

## Chapter 2 PROBLEMS

### Introduction

2.1 What are the typical ranges of resistivity for metals, plastics, and ceramics?

#### Solution:

Typical ranges of resistivity are as follows:

Metals:  $\sim 3$  to  $15 \mu\Omega\cdot\text{cm}$

Plastics and Ceramics:  $10^{10}$ - $10^{20} \mu\Omega\cdot\text{cm}$

Please note that resistivity of alloys is typically much higher. Some ceramics and plastics may exhibit semiconducting or conducting behavior.

2.2 What is the nature of bonding between atoms for most ceramics and plastics?

#### Solution:

Most ceramics the bonding is mixed covalent and ionic. Most plastics the bonding is covalent.

2.3 Compared to metals, what is the advantage in using conducting or semiconducting plastics?

#### Solution:

Low density and flexibility offer key primary advantages. This has led to the area of flexible electronics.

2.4 Which one of these elements shows superconductivity—Ag, Au, or Al?

#### Solution:

At a very low temperature ( $\sim 1.2$  K) aluminum becomes superconducting. Silver and gold do not become superconductors even at very low temperatures. For additional information on superconductivity in elements see, Buzea, C. and Robbie, K. in Superconducting Technology, Vol. 18, (2005), pages R1-R8.

## Ohm's Law

2.5 What is the difference between resistance and resistivity?

### Solution:

Resistance depends on the geometry of a component i.e. how long how wide etc. and also what a component is made from. Resistivity depends upon the composition of the material and its microstructure. Resistivity is largely independent of the size (i.e. length, cross-sectional area etc.) of the component.

2.6 Do all materials obey Ohm's law? Explain.

### Solution:

Most materials do obey Ohm's law. There are a few materials that do not. For example, certain compositions of zinc oxide and other materials have a microstructure in which grain boundaries can show electrical breakdown if the electric field across the material is high enough. Under such conditions, these materials will show non-ohmic behavior.

2.7 Calculate the resistance of an AWG #20 Cu wire one mile in length.

### Solution:

From Table 2.2, the diameter of AWG #20 Cu wire is 0.811814 mm, the length is one mile which is 1600 meters.

The resistance R will be given by:

$$R = \rho \frac{l}{A}$$

Use values of resistivity of  $1.72 \times 10^{-8}$  ohm/met (inverse of conductivity of pure copper  $5.8 \times 10^7$  S/m), length 1609 met, and area of  $5.1761 \times 10^{-7}$  m<sup>2</sup>. This gives a resistance of 5.36 ohms.

Copper wires are usually not made from pure copper (because pure copper is too soft) and in this case we can substitute appropriate value of resistivity. For example, if we say the resistivity

is 1.678 microhm-cm, then the value of resistance will be 32.8 ohms for the same diameter and length.

2.8 What is the length of an AWG #16 Cu wire whose resistance is 21  $\Omega$ ?

**Solution:**

For 16 gauge wire the diameter is 1.627734 mm (See Table 2.2). This gives an area of  $2.08 \times 10^{-6} \text{ m}^2$ . This gives a length of 2534 met.

2.9 If the wire in Problem 2.8 carries a current of 5 A, what is the current density?

**Solution:**

Current density is current per unit area. Area of cross-section is of  $2.08 \times 10^{-6} \text{ m}^2$ . Thus current density will be  $2.40 \times 10^6 \text{ A/met}^2$ .

2.10 Al can handle current densities of  $10^5 \text{ A/cm}^2$  at about 150°C (Gupta 2003). What will be the maximum current allowed in an Al wire of AWG #18 operating at 150°C?

**Solution:**

The wire diameter is (Table 2-2) 1.023696 mm. the area of cross-section is  $8.23 \times 10^{-7} \text{ met.sq}$ . The maximum current will be the product of maximum current density and cross-sectional area and this works out to 823.06 A or 0.823 kA.

2.11 A circuit breaker connects an AWG #0000 Cu conductor wire 300 feet in length. What is the resistance of this wire? If the wire carries 150 A, what is the voltage decrease across this wire?

**Solution:**

From table 2-2, diameter of the wire is 11.684 mm, therefore cross-sectional area is  $1.07 \times 10^{-4} \text{ m}^2$ . Length is 300 feet or 91.44 met (one foot is 0.3048 met). We use conductivity of copper as  $5.8 \times 10^7 \text{ S/m}$ . Thus, the resistance of this wire will be 0.0147 ohms. From Ohm's law the voltage drop across this will be  $150 \times 0.0147 = 2.2 \text{ volts}$

2.12 You may know that a conductor carrying an electrical current generates a magnetic field. A long wire carrying a current generates a magnetic field similar to that generated by a bar magnet. This magnet is known as an electromagnet. Consider a meter of magnetic wire AWG #2. Such wires are usually made from high-conductivity soft-drawn electrolytic Cu, and the conductor is coated with a polymer to provide insulation. What will be the electrical resistance (in ohms) of this wire?

**Solution:**

From Table 2-2, diameter of the AWG # 2 wire is 6.543802 mm, therefore cross-sectional area is  $3.36 \times 10^{-5} \text{ m}^2$ . Length is 1 meter. We use conductivity of copper as  $5.8 \times 10^7 \text{ S/m}$ . Thus, the resistance of this wire will be 0.0005 ohms.

2.13 Ground rods are used for electrical surge protection and are made from materials such as Au, Au-clad steel, or galvanized mild steel. The resistance of the actual rod itself is small; however, the soil surrounding the rod offers electrical resistance (Paschal 2001). The resistance of a ground rod is given by:

$$R = \frac{\rho}{2L\pi} \ln\left(\left[\frac{4L}{a}\right] - 1\right)$$

where  $R$  is the resistance in ohms,  $\rho$  is the resistivity of soil surrounding the ground rod (in  $\Omega$ -cm),  $L$  is the length of ground rod in centimeters, and  $a$  is the diameter of the ground rod. (a) Assuming that the resistivity of a particular soil is  $10^4 \Omega$ -cm, the length of the rod is 10 feet, and the diameter is 0.75 inches, what will be the resistance ( $R$ ) of the ground rod in ohms? (b) Assuming that the ground rod is made from Cu, prove that the resistance of the rod itself is actually very small. (c) What will happen to the resistance of the metallic material as it corrodes over a period of many years?

**Solution:**

- (a) Note that we use soil resistivity which is very high compared to resistivity of copper. The value in ohm-m will be 100 ohm-m. Diameter is 0.75 inches which is  $1.91 \times 10^{-2}$  m. Length is 10 feet which is 3.048 meters. The ground resistance then works out to be 33.731 ohms. This resistance is typical of soils that are more like inorganic clays as opposed to sand or gravel mixtures (which are much higher resistivity).
- (b) Since resistivity of copper is orders of magnitude smaller compared to that of soil, we expect the electrical resistance of the rod itself will be very small.

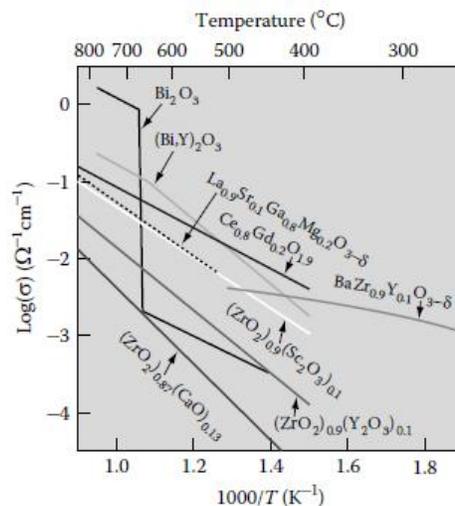
(c) If the metallic rod corrodes, the resistance of the grounding rod will go up and it may not function as intended. This is why copper rods can be electroplated to avoid the risk of oxidation.

2.14 The electrical resistance of pure metals increases with temperature. In many ceramics, the electrical current is carried predominantly by ions (such as oxygen ions in YSZ). Based on the data shown in Figure 2.30, calculate the electrical resistance of a 50- $\mu\text{m}$ -thick YSZ element at 500°C and 800°C. Assume that the cross-sectional area is 1  $\text{cm}^2$  and the length is 1 meter.

**Solution:**

$$R = \rho \frac{l}{A}$$

In figure 2.30 locate the straight line that shows variation of conductivity of YSZ, it is the line that is marked as  $(\text{ZrO}_2)_{0.9}(\text{Y}_2\text{O}_3)_{0.1}$ .



**FIGURE 2.30** The conductivity of different materials used as electrolytes in the development of solid oxide fuel cells. (From Haile, S. M. 2003. *Acta Materilia* 51:5981–6000. With permission.)

At 500 C: the  $\log \sigma = -4$  i.e.  $\sigma = 10^{-4} \Omega^{-1}\text{cm}^{-1}$ . The area is given as  $1 \text{ cm}^2$ . Using these values we get a resistance of  $10^6$  Ohms.

At 800 C: From Figure 2-30,  $\log \sigma = -2$  i.e.  $\sigma = 10^{-2} \Omega^{-1}\text{cm}^{-1}$ , since dimensions do not change significantly (at least that's what we assume here), the resistance will drop by 2 orders of magnitude. The new resistance will be 10,000 Ohms.

2.15 Consider the material calcium oxide–stabilized ZrO<sub>2</sub> (Figure 2.30). Calculate the electrical resistance of a 50- $\mu\text{m}$ -thick YSZ element. Assume that the cross-sectional area is  $1 \text{ cm}^2$  and the length is 1 meter at 500°C and 800°C.

**Solution:**

Same as 2.14

2.16 If a high conductivity at temperatures above 700°C was the only consideration in selecting a material for a solid oxide fuel cell electrolyte, what material would you choose (Figure 2.30)? Besides cost, what additional factors must be considered in the selection of this material?

**Solution:**

The highest conductivity material shown in figure 2.30 is bismuth oxide and this will be chosen if the only consideration were conductivity. Other most important considerations, in addition to cost and long term availability, would be the environmental chemical stability of such a material (i.e. can it withstand oxidizing and reducing conditions) and thermal expansion coefficient relative to other materials that this electrolyte material will be in contact with.

2.17 What is unusual about the change in resistivity as a function of temperature for bismuth oxide ( $B_2O_3$ )?

**Solution:**

Bismuth oxide shows a sharp change in conductivity at around 680 C (see figure 2-30). Often, such large changes in conductivity are associated with some type of phase transformation. As this transformation occurs, there is often a volume change. The volume change leads to build up of stress and usually ceramic materials would fracture if they were taken through such phase transformations repeatedly.

2.18 Calculate the mobility of the electrons in Zn in  $cm^2/V\cdot s$ . Assume that each Zn atom contributes two conduction electrons. The atomic mass of Zn is 65. The resistivity is  $5.9 \mu\Omega\cdot cm$ , and the density is  $7.130 g/cm^3$ .

**Solution:**

One  $cm^3$  of zinc contains 7.13 grams of zinc. One mole i.e. 65 grams of zinc contains an Avogadro's number (i.e.  $6.023 \times 10^{23}$ ) number of atoms. Therefore, one  $cm^3$  of zinc i.e. 7.13 grams of zinc will contain  $6.60677 \times 10^{22}$  atoms. Each zinc atom is assumed to contribute two electrons. Thus, the conduction electron concentration for zinc will be  $2 \times 6.60677 \times 10^{22} = 1.32 \times 10^{23}$  electrons/ $cm^3$ .

The resistivity is given to be  $5.9 \mu\Omega\cdot cm$ , this means conductivity is  $169491.5 S/cm$ .

Now:  $\sigma = n \times \mu_n \times q$ ,  $q: 1.6 \times 10^{-19} C$ . Substituting the values of conductivity, conduction electron concentration, and electronic charge, we get mobility value as  $\mu_n = 8.01 cm^2/V\cdot s$ .

/V-s Assume that the mass of electrons in Au is  $9.1 \times 10^{-31}$  kg. Calculate the mean free path length ( $\lambda$ ) of electrons in Au if the average electron speed is  $10^6$  m/s.

**Solution:**

Approach 1: In equation 2-21, the expression for conductivity is:

$$\sigma = \frac{nq^2\tau}{m}$$

For gold, we use (all quantities in SI units) :  $n=5.91 \times 10^{28}$  electrons/ $m^3$  (Table 2.3, convert from electrons/ $cm^3$ ),  $m: 9.1 \times 10^{-31}$  kg, and  $q: 1.6 \times 10^{-19}$  C, and conductivity of  $4.25 \times 10^7$  S/m (Table 2.1, convert to SI units). This gives us a mean free path of  $2.556 \times 10^{-8}$  m or 255.6 Å. This is consistent with values mentioned in Section 2.10.

The time between collisions ( $\lambda$ ) can then be calculated as mean free path divided by average velocity.

$$\lambda = 2.556 \times 10^{-8} \text{ m} / 10^6 \text{ m/s} = 2.56 \times 10^{-14} \text{ seconds.}$$

Approach 2:

We could also have calculated the time between collisions ( $\lambda$ ) from the mobility value using equation 2.18. In this we use all SI units,

$$\mu = q\tau/m$$

The value of mobility will be  $0.0031 \text{ m}^2/\text{V}\cdot\text{s}$ . Use  $q=1.6 \times 10^{-19}$  C and  $m: 9.1 \times 10^{-31}$  kg. this gives us a value of  $\tau = 1.76 \times 10^{-14}$  s. This is similar to what we calculated before using conductivity and electron concentration values.

Then the mean free path, from equation 2.19 will be  $1.76 \times 10^{-8}$  m or 176 Å. This is similar to the value calculated in approach 1.

2.20 The thermal speed of electrons is about  $10^6$  m/s. However, the drift velocity is rather small because electrons are scattered off by the vibrations of atoms. Calculate the drift velocity of electrons in Cu for an electric field of 1 V/m. Assume the mobility of electrons in Cu is  $32 \text{ cm}^2/\text{V}\cdot\text{s}$ .

**Solution:**

From Equation 2.17,

$$v_{\text{drift}} = \left(\frac{q\tau}{m}\right) \times E$$

Note that the term inside the bracket is mobility. Convert mobility value in SI units as:  $0.0032 \text{ m}^2/\text{V}\cdot\text{s}$ . Electric field is 1 V/m. this means the value of drift velocity is  $0.0032 \text{ m/s}$ . Compare this to thermal velocity which is  $10^6 \text{ m/s}$ . This is because of scattering.

2.21 If the density of Ag is  $10.5 \text{ g/cm}^3$ , what is the concentration of conduction electrons in Ag?

**Solution:**

Atomic mass of silver is 107.868 (See Table 2.3). Thus, 107.868 grams of silver has  $6.023 \times 10^{23}$  atoms. Therefore, in 10.5 grams or 1 cc of silver we have  $5.862 \times 10^{22}$  atoms. For silver, we assume each atom donates one conduction electron. Therefore, the concentration of conduction electrons will also be  $5.862 \times 10^{22} \text{ electrons/cm}^3$ .

2.22 From the information provided in Table 2.3, calculate the expected conductivity of Ag.

**Solution:**

$$\sigma = \mu_n \times q \times n$$

$$\sigma = 57 \frac{\text{cm}^2}{\text{V}\cdot\text{s}} \times 1.6 \times 10^{-19} \text{ C} \times (5.86 \times 10^{22} \text{ electrons/cubic cm})$$

This works out to  $53.4 \times 10^4$  S/cm. The resistivity will be inverse of this i.e.  $1.87 \times 10^{-6}$  ohm/cm. These values are close to what is reported in Table 2.1.

2.23 Au is a face-centered cubic metal with a lattice constant of  $4.080 \text{ \AA}$ . If the atomic mass of Au is 196.9655, calculate the number of conduction electrons per unit volume. Express your answer as number of electrons/cm<sup>3</sup>. Assume that each Au atom donates one conduction electron.

**Solution:**

For metals such as gold (Au) with face center cubic structure we have a total of four atoms per unit cell. Each of the corner atom counts as  $1/8^{\text{th}}$  and each face center atom counts as half.

The volume of the unit cell is  $6.7917 \times 10^{-23} \text{ cm}^3$ . Thus concentration of atoms in atoms/cm<sup>3</sup> :

$$\frac{6.7917 \times 10^{-23}}{4} = 5.88 \times 10^{22}$$

For gold, valence is one, thus concentration of conduction electrons will be the same as concentration of atoms since each atom donates only one electron. Thus, concentration of conduction electrons will be  $5.88 \times 10^{22}$  electrons/cm<sup>3</sup>. This is close to the value listed in Table 2.3.

2.24 A semiconductor is made so that it carries electrical current primarily from the flow of electrons. If the mobility of electrons ( $\mu_n$ ) is  $1350 \text{ cm}^2/\text{V-s}$  and the conduction electron concentration is  $10^{21} \text{ cm}^{-3}$ , what is the electrical conductivity of this material?

**Solution:**

It is stated that the current in this particular semiconductor is predominantly carried by electrons. We use the following equation.

$$\sigma = q \times \mu_n \times q \times n$$

Using values given we get conductivity of  $2.16 \times 10^5 \text{ S/cm}$ .

2.25 A heating element for a flat iron is rated at 1000 W. If the iron works at 220 V, what is the resistance of this heating element?

**Solution:**

Power dissipated is given by  $P = V \times I = I^2 \times R$  (Equation 2.28), using this we get current value of 4.54 Amp, and then using Ohm's law we get resistance value of 48.4 Ohms.

2.26 Electronic components and devices are often tested at 125°C and –55°C to check the high- and low-temperature performances. They can then be compared with the properties observed at room temperature, 25°C. For example, using 25°C as the reference temperature and +125°C as the other temperature,  $\alpha_{125}$  can be written as follows:

$$TCR_{125} = \alpha_{R,125} = \frac{1}{\rho_{125}} \left( \frac{\rho_{125} - \rho_{25}}{125 - 25} \right) \times 10^6 \text{ ppm}/^\circ\text{C}$$

Write an equation to express the temperature coefficient of resistance with  $T = -55^\circ\text{C}$  (note the negative sign) as the other temperature, using 25°C as the base or reference temperature ( $T_0$ ).

**Solution:**

$$TCR_{-55} = \alpha_{R,-55} = \frac{1}{\rho_{-55}} \left( \frac{\rho_{-55} - \rho_{25}}{-55 - 25} \right) \times 10^6 \text{ ppm}/^\circ\text{C}$$

Note that the resistivity at lower temperatures will be lower, thus the numerator in the bracketed expression will be negative and so is the denominator.

2.27 In a circuit, the TCR 125 value for a resistor is 100 ppm/°C. If the resistance ( $R$ ) at 25°C is 1000  $\Omega$ , what is the resistance at 125°C?

**Solution:**

$$TCR_{125} = \alpha_{R,125} = \frac{1}{R_{125}} \left( \frac{R_{125} - R_{25}}{125 - 25} \right) \times 10^6 \text{ ppm}/^\circ\text{C}$$

Since the resistor is the same i.e. geometry remains the same, we can use resistance instead of resistivity. Using values of TCR and  $R_{25}$  in the above equation and solving for  $R_{125}$  we get:

$$100 = \frac{1}{R_{125}} \left( \frac{R_{125} - 1000}{100} \right) \times 10^6$$

This leads to  $R_{125} = 1000/0.99=1010$  Ohms

2.28 Assume that the bus bar discussed in Example 2.11 is heated due to the Joule losses and now operates at 70°C. Calculate the resistance, power loss, and energy consumption for 24 hours and the total energy costs per year. Ignore the change in the length of the Cu bus bar because of thermal expansion.

**Solution:**

The copper conductor discussed in Example 2.11 was shown to have a resistance of  $2.44 \times 10^{-3}$  ohms. From the literature, the temperature coefficient of resistance ( $\alpha_R$ ) for copper is 0.00404 /C, with a reference temperature of 20 C.

The resistance at 70 C will be given by:

$$R = R_0 [1 + \alpha_R (T - T_0)] = 0.00244 [1 + 0.00404(70 - 20)] = 2.93 \times 10^{-3} \text{ ohms.}$$

The voltage drop will be  $V = 1000 \text{ Amp} \times 2.93 \times 10^{-3} \text{ ohms} = 2.93 \text{ volts.}$

Power dissipated =  $I^2 \times R = 2933 \text{ Watts}$  or 2.933 kiloWatts.

Energy lost in 24 hours =  $2.993 \times 24 = 70.4 \text{ kW-h.}$

Assuming cost of electricity is 4 cents/kW-h (See Example 2.11), the cost per day will be \$2.81/day. In one year, the cost will be  $365 \times \$2.81 = \$1027.68$

2.29 Nichrome wire is used for cutting materials such as polystyrene (Styrofoam) into different shapes, including large facades or insulation boards. (a) Calculate the length of an AWG #20 wire that is needed to have a resistance of  $R = 8 \Omega$ . (b) What will be the resistance of this wire if it gets heated to a temperature of  $200^\circ\text{C}$ ? See Tables 2.2 and 2.5.

**Solution:**

(a) From Table 2.5 the resistivity of one type of nichrome alloy is  $100 \mu\Omega\text{-cm}$  or  $0.0001 \text{ ohm-cm}$ . The wire gage is 20 and from Table 2.2 the diameter is  $0.811814 \text{ mm}$  which translates into an area of  $0.0051761 \text{ sq.cm}$ . The resistance is  $8 \text{ Ohms}$ .

The length of the wire is given by:

$$l = \frac{R \times A}{\rho} = \frac{8 \text{ ohm} \times 0.0051761 \text{ sq.cm}}{0.0001 \text{ ohm-cm}} = 414 \text{ cm.}$$

(b) If the wire gets heated to  $200 \text{ C}$ , the new resistance will be given by:

$$R = R_0 [1 + \alpha_R (T - T_0)] = 8 [1 + 400 \times 10^{-6} (200 - 27)] = 8.5536 \text{ ohms.}$$

Note that in Table 2.5 the values of  $\alpha_R$  should be listed as  $400 \times 10^{-6}$ . Also, note that the reference temperature ( $T_0$ ) of nichrome value is  $300 \text{ K}$  which is  $27 \text{ C}$ .

2.30 In the nanoscale region, why does the resistivity of thin films depend upon thickness?

**Solution:**

This is because of surface scattering and surface contaminants (See Section 2.10).

2.31 What elements most affect the resistivity of high-purity Cu?

**Solution:**

Titanium, based on Figure 2.13 data.

2.32 Why does the addition of oxygen in limited concentrations actually *increase* the conductivity of high-purity Cu?

**Solution:**

As mentioned in Section 2.11.2, an optimum level of oxygen can concentrate the total impurity content in form of small precipitates of copper oxide and thus the conductivity goes up after the reduction seen with initial incorporation of oxygen.

2.33 Why does the resistivity of pure metals increase with temperature, whereas that of alloys is relatively stable with temperature?

**Solution:**

In an alloy the concentration of other elements is typically very high and thus the resistivity is increased considerably and is dominated by scattering from these added elements and the effect of phonon scattering is not as strong. For pure metals the resistivity is very low and hence even a small change in temperature brings in considerable amount of phonon scattering.

2.34 Nordheim's coefficient for Au dissolved in Cu is  $C = 5500 \text{ n}\Omega\text{-m}$ . If the resistivity of Cu at

300 K is 16.73 nΩ-m, calculate the resistivity of an Au–Cu alloy containing 1 weight % Au.

**Solution:**

Convert weight fraction ( $w$ ) of gold, which is an impurity in this case, into atomic fraction using equation shown in Example 2.12. Note that Copper is the matrix.

$$x = \frac{M_{\text{Cu}} \times w}{(1 - w)M_{\text{Au}} + wM_{\text{Cu}}}$$

In this use atomic mass of copper as 63.5, that of gold is 197,  $w$ : 0.01 (weight fraction of gold). This gives atomic fraction of gold as  $x = 0.00324534$ .

Using Nordheim's rule (Equation 2.32):

$$\rho_{\text{alloy}} = \rho_{\text{matrix}} + Cx(1 - x)$$

$C$ =Nordheim's coefficient is 5500 nΩ-m, and resistivity of the matrix (Cu, in this case) is 16.73 nΩ-m

Therefore, Resistivity of alloy =  $16.73 + 5500 (0.00324534) (1 - 0.00324534) = 34.52$  nΩ-m.

2.35 What is the electronic configuration for an Mg atom ( $Z = 12$ )?

**Solution:**

$1s^2 2s^2 2p^6 3s^2$ . See Example 2.13

2.36 Draw a schematic of the band diagrams for a typical metal, a semiconductor, and an insulator.

**Solution:**

See Figure 2.27.