Deformation (Chapter 2)

Solution to problem 2-1

a) To calculate the extension or elongation, simply measure the present length (l) from one tip to the other, using a ruler, and then estimate the original length (l_0) of the belemnite. This can be done by restoration (Figure SP2.1) or by adding the lengths of individual belemnite segments.

Elongation = $(l-l_0)/l_0 = (11.56 - 6.22)/6.22 \approx 0.86 = 86\%$.

86% extension means that the belemnite has been stretched by a factor (stretch s) of 1.86.

Note that some of the boudins are barrel shaped, which introduces an uncertainty. If we assume that this shape is an expression of ductile deformation along the margins or corners of the boudins, then restoring a line through the center of the boudinaged belemnite would be the way to go. In the restoration shown here (Figure SP2.1a) I have chosen to balance the gaps and overlaps.

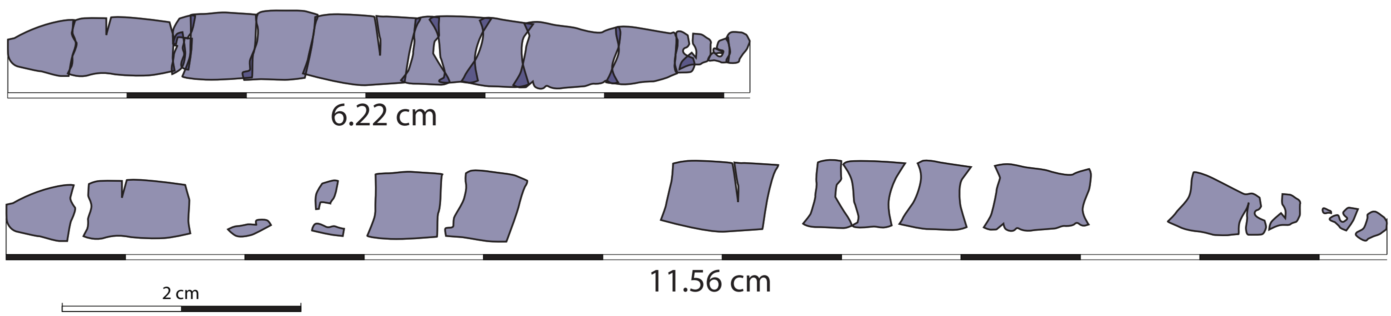


Figure SP2.1a Restored and stretched belemnite of Problem 2-1.

b) This one is similar to what we did above: Measure the current length (l) and then the restored length (l_{0Tr} and l_{0B}) and calculate the extension (=elongation) as above.

Top Triassic extension: $(l-l_{0Tr})/l_{0Tr} = (236.6 - 210)/210 \approx 0.127 = 12.7\%$.

Top Basement extension: $(l-l_{0B})/l_{0B} = (236.6 - 178.5)/178.5 \approx 0.326 = 32.6\%$

Is the extension evenly distributed in the two cases? For the Top Triassic marker, there is clearly more extension (gaps) between the Gullfaks Field and the Viking Graben then elsewhere. For the Top Basement marker the extension is more evenly distributed in the central – eastern part of the section, and lower in the western part (Shetland Platform).

For the North Sea section, how do the two extension estimates compare? Basement is stretched more than the Top Triassic marker. The difference (~20% or 31.5 km extension) is a strain estimate for the late Permian–mid Triassic phase of rifting in this part of the North Sea rift. It tells us that this first phase of rifting involved considerably more extension than the second (Jurassic) and last phase.

How much extension is taken up by the largest 4-5 faults? To answer this question, restore only the largest 4-5 faults and do the same calculation over again. This shows that the 5 largest faults take up 11/26.6 = 41% of the post-Triassic stretching, and 37.6/58.1 = 65% of the total basement extension estimated above (Figure SP2.1c). This tells us that small faults play a role in extension estimates. Since there must be small faults that are not shown in the interpretation (because of limited seismic resolution), there is a component of fault extension that is missing. Hence all of our results underestimate the real extension.

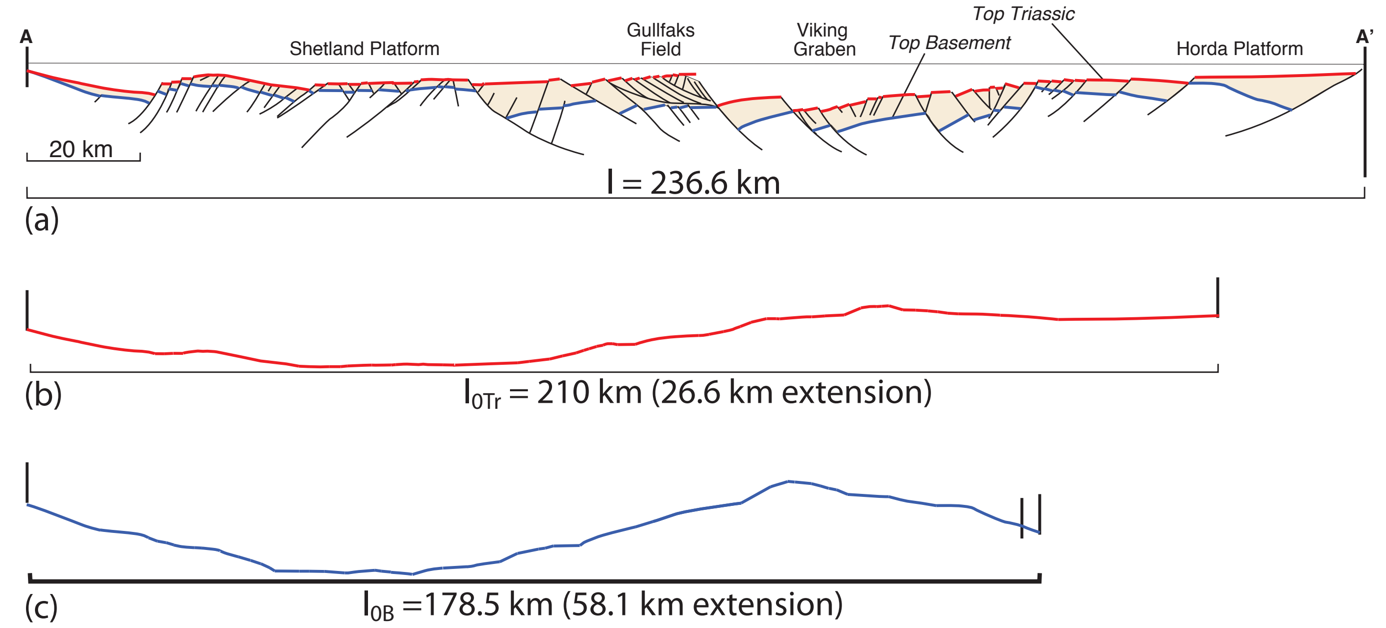


Figure P2.1b (a) Cross-section through the northern North Sea, where post-Triassic strata have been removed. Based on deep seismic line NSDP84-1. (b) Top Triassic restored (fault offsets removed without any rotation of the layering). (c) Top Basement restored.

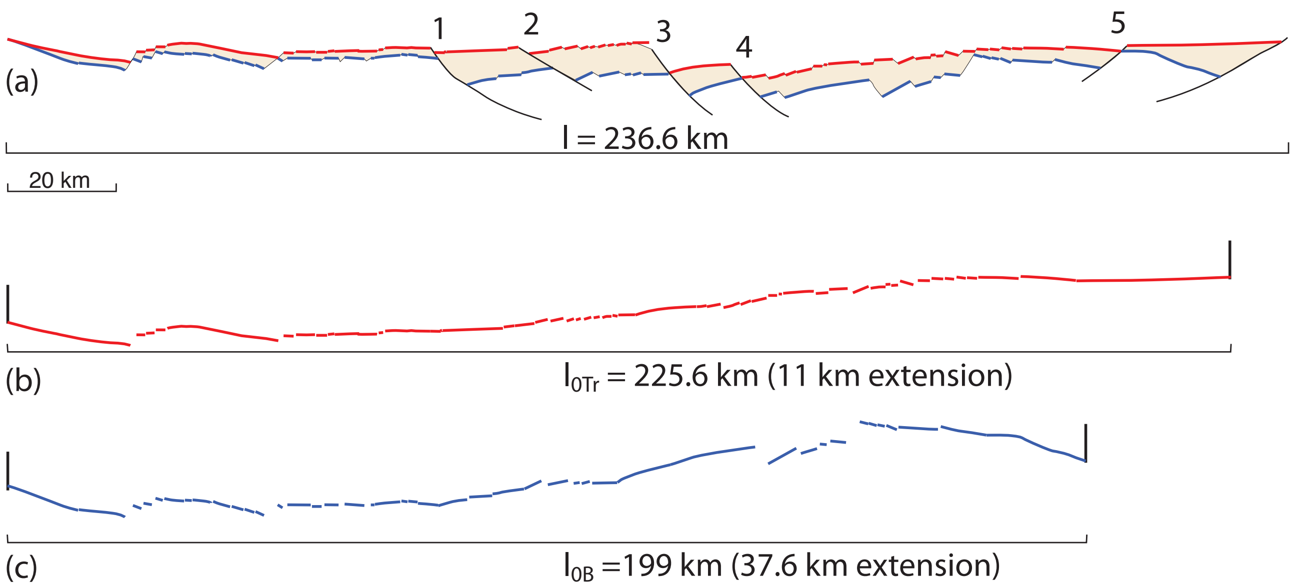


Figure P2.1c Restoration of 5 of the largest fault displacements only.

Is there any other way that we could estimate the extension along the North Sea section? If we knew top MOHO, we could do a constant area restoration, assuming that the prerift crustal thickness was constant and equal to the present thickness at the rift margins.

A comment on uncertainties: The restored line should ideally be more or less planar and horizontal, since this is how sedimentary layers are deposited. We will see from Chapter 20 that this can be fixed, but it requires a choice of deformation such as vertical shear (which does not change our estimate of l_0) or rigid rotation of dipping line segments or fault blocks (which increases l_0).

Problem 2-2.

Bedding is horizontal, and we trace the orientation of the two fossils and measure their angle to bedding (bedding is taken to represent the shear plane). It appears that the two trace fossils have somewhat different orientations, so we get two estimates for the angular shear (38.5 and 53). The shear strains (γ) are tan 38.5° ≈ 0.8 and tan 53°≈1.3.

The difference in strain estimates may be due to different original orientations. However, the skolitos tend to have a fairly consistent bedding-perpendicular initial orientation. Hence it is likely that the shear strain is heterogeneous at the scale of observation. The fact that the lower fossil seems to be tighter may support this interpretation. However, we need to analyze more strain markers in this rock to see if there is a systematic variation in the orientations and thus in strain.

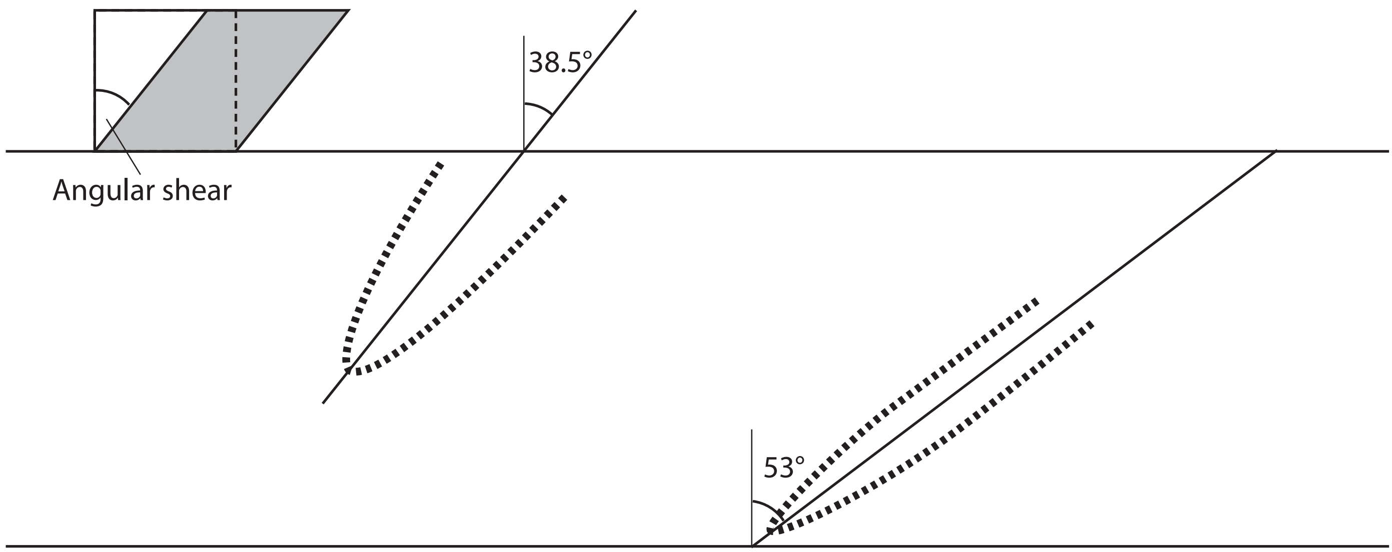


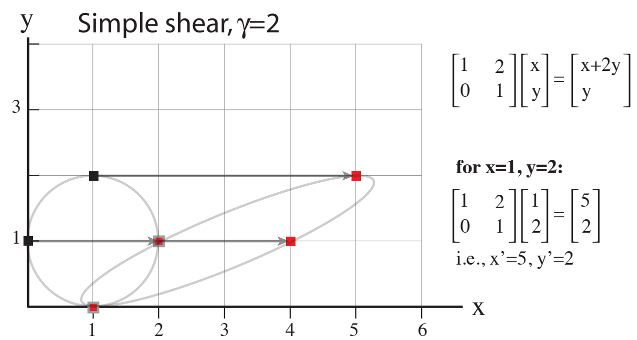
Figure SP2.2 Drawing of the essential features of the pictire and angular observations.

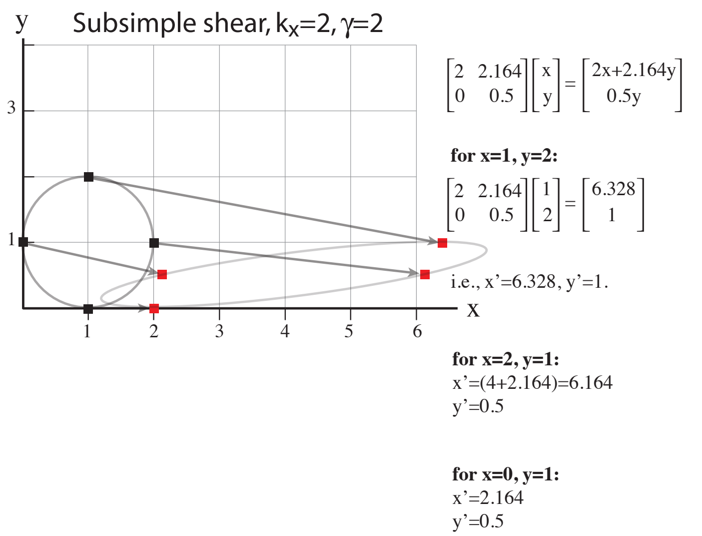
Problem 2-3.

To find the new positions of the four points we have to use the deformation matrix. This easy for simple shear and pure shear (use Equations 2.13 and 2.14 in the textbook), but more difficult for subsimple shear. We then have to calculate Γ from Equation 2.15 (note that the general formula for Γ has a printing error, so use the one for no area change), which in this case (γ=2 and k_x=2) becomes:

$\Gamma = \gamma(k_x-1/k_x)/2(\ln k_x) = 2(2-0.5)/2\ln(0.5) = 3/(-1.386)=-2.164.$

The deformation matrices and calculations of new points are shown in Figure SP2.3.





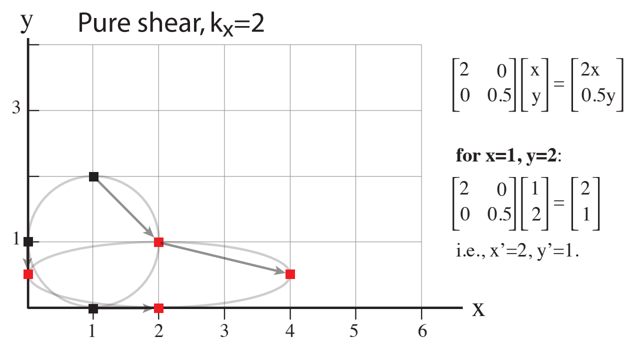


Figure SP2.3 Graphical illustration of the three deformations. An ellipse has been added for clarity.

Problem 2-4.

The matrix

$$\begin{bmatrix} 3 & 0 & 0.25 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

describes a three-dimensional deformation where the off-diagonal term 0.25 relates to a simple shear strain and the diagonal terms (3, 0.5, 0.5) describe a three-dimensional coaxial strain. The pure shear involves extension along the x-axis of the coordinate system, and shortening in the plane orthogonal to x (i.e. along the y- and z-axes of the coordinate system). The simple shear occurs in the x-direction (affects the x-values). To see this, multiply the vector (x,y,z) with the matrix above, which yields (3x+y/4, y/2, z/2). For a unit vector along z (1,0,0), the new vector becomes (3,0,0). The fact that the simple shear and coaxial strain are contained in a single matrix tells us that, mathematically, they are applied simultaneously.

The determinant of this matrix is the product of the diagonal elements, i.e. the pure shear components: Det **D**=3x0.5x0.5= 0.75. The 3 times extension along the x-axis is less than the combined shortening along y and z, which generates a reduction in volume by 25%, all caused by the coaxial component.

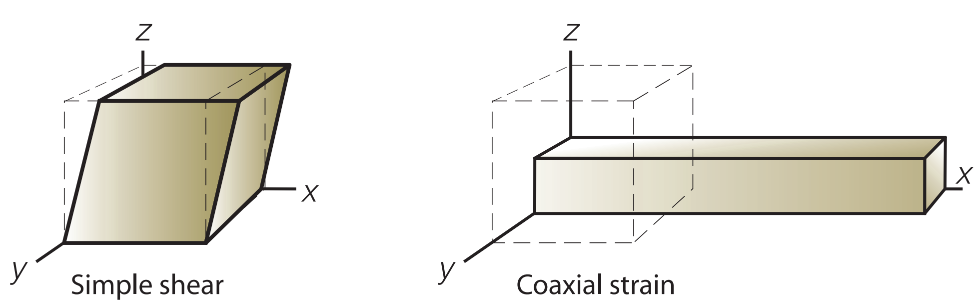


Figure SP2.4 The two components of deformation represented by the deformation matrix in Problem 2-4 in a specified coordinate system.

Problem 2-5.

Since the deformation is taking place in the x-y plane, this is the section we want to study. To write the deformation matrices, we need to remember the premultiplication thing about matrices, which means that the last deformation is represented by the first matrix:

$$\begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix} \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1+\Delta \end{bmatrix} = \begin{bmatrix} k & k\gamma(1+\Delta) \\ 0 & \dfrac{1+\Delta}{k} \end{bmatrix}$$

Problem 2-6

All cases except case (e) are homogeneous, because we see from Figure P2.6 that straight lines remain straight and parallel lines remain parallel. Case (b) does not involve strain, because all the displacement are of equal length and parallel (translation). Neither does (a) involve strain, although this is a little harder to see. It becomes obvious once we realize that the displacement is that of rigid rotation. Remember: Deformation = Rigid rotation + Translation + Strain. The deformation matrix for each of the deformations are shown in Figure SP2-6.

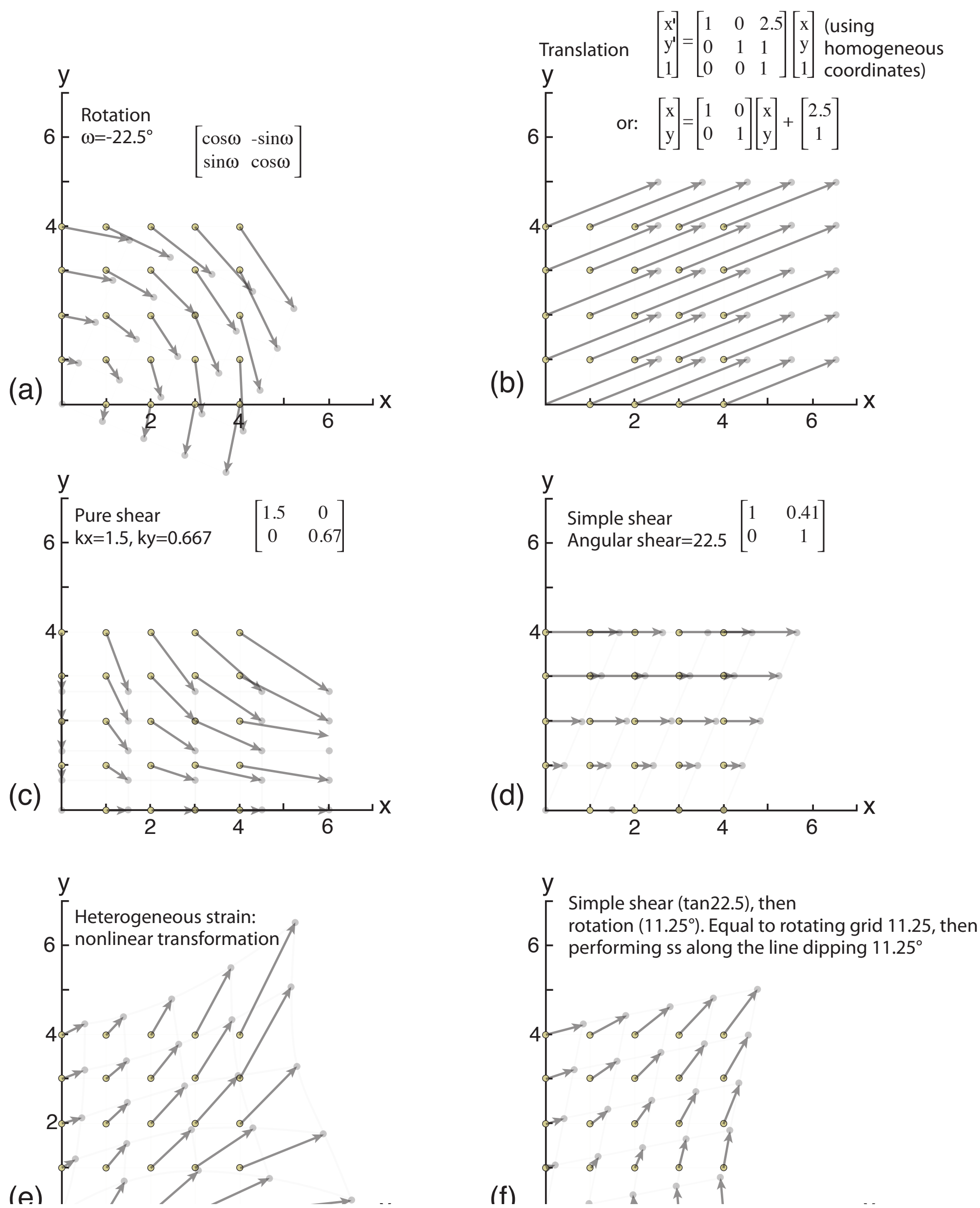


Figure SP2.6 Displacement fields and information about the deformation for Problem 2-6.

Problem 2-7

a) A rock with vertical foliation (strike/dip= 000/90) and vertical lineation (000/90):

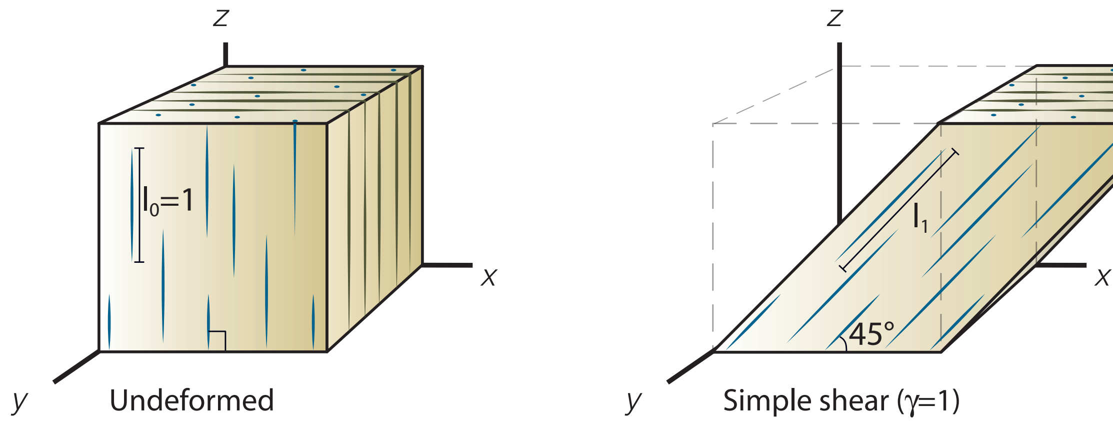


Figure SP2.7 The initial situation and the effect of a shear strain of 1.

b) Because of the special orientation of this lineation (parallel to the z-axis) its angle to z will always be the angular shear strain, which for γ=1 is 45°, and for γ=10 becomes $\tan^{-1}10$= 84.3°, which means that it makes an acute angle of 5.7° to the horizontal shear plane.

To use the deformation matrix **D** to find the line rotation, we first need to find **D**. It is a matrix with no coaxial strain or volume change, which means that the diagonal elements are unity. Then there is one simple shear element, which we denote γ. This element is off-diagonal, and we have to place it where it affects the x-value of any vector that **D** is applied to. Trying and failing shows that this is the upper right-hand corner. We then need to multiply a unit vector **l** representing the undeformed lineation with the deformation matrix **D** (see Appendix A.4). **l**=(001) and the calculation is simple:

$$\begin{bmatrix} 1 & 0 & \gamma \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \gamma \\ 0 \\ 1 \end{bmatrix}$$

With γ=1 and 10 we get the new vectors (1,0,1) and (10,0,1). We find the angles that these vectors make with the x-axis by using the tangent relationship. For γ=1 we get $\tan^{-1}(1/1)$=45°, and for γ=10 we get $\tan^{-1}(10/1)$=5.7°.

c) To find the elongation of a line of unit length parallel to the lineation, we use what we found in b) above, which was that the unit vector (0,0,1) changes to (1,0,1) and (10,0,1) for the two deformations. Since we started out with a vector of unit length the new lengths give us the elongation. For γ=1, the length becomes:

$$\sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$$

and for γ=10, we get a new length that is close to 10 times the original one:

$$\sqrt{10^2 + 0^2 + 1^2} = \sqrt{101} \approx 10$$

d) Nothing happens to the orientation of the foliation during these deformations: It remains vertical and parallel to the x-z plane. You can use Equation A.16 to check this. The pole to the plane is represented by the normal vector **p**=(0,1,0), and we get the new orientation by multiplying this vector with the inverse of the matrix **D**:

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & \gamma^{-1} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\gamma \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

The vector **p** does not change, hence there is no change in the orientation of the plane.

Problem 2-8

a) For subsimple shear with W_k=0.5 (with γ being 1 and 10) we are looking at more complicated calculations. The general matrix for this type of deformation is:

$$\mathbf{D} = \begin{bmatrix} k & 0 & \Gamma \\ 0 & 1 & 0 \\ 0 & 0 & 1/k \end{bmatrix}$$

But how do we find k and Γ? We have to pull k out of the expression for W_k given by Equation 2.29 or A.22. This is a two-dimensional formula, which is fine since we can exclude the y-direction (we have a case of plane strain in the x-z plane). For γ=1 and W_k=0.5 we get :

$0.5 = \cos[\tan^{-1}(2\ln k)]$
$\cos^{-1}0.5 = \tan^{-1}(2\ln k)$
$60 = \tan^{-1}(2\ln k)$
$\tan 60 = 2\ln k$
$0.866 = \ln k$
$k=e^{0.866}$
$k \approx 2.377$

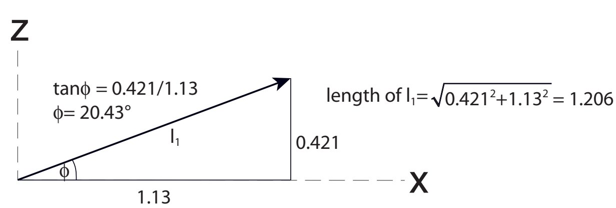
Similarly, for γ=10 we get k ≈ 5769. We can get these k-values from the downloadable Excel-file as well, in which case we would use the sheet called "Wk-based 2D-Strain" and set Wk=0.5 and check the k-values calculated for γ=1 and 10. Now we can calculate Γ like we did for Problem 2.3:

For γ =1:
Γ = γ(k-1/k)/[2(lnk)] = 1(2.377-1/2.377)/[2(ln2.377)] = 1.956/1.7317 = 1.13

For γ =10:
Γ = γ(k-1/k)/2(lnk) = 10(5769-1/5769)/2[8.66] = 57690/17.32= 3330.8

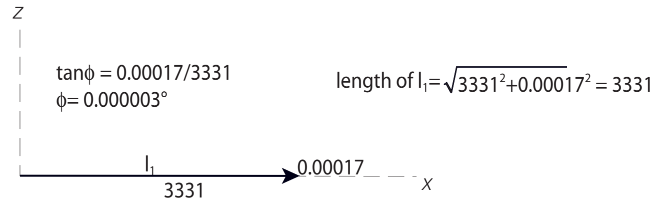
The new line orientation and length for the two cases can now be calculated by multiplying **l** and **D**. For γ =1:

$$\mathbf{Dl} = \begin{bmatrix} 2.377 & 0 & 1.13 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2.377 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.13 \\ 0 \\ 0.421 \end{bmatrix}$$



For γ =10:

$$\mathbf{Dl} = \begin{bmatrix} 5769 & 0 & 3331 \\ 0 & 1 & 0 \\ 0 & 0 & 1/5769 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3331 \\ 0 \\ 0.0001733 \end{bmatrix}$$



Why does the last case give such a long low-angle lineation (almost parallel with the coordinate axis x)? Because for γ =10 requires a huge coaxial component to fulfill W_k=0.5. In this sense, the coaxial component accumulates faster than the simple shear component for a constant-W_k deformation.

As for plane rotations, multiplying the deformation matrices with the pole **p** to the foliation plane has no effect, so the foliation remains vertical:

$$\mathbf{pD} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} k & 0 & \Gamma \\ 0 & 1 & 0 \\ 0 & 0 & 1/k \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

b) The angle α between the flow apophyses in this subsimple shear is governed by W_k and therefore the same for any strain value (it is a flow parameter, not a strain parameter). We can use Equation 2.23:

$\alpha = \cos^{-1} W_k = \cos^{-1} 0.5 = 60°$

or we can use Figure 2.24 for W_k = 0.5 and read off α = 60°.

The angle θ‘ between the long axis (X) of the strain ellipsoid and the x-axis (and shear direction) can be found from Figure 5.12b for γ=1 and k=2.377, which gives us an angle close to 5. γ =10 is outside the range of this graph. We then have to use the Excel spread sheet called “Wk-based 2D-Strain” (which also gives us more precise values for γ=1) and read off θ‘, which for γ=1 and 10 becomes 4.0° and 7.4x10^{-7}.

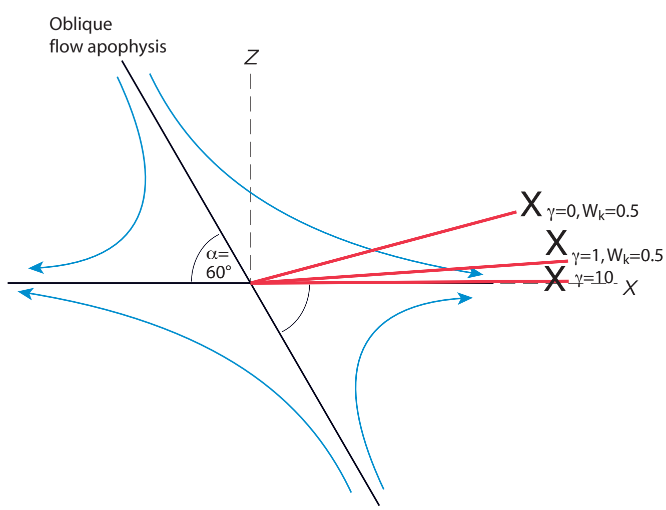


Figure SP2-8 Illustration of flow apophyses (blue lines) and the orientation of the finite strain ellipse (X, red lines) for Wk=0.5 and γ=0, 1, and 10. The value for γ=0 is the orientation of ISA_1.

Problem 2-9

a) The amount of compaction is 20% (porosity is reduced by 50%, while the total volume is reduced by 20%, from 100% to 80%). The vertical elongation Δ therefore becomes -0.2, and the deformation matrix is:

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+\Delta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}$$

b) The strain ellipsoid has two horizontal axes of equal length (X=Y) and a vertical axis (Z) that is 0.8 times the length of the other two. R is the ratio between two principal axes, and there is one R-value for each section containing these axes:

R_{xy}=X/Y=1/1=1, R_{yz}=X/Y=1/0.8=1.25, R_{xy}=X/Z=1/0.8=1.25.

c) The strain ellipsoid is oblate and plot in the lower part (right side) of the Flinn diagram (Figure SP2.9)

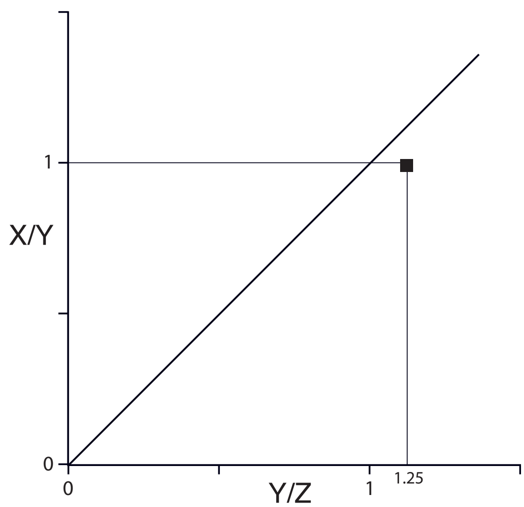


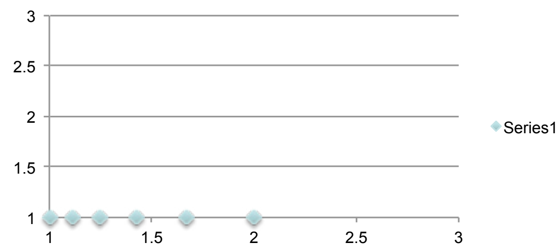
Figure SP2.9 Plot of the compactional strain in the Flinn diagram (linear).

Problem 2-10

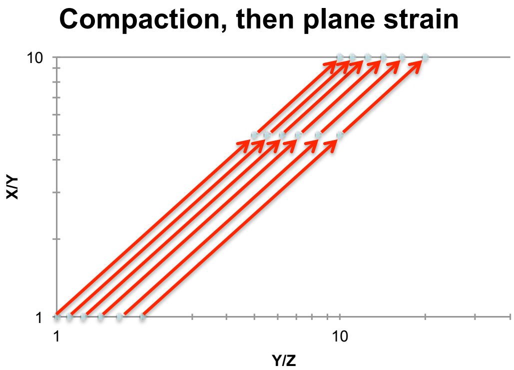
a) Only Z changes length, to $1+\Delta$. 10% compaction gives $Z=1+\Delta=1+(-0.1)=0.9$, $R_{XY}=1$, $R_{YZ}=1.111$.
Do the same for the other compactions.

Spread sheet screen dumps shows setup and Flinn linear plot for the 5 compactions (+ undeformed).

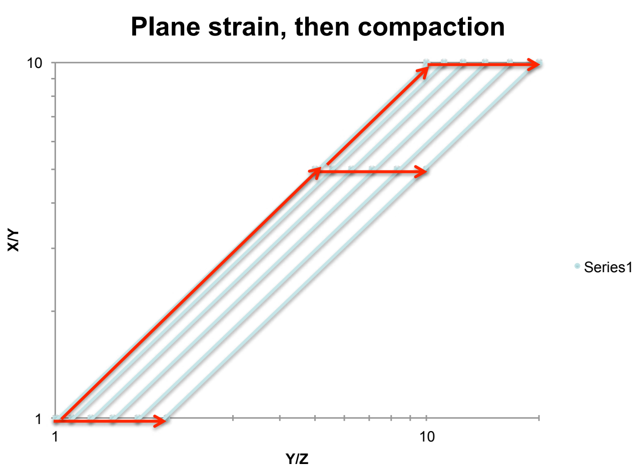
VOLUME CHANGE EXERCISE					
	X	Y	Z	Y/Z	X/Y
undeformed	1	1	1	1	1
10% vol char	1	1	0,9	1,11111111	1
20% vol char	1	1	0,8	1,25	1
	1	1	0,7	1,42857143	1
	1	1	0,6	1,66666667	1
50% vol char	1	1	0,5	2	1



b)



Red lines indicate paths



	X	Y	Z	Y/Z	X/Y	k
Initial	10	1	0,1	10	10	1
10% vol char	10	1	0,09	11,1111111	10	0,89010989
20% vol char	10	1	0,08	12,5	10	0,7826087
30%	10	1	0,07	14,2857143	10	0,67741935
40%	10	1	0,06	16,6666667	10	0,57446809
50%	10	1	0,05	20	10	0,47368421
Initial	5	1	0,2	5	5	1
10% vol char	5	1	0,18	5,55555556	5	0,87804878
20% vol char	5	1	0,16	6,25	5	0,76190476
30%	5	1	0,14	7,14285714	5	0,65116279
40%	5	1	0,12	8,33333333	5	0,54545455
50%	5	1	0,1	10	5	0,44444444
Initial	1	1	1	1	1	#DIV/0!
10% vol char	1	1	0,9	1,11111111	1	0
20% vol char	1	1	0,8	1,25	1	0
30%	1	1	0,7	1,42857143	1	0
40%	1	1	0,6	1,66666667	1	0
50%	1	1	0,5	2	1	0

c) The order does not matter, because these are coaxial deformations:

$$\begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1+\Delta \end{bmatrix}=\begin{bmatrix} k & 0 \\ 0 & \dfrac{1+\Delta}{k} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1+\Delta \end{bmatrix}\begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix}=\begin{bmatrix} k & 0 \\ 0 & \dfrac{1+\Delta}{k} \end{bmatrix}$$

The upper graph to the right shows one of the cases (50% compaction and pure shear case i)). The two paths (depending on the order) end up at the same finite strain.

d) The compaction contours are shown in Flinn diagrams to the right for linear and logarithmic axes.