Exercise 2.2
Example solutions

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Exercise 2.2a Wind Shear

Estimate the wind shear and compare the data with the 1/7th power law. Estimate best power law coefficient to fit the data

Data – Hourly mean wind velocity data at 3 heights – 15m, 40m and 50m (***Ex 2_1 Data.xlsx***)

Then calculate yearly mean velocities for the heights -

For 50m height, the yearly mean wind speed AN50y,

$$AN50y = \frac{\sum_{i=1}^{8760}(AN50h_i)}{8760} = 7.2278\, m/s$$

- $AN50h_i$ represent the wind speed value for the hour i, since one year is considered and therefore we have (24 x 365) 8760 values.
- Similarly, for other heights the yearly average wind speed is calculated. (AN15y, AN40y).
- The four points [0, AN15y, AN40y, AN50y] = [0, 6.4543, 7.0467, 7.2278] are plotted against the height, [0,15,40,50].

Then calculate the wind velocities at different heights,

using the 1/7th power law.

$$u = u_r \left(\frac{z}{z_r}\right)^{1/7}$$

Taking the reference as, $z_r = 15m$, $u_r = 6.4543\ m/s$

Plot the calculated wind velocities against the height.

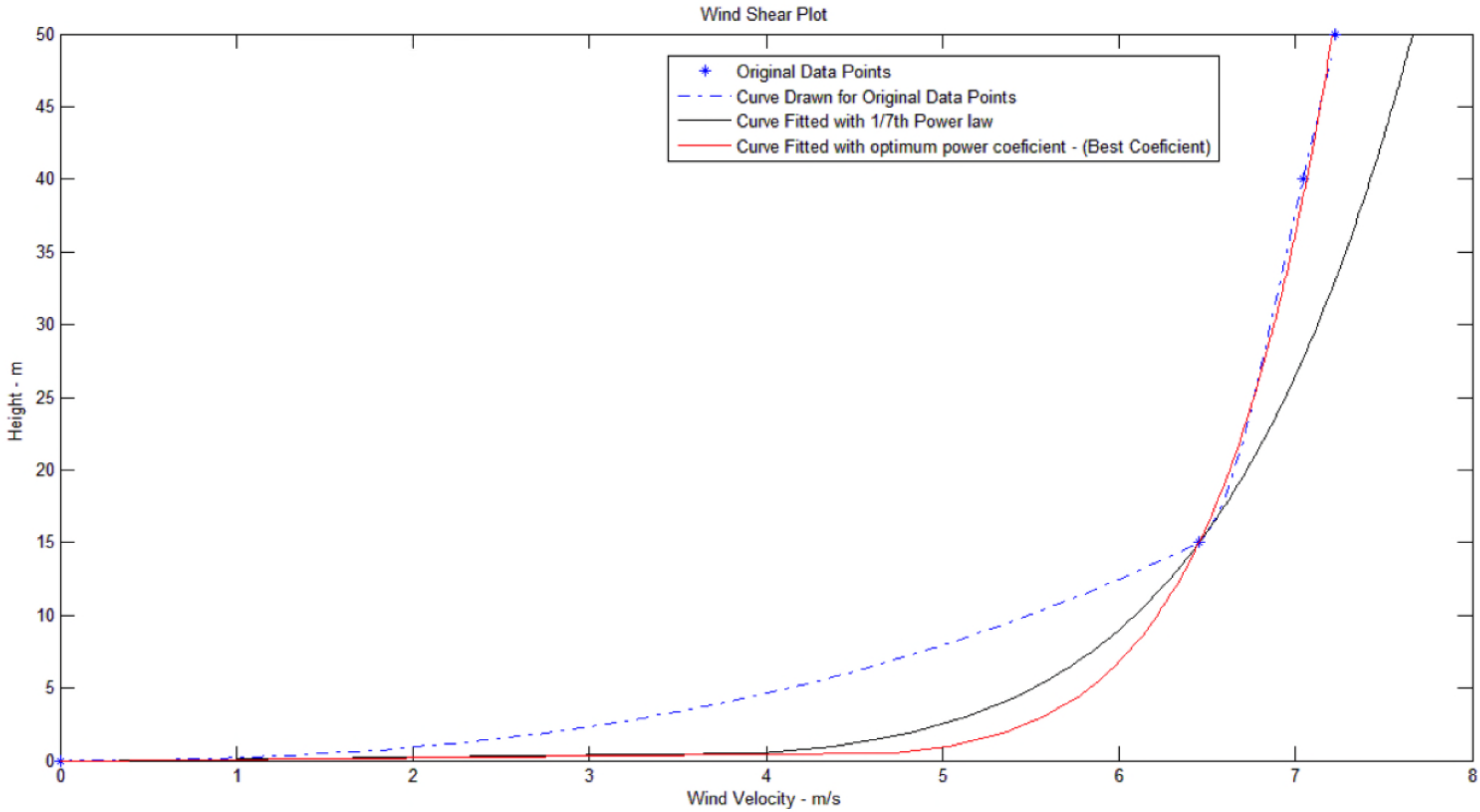
Estimate the best power coefficient to fit the data.

- Consider the general power law, $u = u_r\left(\frac{z}{z_r}\right)^a$
- Take the reference as, $z_r = 15m$, $u_r = 6.4543\ m/s$ and by taking the log in the base 10, the equation becomes,

$$\log 10(\mathrm{u}) = \mathrm{y} = 0.81 + \mathrm{a} \times \log 10(\mathrm{z}) - 1.176 \times \mathrm{a} \qquad \text{(A)}$$

- Original points, $\mathrm{y1} = \log 10(7.0467)$ and $\mathrm{y2} = \log 10(7.2278)$.
- Error function, $\mathrm{S} = (\mathrm{y1} - \mathrm{y11})^2 + (\mathrm{y2} - \mathrm{y22})^2$
- Calculate $\mathrm{y11}$ and $\mathrm{y22}$ from equation A, for $\mathrm{z} = 40$and $\mathrm{z} = 50$ respectively.
- Find the minimum of S for a range of power coefficients, a .
  ($\boldsymbol{S\_\mathbf{min}} = \mathbf{2.2656 \times 10^{-6}}$)
- At the minimum S, by back calculating the corresponding a value is found.

$$\boldsymbol{Optimum\ power\ Coefficient = 0.092\ \cong \frac{1}{10}}$$



Wind Shear Plot
Original Data Points
Curve Drawn for Original Data Points
Curve Fitted with 1/7th Power law
Curve Fitted with optimum power coeficient - (Best Coeficient)
Height - m
Wind Velocity - m/s
0
5
10
15
20
25
30
35
40
45
50
1
2
3
4
5
6
7
8

Ex.2.2b Weibull parameters

Estimate the Weibull Parameters of the wind speed data.

Approach –

- Ex. For 3 m/s bin, calculating Probability Density and Cumulative Density.
- f (3) $= \dfrac{Occurance\ in\ considered\ year}{8760}$
- F (3) = f (1) + f (2) +f (3)
- Similarly, for bins 1 m/s to 25 m/s Probability Density and Cumulative Density are calculated.

Manual calculation of Weibull parameters for the hourly averaged wind speed data.

- Consider equation, $F(U) = 1 - \exp\left[-\left(\frac{U}{c}\right)^{k}\right]$ for calculating Cumulative Density.
- Taking natural log twice, the equation becomes,

$$\ln\left[-\ln\left(Q(U)\right)\right] = k \times \ln(U) - k \times \ln(c)\,, \quad \text{here } Q(U) = 1 - F(U)$$

- Calculate $\ln(U)$ and $\ln\left[-\ln\left(Q(U)\right)\right]$ separately for each wind speed bin. (1 – 25 m/s)

- Plot $\ln\left[-\ln\left(Q(U)\right)\right]$ vs. $\ln(U)$ and a linear function ($y = mx + n$) is fitted to the points.

- When plotted the coefficients of the linear equation becomes, $m = k$ and $n = - k \times \ln(c)$

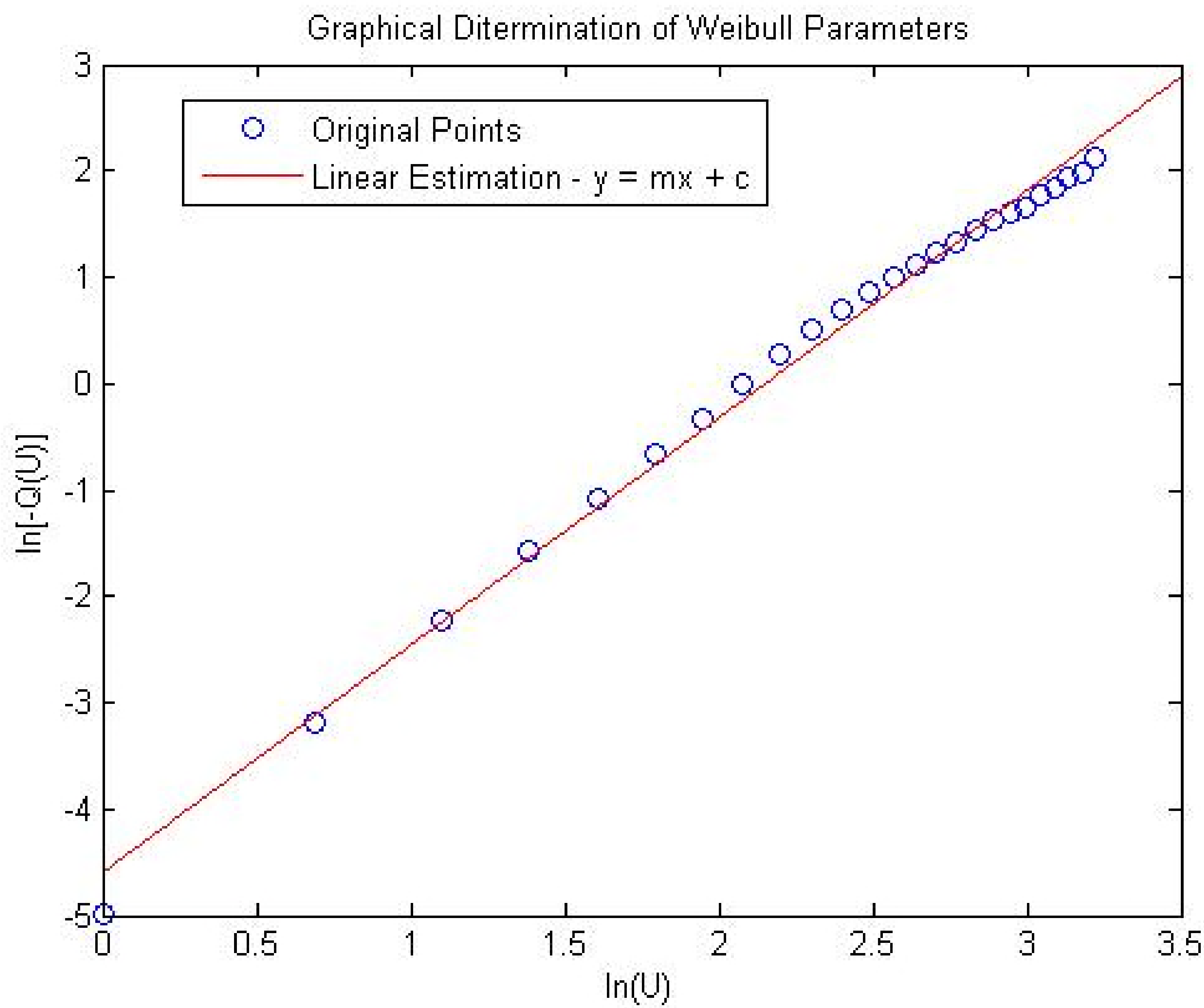


Figure 1 - Graphical Determination of Weibull Parameters for 50m data

From the graph,

m = **k = 2.1364** and

n = $-\mathrm{k}\times\ln(\mathrm{c})$ = - 4.6006 which will give **c = 8.6142**

Using the calculated parameters and considering equations,

$$f(U)=\frac{k}{c}\left(\frac{U}{c}\right)^{k-1}\times exp\left[-\left(\frac{U}{c}\right)^{k}\right]$$

and

$$\mathrm{F(U)}\ =\ 1\ -\ \exp\left[-\left(\frac{\mathrm{U}}{\mathrm{c}}\right)^{\mathrm{k}}\right]$$

the Probability Density and Cumulative Density are calculated and plotted.

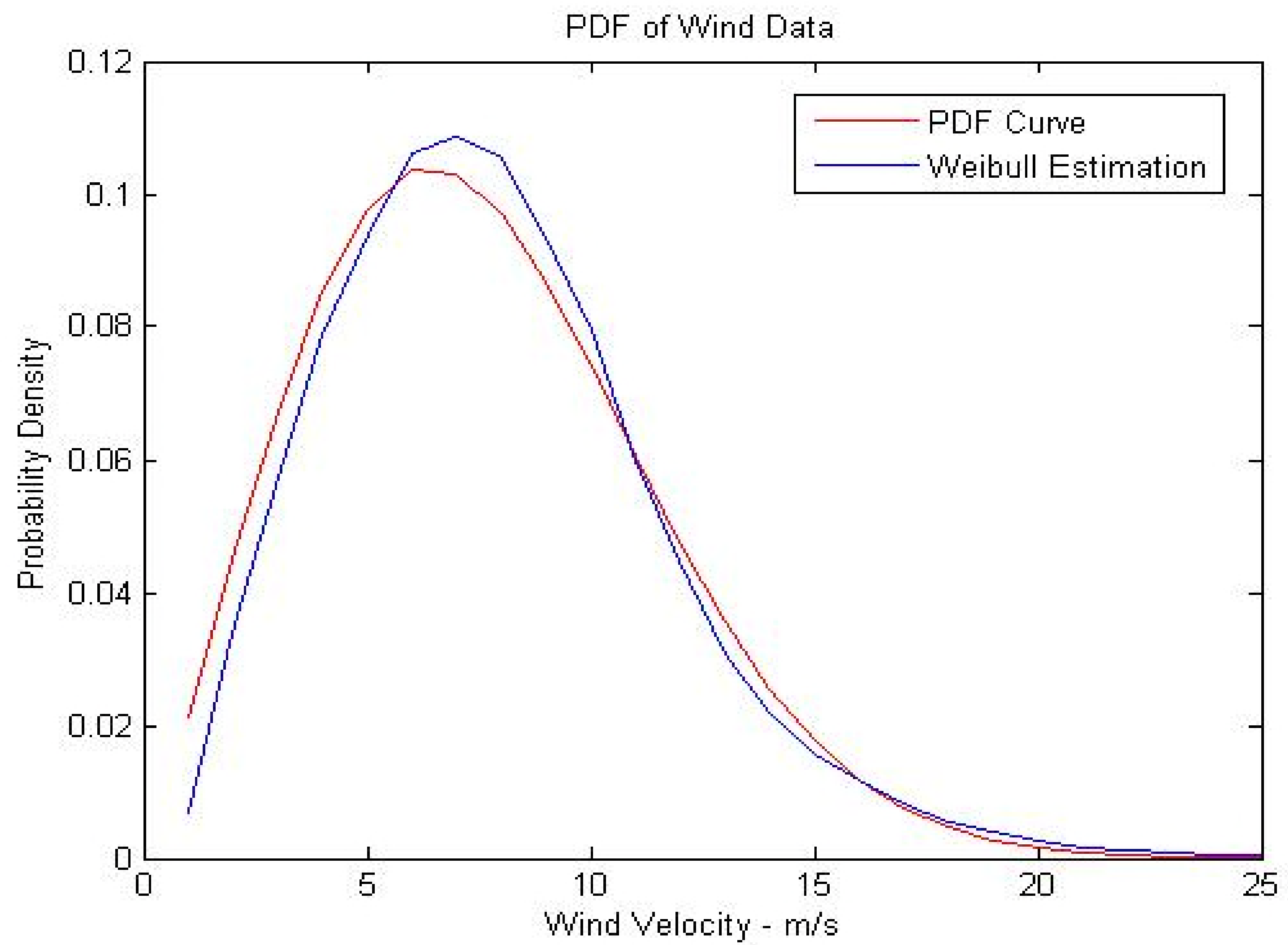


Figure 2 – PDF plot with Weibull Estimation for 50m Data

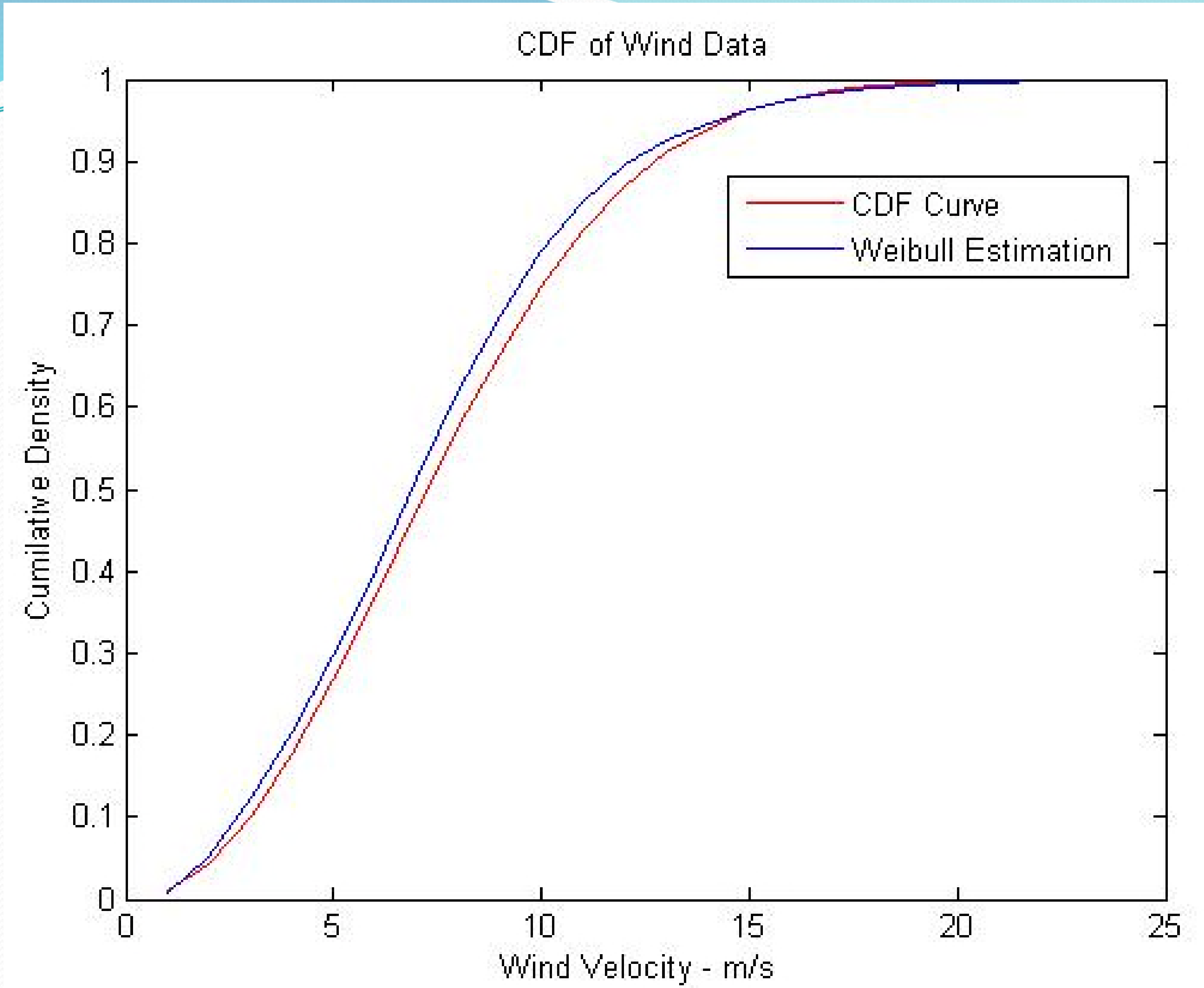


Figure 3 - CDF plot with Weibull Estimation for 50m Data