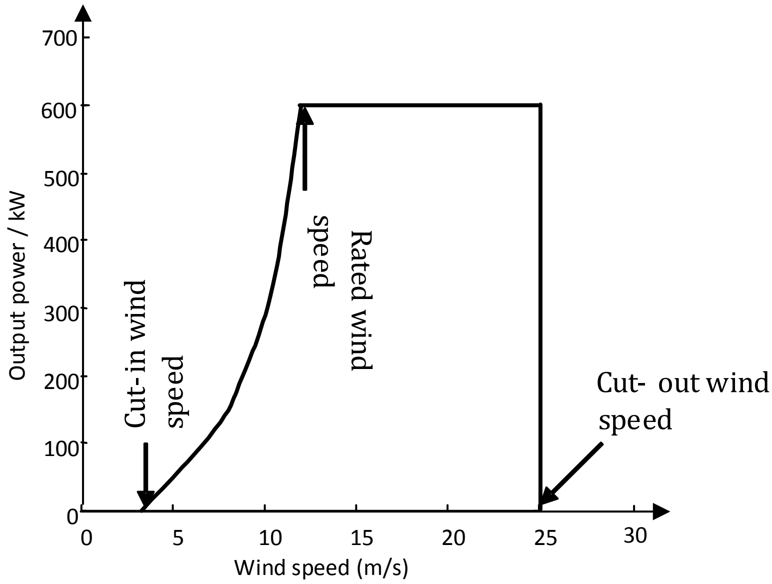
Chapter 2

WIND ENERGY

Take the density of air to be 1.25 kg/m^3

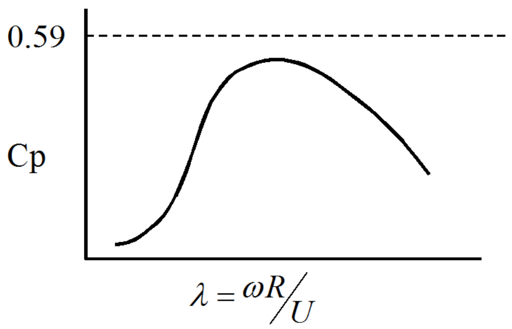
1. Sketch and explain the Power Curve of a wind turbine.

   Solution:



2. Sketch and explain the C_P / λ curve of a wind turbine rotor. Define C_P and λ. What is the Betz Limit?

   Solution:



- Power extracted by the aerodynamic rotor = C_p x Power available where C_p is the power coefficient
- Tip speed ratio is defined as $\lambda = \dfrac{\omega R}{V} = \dfrac{\text{Speeds of rotor tip}}{\text{Free wind speeds}}$
- The maximum value of the power coefficient C_p is 59% and it is called the Betz Limit.

3. Explain using appropriate diagrams and sketches how power may be regulated in a wind turbine by pitch regulation and stall regulation. Why is stall regulation only used for fixed speed wind turbines?

   Solution: See Section 2.6.1

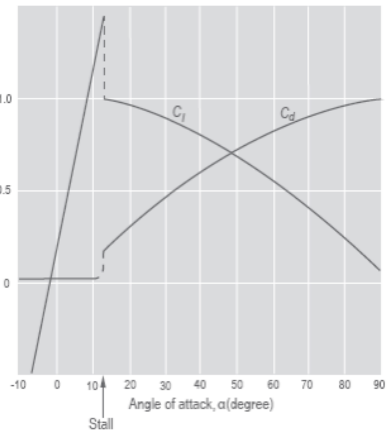


Figure 2.11

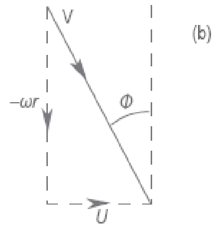


Fig 2.13b

There are two ways of reducing the rotational force F_r and hence the output power of the wind turbine, *pitch and stall regulation.* Consider the airfoil characteristics (Figure 2.11) with the blade operating at an initial angle of attack of 10^0. C_d is very small and so can be ignored but the C_l characteristic is a steeply rising almost straight line. With *pitch regulation* the angle of incidence (*α*) is reduced by mechanically turning the blade about its axis. This reduces C_l , F_r, and hence the power generated by the wind turbine.

For a fixed speed wind turbine, the angular velocity ω of the rotor is held constant by the electrical generator, which is locked on to the 50 or 60 Hz frequency of the power network. Considering the triangle of velocities *V*, *U* and $-\omega r$ The velocity of the blade element ωr is constant and so an increase of the free wind speed *U*, will increase *Φ*. Hence *α* increases as *V* swings round. Consider Figure 2.11 with an initial operating angle of attack of 10^0. Once the free wind speed U has increased sufficiently for the angle of attack to exceed around 13^0 the blade stalls and the lift coefficient and hence the torque decreases. This is *stall regulation* that does not require any physical change in the pitch angle of the blades.

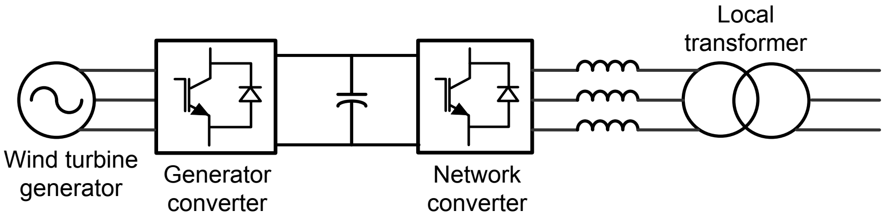
For stall regulation it is necessary to keep the rotor speed of rotation ω constant so that as U increases the angle of attack α also increases and the blade goes into stall.

4. Why are large wind turbines operated at variable speed? Explain the principle of operation of a Full Power Converter variable speed wind turbine.

   Solution:

   - For maximum power extraction, the turbine should operate at a particular value of λ. In order to achieve this the generator speed should change with the wind speed.
   - Under variable speed operation, the mechanical loading on the wind turbine is reduced.
   - As variable speed wind turbines often have a power electronic interface, turbines will be able to meet the grid code requirements with less additional equipment.

In variable speed wind turbines, back-to-back converters are used to extract maximum power from wind. The following figure shows a converter system typically used to control a large full power converter variable speed wind turbine. The generator may be synchronous (wound rotor or permanent magnet) or an induction machine. Operation is possible over a wide speed range.



Full power converter (FPC) variable-speed generator

5. Describe briefly what is meant by the measure-correlate-predict technique for establishing the long-term wind resource at a wind farm site

   Solution: See Section 2.9.1

   The measure-correlate-predict technique takes a series of measurements of wind speed and direction at the mast erected at the wind farm site and correlates them with simultaneous wind speed measurements made at a local meteorological station, which may be at a local airport. The averaging period of the site-measured data is synchronised with that of the meteorological station data. This gives a scatter plot with the mast measurements on one axis and the simultaneous meteorological station measurements on the other. Linear regression is used to establish a relationship between the measured site wind speed and the long-term meteorological wind speed data.

   The coefficients relating the two sets of measurements, sometimes known as speed-up factors, are calculated for the twelve 30^0 directional sectors. Then these coefficients are applied to transfer the long term data record of the meteorological station to the site. The speed up factors allow the long term wind speed record held by the meteorological station to be used as an estimate of what the wind speed at the wind farm site would have been over that period. It is then assumed that this long term wind speed record at the site is representative of the wind speed over the projected life of the wind farm. It is common to use a long term record of 20 years.

   The drawbacks of using the measure-correlate-predict technique include that high site masts are necessary if large wind turbines are planned. There may not be a suitable meteorological station nearby (within say 50 km) or with a similar exposure and wind climate. The data obtained from the meteorological station may not always be of good quality and may include gaps. Therefore, it may be time consuming to ensure that it is properly correlated with the site data. Critically the technique assumes that the previous long-term record provides a good estimate of the future wind resource over the lifetime of the wind farm.

6. Discuss how Weibull parameters may be used to describe hourly mean wind speed data at a site.

   Solution: See Section 2.8.1

It has been found from experience that either the Weibull or the simpler Raleigh distribution (when k=2) can be used to describe the shape of the probabilities of long term records of wind speeds.
The probability density function of a Weibull distribution is given by

$$f(U) = \frac{k}{c}\left(\frac{U}{c}\right)^{k-1} \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

where
c: scale parameter [m/s]
k: shape parameter

and the cumulative distribution function (that the wind speed will be less than or equal to *U*) is given by

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

7. A wind turbine has a rotor diameter of 33 m. At a wind speed of 13 m/s,
   a) What is the power in the wind?
   b) What is the power in the shaft of the turbine if the rotor has a power coefficient C_P of 0.35?
   c) If the speed of rotation of the shaft is 35 r.p.m. calculate the tip speed and torque on the shaft.

   [Answer: a) 1.17 MW; b) 411 kW; c) 60.5 m/s, 112 kNm]

   Solution:

   a) $P_{wind} = \frac{1}{2}\rho A U^3 = \frac{1}{2} \times 1.25 \times \pi \times 16.5^2 \times 13^3 = 1174$ kW

   b) $P_{shaft} = P_{wind} \times C_P = 1174 \times 0.35 = 411$ kW

   c) $\omega = \frac{35}{60} \times 2\pi = 3.67$ rad/sec

   $V_{tip} = \omega R = 3.67 \times 16.5 = 60.5$ m/s

   Torque $= \frac{P}{\omega} = \frac{411}{3.67} = 112$ kNm

8. A wind turbine has a rotor of 55 metres in diameter, mounted at a hub height of 60 metres. The gearbox ratio is 1:28. If it operates at a tip speed ratio (λ) of 8 in a wind speed of 10 m/s, with a power coefficient of 0.35 and a thrust coefficient of 0.55 calculate:
   a) The torque on the generator shaft
   b) The overturning moment at the base of the tower

   [Answer: a) 6.38 kNm; b) 4.9 MNm]

Solution:

a) From $\lambda = \dfrac{\omega R}{U}$, $\omega = \dfrac{U\lambda}{R} = \dfrac{10\times 8}{27.5} = 2.91$ rad/sec

$$P_{wind} = \tfrac{1}{2}\rho A U^3 = \frac{1}{2}\times 1.25\times \pi \times 27.5^2 \times 10^3 = 1.485 \text{ MW}$$

$$P_{shaft} = C_p P_{wind} = 0.35\times 1.485 = 520 \text{ kW}$$

$$Q_{ls\ shaft} = \frac{P}{\omega} = \frac{520}{2.91} = 178.7 \text{ kNm}$$

$$Q_{gen\ shaft} = \frac{178.7}{28} = 6.38 \text{ kNm}$$

b) $Thrust = C_t \dfrac{1}{2}\rho A U^2 = 0.55\times 0.5\times 1.25\times \pi \times 27.5^2 \times 10^2 = 81.67$ kN

$$Moment = 81.67\times 60 = 4.9 \text{ MNm}$$

9. A large horizontal axis wind turbine has a rotor of diameter of 80 m at a hub height of 90 m. It operates at a rotational speed of 15 r.p.m. with a combined efficiency of the gearbox and generator of 95% to produce 2 MW of electrical output power.
   a) If the hub height wind speed is 12 m/s calculate C_p (the Power Coefficient) of the rotor.
   b) Write down the tip speed ratio and hence C_Q (the Torque Coefficient)
   c) Estimate the wind speed at a 10 m high met mast adjacent to the wind turbine

[Answer: a) 0.39; b) 5.23, 0.075; c) 8.8 m/s]

Solution:

a) $P_{wind} = \tfrac{1}{2}\rho A U^3 = \dfrac{1}{2}\times 1.25\times \pi \times 40^2 \times 12^3 = 5.43$ MW

$$P_{shaft} = \frac{2}{0.95} = 2.1 \text{ MW}$$

$$C_p = \frac{P_{shaft}}{P_{wind}} = \frac{2.1}{5.43} = 0.39$$

b) $\lambda = \dfrac{\omega R}{U} = \dfrac{\left(15\times 2\pi/60\right)\times 40}{12} = 5.23$

$$C_Q = \frac{C_p}{\lambda} = \frac{0.39}{5.23} = 0.075$$

c) $U_{hub} = U_{ref}\left(\frac{z_{hub}}{z_{ref}}\right)^{1/7}$

$$U_{ref} = U_{hub} 1 / \left(\frac{z_{hub}}{z_{ref}}\right)^{0.1428} = 12 / \left(\frac{90}{10}\right)^{0.1428} = 8.8 \text{ m/s}$$

10. A horizontal axis wind turbine has a rotor diameter of 20 m, and reaches its full load output at the generator terminals of 150 kW at a wind speed of 13 m/s. The overall efficiency of the gearbox and generator is 94%.

    a) Calculate the power in the air passing through the rotor disk and hence the Power Coefficient (C_p) of the aerodynamic rotor.

    b) The turbine operates at a Tip Speed Ratio (λ) of 4. Calculate the rotational speed of the aerodynamic rotor and hence the torque on the rotor shaft.

    c) The inertia of the complete rotor, gearbox and generator is 50x10^3 kgm^2. Calculate the speed of the rotor 0.5 seconds after the connection to the electrical power network is broken and write down the energy that is absorbed by the brake when it brings the rotor rapidly to rest.

    [Answer: a) 431 kW, 0.37; b) 5.2 rad/sec, 30.7 kNm, c) 5.5 rad/sec, 756 kJ]

    Solution:

a) $P_{wind} = \frac{1}{2}\rho A U^3 = \frac{1}{2}\times 1.25\times\pi\times 10^2\times 13^3 = 431$ kW

$$C_p = \frac{P_{shaft}}{P_{wind}} = \frac{150/0.94}{431} = 0.37$$

b) $\omega = \frac{U\lambda}{R} = \frac{13\times 4}{10} = 5.2$ rad/sec

$$Q_{ls\ shaft} = \frac{P}{\omega} = \frac{150/0.94}{5.2} = 30.7 \text{ kNm}$$

c) KE during operation $= \frac{1}{2} I\omega^2 = \frac{1}{2}\times 50,000\times 5.2^2 = 676$ kJ

Additional KE = $= 150/0.94 \times 0.5 = 79.8$ kJ

Therefore energy to be dissipated in brake = 755.8 kJ

Maximum speed

$$\omega = \sqrt{\frac{2KE}{I}} = \sqrt{\frac{2 \times 755.8}{50}} = 5.5 \text{ rad/sec}$$

11. A wind turbine is rated at 400 kW at 12.5 m/s. If the annual distribution of hourly mean wind speeds at the site may be represented by a Weibull distribution with
    Scale parameter of 8.5
    Shape parameter of 2.5

    Calculate the annual energy production in kWh per year that the wind turbine will produce at rated power.

    [Answer: 254 MWh]

    Solution:

    The time that the wind speed is below 12.5 m/s is calculated from

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^k\right] = 1 - \exp\left[-\left(\frac{12.5}{8.5}\right)^{2.5}\right]$$
$$F(U) = 0.927, \text{ or } 8124 \text{ hours/year}$$

    Assuming that the number of hours the wind turbine operates above cut-out wind speed is negligible, the number of hours the wind turbine operates at rated power
    = 8760 – 8124 = 636 hrs
    Total energy wind turbine will produce = 400 kW x 636 = 254,400 kWh

12. A wind farm consists of 10 wind turbines each with:

Hub height	50 m
Rated power	2 MW
Cut in wind speed	5 m/s
Rated wind speed	13 m/s
Cut-out wind speed	20 m/s

    Long term monitoring wind speeds has been undertaken using an anemometer at 20 m height and the site wind speeds can be represented by a Weibull distribution with:
    Scale parameter 8
    Shape parameter 2 (Raleigh Distribution)

    Assume that the Scale parameter increases with height according to the 1/7 power law.

    Calculate:
    a) The number of hours each year the turbines operate
    b) The annual energy production (in MWh per year) which the wind turbines will produce when operating above rated power.

    [Answer: a) 6414 hours; b) 21.5 GWh]

Note:

The following are likely to be useful

$$\frac{U(z)}{U(h)} = \left(\frac{z}{h}\right)^{\alpha}$$

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

<u>Solution:</u>

a) When the hub height wind speed is 5 m/s, then wind speed at 20 m height

$$= \left(\frac{20}{50}\right)^{1/7} \times 5 = 4.3865 \text{ m/s}$$

The time that the wind speed is below 4.3865 m/s is calculated from

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right] = 1 - \exp\left[-\left(\frac{4.3865}{8.0}\right)^{2}\right]$$

$$F(U) = 0.26, \text{ or } 2274.6 \text{ hours/year}$$

When the hub height wind speed is 20 m/s, then wind speed at 20 m height

$$= \left(\frac{20}{50}\right)^{1/7} \times 20 = 17.546 \text{ m/s}$$

The time that the wind speed is below 17.546 m/s is calculated from

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right] = 1 - \exp\left[-\left(\frac{17.546}{8.0}\right)^{2}\right]$$

$$F(U) = 0.9919, \text{ or } 8688.65 \text{ hours/year}$$

The number of hours each year the turbines operate = 8688.65 – 2274.6 = 6414 hrs

b) When the hub height wind speed is 13 m/s, then wind speed at 20 m height

$$= \left(\frac{20}{50}\right)^{1/7} \times 13 = 11.405 \text{ m/s}$$

The time that the wind speed is below 11.405 m/s is calculated from

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right] = 1 - \exp\left[-\left(\frac{11.405}{8.0}\right)^{2}\right]$$

$$F(U) = 0.869, \text{ or } 7612.2 \text{ hours/year}$$

No of hours wind turbine operates above rated speed = 8688.65 – 7612.2 = 1076.45 hrs

Total energy 10 wind turbines will produce = 10 x 2 MW x 1076.45 = 21.5 GWh

13. Estimate the Weibull parameters of the following set of hourly mean wind speed measured over one year.

Wind speed (m/s)	Hours
0-4	2050
4-8	4000
8-12	2200
12-16	460
16-20	50

[Answer: c 7.2, k 2]

Note: Graph Paper is provided and the following may be useful:

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^k\right]$$

Solution:

$$\ln\left[-\ln\left(1 - F\left(U\right)\right)\right] = k\ln U - k\ln c$$

Wind speed (m/s)	Hours	ln (U)	ln(-ln(1-F(U))
0-4	2050	1.386	-1.24
4-8	4000	2.079	0.11
8-12	2200	2.485	0.99
12-16	460	2.772	1.73
16-20	50	2.995	10

Plotting ln(*U*) versus ln[-ln(1-F(*U*))] gives a graph similar to that of Figure 2.22 and so allows estimation of *c* and *k*.

U	Hours	F(U)	1-F(U)	ln(1-F(U))	ln(U)	ln(-ln(1-F(U)))
4	2200	0.251	0.749	-1.382	1.386	-1.241
8	3700	0.674	0.326	-0.395	2.079	0.113
12	2280	0.934	0.066	-0.069	2.485	0.999
16	550	0.997	0.003	-0.003	2.773	1.736
20	30	1.000	0.000	0.000	2.996	

14. The annual distribution of hub height hourly mean wind speeds at a site may be described by a Weibull distribution with:
    Scale parameter, c, of 8.5
    Shape parameter, k, of 2

    Calculate the gross annual energy yield of a wind turbine with a Power Curve shown in the Table, installed at this site.

    [Answer: 2191 MWh]

Wind speed m/s	Output kW
0-5	0
6	50
7	120
8	280
9	440
10-25	600

The following are likely to be useful

$$f(U) = \frac{k}{c}\left(\frac{U}{c}\right)^{k-1} \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

$$F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

Solution:

	8.5	2		
U	P	f(U)/F(U)	Hours/year	kWh
6	50	0.101	884	44200
7	120	0.098	862	103380
8	280	0.091	800	223996
9	440	0.081	711	312969
10+	600	0.287	2512	1507172
				2191717