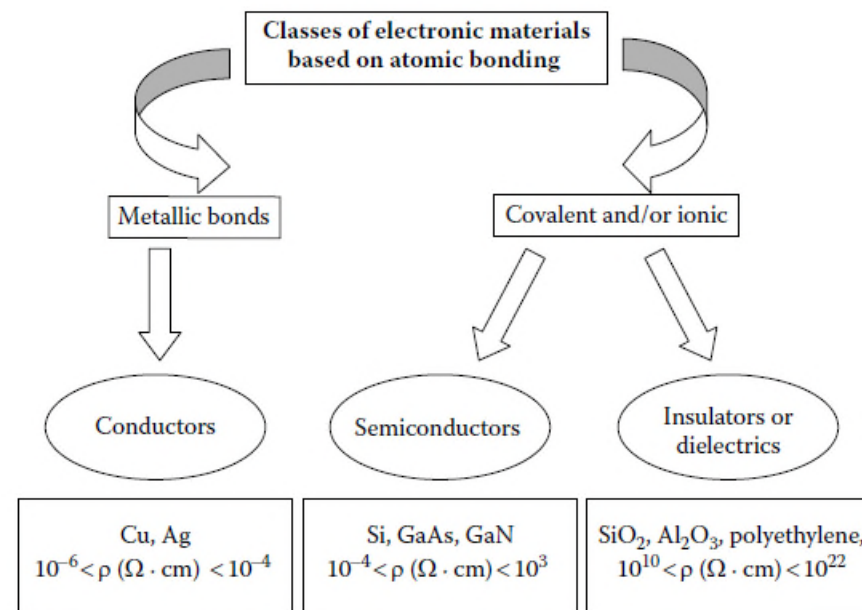
FIGURE 2.2 Classification of materials based on the nature of bonding. Typical ranges of resistivity are also shown.
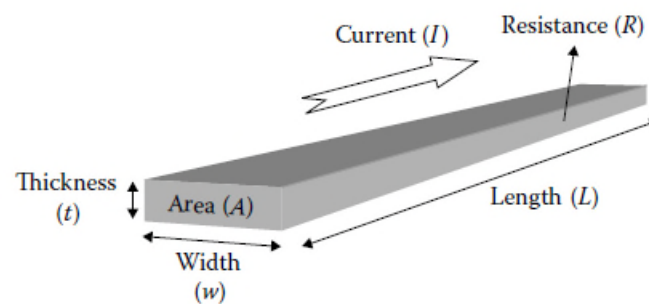
FIGURE 2.3 Geometry of a resistor used in describing Ohm's law.

EXAMPLE 2.1: RESISTIVITY AND CONDUCTIVITY—UNITS AND CONVERSION

The resistivity of a Cu sample is 1.673 $\mu\Omega \cdot cm$.
   a. What is the value of this resistivity in $\Omega \cdot cm$? What is the value in $n\Omega \cdot m$?
   b. The conductivity of an Ag sample is listed as 62.9×10^4 S/cm. What is the resistivity in $\mu\Omega \cdot cm$?

SOLUTION

a. Cu sample resistivity in $\Omega \cdot cm = 1.673 \ \mu\Omega \cdot cm \times 10^{-6} \ \Omega/\mu\Omega = 1.673 \times 10^{-6} \ \Omega \cdot cm$.
Cu sample resistivity in $n\Omega \cdot m = 1.673 \times 10^{-6} \ \Omega \cdot cm \times 10^{9} \ n\Omega/\Omega \times 10^{-2} \ m/cm = 16.73 \ n\Omega \cdot m$.

b. Conductivity of the Ag sample $= 62.9 \times 10^{4}$ S/cm.
Therefore, the resistivity of the Ag sample $= (62.9 \times 10^{4})^{-1} \ \Omega \cdot cm = 1.589 \times 10^{-6} \ \Omega \cdot cm$.
We want the answer in $\mu\Omega \cdot cm$,
Therefore, the resistivity of the Ag sample $= 1.589 \times 10^{-6} \ \Omega \cdot cm \times 10^{6} \ \mu\Omega/\Omega = 1.589 \ \mu\Omega \cdot cm$.

EXAMPLE 2.2: ANTENNAS FOR RADIO FREQUENCY–IDENTIFICATION TECHNOLOGY

Radio frequency–identification (RFID) technology is used for applications such as automatic highway toll-payment systems (e.g., EZ-Pass™), smart cards for access control, libraries, livestock tracking, and many other inventory-control applications (Figure 2.4).

A passive RFID tag is comprised of a conductive antenna, typically made from Cu, with a small computer chip attached to it. The computer chip holds the information. In one process to manufacture antennas, we begin with a thin foil of solid Cu. A pattern is then created by etching away excess

TABLE 2.1
Conductivity/Resistivity Values for Selected Metals

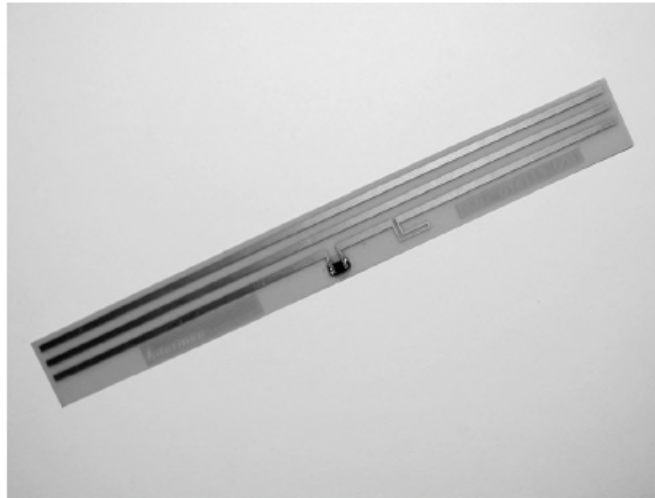| Material	Conductivity (σ) S/cm	Resistivity (ρ) $\mu\Omega \cdot$ cm	Temperature Coefficient of Resistivity (α_R) ($\Omega/\Omega \cdot$ °C)
Aluminum	37.7×10^4	2.65	4.3×10^{-3}
Beryllium	25×10^4	4.0	25×10^{-3}
Cadmium	14.6×10^4	6.83	4.2×10^{-3}
Chromium	7.75×10^4 (at 0°C)	12.9 (at 0°C)	3.0×10^{-3}
Cobalt (magnetic)	16.0×10^4	6.24	5.30×10^{-3}
Copper	59.7×10^4	1.673	4.3×10^{-3}
Gold	42.5×10^4	2.35	3.5×10^{-3}
Iridium	18.8×10^4	5.3	3.93×10^{-3}
Iron (magnetic)	10.3×10^4	9.7	6.51×10^{-3}
Lead	4.84×10^4	20.65	3.68×10^{-3}
Magnesium	22.4×10^4	4.45	3.7×10^{-3}
Mercury	1.0×10^4 (at 50°C)	98.4 (at 50°C)	0.97×10^{-3}
Molybdenum	19.2×10^4 (at 0°C)	5.2 (at 0°C)	5.3×10^{-3}
Nickel (magnetic)	14.6×10^4	6.84	6.92×10^{-3}
Palladium	9.253×10^4	10.8	3.78×10^{-3}
Platinum	10.15×10^4	9.85	3.93×10^{-3}
Rhodium	22.2×10^4	4.51	4.3×10^{-3}
Silver	62.9×10^4	1.59	4.1×10^{-3}
Tantalum	0.74×10^4	13.5	3.83×10^{-3}
Tin	0.90×10^4	11.0 (at 0°C)	3.64×10^{-3}
Titanium	2.38×10^4	42.0	3.5×10^{-3}
Tungsten	18.8×10^4	5.3	4.5×10^{-3}
Zinc	16.9×10^4	5.9	4.1×10^{-3}

FIGURE 2.4 Typical passive radio frequency–identification (RFID) tag showing the antenna made from copper. (Courtesy of Pavel Nikitin, Intermec Technologies Corporation, USA.)

Cu to form the antenna. Other methods of manufacturing antennae include using conductive inks (made from powders of Cu or Ag) or conductive polymers. These approaches offer the possibility of lowering the cost of RFID tags.

a. An antenna is made by etching solid Cu with $\sigma = 5.8 \times 10^7$ S/m. What is its electrical resistance (R)? Assume that the antenna is in the form of a strip that is 20 cm long, 5 mm wide, and 25 μm thick.
b. What will be the resistance of a similar antenna made by screen printing *silver paste* with $\sigma = 1.6 \times 10^6$ S/m?
c. Why do you think the conductivity of Ag ink is *lower* than that of solid Cu?

SOLUTION

a. Let us assume that the current flows along the length direction. Thus, the cross-sectional area will be 5 mm × 25 μm or 0.5 cm × 25 μm × 10^{-4} cm/μm = 125 × 10^{-5} cm². The resistance of the Cu antenna will be (Equation 2.2):

EXAMPLE 2.3: RESISTANCE OF A LONG Cu WIRE

Calculate the resistance of 1000 m of American Wire Gauge (AWG) #18 Cu wire (Table 2.2). Assume $\sigma_{Cu} = 5.8 \times 10^7$ S/m.

SOLUTION

From Table 2.2, for AWG #18, the wire diameter (d) is 0.040303 inches or 1.023696 mm.

Thus, the area of cross section $= \dfrac{\pi d^2}{4} = 8.2306 \times 10^{-7} \, \text{m}^2$

TABLE 2.2
American Wire Gauge Diameter Conversion

American Wire Gauge Number	Conductor Diameter (inches)	Conductor Diameter (mm)
20	0.03196118	0.811814
18	0.040303	1.023696
16	0.0508214	1.290864
14	0.064084	1.627734
12	0.0808081	2.052526
10	0.10189	2.588006
8	0.128496	3.263798
6	0.16202	4.115308
5	0.18194	4.621276
4	0.20431	5.189474
3	0.22942	5.827268
2	0.25763	6.543802
1	0.2893	7.34822
0	0.32486	8.251444
00	0.3648	9.26592
000	0.4096	10.40384
0000	0.46	11.684

EXAMPLE 2.4: RESISTANCE, MASS, AND LENGTH (IN FEET) PER POUND OF Cu AND ALUMINUM (Al) WIRES

a. Calculate the mass and electrical resistance of 1000 feet of Al and Cu wires of AWG #3.
b. How many feet of Cu and Al wire can we get per pound of this wire? Assume that the mass of any insulation can be ignored.
   The densities of Cu and Al are 8930 and 2700 kg/m^3, respectively. Assume $\sigma_{Cu} = 5.8 \times 10^7$ and $\sigma_{Al} = 3.8 \times 10^7$ S/m.

SOLUTION

a. For AWG gauge #3, the diameter is 0.22942 inches or 5.827268 millimeters (Table 2.2). Thus, the area of cross section is

$$\text{Area} = \frac{\pi d^2}{4} = 2.66698 \times 10^{-5} \, \text{m}^2$$

Thus, the resistance of a 1000-foot Cu wire will be (according to Equation 2.2)

$$R_{cu} = \frac{(1000 \text{ ft.}) \times 0.3048 \, \frac{\text{m}}{\text{ft.}}}{(2.66698 \times 10^{-5} \, \text{m}^2) \times \left(5.8 \times 10^7 \, \frac{\text{S}}{\text{m}}\right)} = 0.197 \, \Omega$$

Similarly, the resistance of a 1000-foot Al wire (AWG #3) will be

$$R_{Al} = \frac{(1000 \text{ ft.}) \times 0.3048 \, \frac{\text{m}}{\text{ft.}}}{(2.66698 \times 10^{-5} \, \text{m}^2) \times \left(3.8 \times 10^7 \, \frac{\text{S}}{\text{m}}\right)} = 0.309 \, \Omega$$

EXAMPLE 2.5: Cu INTERCONNECTS FOR INTEGRATED CIRCUITS

Cu, because of its high conductivity ($\sigma = 58 \times 10^4$ S/cm), is widely used in modern-day ICs as a conductor material that provides connections between different components, known as interconnects. If the thickness of Cu is 80 nm, what is its sheet resistance (R_s)? If the width of this Cu conductor is 0.5 μm and the length is 300 μm, what is the resistance?

SOLUTION

The sheet resistance can be calculated as follows:

$$R_s = \frac{\rho}{t} = \frac{1}{\sigma t}$$

$$R_s = \frac{1}{(58 \times 10^4 \, \text{S/cm}) \times (80 \times 10^{-7} \, \text{cm})} = 0.216 \, \Omega/\square$$

From this, we can calculate the resistance as follows:

$$R = R = R_s \frac{L}{W} = 0.216 \times \frac{300 \, \mu\text{m}}{0.5 \, \mu\text{m}} = 129 \, \Omega$$
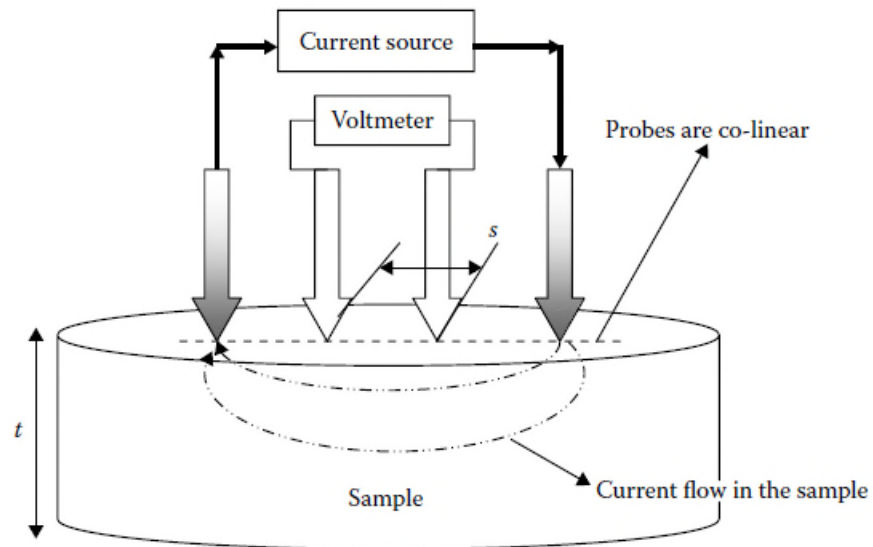
FIGURE 2.5 A schematic of the four-point probe used to measure resistivity. The spacing between probes is *s*, and the sample thickness is *t* (not to scale).
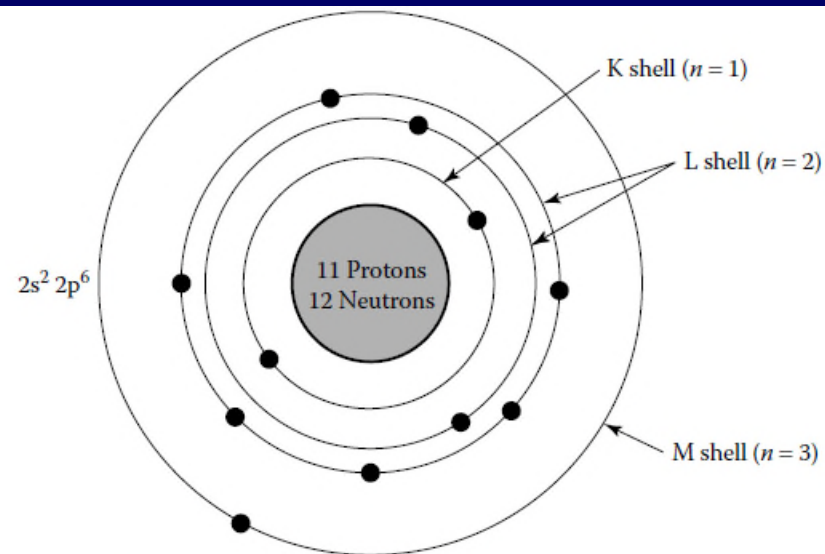
FIGURE 2.6 The structure of a sodium atom (atomic number 11), showing the nucleus surrounded by core electrons and valence electrons. (From Askeland, D. 1989. *The Science and Engineering of Materials*. 3rd ed. Washington, DC: Thomson. With permission.)
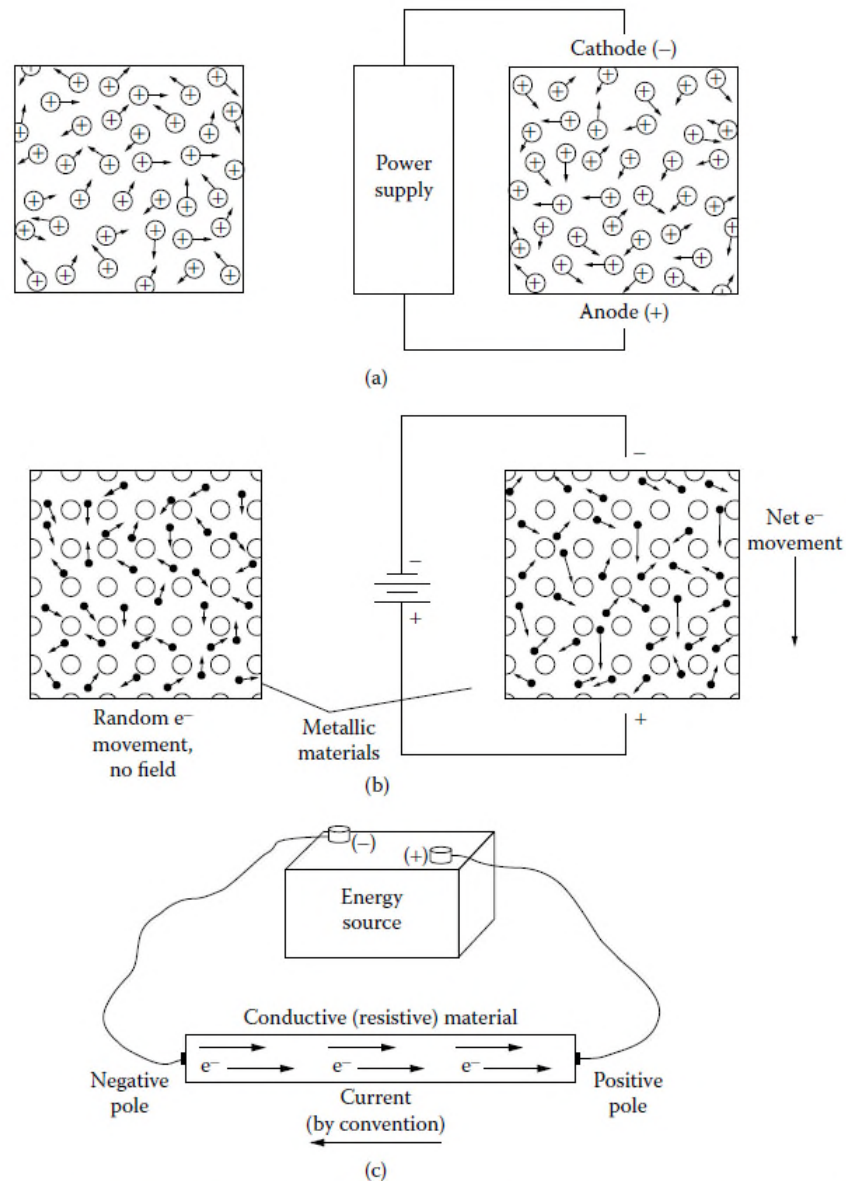
FIGURE 2.7 (a) Random motion of electrons due to thermal energy is similar to the movement of ions in an electroplating solution. (b) The drift of conduction electrons in a pure metal; note that the applied electric field (E) and drift are opposite, and overall, the electrons move within the material toward the anode. (c) The conventional current is said to flow from the positive to the negative terminal. The electron current, i.e. the actual motion of electrons, is in the opposite direction. (From Minges, M. L. 1989. *Electronic Materials Handbook*, Vol. 1, Materials Park, OH: ASM. With permission.)

EXAMPLE 2.6: CARRIER CONCENTRATIONS IN AL AND SI

Calculate the concentration of conduction electrons in (a) Al and (b) Si. Assume that the densities of Al and Si are 2.7 and 2.33 g/cm³. Assuming that the electron mobility values are similar, what will be the expected concentration of electrons in terms of the values of conductivity of Al and Si? The atomic masses for Al and Si are ~27 and 28, respectively.

SOLUTION

a. Because Al has a valence of +3, we assume that each Al atom donates three conduction electrons.

   The atomic mass of Al is 27 g; this means that the mass of 6.023×10^{23} atoms (Avogadro's number) is 27 g.

   A volume of 1 cm^3 is 2.7 g of Al. The number of atoms in this volume will be

$$= \frac{6.023 \times 10^{23} \times \text{density}}{\text{atomic mass}} = \frac{6.023 \times 10^{23} \text{ atoms/mole} \times 2.7 \text{ g/cm}^3}{27 \text{ g/mole}}$$

$$= 6.023 \times 10^{22} \text{ atoms/cm}^3$$

TABLE 2.3

Atomic Mass, Density, Experimentally Measured Electron Mobility, and Electron Concentration for Selected High-Purity Metals

Metal	Atomic Mass (g/mol)	Density (g/cm³)	Mobility of Electrons (μn) (cm²/V·s)	Calculated Number of Carrier Particles (n) # (electrons/cm³)
Silver	107.868	10.5	57	5.86×10^{22}
Copper	63.546	8.92	32	8.43×10^{22} (assumes one electron per atom)
Gold	196.965	19.32	31	5.91×10^{22}
Aluminum	26.981	2.7	13	1.807×10^{23} (assumes three electrons per atom)

EXAMPLE 2.7: CALCULATING THE CONDUCTIVITY OF Aʟ FROM THE MOBILITY VALUES

Assume that the mobility of electrons (μ_n) in Al is 13 cm^2/V·s. The carrier concentration for Al is 1.807×10^{23} electrons (assuming three electrons donated per Al atom). Calculate the expected conductivity of Al.

SOLUTION

From Equation 2.22, the conductivity of Al will be given by the expression

$$\sigma = \left(1.807 \times 10^{23} \frac{electrons}{cm^3}\right) \times \left(13 \frac{cm^3}{V \cdot s}\right) \times \left(1.6 \times 10^{-19} \frac{C}{electron}\right)$$

$$= 37.9 \times 10^4 \ S/cm$$

EXAMPLE 2.8: CARRIER CONCENTRATION IN Cu

The experimentally measured mobility of electrons in copper is 32 cm²/V·s. Calculate the carrier concentration (n) for Cu. Compare this with the values listed in Table 2.3. What does this show about the number of conduction electrons contributed per Cu atom?

SOLUTION

We use the value of Cu conductivity (σ) provided in Table 2.1.

From Equation 2.22,

$$\sigma_{Cu} = 59.7 \times 10^4 \ \text{S/cm} = (n) \times \left(32\frac{\text{cm}^2}{\text{V} \cdot \text{s}}\right) \times (1.6 \times 10^{-19} \text{C})$$
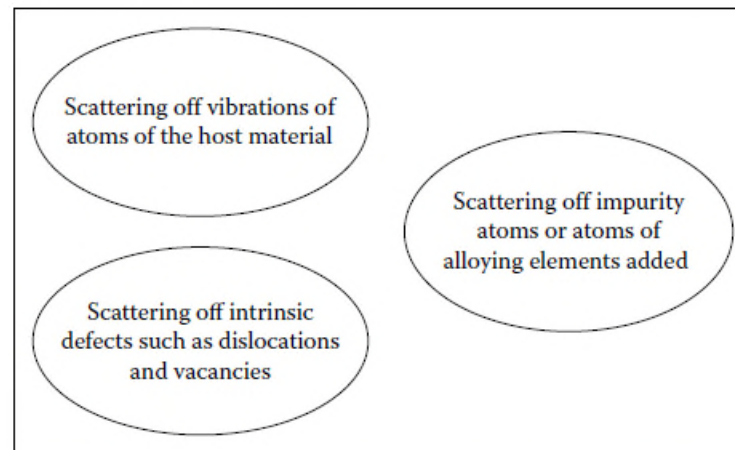
EXAMPLE 2.9: RESISTIVITY OF A SEMICONDUCTOR

A particular single crystal of a semiconducting Si sample, containing a small but deliberately added level of phosphorus (P), provides conductivity with electrons as the majority carriers. Assume that the concentration of P added is 10^{18} atoms/cm^3 and that each P atom provides one electron. Ignore any other possible contributions to the conductivity of this P-doped Si sample. If $\mu_n = 700$ cm^2/V·s, what is the resistivity (ρ) of this semiconductor?

SOLUTION

Because each P atom contributes only one conduction electron, the concentration of P atoms and that of conduction electrons donated by P atoms are equal; therefore, $n = 10^{18}$ electrons/cm^3.

(In the next chapter, we will learn that our assumption about the concentration of conduction electrons contributed by the P atoms, as opposed to those from the Si atoms, is correct.)

Therefore, from Equation 2.22,

$$\sigma = (10^{18}\ \text{electrons/cm}^3) \times \left(700\ \frac{cm^2}{V \cdot s} \right) \times (1.6 \times 10^{-19}\ C)$$

FIGURE 2.8 Schematic showing the sources of the scattering of conduction electrons in a metallic material.

Scattering off vibrations of atoms of the host material

Scattering off impurity atoms or atoms of alloying elements added

Scattering off intrinsic defects such as dislocations and vacancies
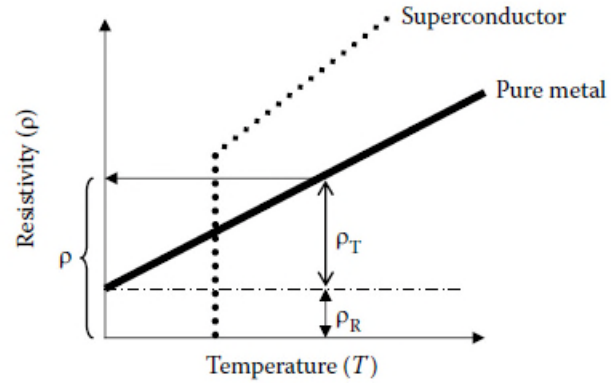
FIGURE 2.9 The temperature dependence of the conductivity of a typical metal and a superconductor.

EXAMPLE 2.10: Pt RESISTANCE–TEMPERATURE DETECTORS

A *resistance–temperature detector* (RTD) makes use of a metal wire (e.g., that of Pt; Figure 2.11). As the temperature changes, the electrical resistance changes; thus, by measuring the change in resistance, we can measure the temperature. A Pt RTD can operate at temperatures in the range of −200°C to +700°C. Assume that the α_R for the Pt material used is 3.93×10^{-3} $\Omega/\Omega \cdot$ °C. The RTD wire is designed so that the resistance at 0°C is 100 Ω. (a) What is the temperature if the resistance measured is 200 Ω? (b) Why is Pt preferred for making RTDs, in comparison to W, Cu, Ag, or Au? (c) In Europe, Pt compositions with $\alpha_R = 0.00385$ $\Omega/\Omega \cdot$ °C are often used. For $R = 200$ Ω, what is the predicted temperature if we use an RTD made from this Pt composition?

TABLE 2.4
Properties of Some Heating-Element Materials

Material	Resistivity at 20°C ($\mu\Omega \cdot cm$)	α_R at 20°C ($\Omega/\Omega \cdot °C$)	Maximum Operating Temperature (°C)	Main Applications
Nickel 80/chromium 20 (Nikorthal™-type alloys)	108	$+14 \times 10^{-3}$	1200	Furnaces, heating elements for domestic appliances
Chromium 22/aluminum 5.8/balance iron (Kanthal™-type alloys)	145	$+3.2 \times 10^{-3}$	1400	Furnaces for heat treatment
Platinum 90/rhodium 10	18.7	–	1550	Laboratory furnaces
Platinum 60/rhodium 40	17.4	–	1800	Laboratory furnaces
Molybdenum	5.7	4.35×10^{-3}	1750	Vacuum furnaces, inert atmosphere
Tantalum	13.5	3.5×10^{-3}	2500	Vacuum furnaces
Graphite	1000	-26.6×10^{-3}	3000	Furnaces requires nonoxidizing atmosphere
Molybdenum disilicide ($MoSi_2$)	40	1200×10^{-3}	1900	Glass industry, laboratory furnaces, ceramic processing
Lanthanum chromite ($LaCrO_3$)	2100	–	1800	Laboratory furnaces
Silicon carbide (SiC), also known as glowbar	1.1×10^5	–	1650	Industrial furnaces
Zirconia	–	–	2200	Laboratory furnaces, becomes an ionic conductor above ~1000°C

TABLE 2.5

Resistivity and Temperature Coefficient Values for Selected Materials

Material	Typical Composition (wt%)	Resistivity ($\mu\Omega \cdot cm$) at 20°C	Temperature Coefficient of Resistivity (α_R) $\Omega/\Omega \cdot$ °C ($T_0 = 300$ K)
Lead (Pb)–tin (Sn) solder	Sn: 63, Pb: 37	14.7	
Brass	Cu: 60, Zn: 40	6.4	1000
	Cu: 70, Zn: 30	8.4	2000
Nichrome	Ni: 58.5, Fe: 22.5, Cr: 16, Mn: 3	100	400
Constantan	Cu: 60, Ni: 40	44.1	+2/+33
Manganin	Cu: 84, Mn: 12, Ni: 4	45	+6/–42 (12°C–100°C)
	Cu: 83, Mn: 13, Ni: 4 (Wire Alloy)	48.2	+15/–15 (15°C–35°C)
	Cu: 86, Mn: 10, Ni: 4 (Shunt Alloy)	38.3	+15/–15 (40°C–60°C)
Palladium–silver alloy	Pd: 60, Ag: 40	42–44	

Data from various sources.

SOLUTION

a.

$$\alpha_R = \frac{1}{\rho_0}\left(\frac{\rho-\rho_0}{T-T_0}\right)$$

$$\therefore 0.00393 = \frac{1}{R}\left(\frac{R-R_0}{T-T_0}\right) = \frac{1}{100}\left(\frac{R-100}{T-0}\right)$$

$$\therefore 0.393T = R-100 \quad \text{or}$$

$$R = 100+0.393T$$

Thus, the variation in resistance is 0.393 Ω/°C.
Because the resistance measured by the RTD is 200 Ω, we rewrite as

$$200 = 100+0.393T \quad \text{or}$$

$$T = \frac{200-100}{0.393} = 254.45°C$$
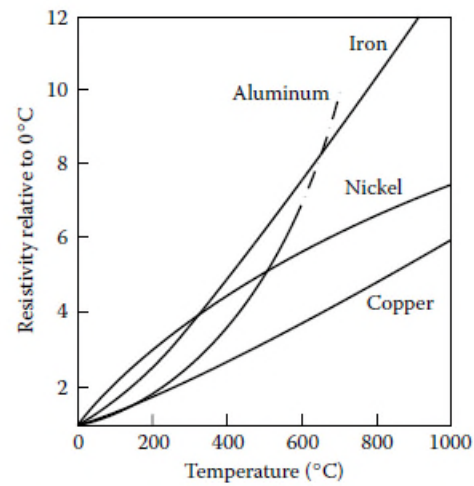
FIGURE 2.10 Relative change in resistivity with temperature (relative to 0°C) for Al, Cu, Ni, and Fe. (From Laughton, M. A., and D. F. Warne, eds. 2003. *Electrical Engineer's Reference Book*. Amsterdam: Elsevier. With permission.)
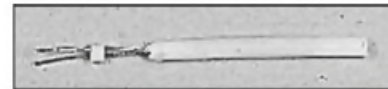
Thick film omega film element

Glass-sealed Biflar winding

Typical RTD probes
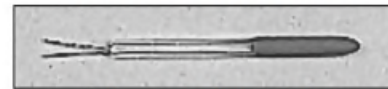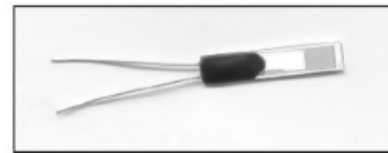
Thin film omega TFD element

FIGURE 2.11 Commercially available resistance–temperature detector devices. (Courtesy of Omega Corporation, San Diego, CA.)

EXAMPLE 2.11: CALCULATION OF JOULE POWER LOSSES IN A Cu BUS BAR AND THE COST OF ELECTRICITY

A Cu *bus bar*, or a conductor used in power transmission, is 300 feet long and has a cross section of ¼ × 4 inches. (a) What is the resistance of this bar at room temperature? (b) Calculate the Joule losses in kilowatts if the current is 1000 A. Assume that the current is DC. (c) What is the energy lost (in kW·h) if the current is carried for 24 hours? (d) Assuming electricity costs 4 cents/kW·h, calculate the dollar value of the energy wasted due to Joule losses on a per-year basis. Assume that the conductivity of the Cu bar is 5.8×10^7 S/m.

SOLUTION

a.

$$R_{\text{Copper bus bar}} = \rho\frac{L}{A} = \frac{300 \text{ ft.} \times 12 \text{ in./ft.} \times 2.54 \text{ cm/in.}}{\left(\frac{1}{4} \times 4\right) \text{in.}^2 \times \left(2.54 \text{ cm/in.}\right)^2 \times \left(5.8 \times 10^7 \text{ S/m} \times 10^{-2} \text{m/cm}\right)} = 2.44366 \times 10^{-3}\,\Omega$$

b. Now, we can calculate the Joule losses as follows:

$$\text{Power lost} = V \times I = I^2 \times R = \left(1000\right)^2 \times 0.00244366\,\Omega = 2443.66 \text{ W}$$

Thus, the power lost is 2.433 kW.

c. The energy lost in 24 hours will be

$$\text{Energy lost} = \text{power} \times \text{time} = 2.443 \text{ kW} \times 24 \text{ h} = 58.6 \text{ kW·h}$$

TABLE 2.6

Conductivity of Different Materials (at 20°C) Based on the IACS Scale

Material	% IACS Conductivity	Material	% IACS Conductivity
Annealed copper	100	Nickel	25
99.999% Copper	102.5	Iron	17
Electrolytic tough-pitch copper ~99.0%	100.2–101.5	Platinum	16
Oxygen-free high-conductivity copper	101		
Silver	104	Tin	13
Aluminum	60	Lead	8

IACS = International Annealed-Copper Standard.

Data from various sources.
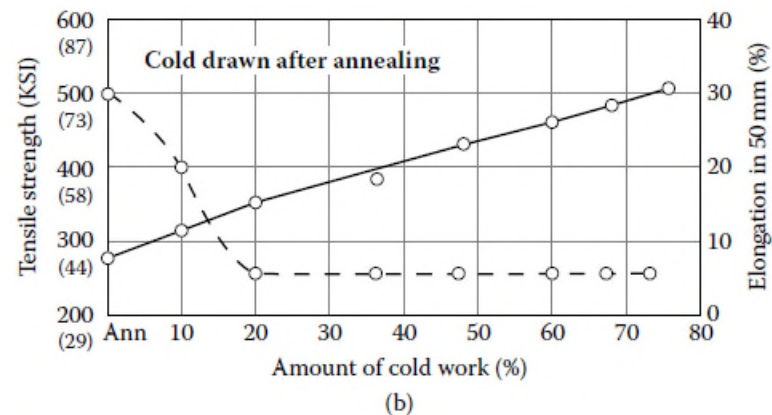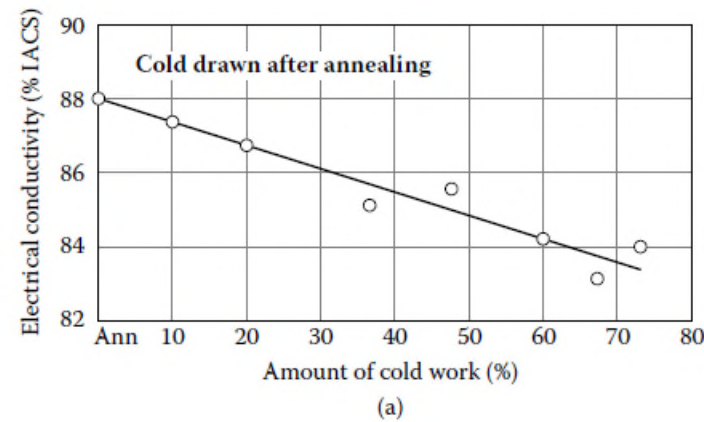
FIGURE 2.12 (a) Electrical conductivity of sterling silver (92.5% Ag–7.5% Cu) shown as % IACS; and (b) increase in tensile strength (solid line) and decrease in percentage of elongation (dotted line) as a function of % cold work. The data are for a 2.3-mm wire, which was cold-drawn after annealing. (From ASM International, eds. *Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, Vol. 2. Materials Park, OH: ASM. With permission.)
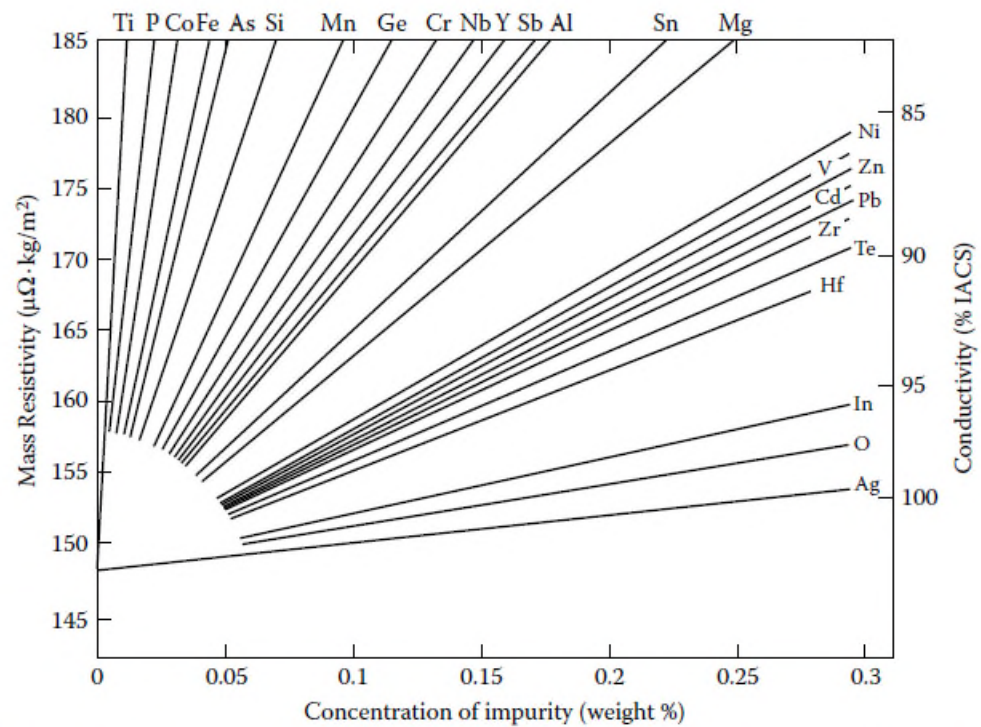
**FIGURE 2.13** Effects of different impurities on the conductivity of copper. (Courtesy of Copper Development Association, New York.)
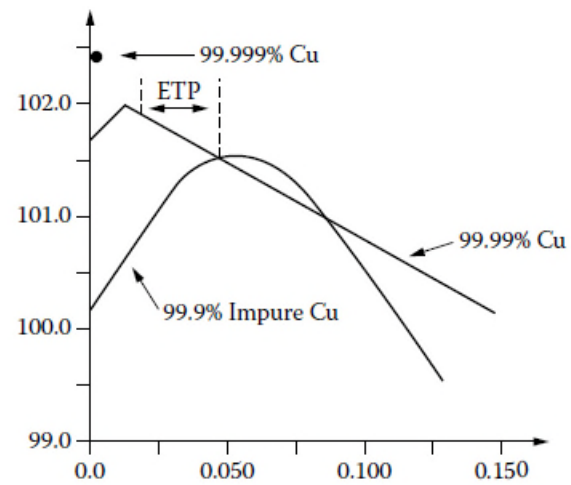
FIGURE 2.14 Effect of oxygen concentration on the conductivity of copper. (Courtesy of Copper Development Association, New York.)
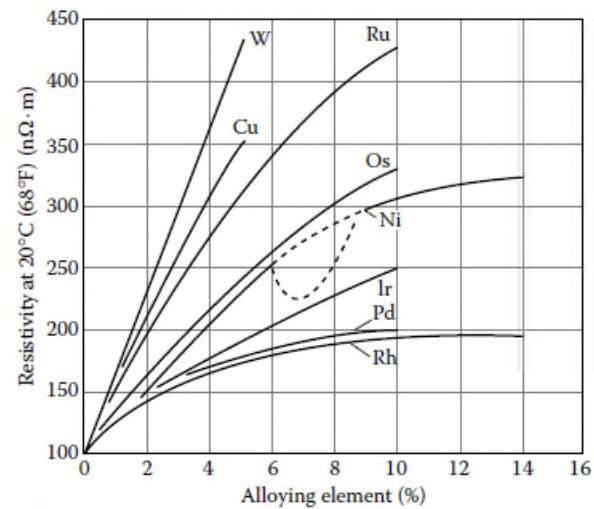
FIGURE 2.15 Effects of additions of various alloying elements on the resistivity of platinum. (From Vines, R. F. and E. M. Wise. 1941. *The Platinum Metals and Their Alloys*. New York: International Nickel Co. With permission.)
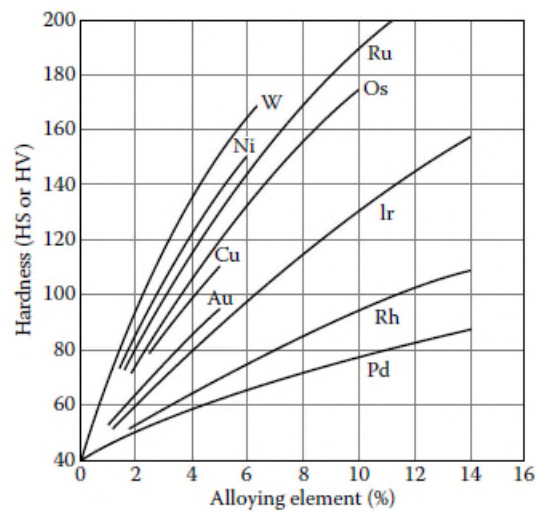
FIGURE 2.16 Increase in the hardness of platinum due to the addition of various alloying elements. (From Lampman, S. R., and T. B. Zorc, eds. 1990. *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials*. Materials Park, OH: ASM. With permission.)
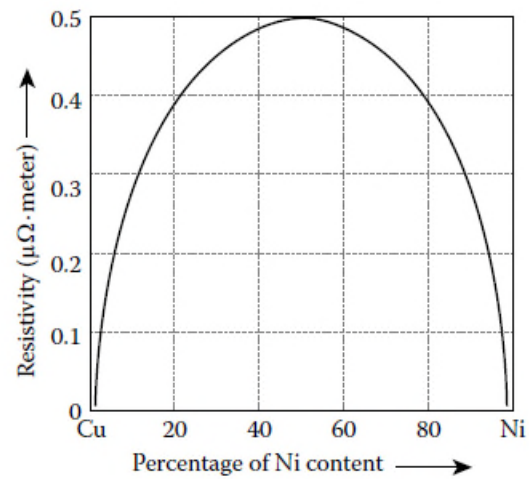
FIGURE 2.17 Variation in the resistivity of copper as a function of nickel concentration. (From Neelkanta, P. 1995. *Handbook of Electromagnetic Materials*. Boca Raton, FL: CRC Press. With permission.)
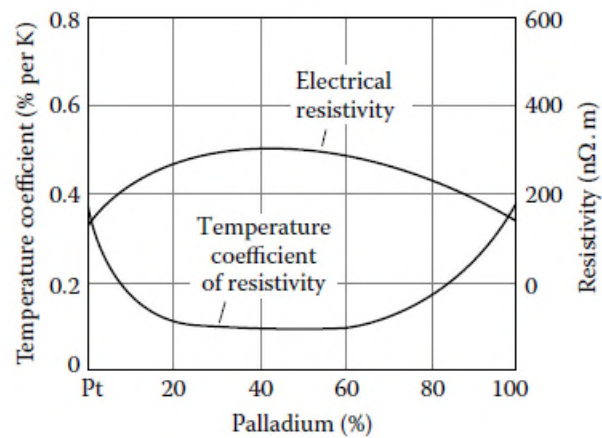
FIGURE 2.18 Electrical resistivity (in nΩ·m) and temperature coefficient of resistivity (%/K) for Pt–Pd alloys. (From Lampman, S. R., and T. B. Zorc, eds. 1990. *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials*. Materials Park, OH: ASM. With permission.)
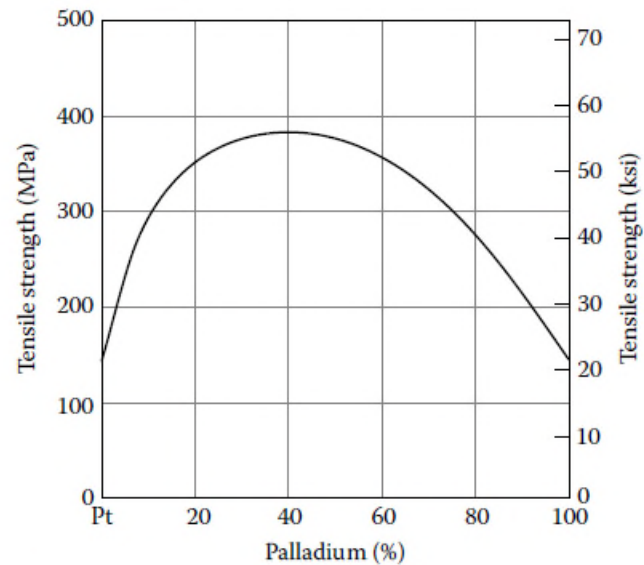
FIGURE 2.19 The change in tensile strength (in MPa on the left y-axis, and in ksi on the right y-axis) of annealed platinum–palladium alloys as a function of palladium concentration. (From Lampman, S. R., and T. B. Zorc, eds. 1990. *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials.* Materials Park, OH: ASM. With permission.)

EXAMPLE 2.12: RESISTIVITY OF A Cu ALLOY USING NORDHEIM'S RULE

Pure Au and Ag are too soft for most applications. As a result, Cu is added as an alloying element to strengthen these metals using solid-solution strengthening. What is the resistivity of an Au alloy containing 1.5 weight % Cu? Assume that Nordheim's coefficient (C) for Cu dissolved in Au is 450 $n\Omega \cdot m$ (Equation 2.32).

SOLUTION

From Table 2.1, we see that the resistivity of Au is 2.35 $\mu\Omega \cdot cm$ or 23.5 $n\Omega \cdot m$. Note that in Equation 2.32, the concentration of the alloy-forming element has to be expressed as an atom fraction, which is achieved by applying the following equation:
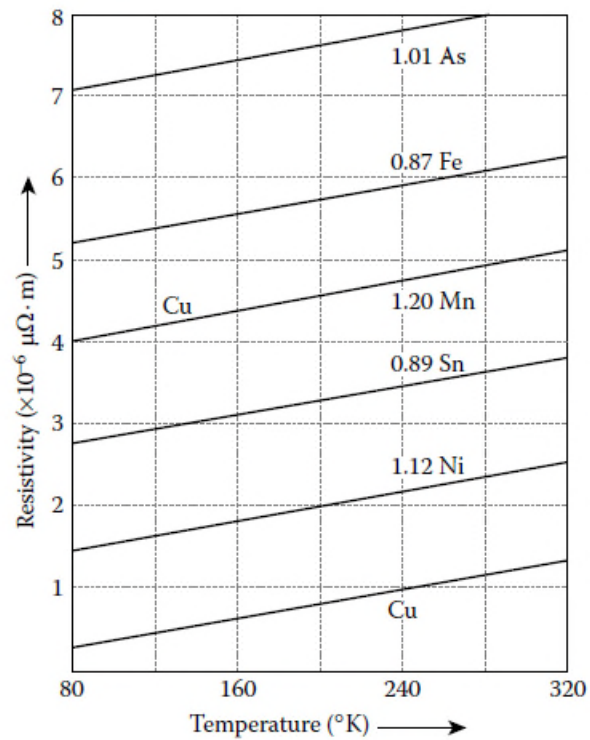
$$x = \frac{M_{Au}w}{(1-w)M_{Cu} + wM_{Au}}$$

FIGURE 2.20 The resistivity of some copper alloys as a function of temperature. The data for resistivity change of copper is also shown. (From Neelkanta, P. 1995. *Handbook of Electromagnetic Materials*. Boca Raton, FL: CRC Press. With permission.)

TABLE 2.7
Quantum Numbers for Electrons

|	
Principal quantum number (n)	$n = 1, 2, 3,$
	$n = 1$ is the K shell, $n = 2$ is the L shell, and so on. The maximum number of electrons for a shell with given n is $2n^2$
Orbital angular-momentum quantum number (l) (also known as the azimuthal quantum number)	$l = 0, 1, \ldots (n-1)$
	$l = 0$ indicates the s subshell, $l = 1$ indicates the p subshell, etc. The maximum number of electrons in the various subshells are s = 2, p = 6, d = 10, f = 14, and g = 18
Magnetic quantum number (m or m_l)	$m = -l, -(l-1), \ldots, 0, (l-1), l$
Spin (s or m_s)	$s = \pm\frac{1}{2}$
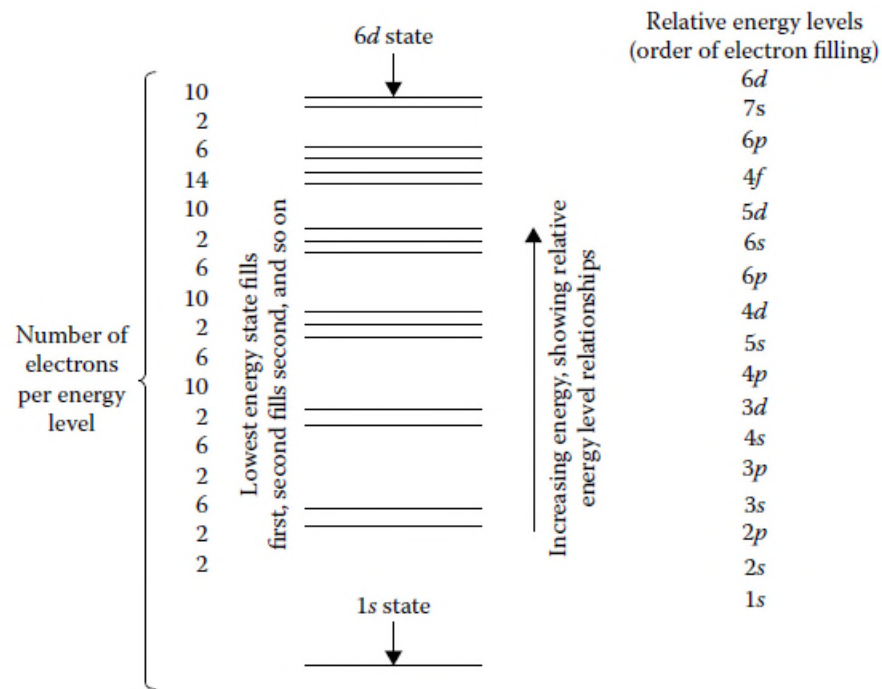
FIGURE 2.21 The order and number of electrons for different elements. (From Minges, M. L. 1989. *Electronic Materials Handbook*. Vol. 1. Materials Park, OH: ASM. With permission.)

EXAMPLE 2.13: ELECTRONIC CONFIGURATION FOR Al

Write down the electronic configuration for Al, whose atomic number (Z) is 13. Explain the meaning of the different symbols used.

Solution

For Al, $Z = 13$; this means that there are 13 electrons in one Al atom.

We start with $n = 1$. In this level, we can have $2(1)^2 = 2$ electrons. For $n = 1$, the only possible value of l is 0. In this s subshell, we can have only two electron energy levels. Thus, the first part of the configuration is $1s^2$ (read as "one s two"). Then, for $n = 2$ (or the L shell), the possible values of l are 0 and 1, that is, s and p sublevels. The total maximum number of electrons in this shell can be $2(2)^2 = 8$. Thus, when $n = 2$, for the s subshell, we can have two electrons; and for the p subshell, we can have six electrons. Thus, the configuration thus far will read $1s^2 2s^2 2p^6$. This accounts for a total of $2 + 2 + 6 = 10$ electrons. We have only three more electrons left. We move to the $n = 3$ level (M shell). We can have up to $2(3)^2 = 18$ electrons in this level, where the possible values of l are 0, 1, and 2 or s, p, and d subshells. We start with the s subshell, place two electrons here, and then move to the p subshell and place one more electron here.

Thus, the electronic configuration for Al will be $1s^2 2s^2 2p^6 3s^2 3p^1$.

TABLE 2.8
Electron Spin States in Iron (Fe)

$n = 1$	$n = 2$		$n = 3$		$n = 4$	
1s	2s	2p	3s	3p	3d	4s
↑	↑	↑↑↑	↑	↑↑↑	↑↑↑↑↑	↑
↓	↓	↓↓↓	↓	↓↓↓	↓	↓

An arrow pointing up (↑) means $s = +1/2$, or spin up; an arrow pointing down (↓) means $s = -1/2$, or spin down.
   Note, the 4s level is filled before the 3d level and two of the 3d electrons are spin-paired.
Source: Edwards-Shea, L. 1996. *The Essence of Solid-State Electronics.* Upper Saddle River, NJ: Prentice Hall.
      With permission.

EXAMPLE 2.14: ELECTRONIC CONFIGURATION FOR FE

Write down the electronic configuration of an Fe (Z = 26) atom.

SOLUTION

There are 26 electrons in an Fe atom. We start with $n = 1$. This orbital can take $2(n = 1)^2$ or two electrons. The only possible value of l is 0, that is, the s subshell. Thus, the electronic configuration until this level will be $1s^2$. For $n = 2$, we can have a maximum of $2(2)^2$ or eight electrons. The possible values of l are 0 and 1; or, we can have s and p subshells. Thus, now the electronic configuration will read $1s^2 2s^2 2p^6$. For $n = 3$, we can have $2(3)^2$ or 18 electrons. However, we have only $26 - 2 - 8 = 16$ electrons remaining. For $n = 3$, we can have possible values of 0, 1, and 2 for l; or s, p, and d subshells. Thus, now the electronic configuration will read as $1s^2 2s^2 2p^6 3s^2 3p^6 3d^8$.

However, according to the filling order stated earlier, the 4s shell will fill *before* the 3d shell. The 4s level will take two electrons. The balance of six electrons will enter the 3d shell.

Thus, the final electronic configuration for iron will be $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^6$.

The d subshell can contain a maximum of 10 electrons, but actually contains only six. Thus, the d subshell is deficient by four electrons. Of the six electrons in the 3d sublevel, two are spin-paired, that is, they have all the same quantum numbers, except the spin quantum numbers (which are +1/2 and −1/2; Table 2.8).

The remaining four of the 3d electrons are *not* spin-paired. These unpaired electrons make it possible for an Fe *atom* to behave like a tiny bar magnet. This behavior contributes to making Fe a magnetic material. We will study this when we discuss magnetic materials (Chapter 9).
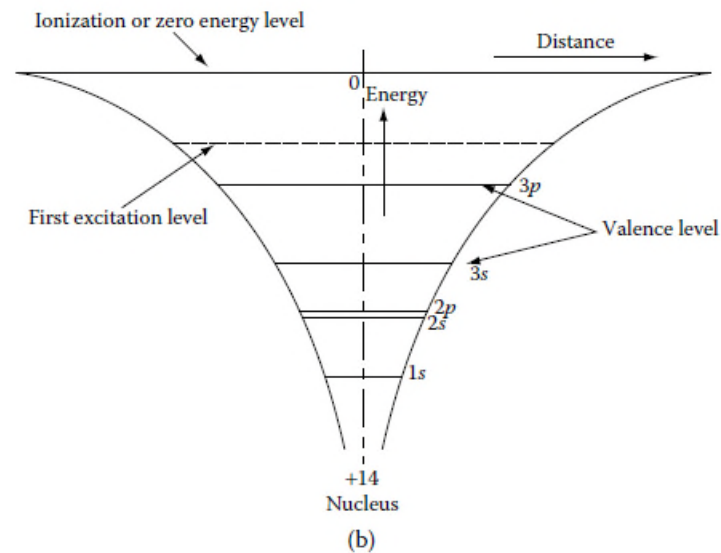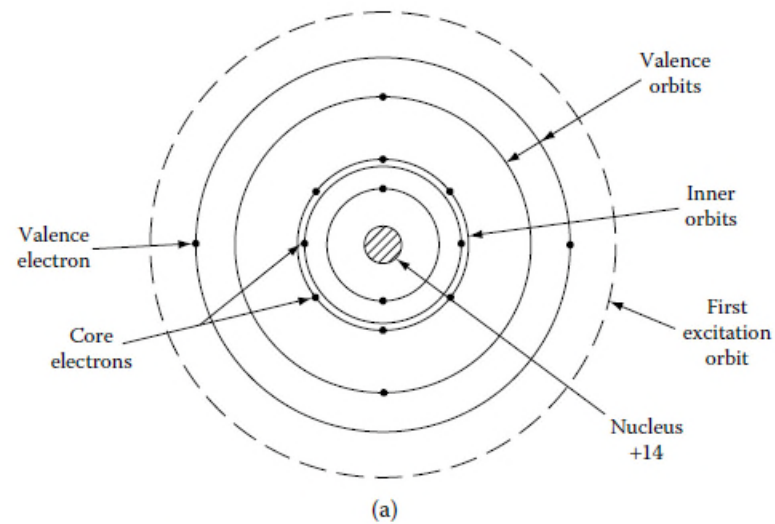
(a)

(b)

FIGURE 2.22 Electronic structure and energy levels in a Si atom: (a) The orbital model of a Si atom showing the 10 core electrons (n = 1 and 2), and the 4 valence electrons (n = 3); (b) energy levels in the Coulombic potential of the nucleus are also shown schematically. (From Streetman, B. G., and S. Banerjee. 2000. *Solid State Electronic Devices*, 5th ed. Upper Saddle River, NJ: Prentice Hall. With permission.)
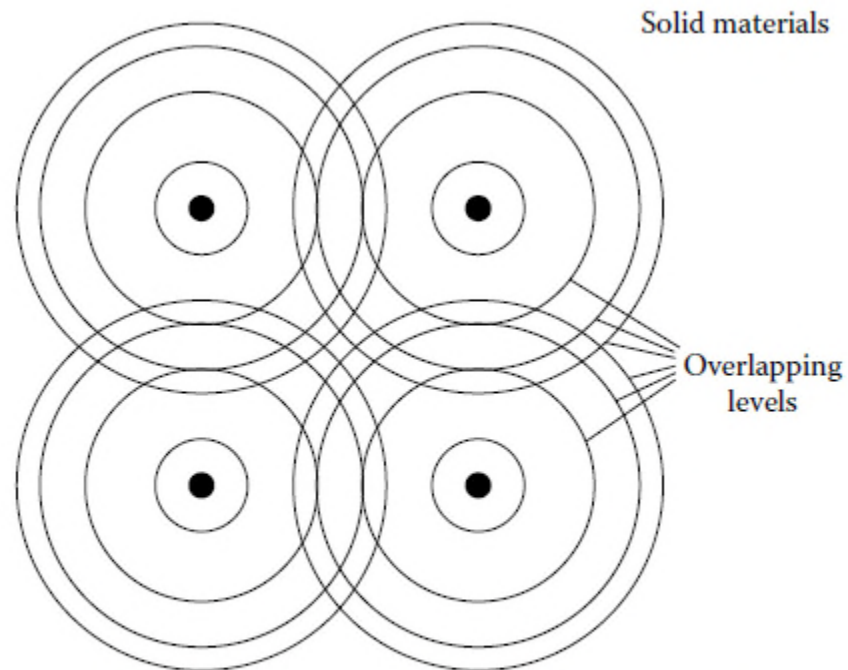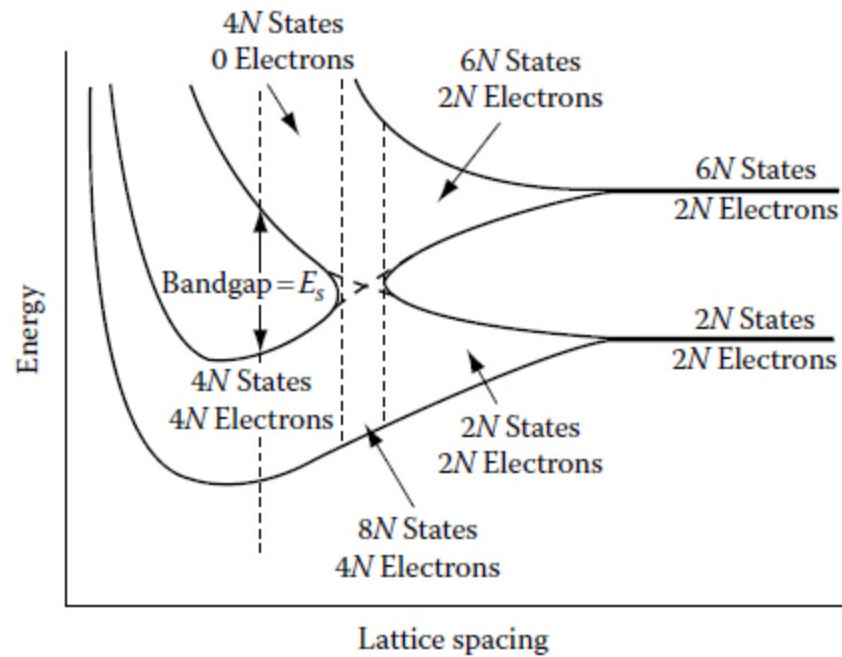
FIGURE 2.23 Overlap of the electron energy levels or wave functions as the atoms come closer. (Adapted from Edwards-Shea, L. 1996. *The Essence of Solid-State Electronics*. Upper Saddle River, NJ: Prentice Hall. With permission.)

FIGURE 2.24 Formation of energy bands in silicon. (From Streetman, B. G., and S. Banerjee. 2000. *Solid State Electronic Devices*, 5th ed. Upper Saddle River, NJ: Prentice Hall. With permission.)
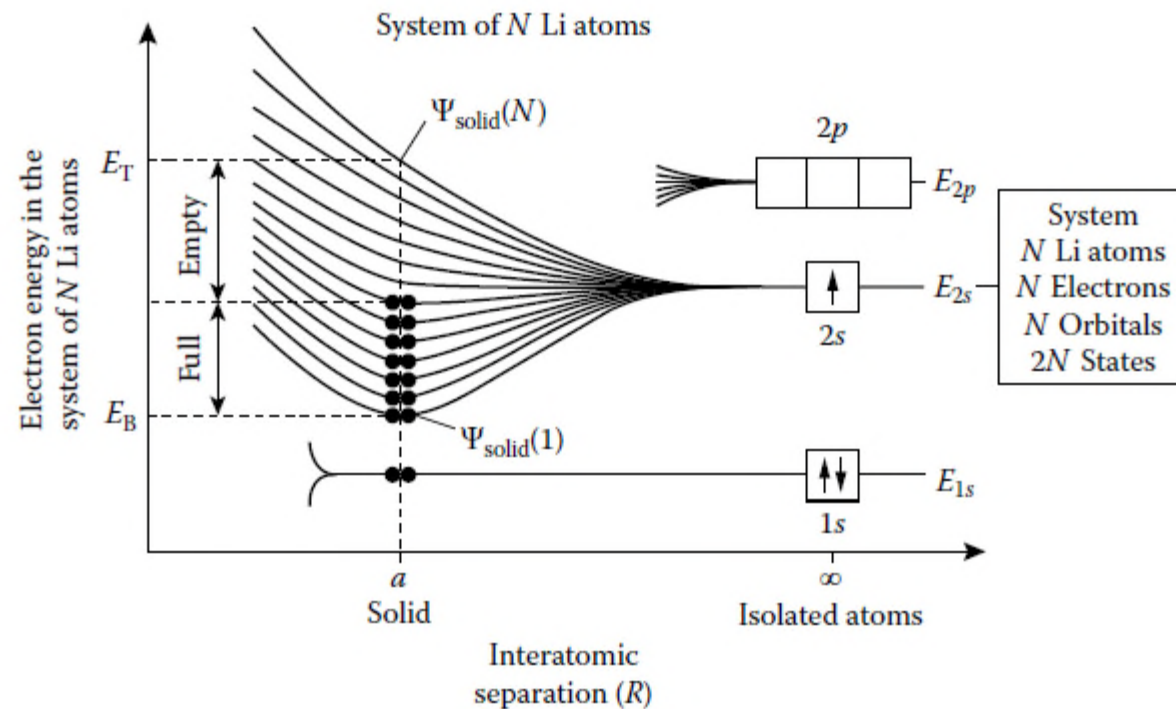
FIGURE 2.25 Formation of energy bands in lithium metal. The 2s band is only half-filled. Note that the 1s level shows very little splitting. (From Kasap, S. O. 2002. *Principles of Electronic Materials and Devices.* New York: McGraw Hill. With permission.)
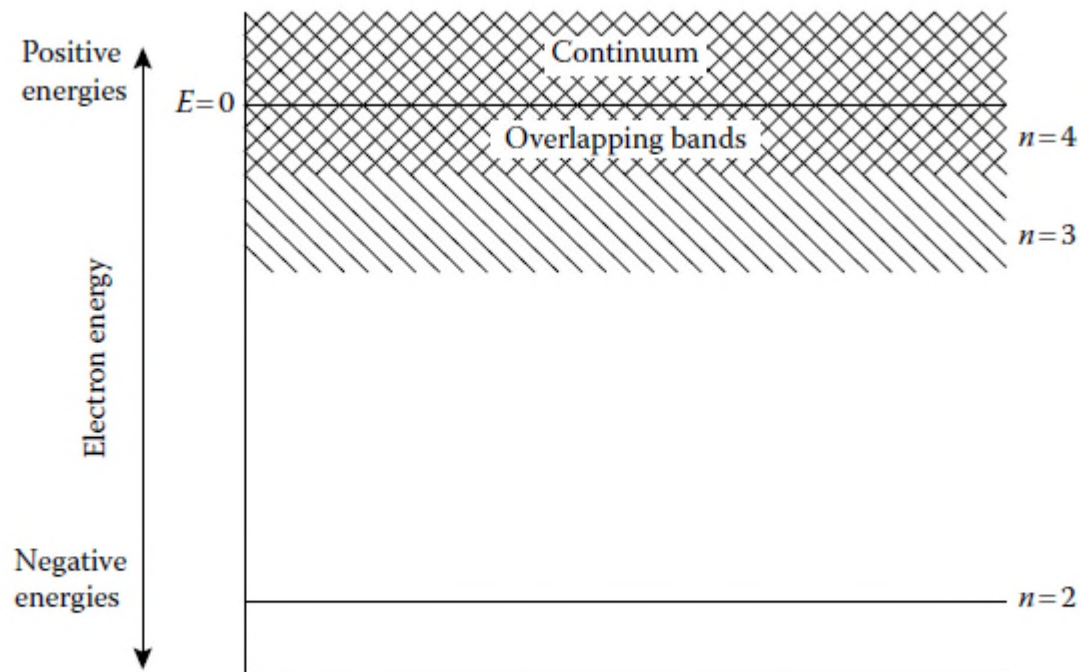
FIGURE 2.26 Schematic of a band diagram for a metal. (From Edwards-Shea, L. 1996. *The Essence of Solid-State Electronics.* Upper Saddle River, NJ: Prentice Hall. With permission.)
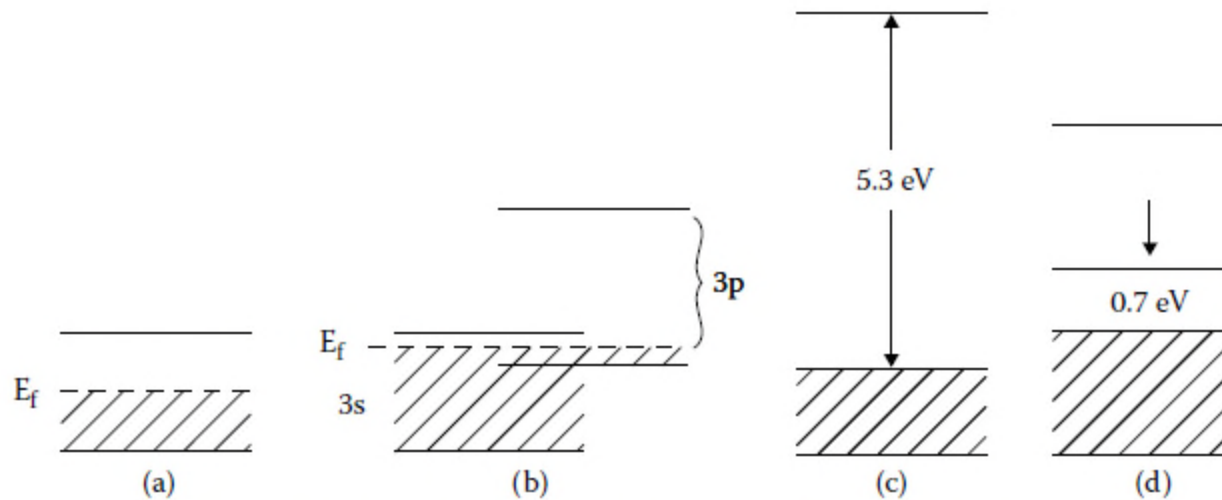
FIGURE 2.27   Band diagrams for conductors: (a) an alkali metal, (b) magnesium (Mg), a bivalent metal, (c) diamond, an insulator and (d) germanium (Ge), a semiconductor. (From Mahajan, S., and K. S. Sree Harsha. 1998. *Principles of Growth and Processing of Semiconductors*. New York: McGraw Hill. With permission.)
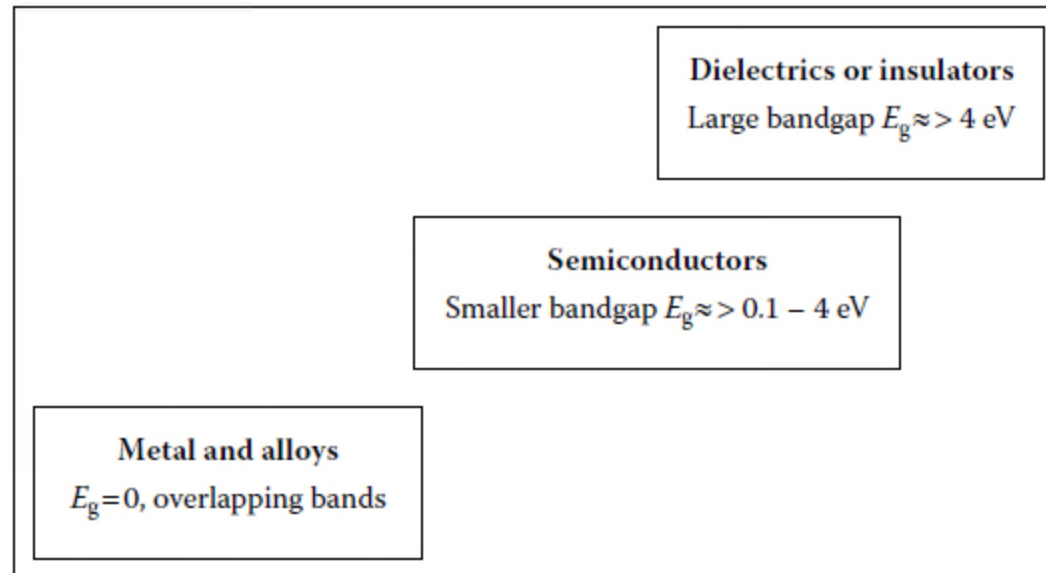
FIGURE 2.28 Classification of materials based on the values of bandgap (E_g) in electron volts (eV).
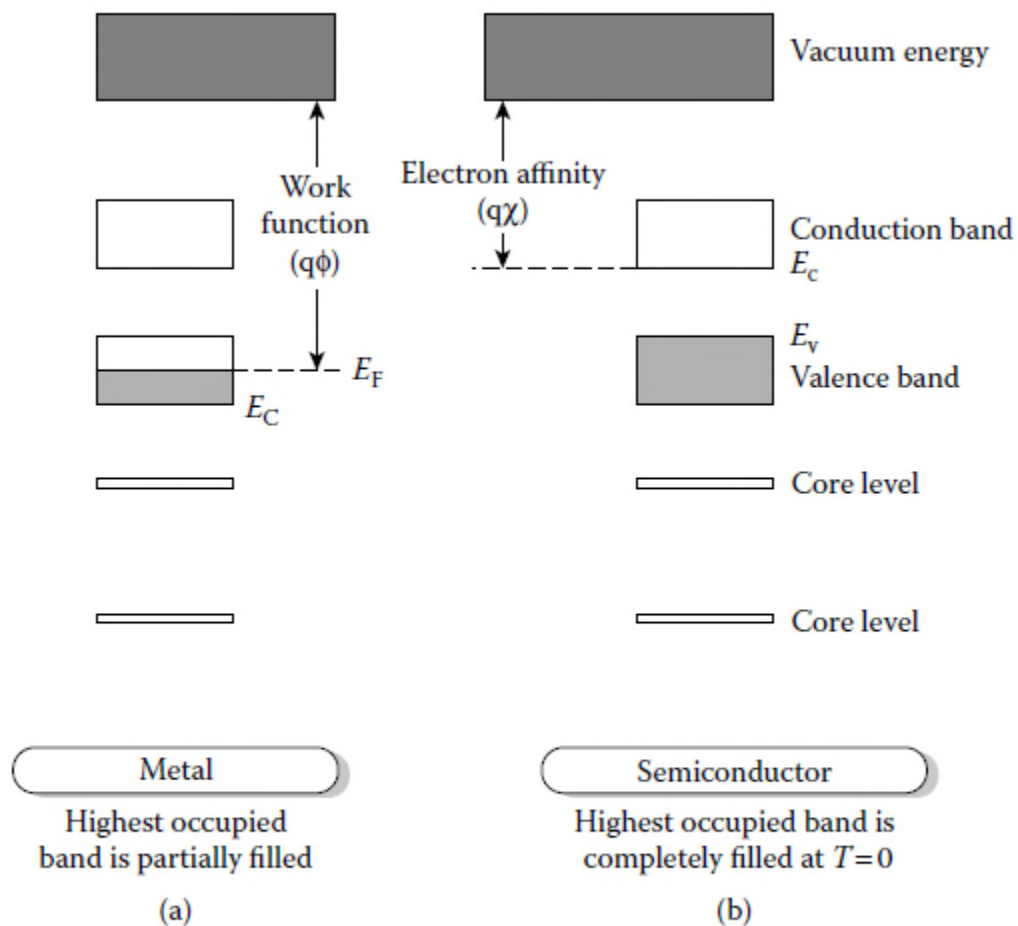
FIGURE 2.29 A schematic of band structure for (a) a typical metal and (b) a semiconductor. The work function is the energy "qϕ." The electron affinity for a metal is shown as the energy "qχ." (Adapted from Singh, J. 2001. *Semiconductor Devices: Basic Principles*. New York: Wiley. With permission.)
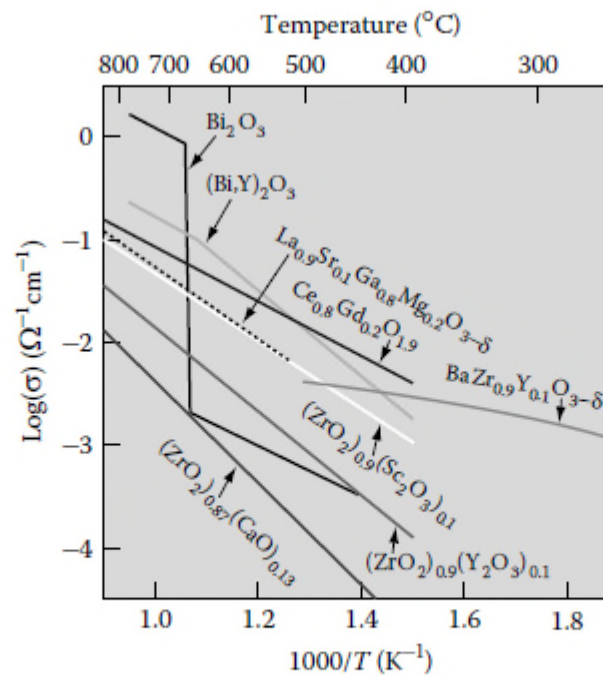
FIGURE 2.30 The conductivity of different materials used as electrolytes in the development of solid oxide fuel cells. (From Haile, S. M. 2003. *Acta Materilia* 51:5981–6000. With permission.)