Renewable Energy Engineering

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Simplification of variance.

Show that

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(U_i - \bar{U}\right)^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} U_i^{\,2} - n\bar{U}^2\right]$$

$$\begin{aligned}\sum_{i=1}^{n}\left(U_i - \bar{U}\right)^2 &= \sum_{i=1}^{n}\left(U_i^2 - 2U_i\bar{U} + \bar{U}^2\right)\\ &= \sum_{i=1}^{n} U_i^2 - 2\bar{U}\sum_{i=1}^{n} U_i + n\bar{U}^2\\ &= \sum_{i=1}^{n} U_i^2 - 2\bar{U}n\bar{U} + n\bar{U}^2\\ &= \sum_{i=1}^{n} U_i^2 - n\bar{U}^2\end{aligned}$$

and so

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n} U_i^2 - n\bar{U}^2$$