CHAPTER 2

P. P 2.1

$$P=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} y^2(t)dt=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} 4\cos^2 \pi t dt=\lim_{T\to\infty}\frac{4}{2T}\int_{-T}^{T}\frac{1}{2}(1+\cos 2\pi t)dt$$

$$=\lim_{T\to\infty}\frac{4}{2T}\frac{2T}{2}=2$$

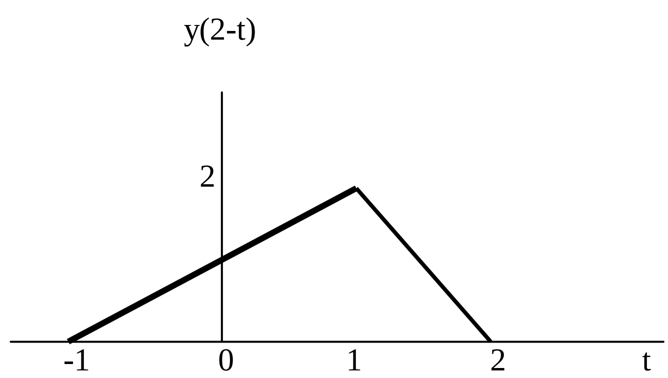
Since 0<P<∞, i.e. the signal has finite power, it is a power signal.

P.P. 2.2

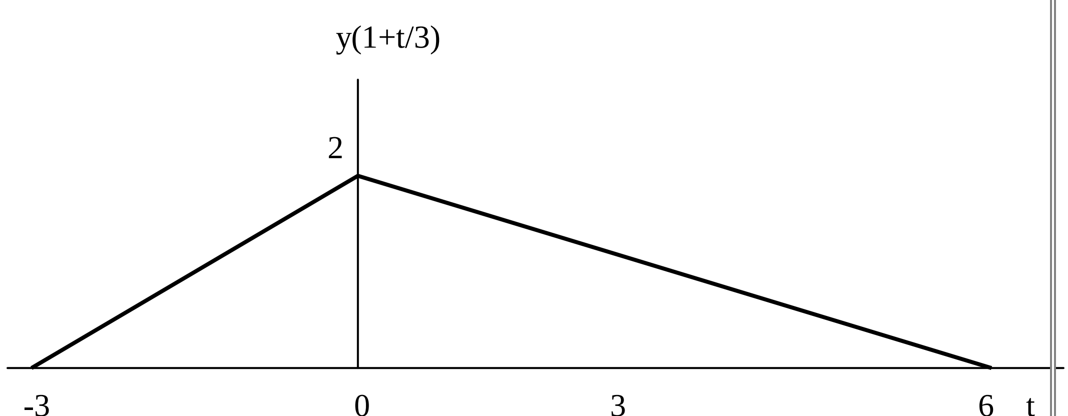
(a) $$t(2-t)=\begin{cases}2(2-t), & 0<2-t<1\\ 3-(2-t), & 1<2-t<3\end{cases}=\begin{cases}4-2t, & 1<t<2\\ 1+t, & -1<t<1\end{cases}=\begin{cases}1+t, & -1<t<1\\ 4-2t, & 1<t<2\end{cases}$$

(b) $$y(1+t/3)=\begin{cases}2(1+t/3), & 0\le 1+t/3<1\\ 3-(1+t/3), & 1\le 1+t/3\le 3\end{cases}=\begin{cases}2+2t/3, & -3\le t\le 0\\ 2-t/3, & 0<t<6\end{cases}$$

These are sketched below.



(a)



(b)

P.P. 2.3

Comparing the given signal with the square wave in Table 2.2, we notice that

$A = 1,\ T = 2, \omega_o = 2\pi / T = \pi$. Hence,

$$f(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)\pi t = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin \pi nt, \qquad n = 2k-1$$

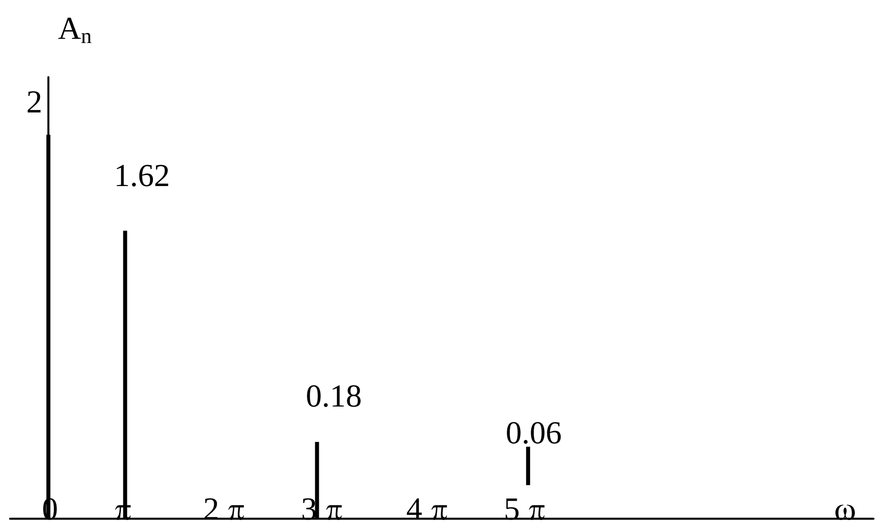
P.P. 2.4

We **c**omparing the given signal with the triangular wave in Table 2.2, we notice that

$A = 4, T = 2, \omega_o = 2\pi / T = \pi$. Hence,

$$h(t) = \frac{4}{2} - \frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(2n-1)\pi t}{(2n-1)^2} = 2 - \frac{16}{\pi^2}\sum_{k=1}^{\infty}\frac{\cos n\pi t}{n^2}, \qquad n = 2k-1$$

The phase spectrum is zero, while the amplitude spectrum is shown below.



P.P. 2.5

$T = 10, \omega_o = 2\pi / T = \pi / 5$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt = \frac{1}{10}\int_{-1}^{1}10e^{-jn\omega_o t}dt = \frac{1}{-jn\omega_o}e^{-jn\omega_o t}\Bigg|_{-1}^{1} = \frac{1}{-jn\omega_o}\left(e^{-jn\omega_o} - e^{jn\omega_o}\right)$$

$$= \frac{2}{n\omega_o}\left(\frac{e^{jn\omega_o} - e^{-jn\omega_o}}{2j}\right) = 2\frac{\sin n\omega_o}{n\omega_o} = 2\frac{\sin n\pi/5}{n\pi/5}$$

$$f(t) = 2\sum_{n=-\infty}^{\infty}\frac{\sin\lambda}{\lambda}e^{j\lambda t}, \qquad \lambda = \frac{n\pi}{5}$$

P.P. 2.6

$C_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \quad \omega_o = \frac{2\pi}{2} = \pi$

$$C_n = \frac{1}{2}\int_{-1}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

u = t and du = dt

dv = $e^{-jn\pi t}$dt which leads to v = $-[1/(2jn\pi)]e^{-jn\pi t}$

$$C_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Bigg|_{-1}^{1} - \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{2n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Bigg|_{-1}^{1}$$

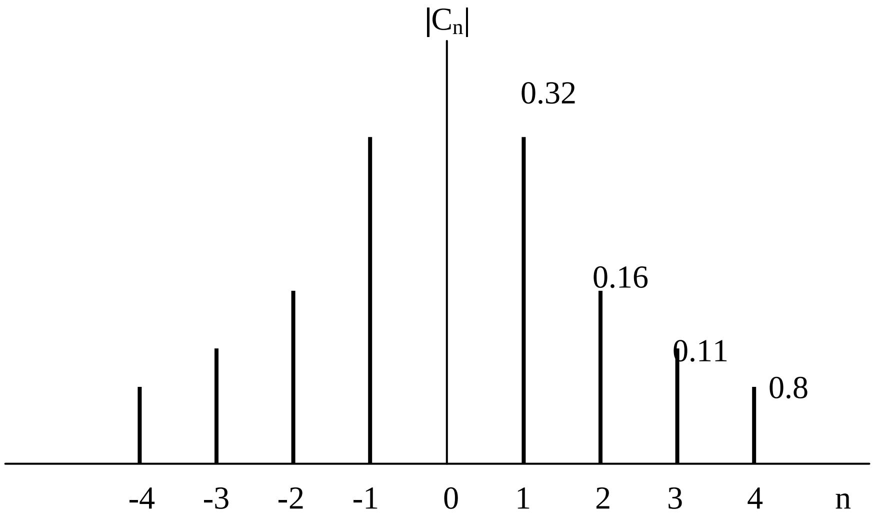
$$= [j/(n\pi)]\cos(n\pi) - [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$C_n = \frac{j(-1)^n}{n\pi} - \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

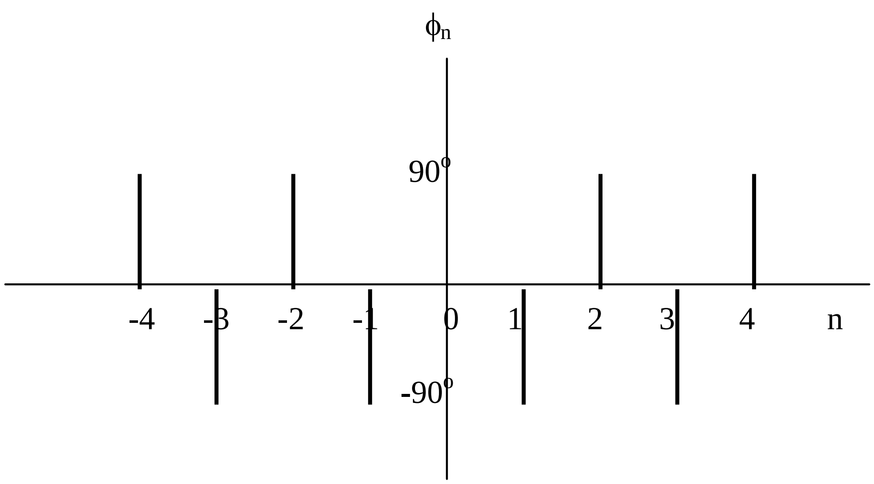
Thus

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_o t} = \sum_{n=-\infty}^{\infty} (-1)^n \frac{j}{n\pi} e^{jn\pi t}$$

The amplitude and phase spectra are shown next.



(a)



(b)

P.P. 2.7

$$P=\frac{1}{T}\int_0^T f^2(t)dt=\frac{1}{2\pi}\int_0^\pi (1)^2 dt=\frac{\pi}{2\pi}=0.5$$

$$C_o=\frac{1}{T}\int_0^T f(t)dt=\frac{1\times\pi}{2\pi}=0.5$$

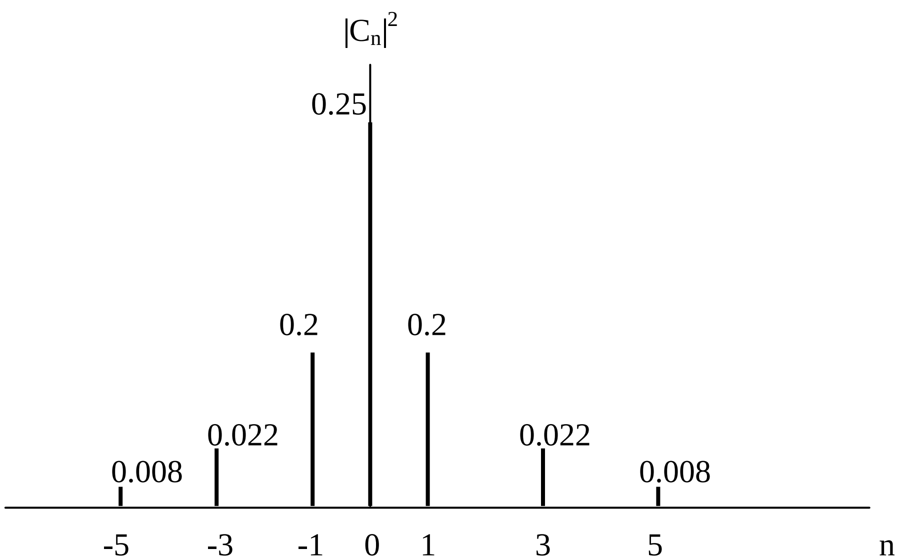
$$\omega_o=2\pi/T=1$$

$$C_n=\frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt=\frac{1}{2\pi}\int_0^\pi (1)e^{-jnt}dt=\frac{1}{2\pi(-jn)}(e^{-jn\pi}-1)$$

$$=\frac{j}{2n\pi}(\cos n\pi-j\sin n\pi-1)=\begin{cases}\dfrac{-j}{n\pi}, & n=1,3,5,7,\ldots\\ 0, & n=2,4,6,\ldots\end{cases}$$

$$|C_n|^2=\begin{cases}\dfrac{1}{n^2\pi^2}, & n=1,3,5,7,\ldots\\ 0, & n=2,4,6,\ldots\end{cases}$$

The power spectrum is shown below.



P.P. 2.8

(a) $X(\omega) = \int_0^1 (1)e^{-j\omega t} = -\frac{1}{j\omega}\left[e^{-j\omega} - 1\right] = \frac{1}{j\omega}\left[1 - e^{-j\omega}\right]$

(b) $\mathcal{F}[\delta(t - t_o) = e^{-j\omega t_o}$

Let $t_o = -3.$

$$X(\omega) = e^{j3\omega}$$

(c) $X(\omega) = 2\mathcal{F}\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right]$

$$= 2\pi\left[\delta(\omega + \omega_o) + \delta(\omega - \omega_o\right]$$

P.P. 2.9

$$X(\omega) = \int_{-\tau}^{0}\left(1 + \frac{t}{\tau}\right)e^{-j\omega t}dt + \int_0^{\tau}\left(1 - \frac{t}{\tau}\right)e^{-j\omega t}dt$$

$$= \int_0^{\tau}\left(1 - \frac{t}{\tau}\right)e^{+j\omega t}dt + \int_0^{\tau}\left(1 - \frac{t}{\tau}\right)e^{-j\omega t}dt$$

$$= 2\int_0^{\tau}\left(1 - \frac{t}{\tau}\right)\cos\omega t dt$$

$$= \frac{2\sin\omega t}{\omega} - \frac{2}{\tau\omega^2}(\cos\omega\tau + \omega\tau\sin\omega\tau - 1)$$

$$= \frac{2}{\tau\omega^2}(1 - \cos\omega\tau)$$

But $\cos 2u = 1 - 2\sin^2 u$

$$1 - \cos\omega\tau = 2\sin^2\omega\tau/2$$

$$X(\omega) = \frac{4}{\tau\omega^2}\sin^2\omega\tau/2 = \tau\frac{\sin^2\omega\tau/2}{(\omega\tau/2)^2}$$

$$= \tau\sin c^2\frac{\omega\tau}{2}$$

P.P. 2.10

$$f(t)=\begin{cases} e^{at}, & t<0 \\ 0, & t>0 \end{cases}$$

$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt=\int_{-\infty}^{0} e^{at}e^{-j\omega t}dt=\int_{-\infty}^{0} e^{(a-j\omega)t}dt$$

$$=\frac{1}{a-j\omega}e^{(a-j\omega)t}\Big|_{-\infty}^{0}=\frac{1}{a-j\omega}$$

The energy spectral density is

$$G(\omega)=|F(\omega)|^2=\frac{1}{a^2+\omega^2}, \qquad \omega=2\pi f \qquad (2.10.2)$$

Thus, the energy contained in the frequency range of $-f_o < f < f_o$ is

$$E=\int_{-f_o}^{f_o}\frac{df}{a^2+(2\pi f)^2} \qquad (2.10.3)$$

The integrand is even so that we can integrate from 0 to f_o and multiply by 2.

If we let $u = 2\pi f/a$ so that $df = adu/2\pi$, eq. (2.10.3) becomes

$$E=\frac{2a}{2\pi a^2}\int_{0}^{2\pi f_o/a}\frac{du}{1+u^2}=\frac{1}{\pi a}\tan^{-1}\frac{2\pi f_o}{a}$$

P.P. 2.11

(a) $X(\omega)=1\left(e^{-j\omega 2}+e^{-j\omega}+e^{j\omega}+e^{j\omega 2}\right)=2\left(\cos\omega+\cos 2\omega\right)$

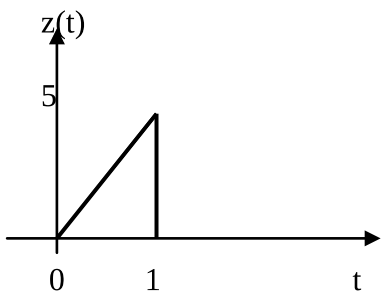
(b) $y(t)=2te^{-t}u(t)$

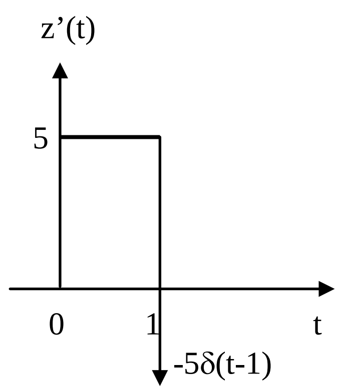
Let $g(t)=2e^{-t}u(t) \longrightarrow G(\omega)=\dfrac{2}{1+j\omega}$

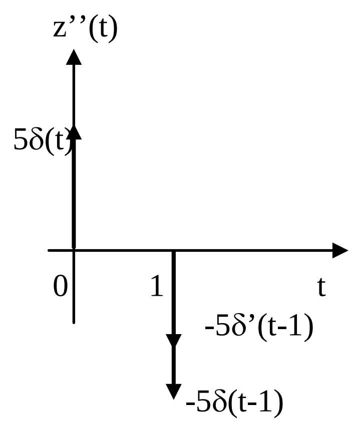
$y(t)=tg(t) \longrightarrow j\dfrac{dG(\omega)}{d\omega}=j2(-1)j(1+j\omega)^{-2}$

$$Y(\omega)=\frac{2}{(1+j\omega)^2}$$

(c) z(t) and its derivatives are shown below.







$$z''(t) = 5\delta(t) - 5\delta(t-1) - 5\delta'(t-1)$$
$$(j\omega)^2 Z(\omega) = 5 - 5e^{-j\omega} - 5j\omega e^{-j\omega}$$
$$Z(\omega) = \frac{-5}{\omega^2}(1 - e^{-j\omega} - j\omega e^{-j\omega})$$

P.P. 2.12
Taking the first derivative gives y’(t) in Fig. A. Taking the second derivative gives y’’(t) as in Fig. B.

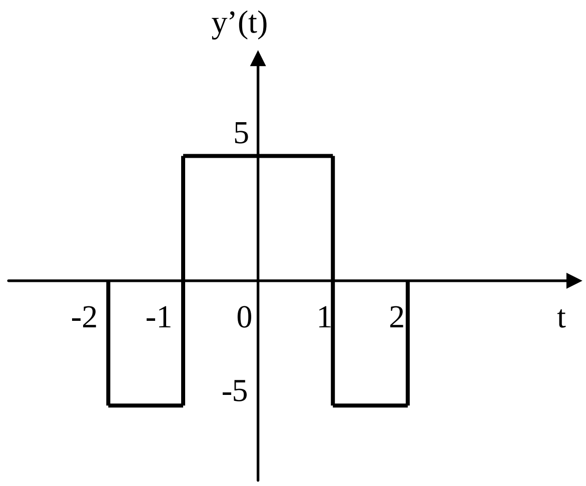


Fig. A

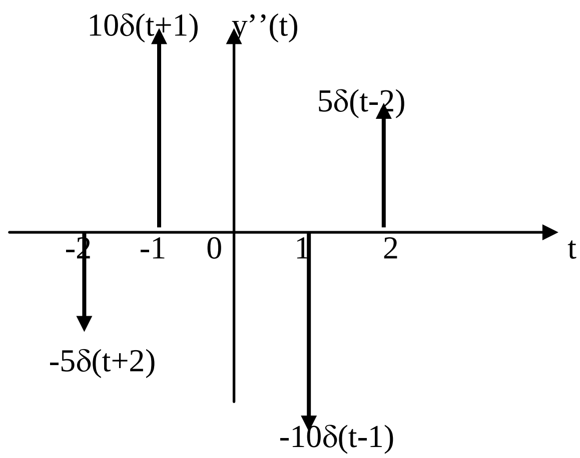


Fig. B

From Fig. B,

$$y''(t) = -5\delta(t+2) + 10\delta(t+1) - 10\delta(t-1) + 5\delta(t-2)$$

Taking the Fourier transform of each term,

$$(j\omega)^2 Y(\omega) = -5e^{j2\omega} + 10e^{j\omega} - 10e^{-j\omega} + 5e^{-j2\omega}$$

$$= -5(e^{j2\omega} - e^{-j2\omega}) + 10(e^{j\omega} - e^{-j\omega})$$

$$-\omega^2 Y(\omega) = 20j\sin\omega - 10j\sin 2\omega$$

or

$$Y(\omega) = \frac{j10}{\omega^2}(\sin 2\omega - \sin\omega)$$

P.P. 2.13

(a) $f(t) = e^{-(t-2)}u(t-2)$

(b) $g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi\delta(\omega)\, e^{j\omega t}}{(5+j\omega)(2+j\omega)} d\omega$

$$= \frac{1}{2}\ \frac{1}{(5)(2)} = \frac{1}{20} = 0.05$$

P.P. 2.14

$-20 = 20\log|H| \quad \longrightarrow \quad H = 10^{-20/20} = 0.1$

$$1 + \left(\frac{\omega}{\omega_c}\right)^2 = \frac{1}{0.1^2} = 100$$

Let $\quad X = \frac{\omega}{\omega_c} = \frac{2\pi f}{2\pi f_c} = \frac{f}{f_c}$

$X^n = 99 \quad \longrightarrow \quad X = 99^{1/n} = 99^{1/3} = 4.6261$

$$f_c = \frac{f}{X} = \frac{50}{4.6261} = 10.81 \text{ MHz}$$

$$H(s) = \frac{\omega_c^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3}$$

where $\omega_{\mathrm{c}} = 2\pi f_c = 2\pi \times 10.81 \times 10^6$ rad/s=6.791×10^7 rad/s

P.P. 2.15

$$H(\omega) = \frac{V_o}{V_i} = \frac{R \parallel \frac{1}{j\omega C}}{j\omega L + R \parallel \frac{1}{j\omega C}} = \frac{\dfrac{\frac{R}{j\omega C}}{R + \frac{1}{j\omega C}}}{j\omega L + R \parallel \frac{1}{j\omega C}} = \frac{\frac{R}{1+j\omega RC}}{j\omega L + \frac{R}{1+j\omega RC}}$$

$$= \frac{R}{R(1-\omega^2 LC) + j\omega L}$$

Since R = 1,

$$H(s) = \frac{1}{s^2 LC + sL + 1} \qquad (s = j\omega)$$

$$H(s) = \frac{1/LC}{s^2 + s/C + 1/LC}$$

We compare this with

$$H(s) = \frac{k}{s^2 + 1.4142\omega_c s + \omega_c^2}$$

$$H(0) = \frac{k}{\omega_c^2} = 1$$

$$\omega_c^2 = \frac{1}{LC}$$

$$\frac{1}{C} = 1.4142 \quad \longrightarrow \quad C = 0.7071 = 707.1 \text{ mF}$$

If $\omega_c = 1, \quad L = 1/C = 1.4142$ H

Prob.2.1

(a) An analog signal is a continuous-time signal in which the variation with time is analogous to some physical phenomenon.
(b) A digital signal is a discrete-time signal that can have a finite number of values (usually binary).
(c) A continuous-time signal takes a value at every instant of time.
(d) A discrete-time signal is defined only at particular instants of time.

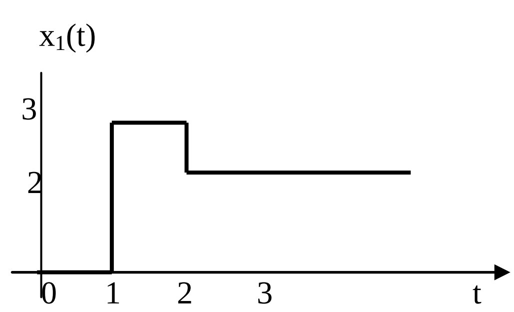
Prob. 2.2

(a) A periodic signal is one that repeats itself every T seconds.
(b) An aperiodic signal is a nonperiodic signal, i.e. it does not repeat itself.
(c) A signal is said to be an energy signal when the total energy E of the signal satisfies the condition $0 < E < \infty$

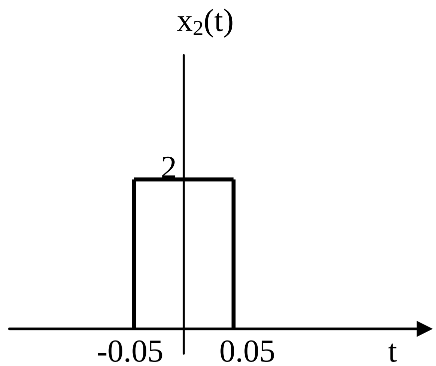
(d)A signal is called a power signal when its average power P satisfies the condition $0 < P < \infty$

Prob. 2.3

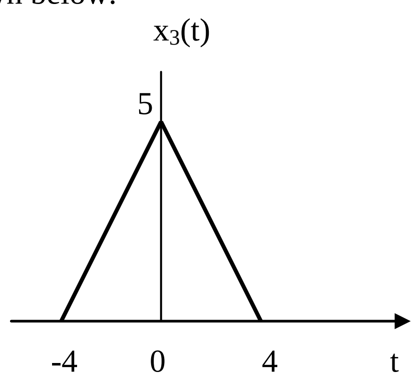
(a) $x_1(t)$ is sketched below.



(b) $x_2(t) = 2\Pi(10t) = 2\Pi(t/0.1)$. It is sketched below.



(c) $x_3(t)$ is shown below.

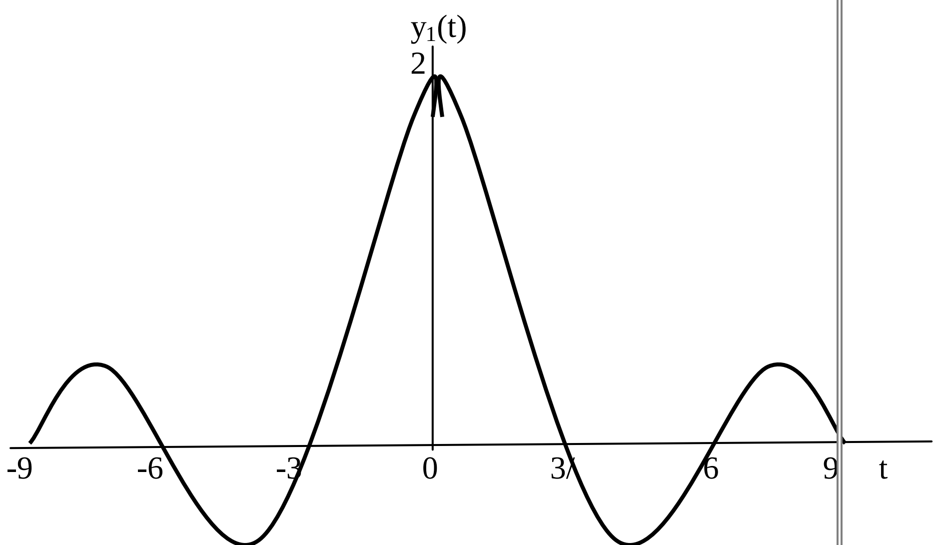


Prob. 2.4

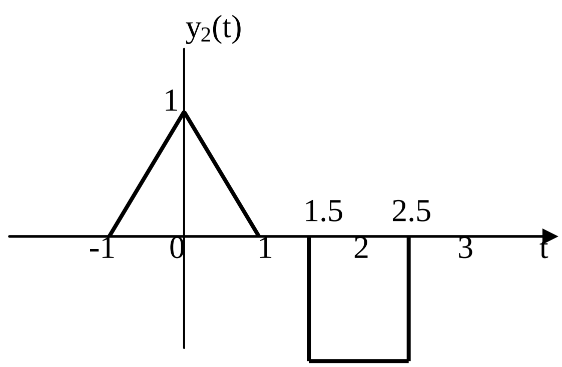
(a) Since sinc (x)= sin(x)/x,

$\sin(\pi t_o / 3) = \sin(n\pi) \quad \longrightarrow \quad t_o = 3n$

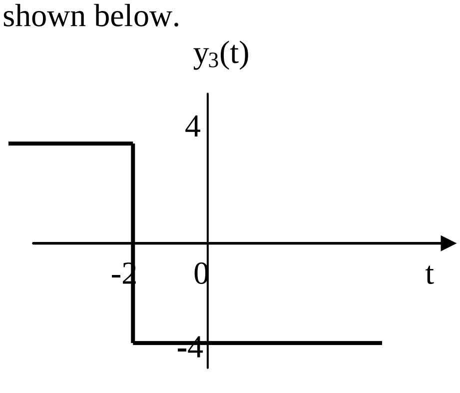
$y_1(t)$ is sketched below.



(b) $y_2(t)$ is sketched below.

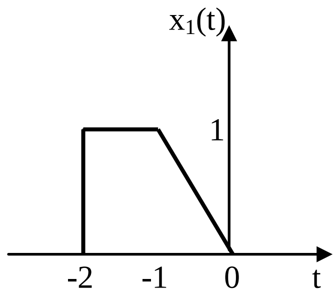


(c) $y_3(t)$ is shown below.

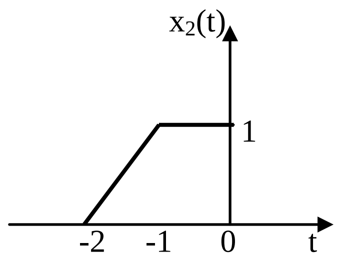


Prob. 2.5

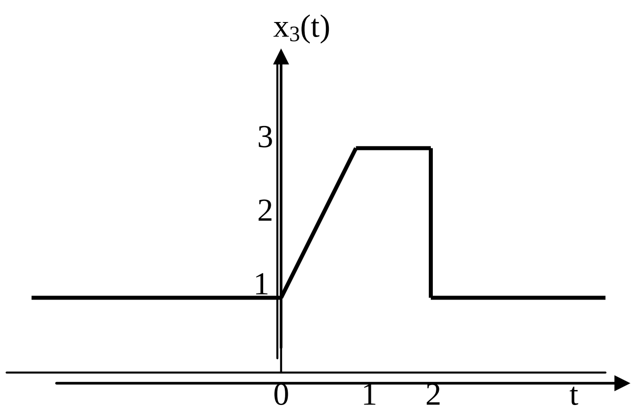
(a) $x_1(t) = x(-t)$ is sketched below.



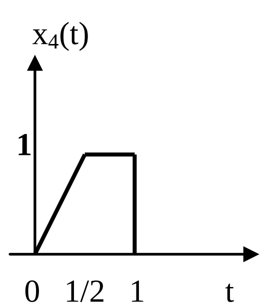
(b) $x_2(t) = x(t+2)$ is sketched below.



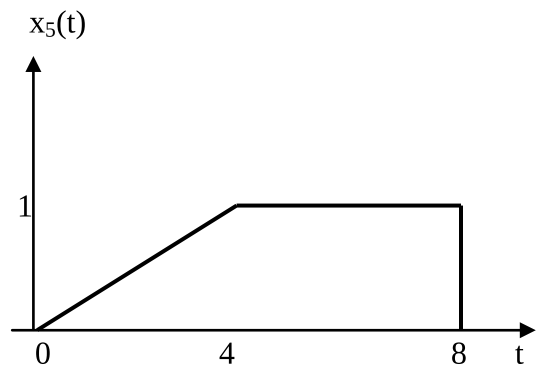
(c) $x_3(t) = 1 + 2x(t)$ is shown below.



(d) $x_4(t) = x(2t)$ is shown below.



(e) $x_5(t) = x(t/4)$ is sketched below.



Prob. 2.6

(a) $$E = \int_{-\infty}^{\infty} (x(t))^2 dt = \int_{-\infty}^{\infty} e^{-2|t|} d = 2\int_{0}^{\infty} e^{-2t} dt = \frac{2}{-2} e^{-2t} \Big|_0^{\infty} = 1 < \infty$$

i.e. $x(t)$ is an energy signal.

(b) $$E = \lim_{T\to\infty} \int_0^T (y(t))^2 dt = \lim_{T\to\infty} \int_0^T t^2 dt = \lim_{T\to\infty} \frac{T^3}{3} = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_0^T (y(t))^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_0^T t^2 dt = \lim_{T\to\infty} \frac{1}{T} \frac{T^3}{3} = \infty$$

i.e. $y(t)$ is neither energy nor power signal.

(c) $$E = \int_{-\tau}^{\tau} (z(t))^2 dt = 2\int_0^{\tau} \left(1 - \frac{t}{\tau}\right)^2 dt = 2\left(t - \frac{t^2}{\tau} + \frac{t^3}{3\tau^2}\right)\Big|_0^{\tau} = 2\left(\tau - \tau + \frac{1}{3}\tau\right) = \frac{2}{3}\tau$$

Hence, $z(t)$ is an energy signal.

Prob. 2.7

It is not possible to generate power signal in a lab because such a signal would have infinite duration and infinite energy. Signals generated in the lab have finite energy and are energy signals.

Prob. 2.8

$$P = \frac{1}{T}\int_0^T f^2(t)dt$$

$$P_g = \frac{1}{T}\int_0^T g^2(t)dt = \frac{1}{T}\int_0^T a^2 f^2(bt+c)dt$$

Let $bt + c = \lambda, \quad dt = d\lambda / b$

$$P_g = \frac{a^2}{b}\frac{1}{T}\int_c^{bT+c} f^2(\lambda)d\lambda = \frac{a^2}{b}P$$

Prob. 2.9

(a) A system is linear when its output is linearly related to its input.
(b) A nonlinear system is one in which the output is not linearly related to its input.
(c) A continuous-time system has input and output signals that are continuous-time.
(d) A discrete-time system has input and output signals that are discrete-time.

Prob. 2.10

(a) A time-varying system is one in which the input-output relationship varies with time.
(b) A time-invariant system is one in which the input-output relationship does not vary with time.
(c) A casual system is one whose output signal (response) does not start before the input signal (excitation) is applied.
(d) A noncausal system is one in which the response depends on the future values of the input.
(e) An analog system is one whose input signal is analog.
(f) A digital system is one whose input is in the form of a sequence of digits.

Prob. 2.11

(a) Linear
(b) Nonlinear
(c) Linear

Prob. 2.12

(a) Linear
(b) Nonlinear
(c) Nonlinear

Prob. 2.13

$$T = 4, \omega_o = 2\pi / T = \pi / 2$$

$$f(t) = \begin{cases} 5, & 0 < t < 1 \\ 10, & 1 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\left[\int_0^1 5dt + \int_1^2 10dt\right] = 3.75$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{4}\left[\int_0^1 5\cos\frac{n\pi t}{2}dt + \int_1^2 10\cos\frac{n\pi t}{2}dt\right] = \frac{-5}{n\pi}\sin\frac{n\pi}{2}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{4}\left[\int_0^1 5\sin\frac{n\pi t}{2}dt + \int_1^2 10\sin\frac{n\pi t}{2}dt\right] = \frac{5}{n\pi}\left(1 - 2\cos n\pi + \cos\frac{n\pi}{2}\right)$$

Prob. 2.14

$T = 4, \omega_0 = 2\pi / T = \pi / 2, \quad b_n = 0$ since $f(t)$ is an even function.

$$f(t) = \begin{cases} 0, & -2 < t < -1 \\ \cos\frac{\pi}{2}t, & -1 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{4}\int_0^1 \cos\frac{\pi}{2}t dt = \frac{1}{2}\frac{2}{\pi}\sin\frac{\pi}{2}t\Big|_0^1 = \frac{1}{\pi}$$

$$a_n = \frac{4}{T}\int_0^T f(t)\cos n\omega_0 t dt = \int_0^1 \cos\frac{\pi}{2}t\cos\frac{n\pi t}{2}dt$$

We apply the trig identity: cosAcosB = ½[cos(A+B) + cos(A-B)]

$$a_n = \frac{1}{2}\int_0^1\left[\cos\frac{\pi}{2}(n+1)t + \cos\frac{\pi}{2}(n-1)t\right]dt$$

For n = 1.

$$a_1 = \frac{1}{2}\int_0^1[\cos\pi t + 1]dt = \frac{1}{2}\left[\frac{\sin\pi t}{\pi} + t\right]\Big|_0^1 = \frac{1}{2}$$

For n > 1,

$$a_n = \frac{1}{\pi(n+1)}\sin\frac{\pi}{2}(n+1) + \frac{1}{\pi(n-1)}\sin\frac{\pi}{2}(n-1)$$

When n = odd, (n+1) and (n-1) are even so that $a_n = 0$.

When n = even, (n+1) and (n-1) are odd so that

$$\sin\frac{\pi}{2}(n+1) = -\sin\frac{\pi}{2}(n-1) = \cos\frac{\pi}{2}n = (-1)^{n/2}$$

$$a_n = \frac{(-1)^{n/2}}{\pi(n+1)} + \frac{-(-1)^{n/2}}{\pi(n-1)} = \frac{-2(-1)^{n/2}}{\pi(n^2-1)}$$

Thus,

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\cos\frac{\pi}{2}t - \frac{2}{\pi}\sum_{n=even}^{\infty}\frac{(-1)^{n/2}}{(n^2-1)}\cos\frac{n\pi}{2}t$$

Prob. 2.15

$$T = 4, \omega_0 = 2\pi/T = \pi/2, \qquad a_0 = 0, \qquad f(t) = 2(t-1)$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{4}\int_0^2 2(t-1)\cos\frac{n\pi}{2}tdt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at - \frac{1}{a}\sin at\bigg|_0^2, \qquad a = \frac{n\pi}{2}$$

$$a_n = \frac{1}{a^2}(\cos n\pi - 1) + \frac{2}{a}\sin n\pi - 0 - \frac{1}{a}\sin n\pi - 0 = \frac{4}{n^2\pi^2}(\cos n\pi - 1)$$

$$b_n = \int_0^2 (t-1)\sin atdt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at + \frac{1}{a}\cos at\bigg|_0^2$$

$$= 0 - \frac{1}{a}(2\cos n\pi - 1) + \frac{1}{a}(\cos n\pi - 1) = \frac{-2}{n\pi}(2 + \cos n\pi)$$

$$f(t) = \sum_{n=1}^{\infty}\left[\frac{4}{n^2\pi^2}(\cos n\pi - 1)\cos\frac{n\pi}{2}t - \frac{2}{n\pi}(2+\cos n\pi)\sin\frac{n\pi}{2}t\right]$$

Prob. 2.16

b_n = 0 since this is an even function

T = 6, $\omega = 2\pi/6 = \pi/3$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{6}\left[\int_1^2 (4t-4)dt + \int_2^3 4 \ dt\right]$$

$$= \frac{1}{3}\left[(2t^2 - 4t)\Big|_1^2 + 4(3-2)\right] = 2$$

$$a_n = \frac{4}{T}\int_0^{T/4} f(t)\cos(n\pi t/3)dt$$

$$= (4/6)\left[\int_1^2 (4t-4)\cos(n\pi t/3)dt + \int_2^3 4\cos(n\pi t/3)dt\right]$$

$$= \frac{16}{6}\left[\frac{9}{n^2\pi^2}\cos\left(\frac{n\pi t}{3}\right) + \frac{3t}{n\pi}\sin\left(\frac{n\pi t}{3}\right) - \frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_1^2 + \frac{16}{6}\left[\frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_2^3$$

$$= [24/(n^2\pi^2)][\cos(2n\pi/3) - \cos(n\pi/3)]$$

Thus $\quad f(t) = 2 + \frac{24}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\cos\left(\frac{2\pi}{n}\right) - \cos\left(\frac{}{}\right)\right]\cos\left(\frac{}{}\right)$

At t = 2,

$$f(2) = 2 + (24/\pi^2)[(\cos(2\pi/3) - \cos(\pi/3))\cos(2\pi/3)$$

$$+ (1/4)(\cos(4\pi/3) - \cos(2\pi/3))\cos(4\pi/3)$$

$$+ (1/9)(\cos(2\pi) - \cos(\pi))\cos(2\pi) + \text{-----}]$$

$$= 2 + 2.432(0.5 + 0 + 0.2222 + \text{-----})$$

f(2) = 3.756

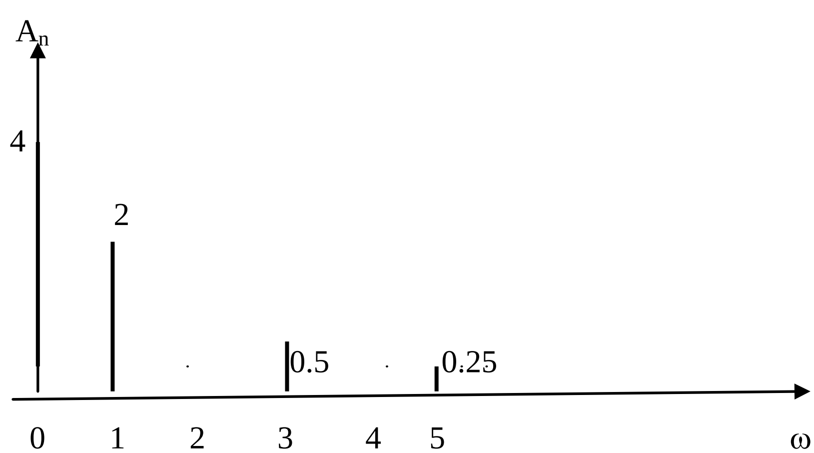
Prob. 2.17

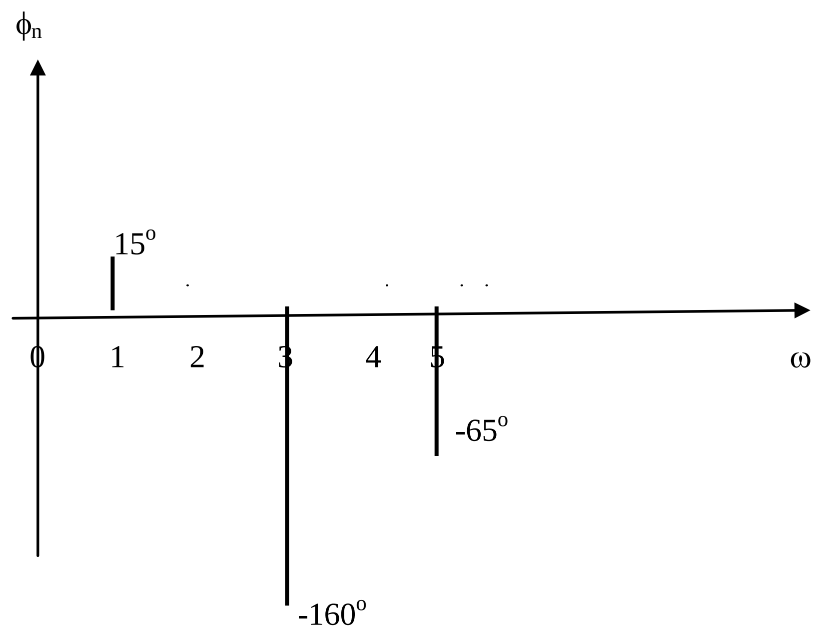
Express each term in the standard form as $a_n\cos(\omega_n t + \theta_n)$.

$$\frac{1}{4}\sin(5t + 25^o) = \frac{1}{4}\cos(5t + 25^o - 90^o) = \frac{1}{4}\cos(5t - 65^o)$$

$$-\frac{1}{2}\cos(3t + 20^o) = \frac{1}{2}\cos(3t - 160^o)$$

Thus, the magnitude and phase spectra are shown below.





Prob. 2.18

$a_n = 20/(n^2\pi^2)$, $b_n = -3/(n\pi)$, $\omega_n = 2n$

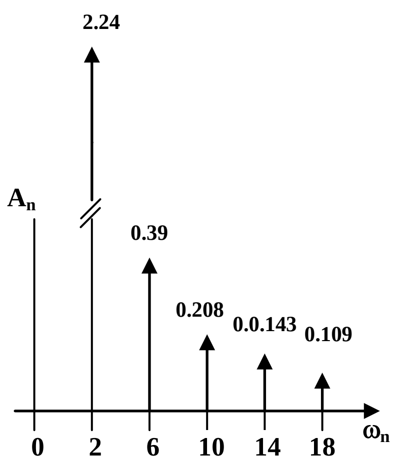
$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\frac{400}{n^4\pi^4} + \frac{9}{n^2\pi^2}}$$

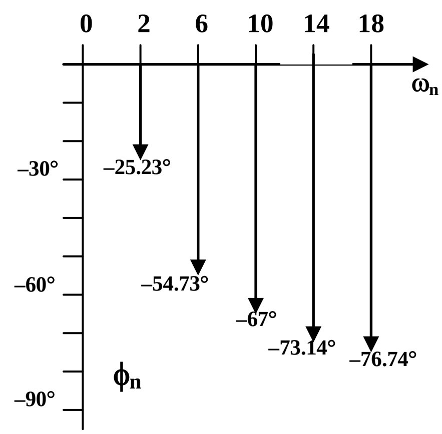
$$= \frac{3}{n\pi}\sqrt{1 + \frac{44.44}{n^2\pi^2}}, \quad n = 1, 3, 5, 7, 9, \text{ etc.}$$

n	A_n
1	2.24
3	0.39
5	0.208
7	0.143
9	0.109

$$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}\{[-3/(n\pi)][n^2\pi^2/20]\} = \tan^{-1}(-nx0.4712)$$

n	ϕ_n
1	–25.23°
3	–54.73°
5	–67°
7	–73.14°
9	–76.74°
∞	–90°





Prob. 2.19

$f(t) = 40\cos 5\pi t - 20\sin(2\pi t + \pi/6)\cos 5\pi t$

But $\quad 2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$f(t) = 40\cos 5\pi t - 10\sin(7\pi t + \pi/6) - 10\sin(-3\pi t + \pi/6)$

Also, $\quad \sin\alpha = \cos(\alpha - 90^o)$

$f(t) = 40\cos 5\pi t - 10\cos(7\pi t - 60^o) + 10\cos(3\pi t - 120^o)$

Thus,

$a_1 = 40, a_2 = -10, a_3 = 10, \omega_1 = 5\pi, \omega_2 = 7\pi, \omega_3 = 3\pi, \theta_1 = 0, \theta_2 = -60^2, \theta_3 = -120^o$

Prob. 2.20

This is an even function.

$b_n = 0, T = 4, \omega_o = 2\pi / T = \pi / 2$

$$a_0 = \frac{\frac{1}{2}(2+4)4}{4} = 3$$

$$f(t) = \begin{cases} 4t, & 0 < t < 1 \\ 4, & 1 < t < 3 \\ 8-4t, & 3 < t < 4 \end{cases}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o t dt, \qquad \alpha = n\omega_0 = n\pi / 2$$

$$a_n = \frac{4}{4}\left[\int_0^1 4t\cos\alpha t dt + \int_1^2 4\cos\alpha t dt\right] = 4\left[\frac{1}{\alpha^2}\cos\alpha t + \frac{t}{\alpha}\sin\alpha t\right]\Bigg|_0^1 - \frac{4}{\alpha}\cos\alpha t\Bigg|_1^2$$

$$= 4\left[\frac{1}{\alpha^2}(\cos\alpha - 1) + \frac{1}{\alpha}\sin\alpha - 0\right] - \frac{4}{\alpha}(\cos 2\alpha - \cos\alpha)$$

$$= \frac{16}{n^2\pi^2}(\cos\frac{n\pi}{2} - 1) + \frac{8}{n\pi}(\sin\frac{n\pi}{2} + \cos\frac{n\pi}{2} - \cos n\pi)$$

$$f(t) = 3 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{2}t$$

Prob. 2.21

(a) Since f(t) is an odd function, $a_0 = 0 = a_n$. $\quad T = \pi, \omega_o = 2\pi / T = 2$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_o t dt = \frac{4}{\pi}\int_0^{\pi/2} t\sin 2nt dt = \frac{4}{\pi}\left[\frac{1}{4n^2}\sin 2nt - \frac{t}{n\pi}\cos 2nt\right]\Bigg|_0^{\pi/2}$$

$$= \frac{4}{\pi}\left[-\frac{\pi}{2n\pi}\cos n\pi\right] = \frac{2}{n\pi}(-1)^{n+1}$$

$$f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin 2nt$$

(b) The total average power is

$$P_T = \frac{1}{T}\int_0^T f^2(t)dt = \frac{2}{\pi}\int_0^2 \frac{4t^2}{\pi^2}dt = \frac{8}{\pi^3}\frac{t^3}{3}\Bigg|_0^{\pi/2} = \frac{1}{3}$$

$$P_4 = a_0^2 + \frac{1}{2}\sum A_n^2 = \frac{1}{2}\sum b_n^2 = \frac{1}{2}\left(\frac{4}{\pi^2} + \frac{4}{4\pi^2} + \frac{4}{9\pi^2} + \frac{4}{25\pi^2}\right) = \frac{2}{\pi^2}(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{25})$$

$$= 0.2026(1.4011) = 0.2839$$

Prob. 2.22

$T = 2\pi, \omega_0 = 2\pi / T = 1$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2 e^{-jnt} dt$$

We integrate by parts twice and obtain

$$\mathrm{C}_n = \frac{2}{n^2}\cos n\pi = \frac{2}{n^2}(-1)^n, \qquad n \neq 0$$

For $n = 0$,

$$C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2 dt = \frac{\pi^2}{3}$$

Hence,

$$f(t) = \frac{\pi^2}{3} + \sum_{n=-\infty, n\neq 0}^{\infty} \frac{2(-1)^n}{n^2} e^{jnt}$$

Prob. 2.23

$$C_n = \frac{1}{T}\int_0^T f(t) e^{-j\omega_0 nt} dt = \frac{1}{T}\int_{-\tau}^{0} (1) e^{-j\omega_0 nt} dt + \frac{1}{T}\int_0^{\tau} (-1) e^{-j\omega_0 nt} dt$$

$$= \frac{1}{T}\frac{e^{-j\omega_0 nt}}{-j\omega_0 n}\bigg|_{-\tau}^{0} - \frac{1}{T}\frac{e^{-j\omega_0 nt}}{-j\omega_0 n}\bigg|_{0}^{\tau} = \frac{1}{-j\omega_0 nT}(1 - e^{-j\omega_0 n\tau}) + \frac{1}{j\omega_0 nT}(e^{-j\omega_0 n\tau} - 1) = \frac{2}{j\omega_0 nT}(e^{-j\omega_0 n\tau} - 1)$$

But $\omega_o = \dfrac{2\pi}{T}$,

$$C_n = \frac{1}{jn\pi}(e^{-j2n\pi\tau/T} - 1), n \neq 0$$

For $n = 0$,

$$C_0 = \frac{1}{T}\int_{-\tau}^{0} e^{-j\omega_0 t} dt + \frac{1}{T}\int_0^{\tau} (-1) e^{-j\omega_0 t} dt = \frac{1}{T}\frac{e^{-j\omega_0 t}}{-j\omega_0}\bigg|_{-\tau}^{0} - \frac{1}{T}\frac{e^{-j\omega_0 t}}{-j\omega_0}\bigg|_{0}^{\tau}$$

But $\omega_0 = \dfrac{2\pi}{T} \longrightarrow \omega_o T = 2\pi$

$$C_0 = \frac{-1}{2\pi}(1 - e^{j\omega_0 \tau}) + \frac{1}{2\pi}(e^{-j\omega_0 \tau} - 1) = \frac{1}{\pi}(\cos\omega_0 \tau - 1)$$

$$f(t) = \frac{1}{\pi}(\cos 2\pi\tau / T - 1) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{1}{jn\pi}(e^{-j2n\pi\tau/T} - 1) e^{jn\pi t/T}$$

Prob. 2.24

$$T = 2, \omega_0 = 2\pi / T = \pi$$

$$C_n = \frac{1}{2}\int_{-1}^{0} 4(1+t)e^{-jn\pi t}dt = 2\left[\frac{e^{-jn\pi t}}{-jn\pi} + \frac{e^{-jn\pi t}}{-n^2\pi^2}(-jn\pi t - 1)\right]\Bigg|_{-1}^{0}$$

$$= 2\left[\frac{(1-e^{jn\pi})}{-jn\pi} - \frac{1}{n^2\pi^2} + \frac{e^{jn\pi}}{n^2\pi^2}(jn\pi - 1)\right] = 2\left[\frac{(1-\cos n\pi - j\sin n\pi)}{-jn\pi} - \frac{1}{n^2\pi^2} + \frac{(\cos n\pi + \sin n\pi)}{n^2\pi^2}(jn\pi -$$

$$= 2\left[\frac{j}{n\pi}(1-\cos n\pi) - \frac{1}{n^2\pi^2} + \frac{(jn\pi - 1)}{n^2\pi^2}\cos n\pi\right]$$

Prob. 2.25

(a) $g(t) = f(t-2) = \sum C_n e^{jn\omega_o(t-2)} = \sum C_n e^{-j2n\omega_0} e^{jn\omega_o t}$

$C_n' = C_n e^{-j2n\omega_0}$

(b) $h(t) = 2\frac{df}{dt} = \sum 2C_n(jn\omega_0)e^{jn\omega_o t}$

$C_n' = 2jn\omega_0 C_n$

(c) $y(t) = \frac{d^2 f}{dt^2} - \frac{df}{dt} = \sum C_n(j^2 n^2\omega_0^2 - jn\omega_0)e^{jn\omega_o t}$

$C_n' = -C_n(n^2\omega_0^2 + jn\omega_0)$

Prob. 2.26

(a) $X(\omega) = \frac{\pi}{(1+j\omega)^2 + \pi^2}$

(b) $Y(\omega) = \frac{1}{3}\left[e^{-j\omega/3} + e^{j\omega/3}\right]$

(c) sgn(t) $\Leftrightarrow$ $\frac{2}{j\omega}$

$$Z(\omega) = \frac{2}{j(\omega+1)}$$

Prob. 2.27

(a) $$X(\omega)=\int_{-1}^{1}4e^{-j\omega t}dt=\frac{4}{-j\omega}e^{-j\omega t}\Big|_{-1}^{1}=\frac{4}{-j\omega}(e^{-j\omega}-e^{j\omega})$$

$$=\frac{8}{\omega}\frac{(e^{j\omega}-e^{-j\omega})}{j2}=\frac{8}{\omega}\sin\omega=8\sin c\omega$$

Or X(ω)=$\mathcal{F}\left[4\Pi(t/\tau)\right]=4\tau\sin c(\omega\tau/2)=8\sin c\omega,\qquad(\tau=2)$

(b) $$Y(\omega)=\int_{0}^{1}2e^{-j\omega t}dt+\int_{1}^{2}(1)e^{-j\omega t}dt=\frac{2}{-j\omega}e^{-j\omega t}\Big|_{0}^{1}+\frac{1}{-j\omega}e^{-j\omega t}\Big|_{1}^{2}$$

$$=\frac{1}{j\omega}(2-e^{-j\omega}-e^{-j2\omega})$$

Or $y'(t)=2\delta(t)-\delta(t-1)-\delta(t-2)$

$j\omega Y(\omega)=(2-e^{-j\omega}-e^{-j2\omega})$

$$Y(\omega)=\frac{1}{j\omega}(2-e^{-j\omega}-e^{-j2\omega})$$

(c) $z''(t)=\delta\left(t+1\right)-2\delta(t)+\delta(t-1)$

$$-\omega^2 Z(\omega)=e^{j\omega}-2+e^{-j\omega}=2\cos\omega-2$$

$$Z(\omega)=\frac{-2}{\omega^2}(\cos\omega-1)$$

But cos2A=1-2sin$^2 A$

$$Z(\omega)=\frac{4\sin^2\omega/2}{\omega}=\sin c^2(\omega/2)$$

Or z(t)=Δ(t) $\longrightarrow$ $Z(\omega)=\sin c^2(\omega/2)$

Prob. 2.28

(a) Taking the first and second derivatives gives

$f''(t)=\delta(t+2)-4\delta(t-1)+3\delta(t-2)$

Taking the Fourier transform of each term:

-$\omega^2 F(\omega)=e^{j2\omega}-4e^{-j\omega}+3e^{-j2\omega}$

$$F(\omega)=\frac{1}{\omega^2}\left(4e^{-j\omega}-3e^{-j2\omega}-e^{j2\omega}\right)$$

(b) $g(t)=2\Delta(t-1)-2\Pi[(t-3)/2]$

$G(\omega)=2e^{-j\omega}\sin c^2(\omega/2)-2e^{-j3}2\sin c\omega$

$$G(\omega)=2e^{-j\omega}\sin c^2(\frac{\omega}{2})-4e^{-j3}\sin c\omega$$

Prob. 2.29

Taking second derivative of the signal gives

$$h''(t) = A\delta'(t) - \frac{A}{\tau}\delta(t) + \frac{A}{\tau}\delta(t-\tau)$$

$$-\omega^2 H(\omega) = Aj\omega - \frac{A}{\tau}(1) + \frac{A}{\tau}e^{-j\omega\tau}$$

$$H(\omega) = \frac{A}{\omega^2\tau}\left[1 - j\omega\tau - e^{-j\omega\tau}\right]$$

Prob. 2.30

(a) Let $h(t) = 1 + m\cos\alpha t$

$$H(\omega) = 2\pi\delta(\omega) + m\pi\left[\delta(\omega+\alpha) + \delta(\omega-\alpha)\right]$$

$$F(\omega) = \frac{1}{2}\left[H(\omega+\beta) + H(\omega-\beta)\right]$$

$$= \pi\left[\delta(\omega+\beta) + \delta(\omega-\beta)\right] + \frac{m\pi}{2}[\delta(\omega+\alpha+\beta) + \delta(\omega+\alpha-\beta)$$

$$+\delta(\omega-\alpha+\beta) + \delta(\omega-\alpha-\beta)]$$

(b) $g(t) = \sin(t) = \dfrac{e^{jt} - e^{-jt}}{2j}, \quad 0 < t < 2\pi$

$$G(\omega) = \int_0^{2\pi} \frac{e^{jt} - e^{-jt}}{2j} e^{-j\omega t} dt = \frac{1}{2j}\int_0^{2\pi} (e^{jt(1-\omega)} - e^{-jt(1+\omega)})dt$$

$$= \frac{1}{2j}\left[\frac{e^{jt(1-\omega)}}{j(1-\omega)} + \frac{e^{-jt(1+\omega)}}{j(1+\omega)}\right]\Bigg|_0^{2\pi} = \frac{-1}{2}\left[\frac{e^{j2\pi(1-\omega)} - 1}{(1-\omega)} + \frac{e^{-j2\pi(1+\omega)} - 1}{(1+\omega)}\right]$$

But $e^{j2\pi} = \cos(2\pi) + j\sin 2\pi = 1$

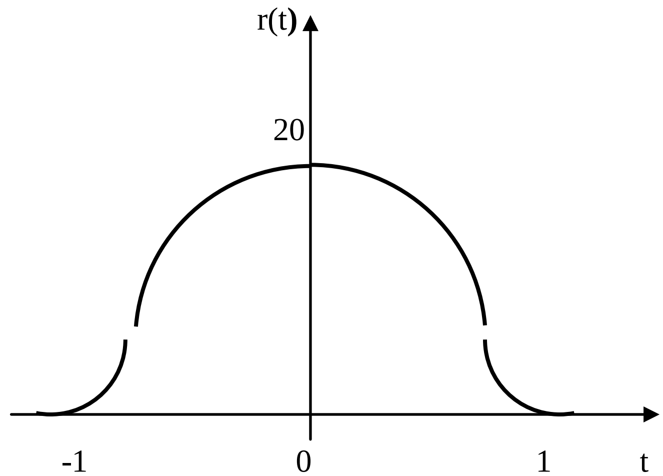
$$G(\omega) = \frac{-1}{2}\left[\frac{e^{-j2\pi\omega} - 1}{(1-\omega)} + \frac{e^{-j2\pi\omega} - 1}{(1+\omega)}\right] = \frac{-1}{2}\frac{(e^{-j2\pi\omega} - 1)(2)}{(1-\omega^2)} = \frac{1 - e^{-j2\pi\omega}}{1-\omega^2}$$

Prob. 2.31

$$P(\omega) = \int_{-\tau/2}^{\tau/2} \cos\frac{\pi t}{2} e^{-j\omega t} dt = \frac{2\tau}{\pi}\frac{\cos(\omega\tau/2)}{1 - \left(\dfrac{\omega\tau}{\pi}\right)^2}$$

Prob. 2.32

(a) r(t) is sketched below.



(b) $$R(\omega) = \int_{-\infty}^{\infty} r(t)e^{-j\omega t}dt = \int_{-1}^{1} 10(1+\cos \pi t)e^{-j\omega t}dt$$

$$= 10\frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-1}^{1} + 5\int_{-1}^{1}(e^{j\pi t}+e^{-j\pi t})e^{-j\omega t}dt$$

$$= \frac{10}{-j\omega}(e^{-j\omega}-e^{j\omega}) + \frac{5e^{j(\pi-\omega)t}}{j(\pi-\omega)} + \frac{5e^{-j(\pi+\omega)t}}{-j(\pi+\omega)}\Bigg|_{-1}^{1}$$

$$= \frac{20}{\omega}\sin\omega + 5\frac{(e^{j(\pi-\omega)}-e^{-j(\pi-\omega)})}{j(\pi-\omega)} - 5\frac{(e^{-j(\pi+\omega)}-e^{j(\pi+\omega)})}{j(\pi+\omega)}$$

$$= 20\sin c\omega + 10\sin c(\pi-\omega) + 10\sin c(\pi+\omega)$$

Prob. 2.33

(a) $$F_1(\omega) = \frac{1}{2}(e^{j\omega a}+e^{-j\omega a}), \qquad a = \pi/4$$

$$f_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{2}\left[e^{j\omega(t+a)}+e^{j\omega(t-a)}\right]d\omega$$

But $$\int_{-\infty}^{\infty} e^{j(t\pm a)\omega}d\omega = 2\pi\delta(t\pm a)$$

$$f_1(\text{t}) = \frac{1}{4\pi}\ 2\pi[\delta(t+a)-\delta(t-a)] = \frac{1}{2}[\delta(t+\pi/4)-\delta(t-\pi/4)]$$

(b) $f_2(t) = e^{-4(t-2)}u(t-2)$

(c) Let $\omega^2 = x$

$$F_3 = \frac{x+2}{x^2+3x+2} = \frac{x+2}{(x+2)(x+1)} = \frac{1}{x+1} = \frac{1}{\omega^2+1}$$

$$f_3(t) = \frac{1}{2}e^{-|t|}$$

Prob. 2.34

$$2\sin\pi\omega = \frac{2}{2j}[e^{j\pi\omega} - e^{-j\pi\omega}]$$

$$f(t) = \frac{1}{2\pi j}\int_{-1}^{1}[e^{j\pi\omega} - e^{-j\pi\omega}]e^{j\omega t}d\omega = \frac{1}{2\pi j}\int_{-1}^{1}[e^{j\omega(\pi+t)} - e^{-j\omega(\pi-t)}]d\omega$$

$$= \frac{1}{2\pi j}\left[\frac{e^{j\omega(\pi+t)}}{j(\pi+t)} + \frac{e^{-j\omega(\pi-t)}}{j(\pi-t)}\right]\Bigg|_0^1 = \frac{2j\sin t}{t^2-\pi^2}$$

Prob. 2.35

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_o-\tau}^{-\omega_o+\tau}\frac{A}{2}e^{j\omega t}d\omega + \frac{1}{2\pi}\int_{-\tau}^{\tau}Ae^{j\omega t}d\omega + \frac{1}{2\pi}\int_{\omega_o-\tau}^{\omega_o+\tau}\frac{A}{2}e^{j\omega t}d\omega$$

$$= \frac{A}{4\pi}\frac{e^{j\omega t}}{jt}\Bigg|_{-\omega_o-\tau}^{-\omega_o+\tau} + \frac{A}{2\pi}\frac{e^{j\omega t}}{jt}\Bigg|_{-\tau}^{\tau} + \frac{A}{4\pi}\frac{e^{j\omega t}}{jt}\Bigg|_{\omega_o-\tau}^{\omega_o+\tau}$$

$$= A\frac{e^{-j\omega_o t}}{2\pi t}\sin\tau t + \frac{A\tau}{\pi}\sin c(\tau t) + A\frac{e^{j\omega_o t}}{2\pi t}\sin\tau t$$

$$f(t) = \frac{A\tau}{\pi}\sin(\tau t)\left[1+\cos(\omega_o t)\right]$$

Prob. 2.36

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}[\int_{-1}^{0}(-2\omega)e^{j\omega t}d\omega+\int_{0}^{1}(2\omega)e^{j\omega t}d\omega]$$

$$=\frac{1}{2\pi}\left[-\frac{2e^{j\omega t}(j\omega t-1)}{\omega^2}\Bigg|_{-1}^{0}+\frac{2e^{j\omega t}(j\omega t-1)}{\omega^2}\Bigg|_{0}^{1}\right]$$

$$=\frac{1}{\pi\omega^2}\left[1-jte^{-jt}-e^{-jt}+jte^{jt}-e^{jt+1}\right]$$

$$=\frac{1}{\pi\omega^2}\left[2-2t\sin t-2\cos t\right]$$

Prob. 2.37

(a) Let $h(t)=g(-2t)$

$$H(\omega)=\frac{1}{2}\frac{20}{(1-\frac{j\omega}{2})}=\frac{20}{2-j\omega}$$

(b) Let $f(t)=tg(t)$

$$F(\omega)=j\frac{dG(\omega)}{d\omega}=20j(-j)(1+j\omega)^{-2}=\frac{20}{(1+j\omega)^2}$$

Let $h(t)=(t+1)g(t+1)u(t+1)=f(t+1)u(t+1)$

$$H(\omega)=e^{j\omega}F(\omega)=\frac{20e^{j\omega}}{(1+j\omega)^2}$$

(c) $\dfrac{dg}{dt} \Leftrightarrow \dfrac{20j\omega}{1+j\omega}$

$$t\frac{dg}{dt}=j\frac{d}{d\omega}\left(\frac{20j\omega}{1+j\omega}\right)=j20\frac{\left[(1+j\omega)j-j\omega(j)\right]}{(1+j\omega)^2}=-\frac{20}{(1+j\omega)^2}$$

(d) Let $y(t)=g(t)\cos\pi t$

$$Y(\omega)=\frac{1}{2}[G(\omega+\pi)+G(\omega-\pi)]=\left[\frac{10}{1+j(\omega+\pi)}+\frac{10}{1+j(\omega-\pi)}\right]$$

Prob. 3.38

(a) Let $f(t)=x(t)e^{-j2t}$

$$F(\omega)=X(\omega+2)=\frac{4+j(\omega+2)}{-(\omega+2)^2+j2(\omega+2)+2}$$

(b) Let $g(t) = x(t)\sin\pi(t-1) = x(t)[\sin\pi t\cos\pi - \cos\pi t\sin\pi]$

Since $\cos\pi$=-1 and $\sin\pi$=0,

$g(t) = -x(t)\sin\pi t$

$$G(\omega) = -\frac{1}{2}\left[X(\omega+\pi) - X(\omega-\pi)\right]$$

$$= \frac{\frac{1}{2}\left[4 + j(\omega-\pi)\right]}{-(\omega-\pi)^2 + j2(\omega-\pi) + 3} - \frac{\frac{1}{2}\left[4 + j(\omega+\pi)\right]}{-(\omega+\pi)^2 + j2(\omega+\pi) + 3}$$

(c) Let $h(t) = x(t) * \delta(t-2) = x(t-2)$

$$H(\omega) = e^{-j2\omega}X(\omega) = \frac{(4 + j\omega)e^{-j2\omega}}{-\omega^2 + j2\omega + 3}$$

(d) Let $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

$$Y(\omega) = \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega) = \frac{4 + j\omega}{j\omega(-\omega^2 + j2\omega + 3)} + \frac{4}{3}\pi\delta(\omega)$$

Prob. 2.39

(a) $u(t+\tau) - u(t-\tau) \quad \Leftrightarrow \quad \dfrac{2\sin\omega\tau}{\omega}$

$$\frac{\sin\omega\tau}{\omega\tau} \quad \Leftrightarrow \quad \frac{1}{2\tau}\left[u(t+\tau) - u(t-\tau)\right]$$

Let τ=1

$$\frac{\sin\omega}{\omega} \quad \Leftrightarrow \quad \frac{1}{2}\left[u(t+1) - u(t-1)\right]$$

$$\mathscr{F}\left[\frac{\sin t}{t}\right] = \frac{1}{2}2\pi\left[u(-\omega+1) - u(-\omega-1)\right]$$

$$\int_{-\infty}^{\infty}\frac{\sin^2 t}{t^2}dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\pi^2\left[u(-\omega+1) - u(-\omega-1)\right]^2 d\omega$$

$$= \frac{\pi}{2}\Big|_{-\omega-1}^{-\omega+1} = \frac{\pi}{2}(2) = \pi$$

(b) Let $f(t) = \frac{4}{t^2+4}$

$$\frac{2a}{t^2+a^2} \quad \Leftrightarrow \quad 2\pi e^{-a|\omega|}, \qquad a = 2$$

$$f^2(t) = \frac{16}{(t^2+4)^2}$$

$$\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega$$

$$\int_{-\infty}^{\infty} \frac{dt}{(t^2+4)^2} = \frac{1}{16}\frac{1}{2\pi}\int_{-\infty}^{\infty} 4\pi^2 e^{-4|\omega|} d\omega = \frac{\pi}{4}\int_{0}^{\infty} e^{-4\omega} d\omega$$

$$= \frac{\pi}{4}\frac{e^{-4\omega}}{-4} = \frac{\pi}{16}$$

Prob. 2.40

(a) $X(\omega) = \frac{1}{a + j\omega}, \qquad a = 3$

$$|X(\omega)|^2 = \frac{1}{a^2+\omega^2}$$

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{d\omega}{a^2+\omega^2} = \frac{2}{2\pi}\frac{1}{3}\tan^{-1}\frac{\omega}{3}\Big|_{0}^{\infty} = \frac{2}{2\pi}\frac{1}{3}\frac{\pi}{2} = \frac{1}{6} = 0.1667 \text{ W}$$

(b) $Y(\omega) = 4\sin c\left(\frac{\omega 4}{2}\right) = 4\sin c(2\omega)$

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} 16\sin c^2(2\omega)d\omega$$

But $\int \frac{\sin^2 x}{x^2}dx = \int \sin c^2 x dx = \frac{\pi}{2}$

Let $2\omega = x \quad \longrightarrow \quad d\omega = \frac{dx}{2}$

$$E = \frac{16}{2\pi}\int \sin c^2 x \frac{dx}{2} = \frac{8}{2\pi}\frac{\pi}{2} = 2$$

Prob. 2.41

$$F(\omega) = 5(2)\sin c\left(\frac{\omega 2}{2}\right) = 10\sin c\omega$$

Applying the time shifting property,

$$G(\omega) = F(\omega)e^{j\omega 2} + F(\omega)e^{-j\omega 2} = 10\sin c\omega\left(e^{j\omega 2} + e^{-j\omega 2}\right)$$

$$G(\omega) = 20\cos(2\omega)\sin c\omega$$

Prob. 2.42

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{j\omega RC}{1 + j\omega RC - \omega^2 LC}$$

$H(0) = 0, \quad H(\infty) = 0 \quad \longrightarrow \quad$ a bandpass filter

Prob. 2.43

$$E_x = \int_{-\infty}^{\infty} x^2(t)dt = \int_0^{\infty} e^{-200\pi t}dt = \frac{1}{200\pi}$$

$$x(t) = e^{-100\pi t} \quad \longrightarrow \quad X(\omega) = \frac{1}{100\pi + j\omega}$$

$$Y(\omega) = H(\omega)X(\omega) = \frac{1}{100\pi + j\omega}, \quad -B < \omega < B$$

$$E_{out} = \frac{1}{3}E_x = \frac{1}{600\pi} = \frac{1}{2\pi}\int_{-B}^{B} |Y(\omega)|^2 \, d\omega = \frac{1}{\pi}\int_0^B \frac{d\omega}{\pi^2 10^4 + \omega^2}$$

$$\frac{1}{600\pi} = \frac{1}{\pi}\left(\frac{1}{100\pi}\tan^{-1}\frac{B}{100\pi}\right) \quad \longrightarrow \quad \frac{B}{100\pi} = \tan\frac{\pi}{6}$$

$$B = 100\pi\tan\frac{\pi}{6} = 5{,}441.4 \text{ rad/s}$$

Prob. 5.44

$$H(s) = \frac{\omega_c^3}{s^3 + 2\omega_c s + 2\omega_c^2 + \omega_c^3} = \frac{1}{s^3 + 2s^2 + 2s + 1} = \frac{1}{(s+1)(s^2+s+1)}$$

$$H(s) = \frac{A}{s+1} + \frac{Bs + C}{s^2 + s + 1}$$

$$1 = A(s^2 + s + 1) + B(s^2 + s) + C(s+1)$$

Equating coefficients,

$s^2:\quad 0 = A + B \quad \longrightarrow \quad B = -A$

$s:\quad 0 = A + B + C = 0 + C \quad \longrightarrow \quad C = 0$

constant: 1=A+C $\longrightarrow$ A=1, B=-1

$$H(s) = \frac{1}{s+1} - \frac{s}{s^2+s+1} = \frac{1}{s+1} - \frac{s}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{s+1} - \frac{(s+1/2)}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1/2}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{s+1} - \frac{(s+1/2)}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1/2}{\frac{\sqrt{3}}{2}} \frac{\frac{\sqrt{3}}{2}}{(s+1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$h(t) = e^{-t} - e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{1}{\sqrt{3}} e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)$$

Prob. 2.45

This is H(s) for third-order Butterworth filter. Hence,

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Prob. 2.46

$$Z_1 = 1 + sL, Z_2 = \frac{1}{sC} \parallel 1 = \frac{\frac{1}{sC}}{1 + \frac{1}{sC}} = \frac{1}{1 + sC}$$

$$V_o = \frac{Z_2}{Z_1 + Z_2} V_s$$

$$H = \frac{V_o}{V_s} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{1}{1+sC}}{1 + sL + \frac{1}{1+sC}} = \frac{1}{s^2 LC + s(L+C) + 2}$$

$$= \frac{1/LC}{s^2 + s(1/C + 1/L) + 2/LC}$$

Comparing this with

$$H(s) = \frac{k}{s^2 + 1.414\omega_c s + \omega_c^2}$$

we get

$$k = 1/LC, \quad \omega_c^2 = \frac{2}{LC}, \qquad 1.414\omega_c = \frac{1}{C} + \frac{1}{L},$$

$$\frac{1}{C} = \frac{L\omega_c^2}{2} = 50L$$

$$14.14 = 50L + \frac{1}{L} \qquad \longrightarrow \qquad 50L^2 - 14.14L + 1 = 0$$

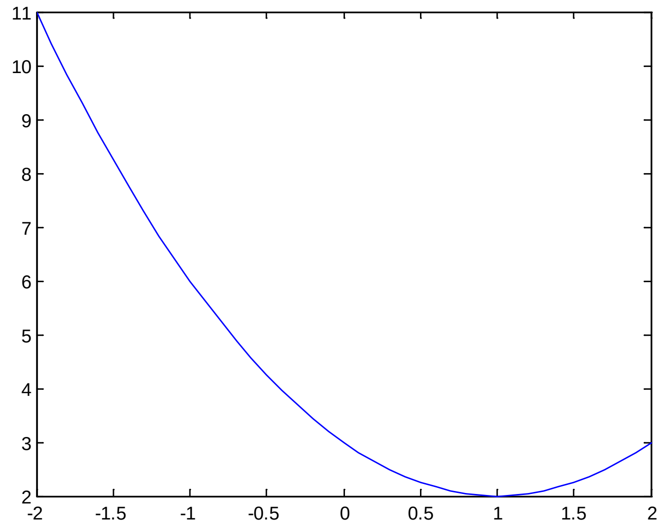
$$L = \frac{14.14 \pm \sqrt{14.14^2 - 4(50)(1)}}{100} \approx 0.1414$$

$$L = 141.4 \text{ mH}, \quad \text{C} = \frac{1}{50\text{L}} = 0.1414 = 141.4 \text{ mF}$$

Prob. 2.47

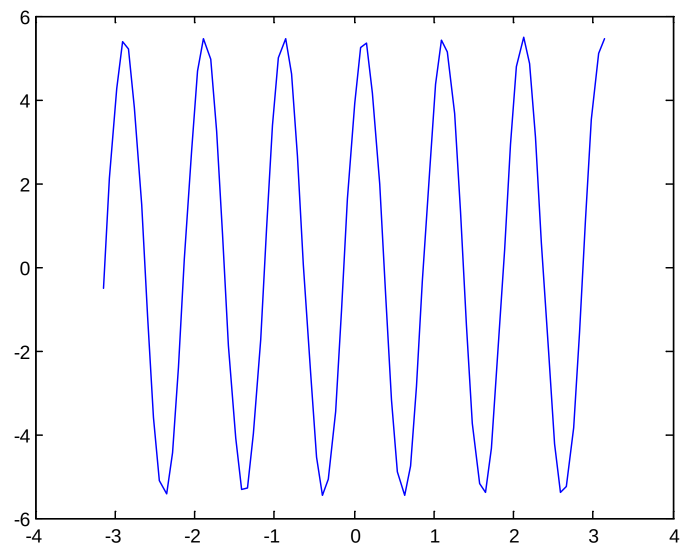
(a) The MATLAB code with the plot is presented below.

```
t=-2:0.1:2;
x = t.*t -2*t +3;
plot(t,x)
```



(b) The MATLAB codes with the plot is shown below.

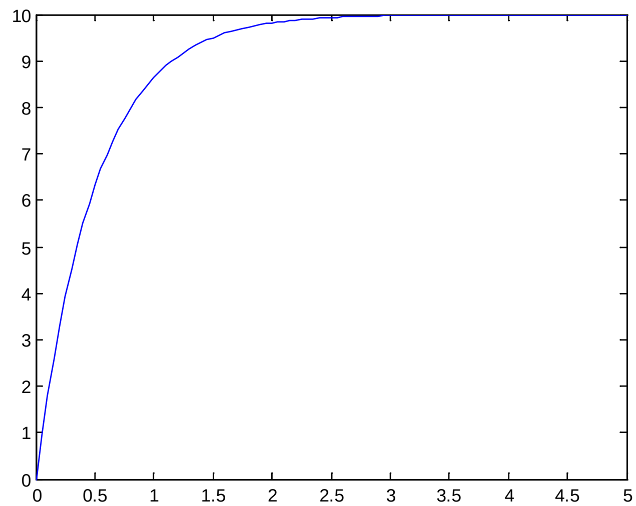
```
t=-pi:pi/40:pi;
y = 4*cos(2*pi*t - 12*pi/180) + 3*sin(2*pi*t);
plot(t,y)
```



(c) The MATLAB code with the plot is shown below.

```
t= 0:0.05:5;
z = 10*( 1 - exp(-2*t));
```

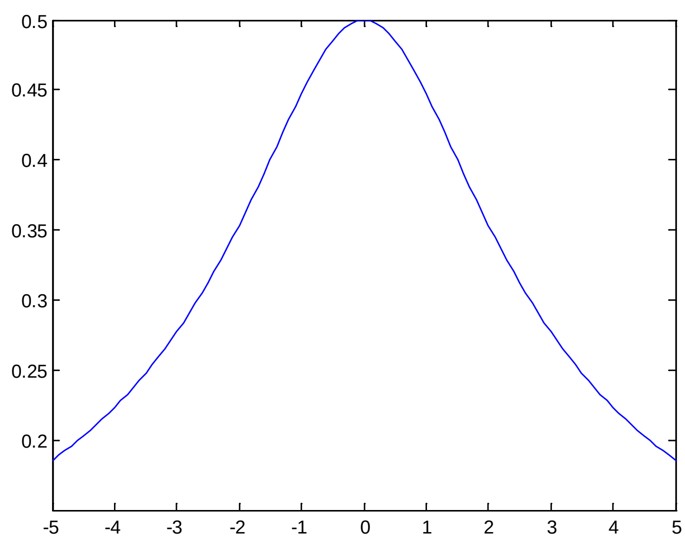
```
plot(t,z)
```



Prob. 2.48

(a) The MATLAB with the plot is shown below.

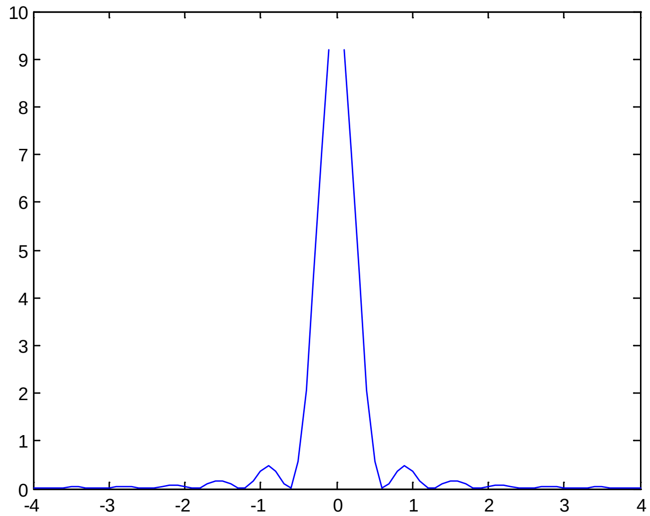
```
w= -5:0.1:5;
F = 1./sqrt( 4 + w.*w);
plot(w,F)
```



(b) The MATLAB with the plot is shown below.

```
w= -4:0.1:4;
G = 10*( sin(5*w).*sin(5*w) )./(25*w.*w);
```

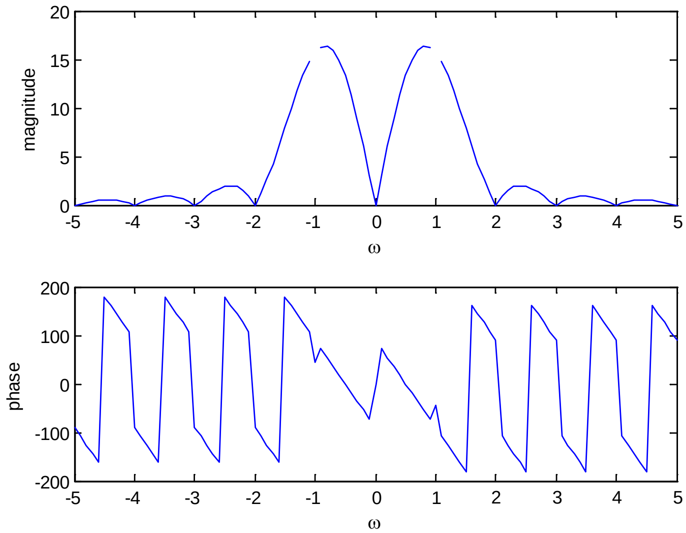
```
plot(w,G)
```



Prob. 4.49

The MATLAB code with the plots is presented below.

```
w= -5:0.1:5;
a=pi*w
F =  10*j.*exp(-j*a).*sin(a) ./(1-w.*w);
FM = abs(F);
FP = angle(F)*180/pi;
subplot(2,1,1); plot(w,FM);
xlabel('\omega'); ylabel('magnitude');
subplot(2,1,2); plot(w,FP);
xlabel('\omega'); ylabel('phase');
```



Prob. 2.50

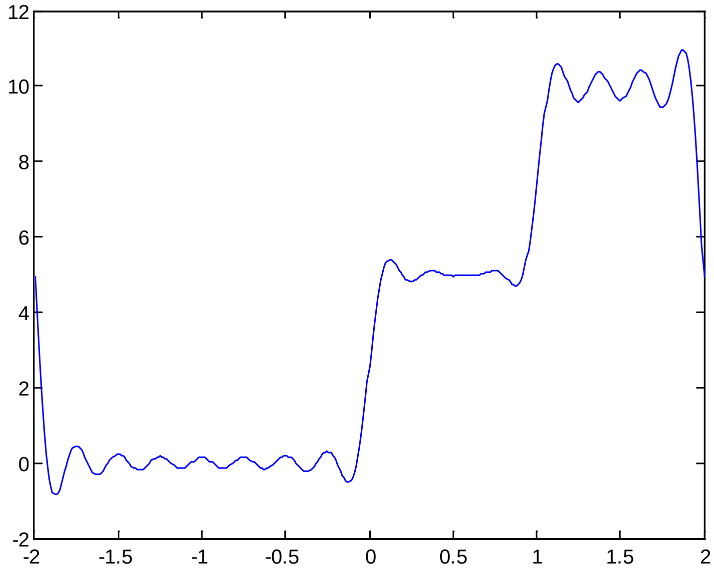
From Prob. 2.13,

$$a_0 = 3.75, a_n = \frac{-5}{n\pi}\sin\frac{n\pi}{2}, b_n = \frac{5}{n\pi}\left(1 - 2\cos n\pi + \cos\frac{n\pi}{2}\right)$$

$$fN = 3.75 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_o t), \qquad \omega_o = \frac{\pi}{2}$$

The MATLAB code with the plot of partial sum is shown below.

```
N=15;
t = -2:0.01:2;
w = pi/2
f0=3.75;
fN = f0*ones(size(t));
for n=1:N
    fac=5/(pi*n);
    an= -fac*sin(w*n);
    bn=fac*(1 - 2*cos(n*pi) + cos(w*n));
    fN = fN +an*cos(n*w*t) + bn*sin(n*w*t);
end
plot(t,fN)
```



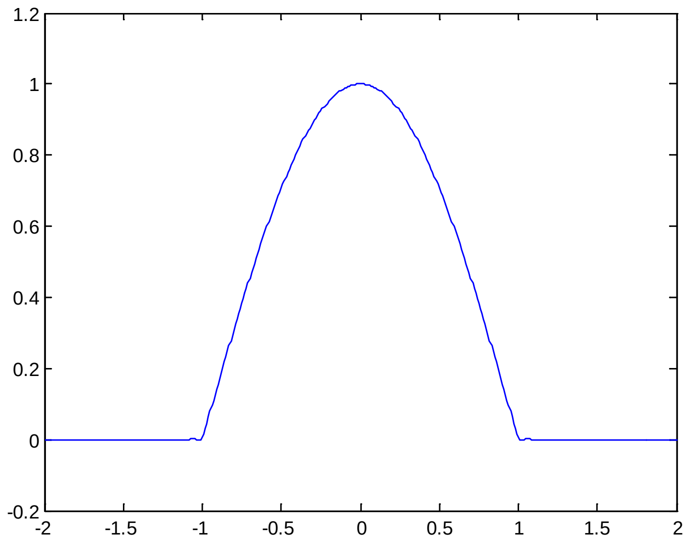
Prob. 2.51

From Problem 2.14,

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\cos\frac{\pi}{2}t - \frac{2}{\pi}\sum_{n=even}^{\infty}\frac{(-1)^{n/2}}{(n^2-1)}\cos\frac{n\pi}{2}t$$

The MATLAB code with the plot is shown below.

```
N = 25;
t = -2:0.01:2;
w = pi/2;
f0 = 1/pi;
fN = f0*ones(size(t));
fN = fN + 0.5*cos(w*t)
fac=-2/pi;
for k=1:N
    n=2*k;
    fac1= (-1)^(n/2);
    fac2 = n^2 -1;
    fN = fN +fac*fac1*cos(n*w*t)/fac2;
end
plot(t,fN)
```



Prob. 2.52

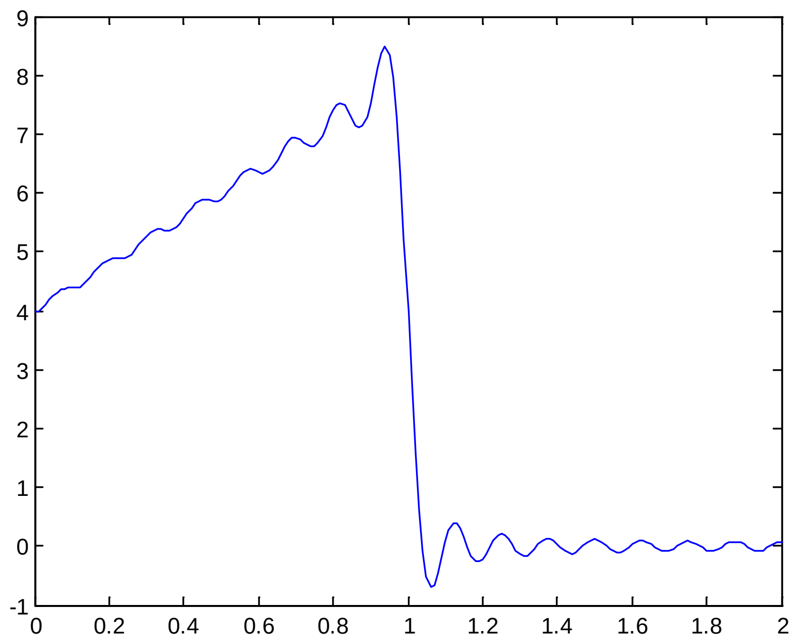
From Problem 2.20,

$$a_n = \frac{16}{n^2\pi^2}(\cos\frac{n\pi}{2} - 1) + \frac{16}{n\pi}\sin\frac{n\pi}{2}$$

$$f(t) = 3 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{2}t$$

The MATLAB code with the plot of partial sum is shown below.

```
N = 31;
t = 0:0.01:2;
w = pi/2;
f0 = 3;
fN = f0*ones(size(t));
for n=1:N
    fac1= 16*( cos(w*n) -1)/(n*n*pi*pi);
    fac2 = 16*sin(w*n)/(n*pi);
    an=fac1 + fac2;
    fN = fN + an*cos(n*w*t);
end
plot(t,fN)
```



Prob. 2.53

The MATLAB code is shown below.

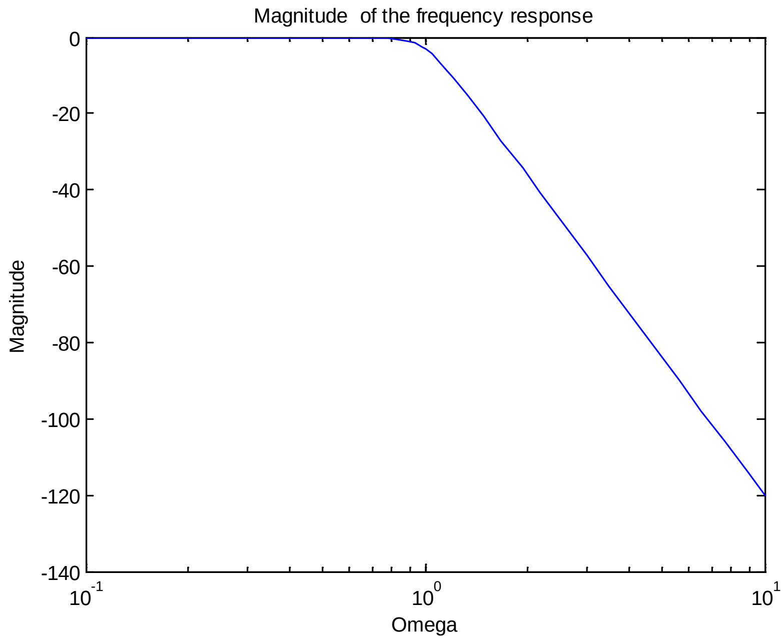
```
den = [ 1  3.236  5.236  5.236 3.236 1];
roots(den)

ans =

  -0.3090 + 0.9510i
  -0.3090 - 0.9510i
  -1.0000 + 0.0000i
  -0.8090 + 0.5879i
  -0.8090 - 0.5879i
```

Prob. 2.54

The MATLAB code with the plot is shown below.

```
[z,p,k]=buttap(6);  %  returns the zeros, poles, and constant k of the
% 6th-order Butterworth  filter
  num=k*poly(z);  % forms the numerator
  den=poly(p);      %  forms the denominator
  [mag,phase,w]=bode(num,den);  % returns magnitude, phase (in
%degrees), and  frequency vector  w (automatically)
  semilogx(w,20*log10(abs(mag)))
% plot(w,mag)    % plots the magnitude verse w
  title('Magnitude  of the frequency response')
```

```
xlabel('Omega')
ylabel('Magnitude')
```



Chapter 3

3.1 (a) $L_{\min} = 0.1\lambda = 0.1\dfrac{c}{f_m} = 0.1\dfrac{3\times10^8 \text{ m/s}}{30\times10^3 \text{ Hz}}$

$L_{\min} = 10^3 \text{ m or } 1 \text{ km}$

(b) $f_c = 100\, f_m = 100\times30\times10^3 \text{ Hz} = 3\times10^6 \text{ Hz}$

$L_{\min} = 0.1\lambda = 0.1\dfrac{c}{f_c} = 0.1\dfrac{3\times10^8 \text{ m/s}}{3\times10^6 \text{ Hz}} = 10 \text{ m}$

3.2 (a) For $f_c = 550 \text{ kHz}$,

$H_{\min} = 0.1\lambda = 0.1\dfrac{c}{f_c} = 0.1\dfrac{3\times10^8 \text{ m/s}}{550\times10^3 \text{ Hz}} = 54.54 \text{ m}$

(b) For $f_c = 1600 \text{ kHz}$,

$H_{\min} = 0.1\lambda = 0.1\dfrac{c}{f_c} = 0.1\dfrac{3\times10^8 \text{ m/s}}{1600\times10^3 \text{ Hz}} = 18.75 \text{ m}$

3.3 $A_c = 10$; Modulation index $= \mu = \dfrac{M_p}{A_c} = \dfrac{A_m}{A_c}$

For waveform sketches, $A_{\max} = A_c + A_m$; $A_{\min} = A_c - A_m$

For From Figure 3.35 (a), $A_{\max} = A_c + A_m = 8$; $A_{\min} = A_c - A_m = 2$

$\therefore A_c = \frac{1}{2}(A_{\max} + A_{\min}) = \frac{1}{2}(8+2) = 5$; $A_m = \frac{1}{2}(A_{\max} - A_{\min}) = \frac{1}{2}(8-2) = 3$

$\mu = \dfrac{A_m}{A_c} = \dfrac{3}{5} = 0.6 \text{ or } 60\%$

(a) $\mu = \dfrac{4}{10} = 0.4$

Sketch is similar to Fig. 3.2 (a) but $A_{\max} = 10 + 4 = 14$ and $A_{\min} = 10 - 4 = 6$.

(b) For $A_m = 10$, $\mu = \dfrac{10}{10} = 1.0$ (100% modulation, not recommended!)

Sketch is similar to Fig. 3.2 (b) but $A_{\max} = 10 + 10 = 20$ and $A_{\min} = 10 - 10 = 0$