

Chapter 2 First-Order and Simple Higher-Order Differential Equations

2.1 Solve the following differential equation:

$$\cos^2 y dx + (1 + e^{-x}) \sin y dy = 0$$

2.2 Solve the following differential equation:

$$\frac{dy}{dx} = \frac{x^3 e^{x^2}}{y \ln y}$$

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2.3 Solve the following differential equation:

$$x \cos^2 y \, dx + e^x \tan y \, dy = 0$$

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2.4 Solve the following differential equation:

$$x (y^2 + 1) dx + (2y + 1) e^{-x} dy = 0$$

2.5 Solve the following differential equation:

$$xy^3 dx + e^{x^2} dy = 0$$

2.6 Solve the following differential equation:

$$x \cos^2 y \, dx + \tan y \, dy = 0$$

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2.7 Solve the following differential equation:

$$xy^3 dx + (y+1)e^{-x} dy = 0$$

2.8 Solve the following differential equation:

$$\frac{dy}{dx} + \frac{x}{y} + 2 = 0$$

2.9 Solve the following differential equation:

$$x \, dy - y \, dx = x \cot \left(\frac{y}{x} \right) dx$$

2.10 Solve the following differential equation:

$$\left[x \cos^2\left(\frac{y}{x}\right) - y \right] dx + x dy = 0$$

2.11 Solve the following differential equation:

$$x \, dy = y(1 + \ln y - \ln x) \, dx$$

2.12 Solve the following differential equation:

$$xy \, dx + (x^2 + y^2) \, dy = 0$$

2.13 Solve the following differential equation:

$$\left[1 + \exp\left(-\frac{y}{x}\right)\right] dy + \left(1 - \frac{y}{x}\right) dx = 0$$

2.14 Solve the following differential equation:

$$(x^2 - xy + y^2) dx - xy dy = 0$$

2.15 Solve the following differential equation:

$$(3 + 2x + 4y)y' = 1 + x + 2y$$

2.16 Solve the following differential equation:

$$y' = \frac{2x + y - 1}{x - y - 2}$$

2.17 Solve the following differential equation:

$$(y + 2)dx = (2x + y - 4)dy$$

2.18 Solve the following differential equation:

$$y' = \sin^2(x-y)$$

2.19 Solve the following differential equation:

$$\frac{dy}{dx} = (x+1)^2 + (4y+1)^2 + 8xy + 1$$

2.20 Solve the following differential equation:

$$(3x^2 + 6xy^2) dx + (6x^2y + 4y^3) dy = 0$$

2.21 Solve the following differential equation:

$$(2x^3 - xy^2 - 2y + 3) dx - (x^2y + 2x) dy = 0$$

2.22 Solve the following differential equation:

$$(xy^2 + x - 2y + 3) dx + x^2y dy = 2(x + y) dy$$

2.23 Solve the following differential equation:

$$3y(x^2 - 1) dx + (x^3 + 8y - 3x) dy = 0, \quad \text{when } x = 0, y = 1$$

2.24 Solve the following differential equation:

$$(x^2 + \ln y) \, dx + \frac{x}{y} \, dy = 0$$

2.25 Solve the following differential equation:

$$2x(3x + y - ye^{-x^2}) dx + (x^2 + 3y^2 + e^{-x^2}) dy = 0$$

2.26 Solve the following differential equation:

$$(3 + y + 2y^2 \sin^2 x) dx + (x + 2xy - y \sin 2x) dy = 0$$

2.27 Solve the following differential equation:

$$(2xy + y^2) dx + (x^2 + 2xy + y^2) dy = 0$$

2.28 Solve the following differential equation:

$$(x^2 - \sin^2 y) dx + x \sin 2y dy = 0$$

2.29 Solve the following differential equation:

$$y(2x - y + 2) \, dx + 2(x - y) \, dy = 0$$

2.30 Solve the following differential equation:

$$(4xy + 3y^2 - x) \, dx + x(x + 2y) \, dy = 0$$

2.31 Solve the following differential equation:

$$y \, dx + x(y^2 + \ln x) \, dy = 0$$

2.32 Solve the following differential equation:

$$(x^2 + 2x + y) \, dx + (3x^2y - x) \, dy = 0$$

2.33 Solve the following differential equation:

$$y^2 dx + (xy + y^2 - 1) dy = 0$$

2.34 Solve the following differential equation:

$$3(x^2 + y^2) \, dx + x(x^2 + 3y^2 + 6y) \, dy = 0$$

2.35 Solve the following differential equation:

$$2y(x + y + 2) \, dx + (y^2 - x^2 - 4x - 1) \, dy = 0$$

2.36 Solve the following differential equation:

$$(2 + y^2 + 2x) \, dx + 2y \, dy = 0$$

2.37 Solve the following differential equation:

$$(2xy^2 - y) \, dx + (y^2 + x + y) \, dy = 0$$

2.38 Solve the following differential equation:

$$y(x + y) \, dx + (x + 2y - 1) \, dy = 0$$

2.39 Solve the following differential equation:

$$2x(x^2 - \sin y + 1)dx + (x^2 + 1) \cos y dy = 0$$

2.40 Consider a homogeneous differential equation of the form

$$M(u) \, dx + N(u) \, dy = 0, \quad u = \frac{y}{x}.$$

If $Mx + Ny = 0$, i.e., $M(u) + N(u)u = 0$, show that $\frac{1}{xM}$ is an integrating factor.

2.41 Solve the following differential equation:

$$(x^2 + y + y^2) \, dx - x \, dy = 0$$

2.42 Solve the following differential equation:

$$(x - \sqrt{x^2 + y^2}) dx + (y - \sqrt{x^2 + y^2}) dy = 0$$

2.43 Solve the following differential equation:

$$y\sqrt{1+y^2} \, dx + (x\sqrt{1+y^2} - y) \, dy = 0$$

2.44 Solve the following differential equation:

$$y^2 dx - (xy + x^3) dy = 0$$

2.45 Solve the following differential equation:

$$y \, dx - x \, dy - 2x^3 \tan \frac{y}{x} \, dx = 0$$

2.46 Solve the following differential equation:

$$(2x^2y^2 + y) \, dx + (x^3y - x) \, dy = 0$$

2.47 Solve the following differential equation:

$$y^2 dx + [xy + \tan(xy)] dy = 0$$

2.48 Solve the following differential equation:

$$(2x^2y^4 - y) \, dx + (4x^3y^3 - x) \, dy = 0$$

2.49 Solve the following differential equation:

$$(x^2y^3 + y) \, dx + (x^3y^2 - x) \, dy = 0$$

2.50 Solve the following differential equation:

$$y(y^2 + 1) \, dx + x(y^2 - x + 1) \, dy = 0$$

2.51 Solve the following differential equation:

$$y^2 dx + (e^x - y) dy = 0$$

2.52 Solve the following differential equation:

$$(x^2y^2 - 2y) \, dx + (x^3y - x) \, dy = 0$$

2.53 Solve the following differential equation:

$$(2x^3y + y^3) \, dx - (x^4 + 2xy^2) \, dy = 0$$

2.54 Solve the following differential equation:

$$(1 + y \cos x) dx - \sin x dy = 0$$

2.55 Solve the following differential equation:

$$(\sin^2 y + x \cot y)y' = 1$$

2.56 Solve the following differential equation:

$$dx - (y - 2xy) dy = 0$$

2.57 Solve the following differential equation:

$$dx - (1 + 2x \tan y) dy = 0$$

2.58 Solve the following differential equation:

$$\frac{dy}{dx} \left(y^3 + \frac{x}{y} \right) = 1$$

2.59 Solve the following differential equation:

$$dx + (x - y^2) dy = 0$$

2.60 Solve the following differential equation:

$$y^2 dx + (xy + y^2 - 1) dy = 0$$

2.61 Solve the following differential equation:

$$y \, dx = (e^y + 2xy - 2x) \, dy$$

2.62 Solve the following differential equation:

$$(2x + 3)y' = y + (2x + 3)^{1/2}, \quad y(-1) = 0$$

2.63 Solve the following differential equation:

$$y \, dx + (y^2 e^y - x) \, dy = 0$$

2.64 Solve the following differential equation:

$$y' = 1 + 3y \tan x$$

2.65 Solve the following differential equation:

$$(1 + \cos x)y' = \sin x(\sin x + \sin x \cos x - y)$$

2.66 Solve the following differential equation:

$$y' = (\sin^2 x - y) \cos x$$

2.67 Solve the following differential equation:

$$xy' - ny - x^{n+2}e^x = 0, \quad n = \text{constant}$$

2.68 Solve the following differential equation:

$$(1+x)\frac{dy}{dx} - y = x(1+x)^2$$

2.69 Solve the following differential equation:

$$(1+y) \, dx + [x - y(1+y)^2] \, dy = 0$$

2.70 Consider the first-order differential equation

$$\frac{dy}{dx} = \alpha(x) F(y) + \beta(x) G(y)$$

If $\frac{G'(y)F(y) - G(y)F'(y)}{F(y)} = a = \text{constant}$, then the transformation

$$u = \frac{G(y)}{F(y)}$$

reduces the differential equation to a first-order linear differential equation. Show that the general solution of the differential equation is given by

$$\frac{G(y)}{F(y)} = \exp\left[a \int \beta(x) dx\right] \left\{ a \int \alpha(x) \exp\left[-a \int \beta(x) dx\right] dx + C \right\}.$$

2.71 The *Riccati equation* is given by $y' = \alpha(x)y^2 + \beta(x)y + \gamma(x)$.

1. If one solution of this equation, say $y_1(x)$, is known, then the general solution can be found by using the transformation $y = y_1 + \frac{1}{u}$, where u is a new dependent variable. Show that u is given by

$$u = e^{-\int P(x) dx} \left[\int Q(x) e^{\int P(x) dx} dx + C \right],$$

where $P(x) = 2\alpha(x)y_1(x) + \beta(x)$ and $Q(x) = -\alpha(x)$.

2. For the differential equation $y' + y^2 = 1 + x^2$, first guess a solution $y_1(x)$ and then use the result of Part 1 to find the general solution $y(x)$.

2.72 Solve the following differential equation:

$$3xy' - 3xy^4 \ln x - y = 0$$

2.73 Solve the following differential equation:

$$\frac{dy}{dx} = \frac{4x^3y^2}{x^4y + 2}$$

2.74 Solve the following differential equation:

$$y(6y^2 - x - 1) dx + 2x dy = 0$$

2.75 Solve the following differential equation:

$$(1 + x)(y' + y^2) - y = 0$$

2.76 Solve the following differential equation:

$$xyy' + y^2 - \sin x = 0$$

2.77 Solve the following differential equation:

$$(2x^3 - y^4)dx + xy^3dy = 0$$

2.78 Solve the following differential equation:

$$y' - y \tan x + y^2 \cos x = 0$$

2.79 Solve the following differential equation:

$$6y^2 \, dx - x(2x^3 + y) \, dy = 0$$

2.80 Solve the following differential equation:

$$xy'^3 - yy'^2 + 1 = 0$$

2.81 Solve the following differential equation:

$$y = xy' + y'^3$$

2.82 Solve the following differential equation:

$$x(y'^2 - 1) = 2y'$$

2.83 Solve the following differential equation:

$$xy'(y' + 2) = y$$

2.84 Solve the following differential equation:

$$x = y' \sqrt{y'^2 + 1}$$

2.85 Solve the following differential equation:

$$2y'^2(y - xy') = 1$$

2.86 Solve the following differential equation:

$$y = 2xy' + y^2y'^3$$

2.87 Solve the following differential equation:

$$y'^3 + y^2 = xyy'$$

2.88 Solve the following differential equation:

$$2xy' - y = y' \ln(yy')$$

2.89 Solve the following differential equation:

$$y = xy' - x^2 y'^3$$

2.90 Solve the following differential equation:

$$y(y - 2xy')^3 = y'^2$$

2.91 Solve the following differential equation:

$$y + xy' = 4\sqrt{y'}$$

2.92 Solve the following differential equation:

$$2xy' - y = \ln y'$$

2.93 Solve the following differential equation:

$$y'' = 2yy'^3$$

2.94 Solve the following differential equation:

$$yy'' = y'^2 - y'^3$$

2.95 Solve the following differential equation:

$$xy''' = (1-x)y''$$

2.96 Solve the following differential equation:

$$y'' = e^x y'^2$$

2.97 Solve the following differential equation:

$$yy'' + y'^2 = 0$$

2.98 Solve the following differential equation:

$$1 + y'^2 = 2yy''$$

2.99 Solve the following differential equation:

$$xy'' = y'(\ln y' - \ln x)$$

2.100 Solve the following differential equation:

$$3yy'y'' - y'^3 + 1 = 0$$

2.101 Solve the following differential equation:

$$y'' - y'^2 - 1 = 0$$

2.102 Solve the following differential equation:

$$x^3 y'' - x^2 y' = 3 - x^2$$

2.103 Solve the following differential equation:

$$2y'' = y'^3 \sin 2x, \quad y(0) = 1, \quad y'(0) = 1$$

2.104 Solve the following differential equation:

$$x \frac{d^2 y}{dx^2} = 2 - \frac{dy}{dx}$$

2.105 Solve the following differential equation:

$$y'' = 3\sqrt{y}, \quad y(0) = 1, \quad y'(0) = 2$$

2.106 Solve the following differential equation:

$$x \frac{d^2 y}{dx^2} = \frac{dy}{dx} + x \sin \left(\frac{1}{x} \cdot \frac{dy}{dx} \right)$$

2.107 Solve the following differential equation:

$$yy'' = y'^2(1 - y' \sin y - yy' \cos y)$$

2.108 Solve the following differential equation:

$$y'' + xy' = x$$

2.109 Solve the following differential equation:

$$xy'' - y'^3 - y' = 0$$

2.110 Solve the following differential equation:

$$y(1 - \ln y)y'' + (1 + \ln y)y'^2 = 0$$

2.111 Solve the following differential equation:

$$xy^2(xy' + y) = 1$$

2.112 Solve the following differential equation:

$$5y + y'^2 = x(x + y')$$

2.113 Solve the following differential equation:

$$y' = \frac{y+2}{x+1} + \tan \frac{y-2x}{x+1}$$

2.114 Solve the following differential equation:

$$y''(e^x + 1) + y' = 0$$

2.115 Solve the following differential equation:

$$xy' = y - xe^{y/x}$$

2.116 Solve the following differential equation:

$$(1 + y^2 \sin 2x) dx - 2y \cos^2 x dy = 0$$

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2.117 Solve the following differential equation:

$$(2\sqrt{xy} - y)dx - xdy = 0, \quad x > 0, \quad y > 0$$

2.118 Solve the following differential equation:

$$y'' + y'^2 = 2e^{-y}$$

2.119 Solve the following differential equation:

$$y' = e^{xy'/y}$$

2.120 Solve the following differential equation:

$$(2x^3y^2 - y)dx + (2x^2y^3 - x)dy = 0$$

2.121 Solve the following differential equation:

$$(y - 1 - xy) \, dx + x \, dy = 0$$

2.122 Solve the following differential equation:

$$xy' - y = x \tan \frac{y}{x}$$

2.123 Solve the following differential equation:

$$y' + \frac{y}{x} = e^{xy}$$

2.124 Solve the following differential equation:

$$yy'' - yy' = (y')^2$$

2.125 Solve the following differential equation:

$$2y \, dx - x [\ln(x^2 y) - 1] \, dy = 0$$

2.126 Solve the following differential equation:

$$y' = \frac{1}{xy + x^3y^3}$$

2.127 Solve the following differential equation:

$$y' = 2\left(\frac{y+2}{x+y-1}\right)^2$$

2.128 Solve the following differential equation:

$$(e^x + 3y^2)dx + 2xydy = 0$$

2.129 Solve the following differential equation:

$$(xy + 2x^3y)dx + x^2dy = 0$$

2.130 Solve the following differential equation:

$$x(y')^2 - 2yy' + 4x = 0$$

2.131 Solve the following differential equation:

$$y''' = 2(y'' - 1) \cot x$$

2.132 Solve the following differential equation:

$$(y + 3x^4y^2)dx + (x + 2x^2y^3)dy = 0$$

2.133 Solve the following differential equation:

$$xy' = y + \sqrt{x^2 - y^2}, \quad x > 0, \quad |y| \leq |x|$$

2.134 Solve the following differential equation:

$$2y(xe^{x^2} + y \sin x \cos x)dx + (2e^{x^2} + 3y \sin^2 x)dy = 0$$

2.135 Solve the following differential equation:

$$\cos y \, dx + \sin y (x - \sin y \cos y) \, dy = 0$$

2.136 Solve the following differential equation:

$$y^3 dx + (3x^2 - 2xy^2) dy = 0$$

2.137 Solve the following differential equation:

$$(y' + 1) \ln \frac{y + x}{x + 3} = \frac{y + x}{x + 3}$$

2.138 Solve the following differential equation:

$$2x^3yy' + 3x^2y^2 + 7 = 0$$

2.139 Solve the following differential equation:

$$\left(x - y \cos \frac{y}{x}\right) dx + x \cos \frac{y}{x} dy = 0$$

2.140 Solve the following differential equation:

$$x^2(xy - ydx) = (x + y)ydx$$

2.141 Solve the following differential equation:

$$(y^4 + xy)dx + (xy^3 - x^2)dy = 0$$

2.142 Solve the following differential equation:

$$(x^2 + 3 \ln y) \, dx - \frac{x}{y} \, dy = 0$$

2.143 Solve the following differential equation:

$$xy'' = y' + x$$

2.144 Solve the following differential equation:

$$ydx + (xy - x - y^3)dy = 0$$

2.145 Solve the following differential equation:

$$y + 2y^3y' = (x + 4y \ln y)y'$$

2.146 Solve the following differential equation:

$$y \ln x \ln y \, dx + dy = 0$$

2.147 Solve the following differential equation:

$$(2x\sqrt{x} + x^2 + y^2) dx + 2y\sqrt{x} dy = 0$$

2.148 Solve the following differential equation:

$$[2x + y \cos(xy)] dx + x \cos(xy) dy = 0$$

2.149 Solve the following differential equation:

$$yy'' - y^2y' - y'^2 = 0$$

2.150 Solve the following differential equation:

$$2y' + x = 4\sqrt{y}$$

2.151 Solve the following differential equation:

$$2y'^3 - 3y'^2 + x = y$$

2.152 Solve the following differential equation:

$$y' - 6x e^{x-y} - 1 = 0$$

2.153 Solve the following differential equation:

$$(1 + y^2)y'' + y'^3 + y' = 0$$

2.154 Solve the following differential equation:

$$(y \sin x + \cos^2 x) dx - \cos x dy = 0$$

2.155 Solve the following differential equation:

$$y(6y^2 - x - 1) dx + 2x dy = 0$$

2.156 Solve the following differential equation:

$$y'(x - \ln y') = 1$$

2.157 Solve the following differential equation:

$$(1 + \cos x)y' + \sin x (\sin x + \sin x \cos x - y) = 0$$

2.158 Solve the following differential equation:

$$x \, dx + \sin^2\left(\frac{y}{x}\right) (y \, dx - x \, dy) = 0$$

2.159 Solve the following differential equation:

$$(2xy^4e^y + 2xy^3 + y) dx + (x^2y^4e^y - x^2y^2 - 3x) dy = 0$$

2.160 Solve the following differential equation:

$$(xy^3 - 1)dx + x^2y^2dy = 0$$