

2 Slope Stability

Planar Block Slides

1. Given: Planar block slide
Find: Algebraic FS .

Solution:

By definition

$$FS = \frac{R}{D}$$

R = resisting forces: Assume Mohr-Coulomb

D = driving forces

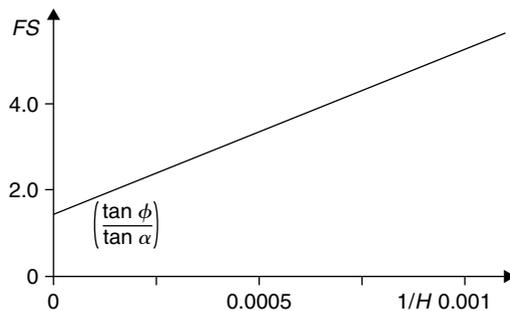
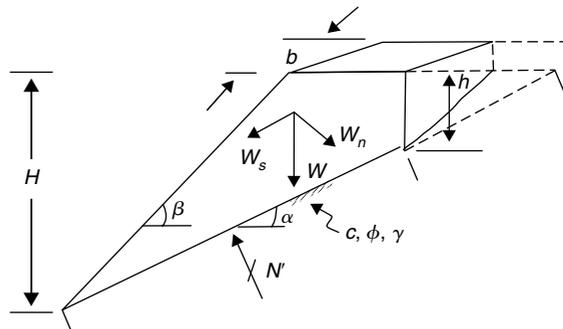
W = weight: $W = W_1 - W_2$

By inspection:

$$W = \frac{\gamma H^2 b}{2} (\cot \alpha - \cot \beta) - \frac{\gamma b^2 b}{2} \cot \alpha$$

$$R = W_n \tan \phi + cA$$

Note: $\Sigma F_n = 0 \quad \therefore 0 = W_n - N' \text{ \& } N' = W_n$



SCHMATIC PLOT (FS vs H^{-1})

Note: almost linear in $1/H$

Also

$$A = b \left(\frac{H}{\sin \alpha} - \frac{b}{\sin \alpha} \right)$$

And $W_n = W \cos \alpha$

$$\therefore R = W \cos \alpha \tan \phi + cA$$

$$D = W \sin \alpha$$

$$FS = \frac{W \cos \alpha \tan \phi + cA}{W \sin \alpha}$$

$$= \frac{\tan \phi}{\tan \alpha} + \frac{cb \left(\frac{H}{\sin \alpha} - \frac{b}{\sin \alpha} \right)}{(\sin \alpha) \left[\left(\frac{\gamma H^2 b}{2} \right) (\cot \alpha - \cot \beta) - \frac{\gamma b^2 b}{2} \cot \alpha \right]}$$

$$= \frac{\tan \phi}{\tan \alpha} + \frac{2c \left(1 - \frac{b}{H} \right)}{(\sin \alpha)^2 \left[(\gamma H) \left[(\cot \alpha - \cot \beta) - \frac{b^2}{H^2} \cot \alpha \right] \right]}$$

$$FS = \frac{\tan \phi}{\tan \alpha} + \frac{2c \left(1 - \frac{b}{H} \right)}{\gamma H (\sin \alpha)^2 \left[\left(1 - \left(\frac{b}{H} \right)^2 \right) \cot \alpha - \cot \beta \right]}$$



2. Given: Problem 1 data
Find: Max β (algebraically).

Solution:

Solve FS for β then set $\frac{df}{d\beta} = 0$

Find β^* stationary pts

Find relative maximum

Find absolute maximum (end pts) or note by inspection that as β increases the FS decreases so β will be max when FS is min, that is, when $FS = 1$, then

$$[\gamma H(\sin \alpha)^2] \left[\left(1 - \left(\frac{b}{H} \right)^2 \right) \cot \alpha - \cot \beta \right] = \frac{2c \left(1 - \frac{b}{H} \right)}{\left(1.0 - \frac{\tan \phi}{\tan \alpha} \right)}$$

$$\cot \beta_{\max} = \left(1 - \left(\frac{b}{H} \right)^2 \right) \cot \alpha - \frac{2c \left(1 - \frac{b}{H} \right)}{\gamma H(\sin \alpha)^2 \left(1.0 - \frac{\tan \phi}{\tan \alpha} \right)}$$

Note: $\frac{\pi}{2} \geq \beta > \alpha$ (physical constraints) & if no tension crack ($b = 0$)

$$\cot \beta_{\max} = \cot \alpha - \frac{2c}{\gamma H(\sin \alpha)^2 \left(1 - \frac{\tan \phi}{\tan \alpha} \right)}$$

3. Given: Problem 1, data allow for water (below bottom of tension crack)
 Find: FS (with water).

Solution:

$$\text{(previous) } FS = \frac{W \cos \alpha \tan \phi + cA}{W \sin \alpha}$$

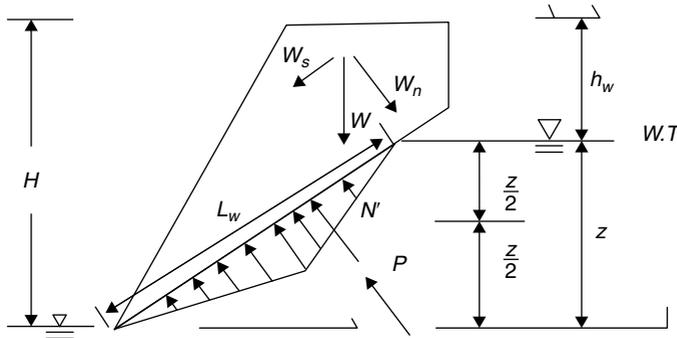
$$\text{where } W = \frac{\gamma H^2 b}{2} (\cot \alpha - \cot \beta) - \frac{\gamma h^2 b}{2} \cot \alpha$$

$$\text{and } A = \frac{b}{\sin \alpha} (H - h)$$

Now

$$FS = \frac{N' \tan \phi + cA}{W \sin \alpha} \quad \leftarrow (3)$$

where $\Sigma F_n = 0$ requires $N' = W_n - P$ with $P =$ water force



as before $W_n = W \cos(\alpha)$

but $P = ?$

$$P = \bar{p} A_w, \quad A_w = b L_w$$

$$\bar{p} = \frac{p_{\max}}{2} (\text{rectangle})$$

$p_{\max} = \gamma_w z/2$ (a linear increase of water pressure with depth below water table is assumed)

$$\frac{z}{2} = \left(\frac{1}{2}\right)(H - h_w)$$

where $h_w =$ water table depth below crest

$$L_w = \frac{z}{\sin \alpha}$$

$$P = \left(\frac{\gamma_w}{2}\right) \left(\frac{z}{2}\right) (b)(L_w)$$

$$P = \frac{\gamma_w b z^2}{4 \sin \alpha}$$

Hence:

$$FS = \frac{(W \cos \alpha - P) \tan \phi + cA}{\underline{\underline{W \sin \alpha}}} \quad \leftarrow (4)$$

where:

$$P = \frac{\gamma_w b z^2}{4 \sin \alpha}; \quad z = H - h_w$$

$$A = \frac{b}{\sin \alpha} (H - h); \quad b = \text{tension crack depth}$$

$$W = \frac{\gamma H^2 b}{2} (\cot \alpha - \cot \beta) - \frac{\gamma b^2 b}{2} \cot \alpha$$

4. Given: Problem data
Find: FS for seismic load.

Solution:

By definition $S = \frac{W}{g} a_s$

where a_s = seismic acceleration

i.e. $a_s = a_0 g$

where a_0 = seismic coefficient

e.g. $a_0 = 0.15$

$$\therefore S = a_0 W$$

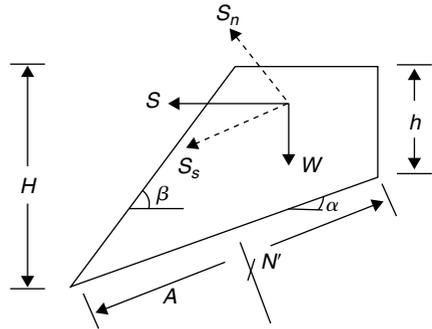
$$\underline{S_n = a_0 W \sin \alpha}$$

$$\underline{S_s = a_0 W \cos \alpha}$$

$$N' = W_n - S_n$$

$$R = N' \tan \phi + cA$$

$$D = W_s + S_s$$



$$\underline{\underline{FS = \frac{(W_n - S_n) \tan \phi + cA}{W_s + S_s}}}$$

← (5)

where $W_s = W \sin \alpha$,

$$S_s = a_0 W \cos \alpha$$

$$W_n = W \cos \alpha$$

$$S_n = a_0 W \sin \alpha$$

$$A = \frac{b}{\sin \alpha} (H - h)$$

$$W = \frac{\gamma b^2 H^2}{2} (\cot \alpha - \cot \beta)$$

$$N' \tan \phi + cA - FS(W_s) = 7.3001(10^7) + 1.4002(10^8) - 1.2156(10^8)$$

$$F = \frac{[9.146(10^7) \text{ lbf}]}{[(\sin 40)(1.1) - \cos 40 \tan 29]}$$
$$= \frac{9.146(10^7)}{0.2824}$$
$$\therefore F = 3.238(10^8) \text{ lbf}$$

$$\sigma = \frac{3.238(10^8)}{(25)(500)(\cot \alpha - \cot \beta)}$$

$$\sigma = 7.346(10^4) \text{ psf} \quad \longleftarrow$$

$$\underline{\underline{\sigma = 510 \text{ psi}}}$$

FS without surcharge.

$$FS = \frac{[N' \tan \phi + cA]}{W_s}$$
$$= \frac{1.317(10^8) \tan 29 + 1.4002(10^8)}{1.719(10^8) \sin 40}$$
$$\underline{\underline{FS = 1.93(1.928)}} \quad \longleftarrow$$

6. Given: Fig. 5 planar block slide and *no* surcharge

Find: H_{\max} .

Solution:

H_{\max} occurs when FS is min i.e., when

$$1 = \frac{R}{D} \quad (1)$$

$$R = N' \tan \phi + cA$$

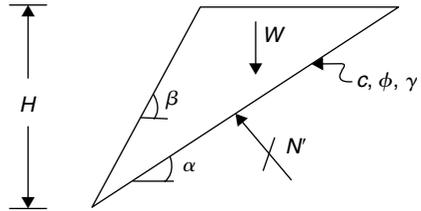
$$D = W_s$$

$$W_s = W \sin \alpha$$

$$W_b = W \cos \alpha = N'$$

$$A = b \frac{H}{\sin \alpha}$$

$$W = \frac{\gamma b H^2}{2} (\cot \alpha - \cot \beta)$$



Solving (1)

$$1 = \frac{\tan \phi}{\tan \alpha} + \frac{Rb \left(\frac{H}{\sin \alpha} \right)}{\frac{\gamma b H^2}{2} (\cot \alpha - \cot \beta) \sin \alpha}$$

$$H_{\max} = \left[\frac{2c}{\gamma (\sin \alpha)^2 (\cot \alpha - \cot \beta)} \right] \left[\frac{1}{1 - \frac{\tan \phi}{\tan \alpha}} \right]$$

$$= \frac{(2)(50)(144)}{(156)(\sin 40)^2 (\cot 40 - \cot 50) \left(1 - \frac{\tan 29}{\tan 40} \right)}$$

$$\underline{\underline{H_{\max} = 1,867 \text{ ft}}}$$

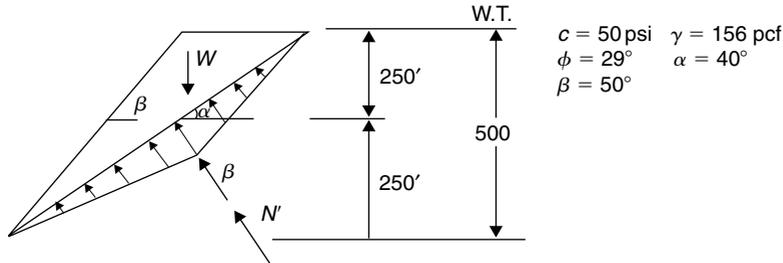
←

$$\begin{aligned}FS &= FS^\circ + \frac{(N_b \tan \phi + T_b)(0.6)}{W \sin \alpha} \\&= 1.928 + \frac{(0.6) \left[(10^6) \left(\frac{1}{\sqrt{2}} \right) (4.956) \tan 29 + 10^6 \left(\frac{1}{\sqrt{2}} \right) \right]}{(\sin 40) 1.719(10^8)} \\&= 1.928 + \frac{5.447(10^6)}{1.105(10^8)} 0.6 \\&= 1.928 + (0.049)(0.6) \\&= \underline{\underline{FS = 1.958, \Delta F = 0.30}}\end{aligned}$$

8. Given: Planar block slide data and water table at crest

Find: FS .

Solution:



$$FS = \frac{R}{D}$$

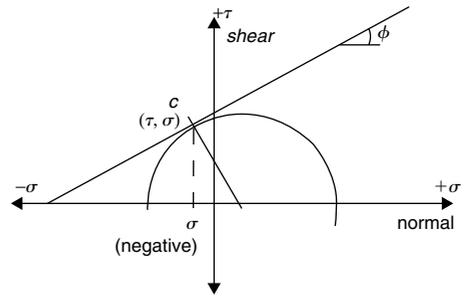
$$R = N' \tan \phi + cA$$

$$D = W_s$$

$$N' = W_n - p$$

$$W = \frac{\gamma b H^2}{2} (\cot \alpha - \cot \beta)$$

$$\left. \begin{array}{l} W_n = 1.317(10^8) \text{ lbf} \\ W_s = 1.105(10^8) \text{ lbf} \\ cA = 1.400(10^8) \text{ lbf} \end{array} \right\} \text{ (prior calculations)}$$



Note: (normal load is negative)

$$P = \bar{p} A_w$$

$$A_w = \frac{bH}{\sin \alpha}$$

$$\bar{p} = \frac{p_{\max}}{2}$$

$$p_{\max} = \frac{\gamma_w z}{2}$$

$$\frac{z}{2} = 250'$$

$$P = (67.4) \left(\frac{250}{2} \right) (25) \left(\frac{500}{\sin 40} \right)$$

$$P = 1.5168(10^8)$$

$$R = N' \tan \phi + cA$$

$$= (1.317 - 1.517)10^8 \tan 29 + 1.400(10^8)$$

$$R = -1.109(10^7) + 1.4(10^8)$$

$$D = 1.105(10^8)$$

$$FS = \frac{1.289(10^8)}{1.105(10^8)}$$

$$FS = 1.167$$

9. Given: Planar block slide in sketch with cohesion destroyed

Find: acceleration a

Solution:

Assume $\sigma = 0$ (no surcharge)

$$\text{then } FS = \frac{R}{D}$$

But also for the slide mass center

$$F = ma$$

$$m = \frac{W}{g}$$

$$F = D - R$$

$$\therefore D(1 - FS) = \left(\frac{W}{g}\right)a$$

$$\therefore a > 0 \text{ if } FS < 1$$

$$\begin{aligned} FS &= \frac{N' \tan \phi}{W_s} \\ &= \frac{W \cos \alpha \tan \phi}{W \sin \alpha} \\ &= \frac{\tan \phi}{\tan \alpha} \\ &= \frac{\tan 29}{\tan 40} \\ FS &= 0.66 \end{aligned}$$

acceleration – yes downhill = parallel to slide surface

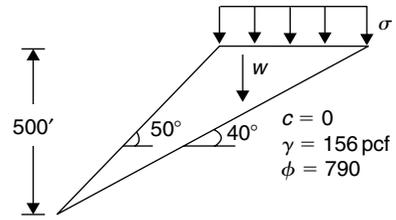
$$FS = 0.66$$

$$\text{then } D(1 - 0.66) = \frac{W}{g}a$$

$$D = W \sin \alpha$$

$$\begin{aligned} \therefore a &= g \sin \alpha (1 - 0.66) \\ &= 32.2(\sin 40)(0.34) \end{aligned}$$

$$\underline{\underline{a = 7.04 \text{ ft/s}^2}}$$



← tangential direction
(parallel to failure surface)

10. Given: Planar block slide

Find: α for $FS = 1.5$.

Solution:

(Free body)

$$\Sigma_n F = 0 \quad W_n - N' - P = 0$$

$$N' = W_n - P$$

$$W_s = W \sin \alpha \quad W_n = W \cos \alpha$$

$$W = \frac{\gamma b H^2}{2} (\cot \alpha - \cot \beta)$$

$$= (150)(1) \left(\frac{475}{2} \right)^2 (\cot 35 - \cot 45)$$

$$= 7.728$$

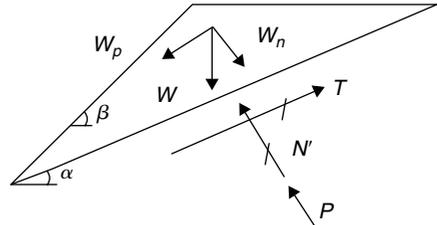
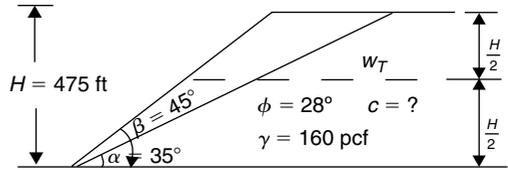
$$W = 7.252(10^6) \text{ lbf}$$

$$W_n = 7.252(10^6) \cos 35$$

$$W_n = 5.941(10^6) \text{ lbf}$$

$$W_s = 7.252(10^6) \sin 35$$

$$W_s = 4.156(10^6) \text{ lbf}$$



Water force must check for water head if $\tan \beta > 2 \tan \alpha$, then $h_w = \frac{z}{2}$ else

$$h_w = \frac{z}{2} \left(-1 + \frac{\tan \beta}{\tan \alpha} \right)$$

check

$$\tan 45 > 2 \tan 35$$

$$(1) > 2(0.700)$$

no.

$$\therefore h_w = \left(\frac{475}{2} \right) \left(\frac{1}{2} \right) \left(-1 + \frac{\tan 45}{\tan 35} \right)$$

$$\underline{\underline{h_w = 50.84 \text{ ft}}}$$

$$p_{\max} = (62.4)(50.84)$$

$$\bar{p} = \frac{p_{\max}}{2} = \frac{(62.4)(50.84)}{2}$$

$$\bar{p} = 1.586(10^3) \text{ psf}$$

$$P = \bar{p}A$$

$$= 1.586(10^3)(1) \left(\frac{H_w}{\sin \alpha} \right)$$

$$= 1.586(10^3)(1) \left(\frac{\frac{475}{2}}{\sin 35} \right)$$

$$\underline{\underline{P = 6.567(10^5) \text{ lbf}}}$$

$$N' = 5.941(10^6) - 0.6567(10^6)$$

$$\underline{N' = 5.284(10^6)}$$

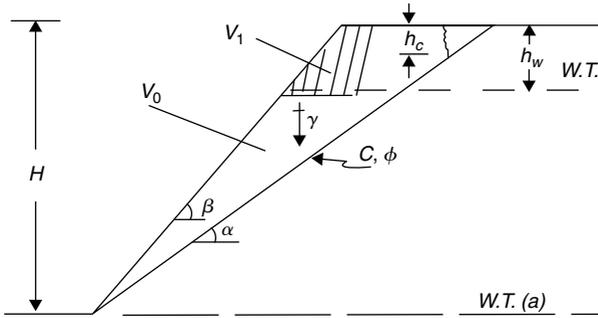
$$FS = \frac{N' \tan \phi + cbL}{W_s}$$

$$cbL = (1.5)(4.156)(10^6) - \tan 28(5.284)10^6$$
$$= 3.424(10^6)$$

$$c = \frac{3.424(10^6)}{\frac{(1)(475)}{\sin 35}}$$

$$\underline{\underline{c = 4.135(10^3) \text{ psf}}}$$

11. Given: Planar block slide in sketch:



Find:

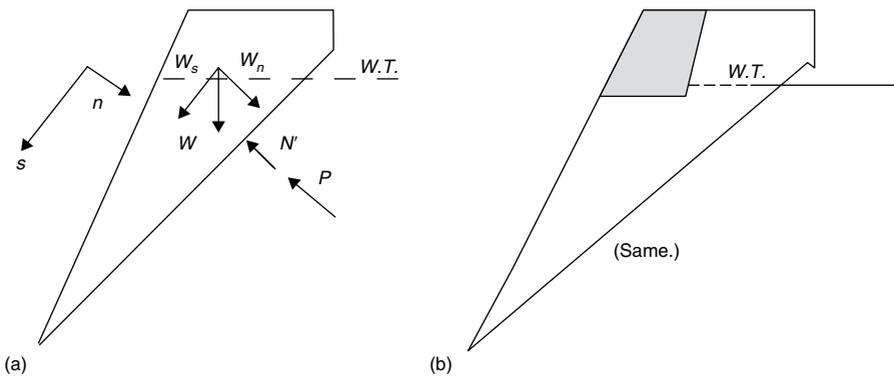
- (a) Formula for FS with relieving bench V_1
- (b) Formula for FS with toe berm

Solution:

By definition: $FS = \frac{R}{D}$

R = resisting forces

D = driving forces



Assume M-C criterion

$$FS_a = \frac{N' \tan \phi + C}{W \sin \alpha}$$

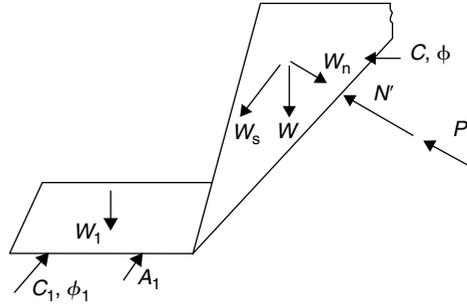
$$\therefore FS_a = \frac{\tan \phi}{\tan \alpha} + \frac{C}{\gamma V_0 \sin \alpha}$$

$$FS_b = \frac{N' \tan \phi + C}{W \sin \alpha}$$

$$FS_b = \frac{\tan \phi}{\tan \alpha} + \frac{C}{\gamma (V_0 - V_1) \sin \alpha}$$

$$\begin{aligned}
 C &= cA, \\
 A &= \text{area of failure surface} \\
 N' &= (W \cos \alpha - P) \\
 W &= \gamma V_0 \\
 \therefore FS_a &= \frac{(W \cos \alpha - P) \tan \phi + C}{W \sin \alpha}
 \end{aligned}$$

But $P = 0$ when W.T. is below to toe.
 The first terms are the same, but the second term in FS_b is greater because $V_0 - V_1$ is less than V_0 is FS .



$$FS = \frac{(N' \tan \phi + C) + W_1 \tan \phi + C_1}{W_s} \quad (a)$$

$$FS(\text{with berm}) = FS(\text{without berm}) + \frac{W_1 \tan \phi_1 + c_1 A_1}{W_s} > FS(\text{without berm}) \quad (b)$$

The added resistance $W_1 \tan \phi_1 + C_1$, comes without added driving force and thus increases the FS .

12. Given: sketch, data

No tension crack, no benches, $\alpha = 29^\circ$, $\gamma = 156$ pcf, $\beta = 50^\circ$, persistence = 0.87

$c_r = 64,800$ psf $c_j = 1,620$ psf

$\phi_r = 32^\circ$ $\phi_j = 25^\circ$

Find:

(a) H_{\max} when WT at crest

(b) H_{\max} when WT at toe

Solution:

$$FS = \frac{R}{D}$$

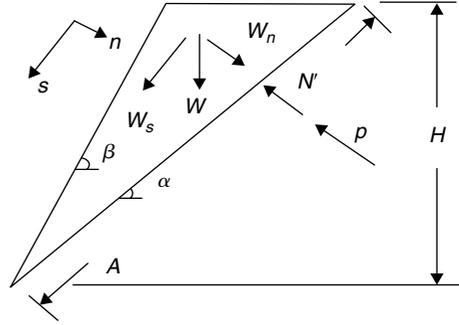
$$D = W \sin \alpha$$

$$R = N' \tan \phi + C$$

$$N' = (W_n - p)$$

$$C = cA$$

$$W_n = W \cos \alpha$$



(b) When depressurized $P = 0$

$$W = \frac{\gamma H^2}{2} (\cot \alpha - \cot \beta)$$

$$A = \frac{H}{\sin \alpha}$$

$$FS = \frac{W \cos \alpha \tan \phi}{W \sin \alpha} + \frac{\frac{cH}{\sin \alpha}}{W \sin \alpha}$$

$$FS = \frac{\tan \phi}{\tan \alpha} + \frac{\frac{cH}{\sin \alpha}}{\frac{\gamma H^2}{2} (\cot \alpha - \cot \beta) \sin \alpha}$$

$$\therefore \left(FS - \frac{\tan \phi}{\tan \alpha} \right) \left(\frac{\gamma}{2} \right) \left(\frac{\cot \alpha - \cot \beta}{\frac{c}{\sin^2 \alpha}} \right) = \frac{1}{H}$$

By inspection $\frac{1}{H}$ is minimum when FS is minimum

$\therefore H_{\max}$ occurs of $FS = 1$.

need rock mass c , ϕ

$$c = (1 - p)c_r + pc_j$$

$$= (1 - 0.87)64,800 + 0.87(1,620)$$

$$= 8,424 + 1,409$$

$$\underline{c = 9,833 \text{ psf}}$$

$$\tan \phi = (1 - p) \tan \phi_r + p \tan \phi_j$$

$$= (1 - 0.87) \tan 32 + 0.27 \tan 25$$

$$\tan \phi = 0.08123 + 0.4057$$

$$= 0.4829$$

$$\underline{\underline{\phi = 25.96 \approx 26^\circ}}$$

$$\underline{\underline{[0.5502] \left[\left(\frac{156}{2} \right) \frac{\cot 29 - \cot 50}{(9,833) \left(\frac{1}{\sin^2 29} \right)} \right] = \frac{1}{H}}}$$

$$9.899(10^{-4}) = \frac{1}{H}$$

$$\underline{\underline{H = 1,010 \text{ ft}}}$$

← $H_{\max \text{ wet}}$

13. Given: Planar block slide

$$\gamma = 158 \text{ pcf}$$

$$\left. \begin{array}{l} \phi_n = 38^\circ \\ c_r = 1,000 \text{ psi} \\ A_j/A = 0.93 \end{array} \right\} \text{Mohr-Coulomb}$$

$$\left. \begin{array}{l} \phi_j = 27^\circ \\ c_j = 15 \text{ psi} \end{array} \right\}$$

bench height = 55 ft

b = breadth

Find: H_{\max} .

Solution:

H_{\max} occurs when $FS = 1$

$$FS = \frac{R}{D}$$

$$R = W'_n \tan \phi + cLb$$

$$D = W_s$$

$$\therefore W_s = W_n \tan \phi + cLb$$

$$W \sin \alpha = (W \cos \alpha - P) \tan \phi + cLb$$

$$\begin{aligned} W &= \frac{\gamma H^2}{2} (\cot \alpha - \cot \beta) \\ &= (158) \left(\frac{1}{2} \right) H^2 (\cot 32 - \cot 49) \\ \underline{W} &= \underline{57.75 H^2} \end{aligned}$$

$$\begin{aligned} 57.75 H^2 \sin 32 &= 57.75 H^2 \cos 32 \tan \phi - P \tan \phi + \frac{cH(1)}{\sin 32} \\ \underline{\underline{30.60}} &= \underline{\underline{48.98 \tan \phi + \frac{c}{H} 1.887 - \frac{P \tan \phi}{H^2}}} \end{aligned}$$

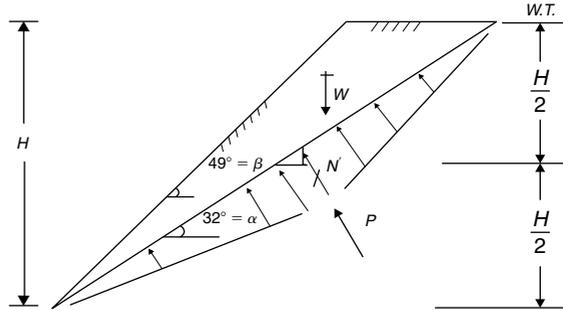
need, ϕ , c for rock mass

$$\begin{aligned} c &= (1 - 0.93)c_r + 0.93c_j \\ &= 0.07(1,000) + 0.93(10.0) \\ \underline{\underline{c}} &= \underline{\underline{79.3 \text{ psi (11,719 psf)}}} \end{aligned}$$

$$\begin{aligned} \tan \phi &= 0.07 \tan 38 + 0.93 \tan 27 \\ \underline{\underline{\tan \phi}} &= \underline{\underline{0.5286}} \quad \phi = 27.9^\circ \end{aligned}$$

return

$$30.60 = 48.98(0.5286) + \frac{(11,419)(1.887)}{H} - \frac{P}{H^2}(0.5286)$$



need: $P = \bar{p}Lb$

$$= \left(\frac{\gamma_w H}{4} \right) \left(\frac{H}{\sin \alpha} \right) \quad (1)$$

$$\frac{P}{H^2} = \frac{(62.4)}{4} \frac{H^2}{\sin \alpha}$$

$$30.60 = 25.89 + \frac{1}{H}(22,812) - \frac{62.4}{4 \sin 32}(0.5286)$$

$$\frac{1}{H} = \frac{30.60 - 25.89 + 15.56}{21,549}$$

$$\underline{\underline{H = 1,065 \text{ ft}}}$$

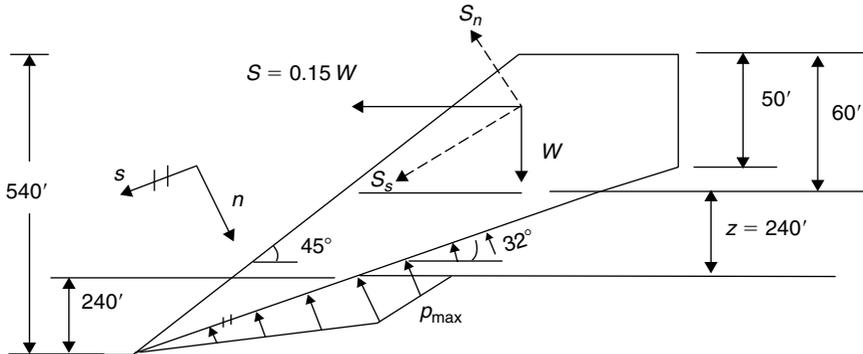
14. Given: Planar block slide data

$$FS(\min) = 1.05$$

$$\text{Bench height} = 60'$$

$$\text{Persistence} = 79\% \left(\frac{A_j}{A} \right)$$

$$W_t = 1.351(10^7) \text{ lbf per ft of thickness.}$$



$$\phi_r = 33^\circ \quad c_r = 2,870 \text{ psi} \quad \phi_j = 28^\circ \quad c_j = 10.0 \text{ psi} \quad \gamma = 158 \text{ pcf}$$

Find: If $FS = 1.05$ possible.

Solution:

Assume water distribution as $p = \gamma_w z$ where z is 1/2 distance (vertical) to toe

$$z = \frac{(540 - 60)}{2} \quad p_{\max} = \frac{(62.4)(240)}{144}$$

$$z = 240 \text{ ft,} \quad p_{\max} = 104 \text{ psi}$$

$$\bar{p} = \frac{p_{\max}}{2}$$

$$\bar{p} = 52 \text{ psi}$$

$$P = \bar{p}A$$

$$= \bar{p}(1) \left(\frac{480}{\sin(32)} \right) (144)$$

$$P = 6.783(10^6) \text{ lbf}$$

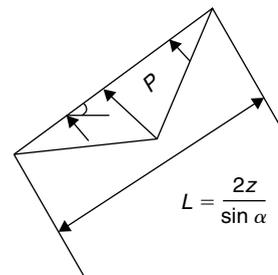
$$W_n = W \cos \alpha$$

$$= 1.351(10^7) \cos 32$$

$$W_n = 1.146(10^7) \text{ lbf}$$

$$W_s = 1.351(10^7) \sin 32$$

$$W_s = 7.159(10^6) \text{ lbf (no seismic force)}$$



$$\begin{aligned}
 W'_n &= W_n - P \\
 &= 1.146(10^7) - 6.783(10^6) \\
 W'_n &= 4.6745(10^8) \text{ lsf (no seismic force)}
 \end{aligned}$$

$$\begin{aligned}
 W'_n &= W_n - P - S_s \\
 S_n &= S \sin \alpha & S_s &= S \cos \alpha \\
 &= 0.15 W \sin 32 & &= 0.15 W \cos \alpha \\
 &= 0.15 W_s & &= 0.15 W_n \\
 &= 0.15 (7.159)(10^6) & &= 0.15 (1.146)10^7 \\
 S_n &= 1.074(10^6) \text{ lbf} & S_s &= 1.719 (10^6) \text{ lbf}
 \end{aligned}$$

$$\begin{aligned}
 W'_n &= 4.6745 - 1,074(10^6) \\
 W'_n &= 3,600(10^6) \text{ lbf} \quad \text{with seismic force}
 \end{aligned}$$

need: c, ϕ

$$\begin{aligned}
 c &= 0.79 c_j + 0.21 c_r \\
 &= (0.79)(10) + (0.21)(2870) \\
 &= 7.9 + 603 \\
 c &= 610.9 \text{ psi}
 \end{aligned}$$

$$\begin{aligned}
 \tan \phi &= (0.79) \tan 28 + (0.21) \tan 33 \\
 \tan \phi &= 0.556 \\
 \phi &= 29.1^\circ
 \end{aligned}$$

$$\begin{aligned}
 FS &= \frac{W'_n \tan \phi + cA}{W_s + S_s}, \\
 &= \frac{3.60(10^6) \tan 29.1 + (611)(144)(92.5)(1)}{7.159(10^6) + (1.719)(10^6)} \\
 &= \frac{2.004(10^6) + 8.139(10^7)}{8.878(10^6)} \\
 &= \frac{81.72(10^6)}{8.878(10^6)} \\
 FS &= 9.39 \quad \text{Yes.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Note:} \\
 L &= \frac{540 - 50}{\sin 32}
 \end{aligned}$$

Note: The actual water pressure is less because of the slope and failure surface angles, so FS is even higher.

15. Given: Planar block slide Mohr–Coulomb failure

Find: β_{\max} (dry).

Solution:

$$F_S = \frac{R}{D}$$

$$= \frac{N' \tan \phi + cA}{W_s}$$

$$N' = W_n$$

$$W_n = W \cos \alpha$$

$$A = \frac{H}{\sin \alpha} \quad (1 \text{ ft thick})$$

$$W_s = W \sin \alpha$$

$$\therefore F_S = \frac{\tan \phi}{\tan \alpha} + \frac{cH}{(\sin \alpha)^2 W}$$

$$\text{But } W = \frac{\gamma H^2}{2} (1)(\cot \alpha - \cot \beta)$$

So F_S is min when β is max.

$$F_{S_{\min}} = 1.0$$

$$\therefore 1 = \frac{\tan \phi}{\tan \alpha} + \frac{cH}{\sin^2(\alpha) W}$$

$$1 = \frac{\tan 30}{\tan 34} + \frac{(1,440 \text{ psf})(613 \text{ ft})(1 \text{ ft})}{\sin^2(34) W}$$

$$1 = 0.856 + \frac{2.823(10^6)}{W}$$

$$W = \frac{2.823(10^6)}{0.144}$$

$$W = 19.598(10^6) \text{ lbf}$$

$$W = \frac{\gamma H^2}{2} (\cot \alpha - \cot \beta)$$

$$\cot \alpha - \cot \beta = \frac{(2)(19.598)(10^6)}{(162)(613)^2}$$

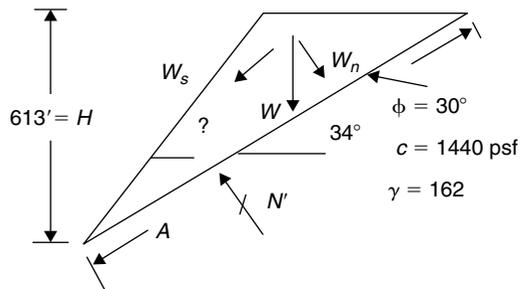
$$\cot \alpha - \cot \beta = 0.64387$$

$$\cot \beta = \cot 34 - 0.64357$$

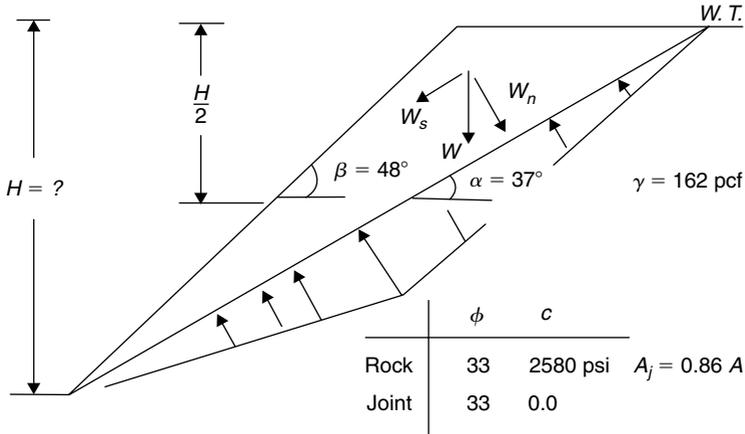
$$\cot \beta = 0.83869$$

$$\underline{\underline{\beta_{\max} = 50^\circ}}$$

← β_{\max}



16. Given: Slope, sketch, no seismic load, etc.



Find: H at $FS = 1.15$.

Solution:

$$FS = \frac{F_R}{F_D}$$

$$F_D = W_s$$

$$= W \sin \alpha$$

$$F_D = \frac{\gamma H^2}{2} (b)(\cot \alpha - \cot \beta) \sin(\alpha_j)$$

$$= H^2 \frac{(162)(1)}{2} (\cot 37 - \cot 48) \sin(37)$$

$$F_D = H^2 (81)(0.4266) \sin(37)$$

$$\underline{F_D = 20.80 H^2}$$

$$F_R = W'_n \tan \phi + cL$$

$$W'_n = W_n - P$$

$$W_n = W \cos \alpha$$

$$= H^2 (81)(0.4266) \cos(37)$$

$$\underline{W_n = 27.60 H^2}$$

$$P = \bar{p}A \quad \bar{p} = \frac{p_{\max}}{2}, \quad A = Lb$$

$$= \left(\gamma_w \frac{H}{2} \right) \left(\frac{1}{2} \right) (1) \left(\frac{H}{\sin \alpha} \right)$$

$$= \left(\frac{62.4}{4} \right) H^2 \frac{1}{\sin 37}$$

$$\underline{P = 25.92 H^2}$$

$$\begin{aligned}
 cL &= \left(c_r \frac{A_n}{A} + c_j \frac{A_j}{A} \right) \left(\frac{H}{\sin \alpha} \right) \\
 &= (0.14)(2,580)(144) \left(\frac{H}{\sin 37} \right) \\
 \underline{cL} &= \underline{8.643(10^4)H}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{F_R}{F_D} &= \frac{(27.60 H^2 - 25.92 H^2) \tan(33) + 8.643(10^4)H}{20.80 H^2} \\
 \frac{F_R}{F_D} &= 0.0525 + \frac{4.155(10^3)}{H}
 \end{aligned}$$

$$\text{But } F_S = \frac{F_R}{F_D}$$

$$1.15 = 0.0525 + \frac{4.155(10^3)}{H}$$

$$\frac{1}{H} = 2.641(10^{-4})$$

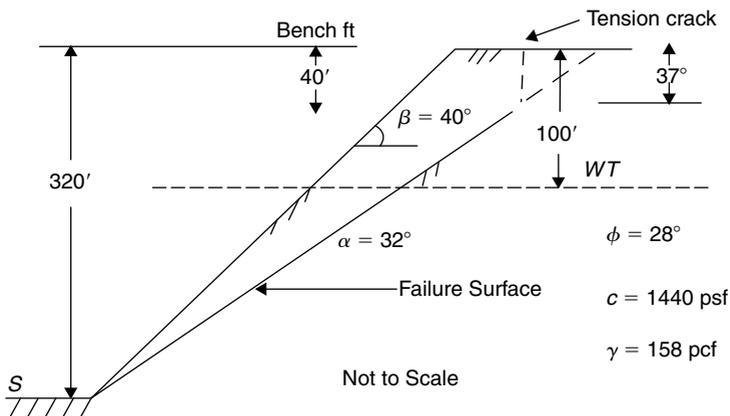
$$\underline{\underline{H = 3,786 \text{ ft}}}$$

← depth at FS = 1.15

17. Given: Sketch of the potential slope failure shown in the sketch,

Find:

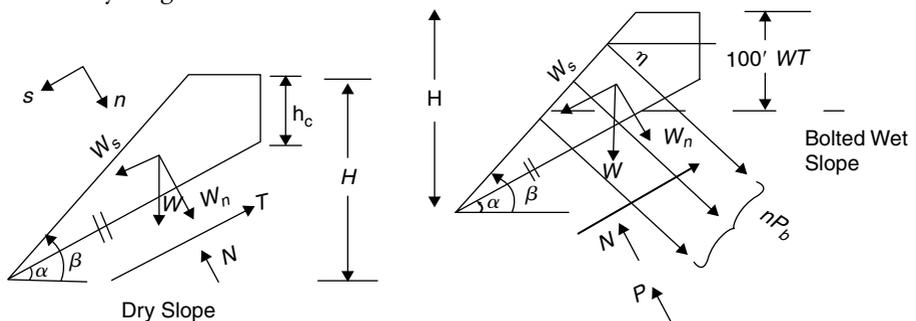
- (a) Factor-of-safety of a cable bolted slope when the water table is drawn down 100 ft
 Bench height = 40' (vertical bolt spacing)
 Horizontal bolt spacing = 20'
 Bolt angle = 5° down
 Bolt loading = 700 kips/hole
- (b) Factor of safety of the same slope but without bolts when the water table is drawn below the toe
- (c) Reasons for preferring one over the other.



Sketch for problem with given data.

Solution:

Free body diagram.



(a) Factor of safety of a bolted slope with water table

$$\eta = \text{bolting angle} = -5^\circ$$

$$n = \text{number of holes per row}$$

$$= \frac{320'}{40'} = 8$$

$$\Sigma F_n = 0$$

$$N = W \cos(\alpha) - P_s + nP_b \sin(\alpha - \eta)$$

$$T = N \tan(\phi) + c \left[\frac{H - h_c}{\sin(\alpha)} \right] b$$

$$R = T + nP_b \cos(\alpha - \eta)$$

$$D = W \sin(\alpha)$$

$$FS = \frac{R}{D}$$

$$= \frac{[W \cos(\alpha) - P_s + nP_b \sin(\alpha - \eta)] \tan(\phi) + c \left[\frac{H - h_c}{\sin(\alpha)} \right] b + nP_b \cos(\alpha - \eta)}{W \sin(\alpha)}$$

$$FS = \frac{\tan(\phi)}{\tan(\alpha)} - \frac{P_s \tan(\phi)}{W \sin(\alpha)} + \frac{c(H - h_c)b}{W \sin^2(\alpha)} + \frac{nP_b [\sin(\alpha - \eta) \tan \phi + \cos(\alpha - \eta)]}{W \sin \alpha}$$

$$= \frac{\tan(\phi)}{\tan(\alpha)} - \frac{P_s \tan(\phi)}{W \sin(\alpha)} + \frac{c(H - h_c)b}{W \sin^2(\alpha)} + \frac{nP_b \cos(\alpha - \eta - \phi)}{W \sin \alpha \cos \phi}$$

$$\text{using—} \sin(A) \sin(B) + \cos(A) \cos(B) = \cos(A - B)$$

$$FS = FS_{\text{wet unbolted}} + \Delta FS_{\text{bolt}}$$

Assume $e = 18\%$

$$G = \frac{\gamma_{\text{dry}}}{\gamma_w} (1 + e)$$

$$G = \frac{158}{62.4} (1.18) = 2.99 \rightarrow \text{grain SG gravity}$$

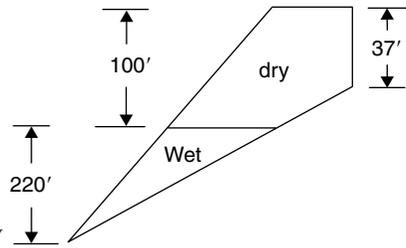
$$\begin{aligned} \gamma_{\text{wet}} &= \left(\frac{2.99 + 0.18}{1.18} \right) 62.4 = \left(\frac{G + Se}{1 + e} \right) \gamma_w \\ &= 168 \text{ pcf} \end{aligned}$$

$$W_{\text{wet}} = \gamma_{\text{dry}} V_{\text{dry}} + (\gamma_{\text{wet}} - \gamma_{\text{dry}}) V_{\text{wet}}$$

$$\begin{aligned} V_{\text{wet}} &= \frac{(H_w)^2}{2} [\cot(\alpha) - \cot(\beta)] b \\ &= \frac{(220)^2}{2} [\cot(32) - \cot(40)] 1 \\ &= 9,888 \text{ ft}^3 \end{aligned}$$

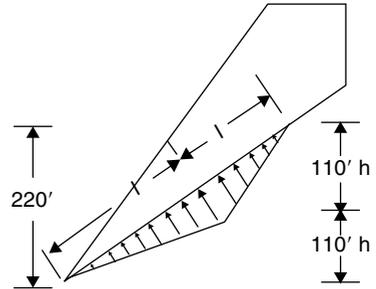
$$\begin{aligned} V_{\text{dry}} &= \frac{H^2}{2} [\cot(\alpha) - \cot(\beta)] b - \left[\frac{1}{2} h_c^2 \cot(30) \right] \\ &= \frac{320^2}{2} [\cot(32) - \cot(40)] 1 - \left[\frac{37^2}{2} \cot(32) \right] \\ &= 1,9824 \text{ ft}^3 \end{aligned}$$

$$\begin{aligned} W_{\text{wet}} &= (158)1,9824(20) + (168 - 158)9,888(20) \\ &= 64,621,440 \text{ lbs} \end{aligned}$$



$$\begin{aligned}
 P_s &= 2 \int_0^l P b \, dl \\
 &= 2 \int_0^l b \gamma_w l \sin(\alpha) \, dl \\
 &= 2 \frac{\gamma_w h l b}{2} = \gamma_w h l b
 \end{aligned}$$

$$= (62.4)(110) \left[\frac{110}{\sin(32)} \right] (20) = \underline{28,496,420 \text{ lbf}}$$



$$\begin{aligned}
 FS &= \frac{\tan(28)}{\tan(32)} - \frac{(28,496,420) \tan(28)}{(64,621,440) \sin(32)} + \frac{1,440(320 - 37)(20)}{(64,621,440) \sin^2(32)} \\
 &\quad + \frac{(8)(700 \times 10^3) \cos(32 + 5 - 28)}{(64,621,440) \sin(32) \cos(28)} \\
 &= 0.851 - 0.442 + 0.449 + 0.183
 \end{aligned}$$

$$\underline{FS = 1.041}$$

← FS wet, bolted slope

(b) Factor of safety of a dry slope.

$$\begin{aligned}
 W &= \gamma_{\text{dry}} V_{\text{dry}} \\
 &= (158)19,824(20) = \underline{6,26,43,840 \text{ lbs}}
 \end{aligned}$$

$$\Sigma F_n = 0$$

$$N = W \cos(\alpha)$$

$$T = N \tan \phi + c(H - h_c) \frac{b}{\sin(\alpha)}$$

$$R = T$$

$$= [W \cos(\alpha)] \tan(\phi) + c(H - h_c) \frac{b}{\sin(\alpha)}$$

$$D = W \sin(\alpha)$$

$$\begin{aligned}
 FS_{\text{dry}} &= \frac{R}{D} = \frac{W \cos(\alpha) + \tan(\phi) + c(H - h_c) \frac{b}{\sin(\alpha)}}{W \sin \alpha} \\
 &= \frac{\tan(\phi)}{\tan(\alpha)} + \frac{c(H - h_c)b}{W \sin^2(\alpha)} \\
 &= 0.851 + \frac{1440(320 - 37)(20)}{6,26,43,840 \sin^2(32)} \\
 &= 0.851 + 0.463
 \end{aligned}$$

$$\underline{FS = 1.314}$$

← FS dry, unbolted

Wedge Failures

18. Given: Wedge data

$$A: \alpha = 0^\circ \quad \delta = 60^\circ$$

$$B: \alpha = 90^\circ \quad \delta = 60^\circ$$

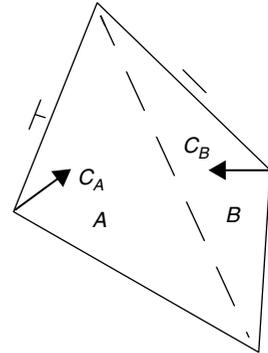
Find: normals C_A, C_B .

Solution:

	x	y	z
C	$\sin \delta \sin \alpha$	$\sin \delta \cos \alpha$	$\cos \delta$
$A:$	$\left(\frac{\sqrt{3}}{2}\right)(0)$	$\left(\frac{\sqrt{3}}{2}\right)(1)$	$\left(\frac{1}{2}\right)$
$B:$	$\left(\frac{\sqrt{3}}{2}\right)(1)$	$\left(\frac{\sqrt{3}}{2}\right)(0)$	$\left(\frac{1}{2}\right)$

$$\therefore \left. \begin{array}{l} \vec{C}_A = \left(0, \frac{\sqrt{3}}{2}, \frac{1}{2}\right) \\ \vec{C}_B = \left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right) \end{array} \right\} \text{direction cosines}$$

$$\left. \begin{array}{l} \vec{C}_A: (90^\circ, 30^\circ, 60^\circ) \\ \vec{C}_B: (30^\circ, 90^\circ, 60^\circ) \end{array} \right\} \text{direction angles}$$



19. Given: The wedge from problem 18,

$$c_A = \left(0, \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$c_B = \left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$$

Find: Dip direction and dip of the line of intersection \vec{s} .

Solution:

$\vec{s} = \vec{c}_A \times \vec{c}_B$	form "determinant"		
	<i>i</i>	<i>j</i>	<i>h</i>
c_A :	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
c_B :	$\frac{\sqrt{3}}{2}$	0	$\frac{1}{2}$

$$\vec{s} = \left(\frac{\sqrt{3}}{4} \frac{\sqrt{3}}{4} - \frac{3}{4}\right) \quad \text{dir. numbers}$$

$$|s| = \left[\left(\frac{\sqrt{3}}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2 + \left(-\frac{3}{4}\right)^2 \right]^{1/2}$$

$$|s| = \frac{\sqrt{15}}{\sqrt{16}} = \left(\frac{\sqrt{3}}{4}\right) \left(\frac{\sqrt{5}}{1}\right)$$

direction cosines:

	S_x	S_y	S_z	$\alpha_s = \underline{45^\circ}$
\vec{S}	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{\sqrt{3}}{\sqrt{5}}$	$\delta_s = \underline{50.8^\circ}$

$\leftarrow \alpha_s, \delta$

$$\tan \alpha_s = \frac{S_x}{S_y}$$

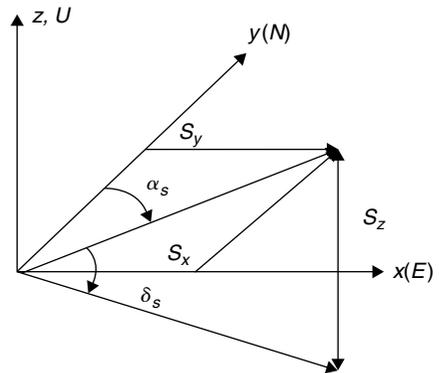
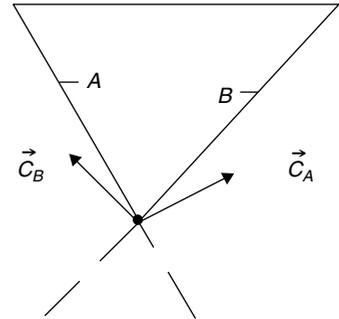
$$\tan \alpha_s = \frac{1/\sqrt{5}}{1/\sqrt{5}}$$

$$\tan \alpha_s = 1$$

$$\therefore \underline{\alpha_s = 45^\circ}$$

$$\tan \delta_s = \frac{-S_z}{[S_x^2 + S_y^2]^{1/2}}$$

$$\tan \delta_s = \frac{+\frac{\sqrt{3}}{\sqrt{5}}}{\sqrt{\frac{2}{5}}} = +\frac{\sqrt{3}}{\sqrt{2}} \therefore \delta_s = \underline{50.8^\circ}$$

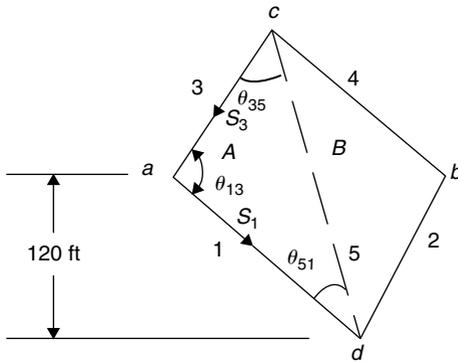


20. Given: Table 2.7 data and $\overline{ad} = 120$ ft

Find: Joint plane areas A_A, B_A (no tension crack).

Solution:

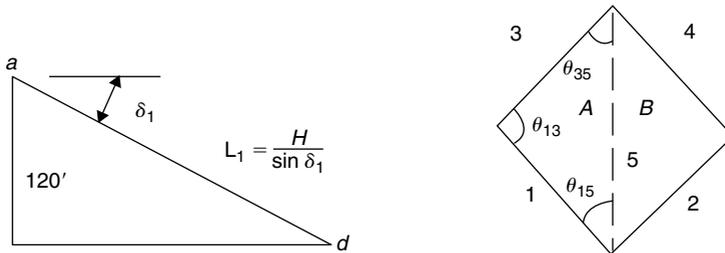
Vectors S_1, S_3 along lines 1 and 2 can be found from intersections of (1) Joint plane A and Face F, and (2) Joint Plane A and Upland U, since normals to F and U can be found from their dip directions and dips. The normal to A is known from problem 18.

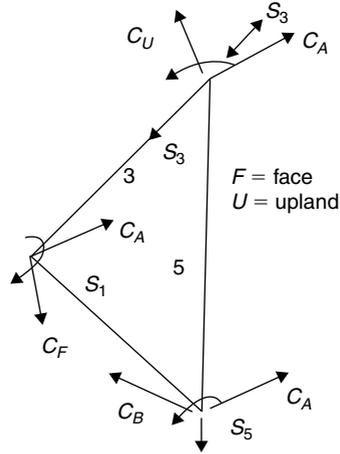


- The angle θ_{13} can be found from $S_1 \cdot S_3 = |S_1||S_3| \cos \theta_{13}$ $|S_1| = 1$ $|S_3| = 1$ by normalization
- Similarly angles θ_{35} & θ_{51} can be found
- The distance L_1 can be found from the dip of line 1 (S_1) and $H = 120'$.
- The distance L_3 can be found.

From the sine law

$$\frac{L_3}{\sin \theta_{15}} = \frac{L_1}{\sin \theta_{35}} = \frac{L_5}{\sin \theta_{13}}$$





Need vectors S_1, S_3, S_5 that are formed by planes of intersection e.g. $S_5 = \vec{C}_A \times \vec{C}_B$ (as in problems 18 & 19) Need direction cosines of C_A, C_F, C_U, C_B

	$C: \sin \delta \sin \alpha$	$\sin \delta \cos \alpha$	$\cos \delta$	
(problem 18)	$C_A = 0$	0.8660	0.5000	$ C_A = 1$
(Problem 18)	$C_B = 0.8660$	0	0.5000	$ C_B = 1$
	$C_F \sin 85 \sin 45$	$\sin 85 \cos 45$	$\cos 85$	
	$C_F = 0.7044$	0.7044	0.08716	$ C_F = 1$
	$C_U = 0.06163$	0.06163	0.9962	$ C_U = 1$

CROSS PRODUCTS

$C_A \times C_B =$	0.4330 (0.4472)	0.4330 (0.4472)	-0.7500 (-0.7746)	$ S_5 = 0.9682$ normalized
$C_A \times C_F =$	-0.2767 (-0.3657)	0.3522 (0.4654)	-0.6100 (-0.8060)	$ S_1 = 0.7568$ normalized
$C_A \times C_Z =$	0.8321 (0.9973)	0.0308 (0.03692)	-0.0534 (-0.06400)	$ S_3 = 0.8344$ normalized

dir. cos

	x	y	z
S_5	0.4472	0.4472	-0.7746
S_1	-0.3657	0.4654	-0.8060
S_3	0.9973	0.03692	-0.06400

DOT PRODUCTS

$$S_{13} = |S_1||S_3| \cos \theta_{13} = S_{1x}S_{3x} + S_{1y}S_{3y} + S_{1z}S_{3z}$$

$$S_{13} = -0.2959 \quad \therefore \theta_{13} = 107^\circ \quad (\text{or } 73^\circ \text{ alt. sol.})$$

$$S_{15} = 0.6689 \quad \therefore \theta_{15} = 48.0^\circ \quad [\text{Look of direction of } S_3]$$

$$S_{35} = 0.5102 \quad \therefore \theta_{35} = 59.3^\circ$$

Note: $\theta_{15} + \theta_{35} + \theta_{13} = 180^\circ \therefore \theta_{13} = 73^\circ$

Sine law for length

$$\frac{L_1}{\sin \theta_{35}} = \frac{L_3}{\sin \theta_{15}}$$

$$L_1 = \frac{H}{\sin \delta_1}$$

need dip of L_1 , δ_1

$$\begin{aligned} \tan \delta_1 &= \frac{-S_{1z}}{[S_{1x}^2 + S_{1y}^2]^{1/2}} \\ &= \frac{-(-0.8060)}{[0.3657^2 + 0.4634^2]^{1/2}} \end{aligned}$$

$$\tan \delta_1 = 1.3617$$

$$\therefore \delta_1 = 53.70^\circ$$

$$L_1 = \frac{120}{\sin 53.7}$$

$$\underline{L_1 = 148.9 \text{ ft}}$$

$$L_3 = (148.9) \frac{(\sin 48)}{\sin(59.3)}$$

$$\underline{L_3 = 128.7 \text{ ft}}$$

$$\begin{aligned} A_A &= \frac{1}{2} L_1 L_3 |S_1||S_3| \sin \theta_{13} \\ &= \left(\frac{1}{2}\right) (148.9)(128.7) \sin 73 \end{aligned}$$

$$\underline{A_A = 9,161 \text{ sq ft}}$$

← A_A

by symmetry of this problem

$$\underline{B_A = 9,161 \text{ sq ft}}$$

← B_A

Computer check using WEDGE (course download) $\underline{A_A = B_A = 9,161 \text{ sq ft o.k.}}$

20. (Alternative)

Given: Data in Table where the vertical distance between \underline{a} and \underline{d} is 120 ft,

Find: Areas of A and B (without a tension crack)

Solution:

This is a lengthy calculation best done with the aid of a computer program:
Using WEDGE from course downloads

$$A(\text{area}) = B(\text{area}) = \underline{\underline{9,160 \text{ ft}^2}} \quad \leftarrow$$

Using SWEDGE from ROCSCIENCE (same results)

Given: Data in Table & vertical distance $\overline{ad} = 120 \text{ ft}$, $\gamma = 158 \text{ pcf}$ water below
toe slope tension crack offset = 90 ft

Find: A , B areas and FS_{dry} and volume.

Solution:

Using WEDGE course download.

$$\underline{\underline{A = B = 8,072 \text{ ft}^2}}$$

$$\underline{\underline{FS_{\text{dry}} = 1.28}}$$

$$\underline{\underline{\text{Volume} = 11,683 \text{ yds}^3.}}$$

Same results using SWEDGE from ROCSCIENCE.

21. (no tension crack)

WEDGE	DIP DIRECTION DEGREES	DIP ANGLE DEGREES
PLANE A	0.0	60.0
PLANE B	90.0	60.0
LINE OF INTERSECTION	45.0	50.8
SLOPE FACE	45.0	85.0
UPLAND	45.0	5.0
TENSION CRACK	45.0	75.0
EXTERNAL LOAD	0.0	0.0

WEDGE HEIGHT (LEFT SIDE)	120.0 FT.
LENGTH PLANE A TRACE (UPLAND)	128.8 FT.
TENSION CRACK OFFSET (PLANE A TRACE)	128.8 FT.
EXTERNAL LOAD MAGNITUDE	0.0 KIPS
ROCK UNIT WEIGHT	158.0 PCF.

	FRICITION ANGLE DEGREES	COHESION PSF.
PLANE A	32.0	1080.0
PLANE B	37.0	1640.0

	AREA SQ. FT.	WATER FORCE KIPS
PLANE A	9160.1	0.0
PLANE B	9160.1	0.0
TENSION CRACK	0.0	0.0

WEDGE VOLUME = 3,27,142.6 CU. FT. = 12116.4 CU. YD.
 WEDGE WEIGHT = 51,688.5 KIPS

FACTOR OF SAFETY = 1.33

Checks Table 18.

WEDGE	DIP DIRECTION DEGREES	DIP ANGLE DEGREES
PLANE A	0.0	60.0
PLANE B	90.0	60.0
LINE OF INTERSECTION	45.0	50.8
SLOPE FACE	45.0	85.0
UPLAND	45.0	5.0
TENSION CRACK	45.0	75.0
EXTERNAL LOAD	0.0	0.0

WEDGE HEIGHT (LEFT SIDE)	120.0 FT.
LENGTH PLANE A TRACE (UPLAND)	128.8 FT.

TENSION CRACK OFFSET (PLANE A TRACE) 90.0 FT.
 EXTERNAL LOAD MAGNITUDE 0.0 KIPS
 ROCK UNIT WEIGHT 158.0 PCF.

	FRICITION ANGLE DEGREES	COHESION PSF.
PLANE A	32.0	1080.0
PLANE B	37.0	1640.0

	AREA SQ. FT.	WATER FORCE KIPS
PLANE A	8071.9	0.0
PLANE B	8071.9	0.0
TENSION CRACK	1312.0	0.0

WEDGE VOLUME = 315429.3 CU. FT. = 11682.6 CU. YD.

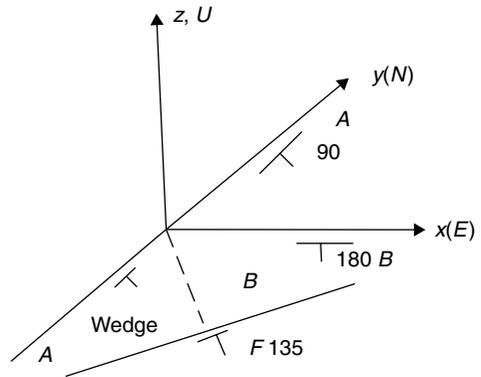
WEDGE WEIGHT = 49837.8 KIPS

FACTOR OF SAFETY = 1.28

← F.S.

22. Given: Wedge data

Wedge forms as shown in sketch. Line of intersection has dip direction of 135° and is in the same direction as the face dip direction. When the face dips $\pm 90^\circ$ to line of intersection, then kinematic failure is impossible. Thus, the range of concern is for face dip directions. ($45^\circ, 225^\circ$).



23. Given: Wedge data

K_1 joints $\alpha = 110^\circ \quad \delta = 38^\circ$

K_2 joints $\alpha = 147^\circ \quad \delta = 42^\circ$

$n_1: (0.5785, -0.2106, 0.7880)$

$n_2: (\sin \delta \sin \alpha, \sin \delta \cos \alpha, \cos \delta)$
 $\sin(42) \sin(147) \sin(42) \cos(147) \cos(42)$

$n_2: (0.3644, -0.5662, 0.7431)$

$\vec{n}_1 \times \vec{n}_2 = \vec{S}(0.2857 \quad -0.1427 \quad -0.2479)$

$|s| = [(0.2857)^2 + (-0.1427)^2 + (-0.2479)^2]^{1/2}$

$|s| = 0.4043$

dir. cos: $\vec{s} = (0.7067, -0.3530, -0.6132)$

$$\tan \alpha_s = \frac{s_x}{s_y} \qquad \cos \delta' = -0.6132$$

$$= \frac{0.7067}{-0.3530} \qquad \delta' = 127.8^\circ$$

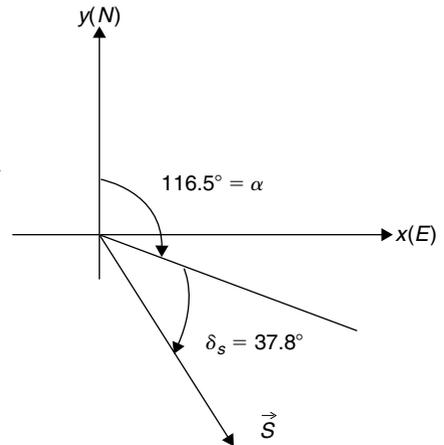
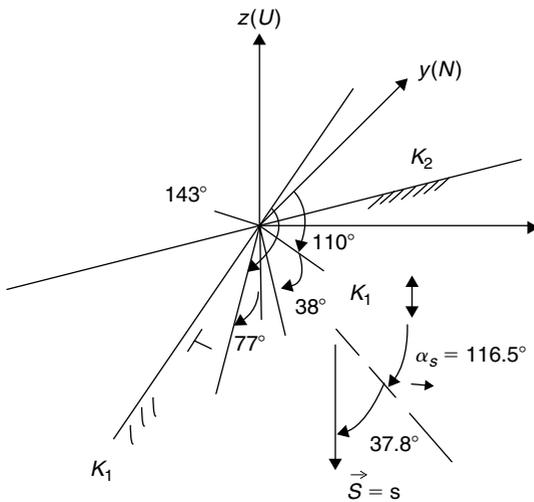
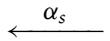
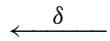
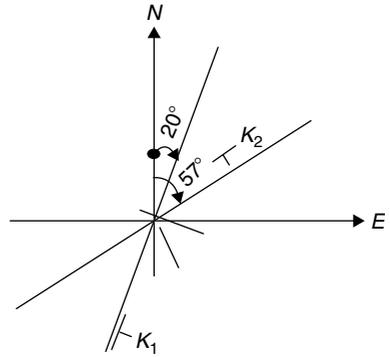
$$\qquad \qquad \qquad \delta = \delta' - 90^\circ$$

$\tan \alpha_s = -0.2002 \qquad \underline{\underline{\delta_s = 37.8^\circ}}$

$\alpha_s = -63.5^\circ$ or 116.5°

$s_x > 0 \quad s_y < 0 \quad \therefore$ in 4th quadrant

$\alpha_s = 116.5^\circ$



24. Given: Wedge data

Find:

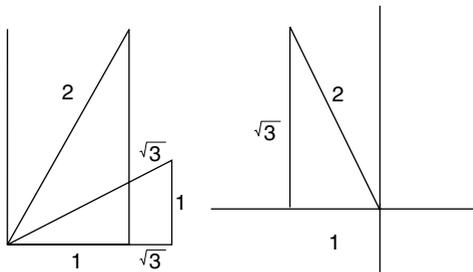
- dip direction and dip of lines of intersection of A & B
- Range of azimuths that are safe
- Length of line formed by A & F (Face)

Solution:

- Need normals to A & B

Notes:

	$C_x = \sin \delta \sin \alpha$	$C_y = \cos \delta \cos \alpha$	$C_z = \cos \delta$
	x	y	z
A:	$\frac{\sqrt{3}}{2} \frac{1}{2}$	$\frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2}$	$\frac{1}{2}$
B:	$\frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2} \left(-\frac{1}{2}\right)$	$\frac{1}{2}$
S:	$\left[\left(\frac{3}{4}\right) \left(\frac{1}{2}\right) + \frac{\sqrt{3}}{4} \frac{1}{2} \right]$	$-\left(\frac{\sqrt{3}}{4} \frac{1}{2} - \frac{3}{4} \frac{1}{2} \right)$	$\left[\frac{\sqrt{3}}{4} \left(-\frac{\sqrt{3}}{4}\right) - \frac{3}{4} \cdot \frac{3}{4} \right]$
S:	$\frac{3 + \sqrt{3}}{8}$	$\frac{3 - \sqrt{3}}{8}$	$-\left(\frac{9 + 3}{16}\right)$
S:	0.9915	0.1585	-0.7500



$$\sin \alpha = \frac{0.5915}{(0.5915^2 + 0.1585^2)^{1/2}} = \frac{S_x}{(S_x^2 + S_y^2)^{1/2}}$$

$$\sin \alpha = 0.9659$$

$$\underline{\alpha = 75^\circ}$$

$\leftarrow \alpha_{AB}$

$$\sin \delta = \frac{-S_z}{(S_x^2 + S_y^2 + S_z^2)^{1/2}}$$

$$= \frac{0.75}{(0.5915^2 + 0.1585^2 + 0.75^2)^{1/2}}$$

$$\sin \delta = 0.7746$$

$$\underline{\delta = 50.8^\circ}$$

$\leftarrow \delta_{AB}$

(b) The face dip direction is 75° if a line of intersection does not penetrate the face, then sliding cannot occur. Thus $75^\circ \pm 90^\circ$ defines range of dip direction that are unsafe ($-15^\circ, +165^\circ$)

\therefore safe $(165^\circ, 345^\circ)$

← safe α'_s

(c)

	x	y	z
$A:$	$\frac{\sqrt{3}}{2} \frac{1}{2}$	$\frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$F:$	$\sin 85 \sin 75$	$\sin 85 \cos 75$	$\cos 85$
$F:$	0.9623	0.2578	0.0872
$S_{AF}:$	0.0654 – 0.1289	–(0.03776 – 0.48115)	0.1116 – 0.7217
$S_{AF}:$	–0.0635	0.44339	–0.6101

$$\sin \alpha = \frac{-0.0635}{[(-0.0635)^2 + (0.44339)^2]^{1/2}}$$

$$\sin \alpha = -0.1418$$

$$\alpha = \underline{\underline{-8.2^\circ}} \quad \text{quadrant!}$$

$$\sin \delta = \frac{-(-0.6101)}{(0.0635^2 + 0.4433^2 + 0.6101^2)^{1/2}}$$

$$\sin \delta = 0.8062$$

$$\delta = \underline{\underline{53.7^\circ}}$$

$$L = \frac{H}{\sin \delta}$$

$$= \frac{85}{0.8062}$$

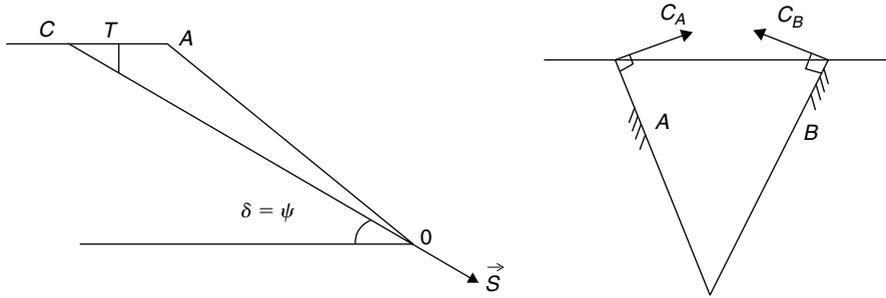
$$\underline{\underline{L = 105.5 \text{ ft}}}$$

25. Given: Wedge slide data and required $FS = 1.10$

Find: If can obtain $FS = 1.10$.

Solution:

Try quick screening for sliding down line of intersection assuming zero cohesion and lowest $\phi(28^\circ)$



Direction cosines (from T.2)

	x	y	z
\vec{C}_A	0.7501	0.4321	0.5000
\vec{C}_B	-0.4337	-0.7501	0.5000

$$\begin{aligned} \vec{S} &= \vec{C}_A \times \vec{C}_B = (S_x, S_y, S_z) \\ &= (0.4321)(0.5000) - (-0.7501)(0.5000) \\ &\quad - (0.7501)(0.5000) + (-0.4333)(0.5000) \\ &\quad + (0.7701)(-0.7501) - (-0.4337)(0.4321) \end{aligned}$$

$$\vec{S} = (0.5911, -0.5911, +0.3753)$$

$$\tan \alpha = \frac{0.5911}{(-0.5911)}$$

$$\alpha = 135^\circ \text{ dip direction}$$

$$\tan \delta = \frac{S_z}{\sqrt{S_x^2 + S_y^2}}$$

$$= \frac{0.3753}{[(0.5911)^2 + (0.5911)^2]^{1/2}}$$

$$\tan \alpha = 0.44895$$

$$\delta = 24.2^\circ$$

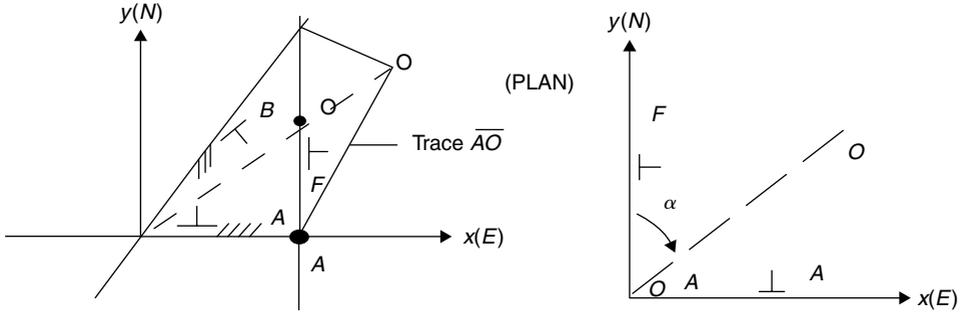
$$\begin{aligned} FS(\text{dry}, C = 0, \phi = \phi_{\min}) &= \frac{\tan \phi}{\tan \delta} \\ &= \frac{\tan 28}{\tan 24.2} \\ &= \underline{\underline{FS = 1.18}} \end{aligned}$$

$\xleftarrow{\text{yes}}$
(>1.10)

(added cohesion and added friction will increase this FS)

26. Given: Wedge data in table, sketch
 Find: dip, dip direction, length of \overline{AO}

Solution:



Note:

	x	y	z
C_a	$\sin \alpha_a \sin \delta_a$	$\cos \alpha_a \sin \delta_a$	$\cos \delta_a$
C_b	$\sin \alpha_f \sin \delta_f$	$\cos \alpha_f \sin \delta_f$	$\cos \delta_f$

$$\alpha_a = 0^\circ \quad \delta_a = 45^\circ$$

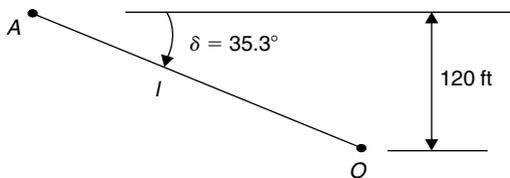
$$\alpha_f = 90^\circ \quad \delta_f = 45^\circ$$

S	S_x	S_y	S_z
C_a	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
C_b	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$
S_s	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

$$\tan \alpha_{AO} = \frac{S_x}{S_y} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1 : \underline{\alpha_{AO} = 45^\circ} \quad \leftarrow \alpha = 45^\circ$$

$$\tan \delta_{AO} = \frac{S_z}{\sqrt{S_x^2 + S_y^2}} = \frac{-\frac{1}{2}}{\sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2}} = -\frac{1}{\sqrt{2}} : \underline{\delta_{AO} = -35.3^\circ} \quad \leftarrow \delta = 35.3^\circ$$

length



$$l \sin 35.5 = 120'$$

$$\underline{l = 208 \text{ ft}}$$

\leftarrow length

27. Given: Wedge data

Find:

- Dip direction and dip of lines of intersection of A & B
- Range of azimuths that are safe
- Length of line formed by A & F (face)

Solution:

- Need normals to A & B

Notes:

	$C_x = \sin \delta \sin \alpha$	$C_y = \sin \delta \cos \alpha$	$C_z = \cos \delta$
	x	y	z
A:	$\frac{\sqrt{3}}{2} \frac{1}{2}$	$\frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2}$	$\frac{1}{2}$
B:	$\frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2} \left(-\frac{1}{2}\right)$	$\frac{1}{2}$
S:	$\left[\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{4} \frac{1}{2}\right]$	$-\left(\frac{\sqrt{3}}{4} \frac{1}{2} - \frac{3}{4} \frac{1}{2}\right)$	$\left[\frac{\sqrt{3}}{4} \left(-\frac{\sqrt{3}}{4}\right) - \frac{3}{4} \cdot \frac{3}{4}\right]$
S:	$\frac{3 + \sqrt{3}}{8}$	$\frac{3 - \sqrt{3}}{8}$	$-\left(\frac{9 + 3}{16}\right)$
S:	0.5915	0.1585	-0.7500

$$\sin \alpha = \frac{0.5915}{(0.5915^2 + 0.1585^2)^{1/2}} = \frac{S_x}{(S_x^2 + S_y^2)^{1/2}}$$

$$\sin \alpha = 0.9659$$

$$\underline{\alpha = 75^\circ}$$

← α_{AB}

$$\sin \delta = \frac{-S_z}{(S_x^2 + S_y^2 + S_z^2)^{1/2}}$$

$$= \frac{0.75}{(0.5915^2 + 0.1585^2 + 0.75^2)^{1/2}}$$

$$\sin \delta = 0.7746$$

$$\underline{\delta = 50.8^\circ}$$

← δ_{AB}

- The face dip direction is 75° if a line of intersection does not penetrate the face, then sliding cannot occur. Thus $75^\circ \pm 90^\circ$ defines range of dip direction that are unsafe ($-15^\circ, +165^\circ$)

$$\therefore \text{safe } \underline{(165^\circ, 345^\circ)}$$

← Safe α'_s

(c)

	x	y	z
A:	$\frac{\sqrt{3}}{2} \frac{1}{2}$	$\frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2}$	$\frac{1}{2}$
F:	$\sin 85 \sin 75$	$\sin 85 \cos 75$	$\cos 85$
F:	0.9623	0.2578	0.0872
S_{AF} :	$0.0654 - 0.1289$	$-(0.03776 - 0.48115)$	$0.1116 - 0.7217$
S_{AF} :	-0.0635	0.44339	-0.6101

$$\sin \alpha = \frac{-0.0635}{[(-0.0635)^2 + (0.4433)^2]^{1/2}}$$

$$\sin \alpha = -0.1418$$

$$\underline{\underline{\alpha = -8.2^\circ}} \quad \text{quadrant!}$$

$$\sin \delta = \frac{-(-0.6101)}{(0.0635^2 + 0.4433^2 + 0.6101^2)^{1/2}}$$

$$\sin \delta = 0.8062$$

$$\underline{\underline{\delta = 53.7^\circ}}$$

$$L = \frac{H}{\sin \delta}$$

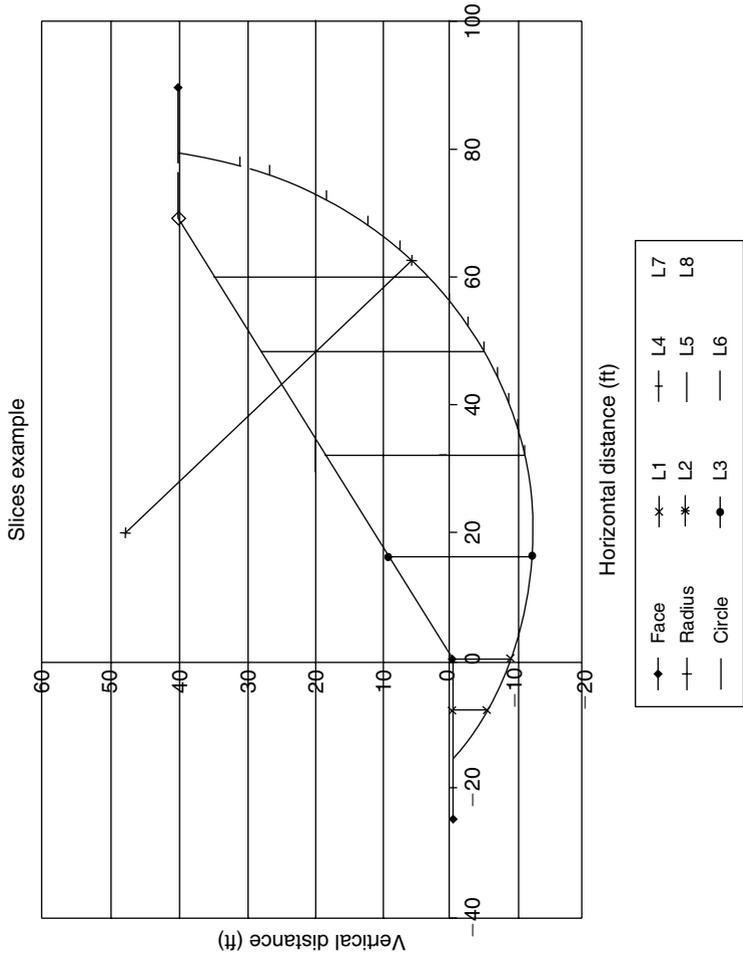
$$= \frac{25.9}{0.8062}$$

$$\underline{\underline{L = 32.13 \text{ m}}}$$

Rotational Slides

28.

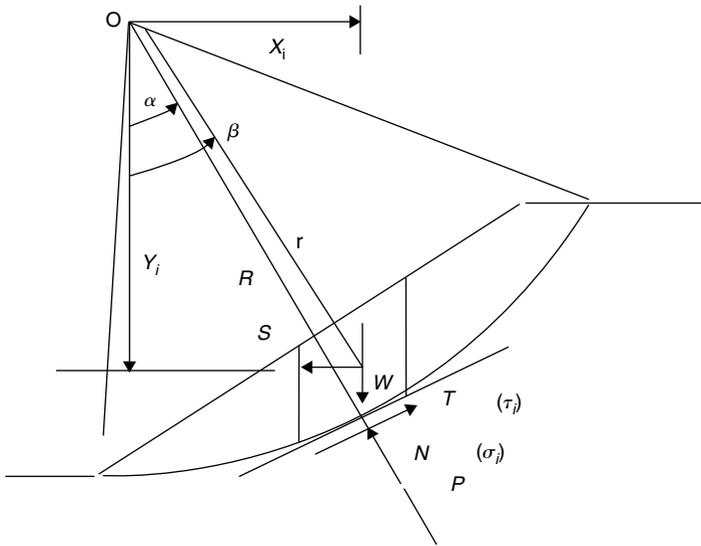
Slices Problem													
30 deg slope, dips below toe													
$b = 1 \text{ ft}$ spwt = 100 pcf													
$R = 60 \text{ ft}$													
coordinates													
Slice	$x(\text{ft})$	$b(\text{ave})\text{ft}$	vol(ft ³)	wt(lbf)	δy	δx	chord slope angle = deg	rads	$W_n =$	$W_s =$	water P(slice)	$\phi = \text{deg}$ $N' \tan(\phi)$	$C = \text{psf}$ $cLb = \text{lbf}$
	-16	0.00											
1	-8	5.07	20.26	2,026	-5.07	8.00	-32.34	-0.56451	1,712	-1,084	1,497	70	9.47 6,818
2	0	8.57	54.54	5,454	-3.50	8.00	-23.64	-0.41268	4,996	-2,187	3,715	416	8.73 6,288
3	16	21.10	237.38	23,738	-3.30	16.00	-11.65	-0.20328	23,249	-4,792	15,124	2,640	16.34 11,762
4	32	29.26	402.94	40,294	1.08	16.00	3.86	0.06732	40,202	2,711	25,200	4,874	16.04 11,546
5	48	32.78	496.33	49,633	5.72	16.00	19.68	0.34344	46,735	16,713	32,892	4,498	16.99 12,234
6	60	31.36	384.85	38,485	8.34	12.00	34.81	0.60762	31,596	21,972	29,250	762	14.62 10,524
7	69.28	26.23	267.21	26,721	10.49	9.28	48.51	0.84674	17,701	20,017	25,171	-2,427	14.01 10,086
8	77.28	9.86	144.35	14,435	16.37	8.00	63.95	1.11614	6,339	12,968	20,510	-4,605	18.22 13,116
9	79.47	0.00	10.80	1,080	9.90	2.19	77.53	1.35316	233	1,054	3,120	-938	10.14 7,303
Total = 2,01,865													
Totals 1,72,764 75,435 dry = 56,134													
FS = 1.471 wet 2.040 dry -8,064 wet = 13,261													
RocSci: Simplified Bishop = 2.318 1.58													
"Slide" Janbu = 2.000 1.425													
MR = 6.6601E+06 9.2325E+06													
MD = 4.5261E+06 4.5261E+06													
WET DRY													



D part W.T. 1 with 140c60 ft relieving bench (crest)												
	SCALE =	1" = 40'	1 FT THICK									
	(SQ IN)	(SQ FT)	(PCF)	(LBF)								
SEC	AREAS	AREAS	SPWT	ANGLE	W _s	W _N	P	W _{N'}	TAN φ	R	c	cL
1	0.294	110	51,744	74	49,739.53	14,262.58	25,958.4	-11,695.8	0.726543	0	0	0
2	1.211		207,936	57	174,389.8	113,250.1	161,304	-48,053.9	0.649408	0	0	0
KJ-OLD	0.326	125	65,200									
A-YOU	0.074	110	13,024									
TOT-KJ	0.885	110	155,760									
3	3.407		456,132.8	38	280,823.4	359,437.6	349,752	9,685.551	0	0	2,300	59 135,700
B-YOU	1.051	110	184,976									
B'-YOU	1.6788	110	295,468.8									
B'-OLD	1.1192	125	223,840									
JG-MED	0.609	125	121,800									
4	7.654		907,600	13	204,165.6	884,338.3	652,579.2	231,759.1	0	0	2,800	83 232,400
C-YOU	3	110	528,000									
C'-OLD	2	125	400,000									
C''-MED	1.929	125	385,800									
GD LEAN	0.725	105	121,800									
5	7.654			0								
D-YOU		110	Total driving =		709,118.3		Total friction =		0		Total cohesion =	112,000
D'-OLD		125										480,100
D''-MED		125										
LEAN		105			FS = R/D =	1.05	L =	377.8203	ft			

L = 378 ft

30. Given: Slices analysis with S loads



Show:
$$FS = \frac{R \sum_1^n (N'_i \tan \phi + C_i)}{R \sum_1^n (W_i \sin \alpha_i + S_i \frac{r_i}{R} \cos \alpha_i)}$$

when $N'_i = W_i \cos \alpha_i - P_i - S_i \sin \alpha_i$.

Solution:

Overall equilibrium

$$\begin{aligned} \sum_1^n (W_i x_i + S_i y_i) &= \sum_1^n \tau_i A_i R \\ \sum [W_i R \sin(\alpha_i) + S_i R_i \cos(\beta_i)] & \\ &= \sum \tau_i \max \frac{A_i R}{f s_i} \end{aligned}$$

- assume $f s_i = f s_j = f s = FS$
 - and MC strength so
- $$\tau_i \max = \sigma'_i \tan \phi_i + c_i$$

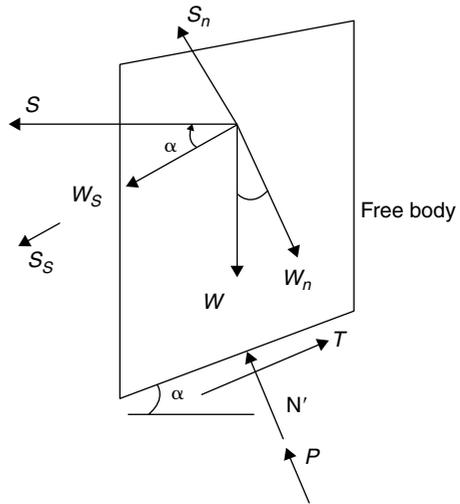
$$\begin{aligned} \therefore \sum (W_i R \sin \alpha_i + S_i R_i \cos \beta_i) & \\ &= \frac{\sum (\sigma'_i + \tan \phi_i + c_i) A_i R}{FS} \end{aligned}$$

$$\therefore FS = \frac{\sum_1^n (N'_i \tan \phi_i + C_i) R}{\sum_1^n [W_i \sin \alpha_i + S_i (\frac{r_i}{R}) \cos \beta_i]}$$

when $N'_i = \sigma'_i A_i$ & $C_i = c_i \cdot A_i$

- Normal force equilibrium
- Neglect side forces

$$\begin{aligned} \sum_n F &= 0 \\ 0 &= W_n - N' - P - S_n \\ \underline{\underline{N'_i}} &= \underline{\underline{W_i \cos \alpha_i - P_i - S_i \sin \alpha_i}} \text{ (each slice } i) \end{aligned}$$

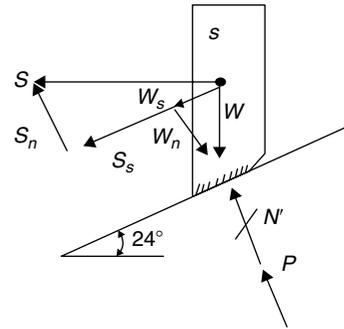


31. Given: Figure and table data

$$R = 300 \text{ ft} \quad H = 120 \text{ ft} \quad \gamma = 95 \text{ psf} \quad C = 367 \text{ psf} \quad \phi = 16^\circ \quad \beta = 29^\circ$$

Find:

- (1) Seismic force on slice 7, $a_s = 0.15$
- (2) Water force on bottom of 7
- (3) FS of 7
- (4) FS slip surface, dry & no seismic force.



Solution:

$$\begin{aligned}
 (1) \quad S &= ma \\
 &= \frac{W}{g} 0.15g \\
 S &= 0.15W \\
 S &= 0.15(10.29)(10^5) \\
 \underline{S} &= \underline{1.544(10^5) \text{ lbf}} \quad \leftarrow (1)
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad P &= \bar{p}A \quad \bar{p} = rH \\
 &= (62.4)(82)(75)(1) \\
 \underline{P} &= \underline{3.838(10^5) \text{ lbf}} \quad \leftarrow (2)
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad FS(7) &= \frac{R(7)}{D(7)} \\
 D &= W_s + S_s \\
 &= (10.29)(10^5) \sin 24 + 1.544(10^5) \cos 34 \\
 \underline{D} &= \underline{5.596(10^5) \text{ lbf}} \\
 R &= N' \tan \phi + cL \\
 N' &= W_n - P - S_s \\
 &= 10.29(10^5) \cos 24 - 3.838(10^5) - 1.544(10^5) \sin 24 \\
 \underline{N'} &= \underline{4.934(10^5) \text{ lbf}} \\
 FS(7) &= \frac{4.934(10^5) \tan 16 + (367)(75)(1)}{5.596(10^5)} \\
 \underline{FS(7)} &= \underline{0.302} \quad \leftarrow (3)
 \end{aligned}$$

Note: (Slopes 1-5 are neg.)

$$\begin{aligned}
 (4) \quad FS &= \frac{\Sigma M_R}{\Sigma M_D} \quad (\text{dry, no seismic force}) \\
 &= \frac{\Sigma_1^{10} W_r \tan \phi + \Sigma_1^{10} C + \Sigma_1^5 W_s}{\Sigma_6^{10} W_s} \\
 &= \frac{52.67(10^5) \tan 16 + (367)(540) + 5.97(10^5)}{12.93(10^5)} \\
 \underline{FS} &= \underline{1.78} \quad \leftarrow (4)
 \end{aligned}$$

32. This amounts to part 4 of 31.

$$\begin{aligned}\underline{FS} &= \underline{1.78} \\ FS &= \frac{\Sigma M_R}{\Sigma M_D} \\ FS &= \frac{\sum_1^{10} R(W \cos \alpha \tan \phi + cL) + \sum_1^5 R W \sin \alpha}{\sum_6^{10} R W \sin \alpha}\end{aligned}$$

\therefore as in 31(4).

33. Given: Circular arc failure data in Fig. 2.33 and Table 2.14.
Find: $FS(2)$, $FS(7)$, show total FS .

Solution:

$FS = \Sigma M_R / \Sigma M_D$, R = radius of slips circle

Slice 2

$$FS = \frac{R[(W'_n \tan \phi) + C]}{RW_s}$$

$$\begin{aligned} P &= \gamma_w WL(1) \\ &= (62.4)(1)(52)(30) \\ \underline{P} &= \underline{9.734(10^4) \text{ lbf}} \end{aligned}$$

$$\begin{aligned} C &= cL(1) \\ &= (367)(30)(1) \\ \underline{C} &= \underline{1.101(10^4) \text{ lbf}} \end{aligned}$$

normal equilibrium:

$$\begin{aligned} W'_n &= W \cos \alpha - P \\ &= 27.78(10^4)(\cos 28) - 9.734(10^4) \\ &= 24.49(10^4) - 9.734(10^4) \\ \underline{W'_n} &= \underline{14.76(10^4) \text{ lbf}} \end{aligned}$$

$$\begin{aligned} W_s &= W \sin \alpha \quad \text{But is resisting!} \\ \therefore FS &= \frac{[(W'_n \tan \phi + C)R + RW_s]}{0} \\ \underline{FS} &\rightarrow \underline{\infty \text{ Slice 2}} \end{aligned}$$

← Slice 2

Slice 7

$$\begin{aligned} P &= (62.4)(1)(75)(82) \\ \underline{P} &= \underline{3.838(10^5) \text{ lbf}} \\ W'_n &= 10.29(10^5) \cos(24) - 3.838(10^5) \\ \underline{W'_n} &= \underline{5.563(10^5) \text{ lbf}} \end{aligned}$$

$$\begin{aligned} C &= (367)(75)(1) \\ \underline{C} &= \underline{2.753(10^4) \text{ lbf}} \end{aligned}$$

$$\begin{aligned} W_s &= 10.29(10^5) \sin 24 \\ W_s &= 4.185(10^5) \text{ lbf} \end{aligned}$$

$$\begin{aligned}
 M_R &= R(W'_n \tan \phi + C) \\
 &= R(5.563(10^5) \tan 16 + 0.275(10^5)) \\
 \underline{M_R} &= \underline{R(1.870)10^5 \text{ lbf}}
 \end{aligned}$$

$$\begin{aligned}
 M_D &= RW_n \\
 \underline{M_D} &= \underline{R(4.185)(10^5)}
 \end{aligned}$$

$$\begin{aligned}
 FS &= \frac{R(1.870)(10^5)}{R(4.185)(10^5)} \\
 \underline{FS} &= \underline{0.447}
 \end{aligned}$$

← Slice 7

Algebraically

$$\begin{aligned}
 FS &= \frac{\Sigma M_R}{\Sigma M_D} \\
 FS &= \frac{\sum_{i=1}^{10} R(W'_n \tan \phi + C)_i + \sum_{i=1}^5 R(W_s)_i}{\sum_{i=6}^{10} R(W_s)_i}
 \end{aligned}$$

34. Given: Circular failure in sketch
Find: Expression for safety factor

Solution:

By definition

$$FS = \frac{M_R}{M_D} = \left(\frac{\text{Moments resistance}}{\text{Moments driving}} \right)$$

$$M_R = \sum_{\text{slices}} (\text{forces})(\text{lever arm}) \quad R$$

$$M_D = \sum_{\text{slices}} (\text{forces})(\text{lever arm}) \quad D$$

lever arm = R
since circular failure

$$M_R = R \sum_{i=1}^5 (N \tan \phi + C)_i + \sum_{i=1}^2 RT_i$$

$$M_D = R \sum_{i=1}^5 T$$

T = Tangential component of weight

N = Normal component of weight

α = inclination of slice bottom from horizontal

$N = W \cos(\alpha)$

$T = W \sin(\alpha)$

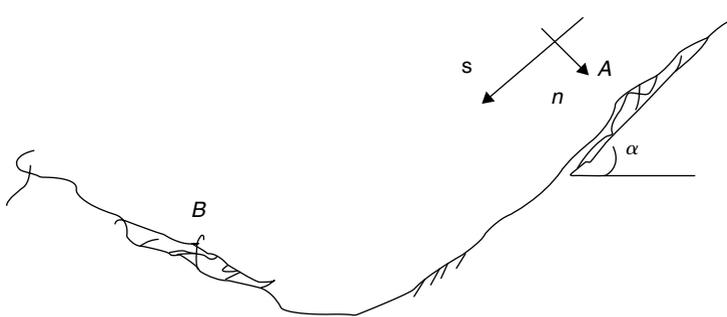
Note: No water table shown, assume dry

$$FS = \frac{\sum_{i=1}^5 R(N \tan \phi + C)_i + \sum_{i=1}^2 RT_i}{\sum_{i=3}^5 RT_i}$$

where $cA = C$ and A = area of slice base

Dynamics, Toppling

35. Given: Slope situation



Find: a , v , s , of mass center.

Solution:

Mass center obeys

$F = m\ddot{s}$ where s = down hill distance
 \dot{s} = velocity
 \ddot{s} = acc. of mass center

- Choose origin of coordinates n, s at mass center starting point A

At time $t = 0$, $s = \dot{s} = \ddot{s} = 0$

- Downhill forces = D , Resisting up hill forces = F

So $F = D - R$

D = downhill component of weight

Free body of slide mass

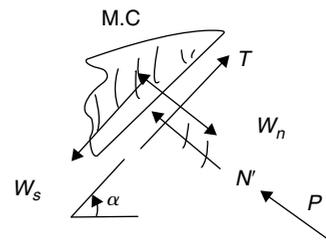
$D = W \sin \alpha$

$N = W \cos \alpha$

$P = 0$ dry assumption

- Frictional resistance

$F_f = N' \tan \phi$



$$\therefore W \sin \alpha - W \cos \alpha \tan \phi = \frac{W}{g} \ddot{s}$$

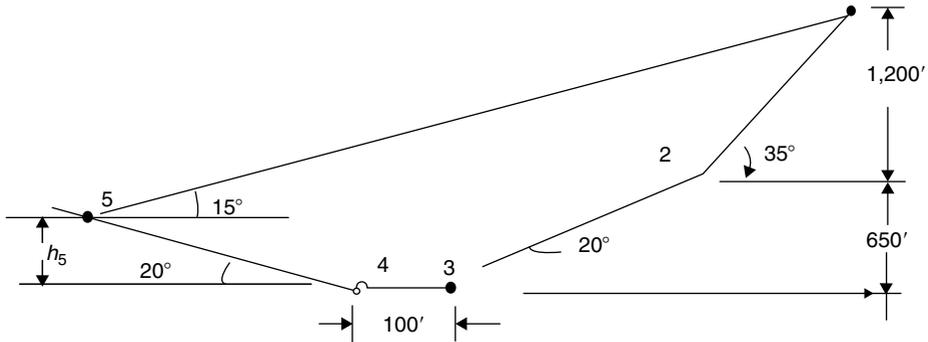
$$W \sin \alpha - W \cos \alpha \tan \phi = \frac{W}{g} \ddot{s}$$

$$\ddot{s} = g \left[\frac{\sin(\alpha - \phi)}{\cos \phi} \right] \quad \left. \begin{array}{l} (\alpha \geq \phi) \\ \text{(If } \phi > \alpha \text{ then no sliding occurs and } \ddot{s} = 0) \quad (\alpha \leq \phi) \end{array} \right\} \leftarrow$$

At constant slope

$$\begin{aligned} \dot{s} &= \ddot{s}t + \dot{s}(0) \\ \therefore \dot{s} &= g \left[\frac{\sin(\alpha - \phi)}{\cos \phi} \right] t \quad (\alpha \geq 0) \\ \therefore s &= g \left[\frac{\sin(\alpha - \phi)}{\cos \phi} \right] \frac{t^2}{2} \quad (\alpha \geq 0) \end{aligned} \left. \vphantom{\begin{aligned} \dot{s} &= \ddot{s}t + \dot{s}(0) \\ \therefore \dot{s} &= g \left[\frac{\sin(\alpha - \phi)}{\cos \phi} \right] t \quad (\alpha \geq 0) \\ \therefore s &= g \left[\frac{\sin(\alpha - \phi)}{\cos \phi} \right] \frac{t^2}{2} \quad (\alpha \geq 0) \end{aligned}} \right\}$$

36. Given: Profile $\phi = 15^\circ$



Find: h_5

Solution:

(1) Draw ϕ -line

(2) From notes on dynamics of sliding, $0 = Wh - Wd \tan \phi$

$$\tan \phi = \frac{h}{d}$$

$$h = 1,200 + 650 - h_5$$

$$d = d_1 + d_2 + 100 + d_5$$

$$\tan 35^\circ = \frac{1,200}{d_1}$$

$$\tan 70^\circ = \frac{650}{d_2}$$

$$\tan 70^\circ = \frac{h_5}{d_5}$$

$$\therefore d = \frac{1,200}{\tan 35^\circ} + \frac{650}{\tan 20^\circ} + 100 + \frac{h_5}{\tan 20^\circ}$$

$$\tan 15^\circ = \frac{(1,200 + 650 - h_5)}{\left(\frac{1,200}{\tan 35^\circ} + \frac{650}{\tan 20^\circ} + 100 + \frac{h_5}{\tan 20^\circ}\right)}$$

$$(0.260) \left(1,713.0 + 1,786 + 100 + \frac{h_5}{\tan 20^\circ}\right) = 1,850 - h_5$$

$$964 + \frac{0.268 h_5}{\tan 20^\circ} = 1,850 - h_5$$

$$1,738 h_5 = 886$$

$$\underline{\underline{h_5 = 510}}$$

← h_5

37. Given: Topping rock block

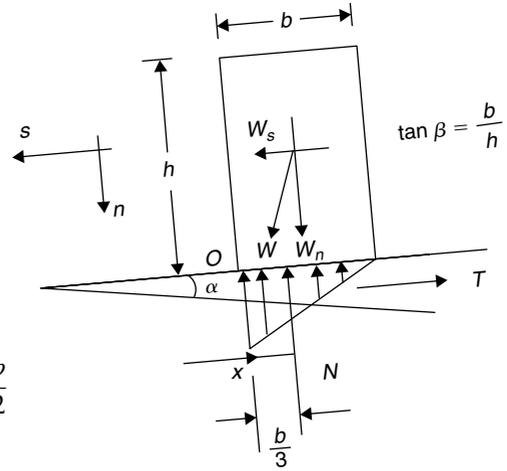
Show: $\tan(\alpha) < \frac{1}{3} \tan \beta$ for stability against toppling.

Solution:

Equilibrium requires

$$\begin{aligned} \Sigma_n F &= W & W \cos \alpha &= N \\ \Sigma_s F &= 0 & W \sin \alpha &= T \\ \Sigma_o M &= 0 & 0 &= W_s \frac{b}{2} + Nx - N \frac{b}{2} \end{aligned}$$

where x = distance from 0 to N



$$\begin{aligned} \therefore W \sin \alpha \frac{b}{2} + W \cos \alpha \left(x - \frac{b}{2} \right) &= 0 \\ \tan \alpha \frac{b}{2} &= \frac{b}{2} - x \\ x &= \frac{b}{2} - \frac{b}{2} \tan \alpha \end{aligned}$$

but with a triangular stress distribution

$$\begin{aligned} x &\geq \frac{b}{3} \\ \therefore \frac{b}{2} - \frac{b}{2} \tan \alpha &\geq \frac{b}{3} \\ \frac{b}{6} &\geq \frac{b}{2} \tan \alpha, \quad \frac{b}{h} = \tan \beta \\ \therefore \tan \alpha &\leq \frac{1}{3} \tan \beta \end{aligned}$$

← (for stability w.r.t. toppling)

38. Given: Rock block on slope
Find: Dimension necessary to prevent toppling.

To prevent toppling, must be in equilibrium

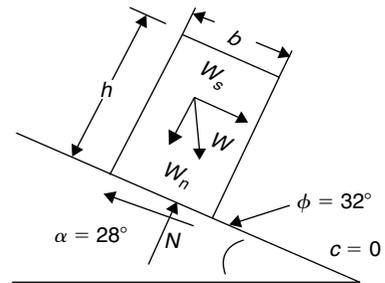
$$\Sigma F = 0: N = W_n = W \cos \alpha$$

$$T = W_s = W \sin \alpha$$

$$\Sigma M = 0 \quad Nx - T \frac{h}{2} = 0$$

$$\therefore x = \left(\frac{h}{2}\right) \left(\frac{T}{N}\right)$$

$$x = \left(\frac{h}{2}\right) (\tan \alpha) \quad \& \quad x < \frac{b}{2}$$



$$\therefore \left(\frac{h}{2}\right) \tan \alpha \leq \frac{b}{2}$$

$$\frac{b}{h} \geq \tan \alpha$$

$$b \geq h \tan 28$$

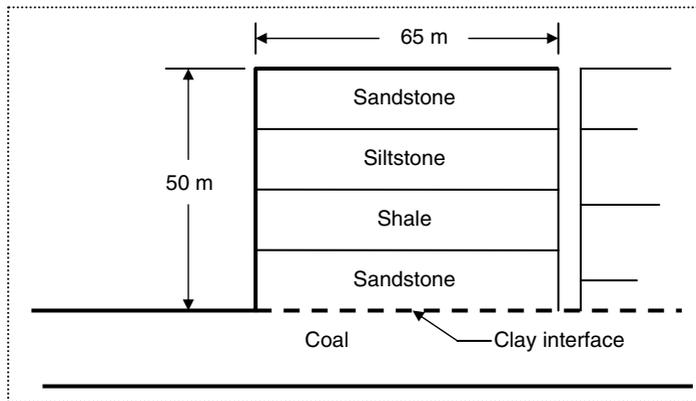
$$\underline{\underline{b \geq h(0.5317)}}$$

square base $\therefore b' \text{ into page} = b.$

e.g. if $h = 10'$ $b = b' \geq 5.317 \text{ ft}$

Added Problems

39. Consider a “highwall” in flat sedimentary strata as shown in the sketch. Three sets of joints are present. One set, J1, is composed of bedding plane joints. The other two sets, J2 and J3, are at right angles to the bedding and to each other as is often the case. The vertical joints J2 and J3 allow vertical separation of a large block of ground. For each meter of block into the page, estimate the acceleration (from blasting) that will just move the block forward as it slips along the clay interface. Note: the cohesion of the clay interface is 100 kPa; the friction angle is zero.



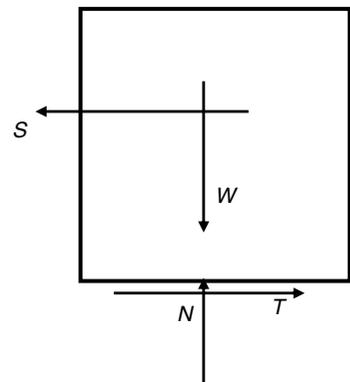
Given: large rock block, material properties of the interface
Find: acceleration a necessary to move the block.

Solution:

Draw a free body diagram with inertial force.

Thus, W = weight, N = normal reaction, S = inertial force, T = tangential reaction.
Now consider a safety factor with respect to sliding. Thus,

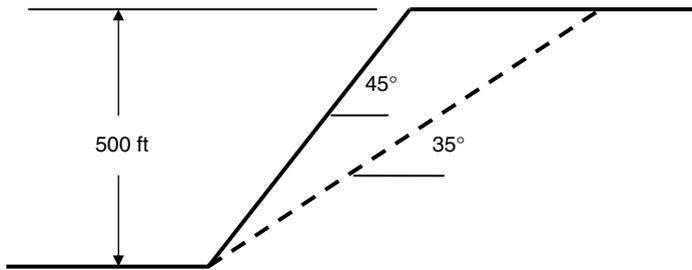
$$\begin{aligned}
 FS &= R/D \\
 &= (N \tan(\phi) + cA)/S \\
 &= (0 + cA)/S \\
 &= (0.1)(65)(1)/(W/g)a \\
 &= (0.1)(65)(1)/(22.6)(65)(50)(a/g) \\
 FS &= 0.0885/(a/g)
 \end{aligned}$$



where a reasonable specific weight of 22.6 kN/m^3 is assumed. The ratio (a/g) is a seismic coefficient. At the instant of slip, $FS = 1$, so $(a/g) = 0.0885$.

Thus, $a = (0.0885)(9.807)$, so $\underline{a = 0.868 \text{ m/s}^2}$

40. Suppose the cohesion of the failure surface in the sketch is diminished with each blast by fracturing of the interface, so the persistence p , defined as the ratio of fracture area to total area, decreases by steps with each blast. Further suppose blasting occurs twice a day and the slope moves downhill 0.1 inch per blast. Is there a day when the slope continues to accelerate after a blast? Note: cohesion and angle of friction of the failure surface are (900 psi, 40°). The fractured portion of the failure surface has cohesion and friction angle (0 psi, 30°). Specific weight is 150 pcf.



Given: slope data

Find: days to failure.

Solution:

By inspection, the failure surface inclination is less than the friction angle before any blast damage has occurred (40 versus 35 degrees), so the slope is safe at the outset. However, the friction angle of damaged, fractured failure surface is less than the failure surface slope (30 versus 35 degrees) and is also cohesionless, so failure will occur when blast damage extends the length of the slope. Blasting occurs twice per day, so the fracturing is 0.2 inch per day. The slope length is $500/\sin(35) = 872$ ft or 10,461 inches. Days to failure are no more than $10,461/0.2 = 52,300$ or about 14.3 yrs.

When the factor of safety is unity, then the next blast will accelerate the slope. In this situation

$$FS = R/D = (W \cos(35) \tan(\phi) + cA)/W \sin(35)$$

The weight of the slide mass remains constant as do the area and inclination of the potential failure surface. However, the cohesion and properties of the failure surface change with persistence. These properties may be estimated with the help of the Terzaghi jointed rock mass model. Thus, from equations 2.4 in the text

$$\begin{aligned} \tan(\phi) &= (1 - p) \tan(\phi_r) + (p) \tan(\phi_j) \\ c &= (1 - p)c_r + (p)(c_j) \end{aligned}$$

Thus,

$$\begin{aligned}\tan(\phi) &= (1 - p)\tan(40) + (p)\tan(30) = 0.8391 - 0.2617p \\ c &= (1 - p)(900) + (p)(0) = 900 - 900p \text{ (psi)}\end{aligned}$$

After substitution into the factor of safety equation and setting $FS = 1$,

$$W \sin(35) = W \cos(35)(0.8391 - 0.2617p) + (900 - 900p)A$$

that may be solved for persistence p after computing W and A .

$$\begin{aligned}W &= (\gamma H^2/2)(\cot(\alpha) - \cot(\beta)) \\ &= (150)(500)(500/2)(\cot(35) - \cot(45)) \\ W &= 8.028(10^6) \text{ lbf} \\ A &= H/\sin(\alpha) = 500/\sin(35) = 871.7 \text{ ft}\end{aligned}$$

After substitution

$$\begin{aligned}(8.028)(10^6)(0.5736) &= (8.028)(10^6)(0.8192)(0.8391 - 0.2617p) \\ &\quad + (900 - 900p)(871.7)(144)\end{aligned}$$

and solving: $p = 0.993$ indicating the entire failure surface will sustain blast damage before slope failure.

Check so far:

$$\begin{aligned}\tan(\phi) &= (1 - 0.993)\tan(40) + (0.993)\tan(30) = 0.5792 \\ c &= (1 - 0.993)(900) + (0.993)(0) = 6.3 \text{ psi}\end{aligned}$$

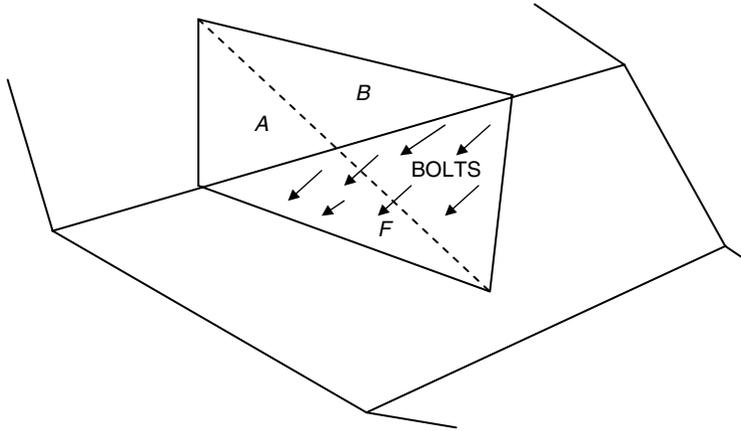
$$\begin{aligned}8.028(10^6) \sin(35) &= 8.028(10^6) \cos(35) \tan(30) + 6.3(144)(500/\sin(35)) \\ 4.605(10^6) &= 3.797(10^6) + 7.908(10^5) \\ &= 4.588(10^6)\end{aligned}$$

which is a reasonably close check, although not exact.

The time to failure is therefore as before, **14.3 yrs.**

Comment: Slopes sometimes appear to fail for no apparent reason. Day after day, operations are much the same, so the question that arises is what has changed? Why the failure? One answer is blasting causes slope acceleration but with passing of the transient, safety returns. However, the some loss of cohesion occurs with each blast. Eventually, the slope may fail as cohesion is destroyed entirely.

41. Bolting reinforcement of a potential wedge failure is anticipated. Bolts will be installed perpendicular to the face of the wedge as indicated in the sketch. All bolts will be tensioned the same amount and installed on a square pattern $S \times S$ m. Derive a formula for the wedge factor of safety with respect to sliding down the line of intersection of joints A and B that includes the effect of bolting. Assume there are m_A and m_B bolts intersecting planes A and B .



Given: Wedge failure data (dip and dip direction, cohesion and friction angles, specific weight, upland, face and foreland dip and dip directions, water table) and bolting data (spacing, number, tension)
Find: Bolted wedge safety factor formula.

Solution:

The FS expression without bolting is equation 2.23 in the text. Thus,

$$FS_w = R/D = (R_A + R_B)/D$$

Bolting will improve the resistance by mobilizing more frictional resistance via normal force (N) components and directly by uphill components (T) of bolt tension in much the same way as in planar block slides. Thus,

$$FS_w = R/D = (R_A + N_A \tan(\phi_A) + T_A + R_B + N_B \tan(\phi_B) + T_B)/D$$

An actual computation requires procedures for computing the bolting force components. Because all bolts have the same direction and tension, the total forces in this formula are simply the number of bolts times the force per bolt, that is,

$$F_A (\text{bolt}) = m_A f(\text{bolt}), \quad F_B (\text{bolt}) = m_B f(\text{bolt})$$

These forces are directed normal to the face, that is, parallel to the face normal, n_F . With the normal to the joint faces, n_A and n_B . These normal directions may be obtained from Table 2.1 in the text.

Projection of the bolt forces onto the normal directions of the joint planes is achieved with the vector dot product $\vec{n}_F \cdot \vec{n}_A = \cos(\theta_{AF})$, and similarly, $\vec{n}_F \cdot \vec{n}_B = \cos(\theta_{BF})$. Thus,

$$N_A = F_A \cos(\theta_{AF}) \quad \text{and} \quad N_B = F_B \cos(\theta_{BF})$$

The uphill components of the bolt forces should point up the line of intersection. If a unit vector \vec{s} down the line of intersection has been computed in the process of determining the dip and dip direction of the line of intersection, then the vector dot product may again be used, such that

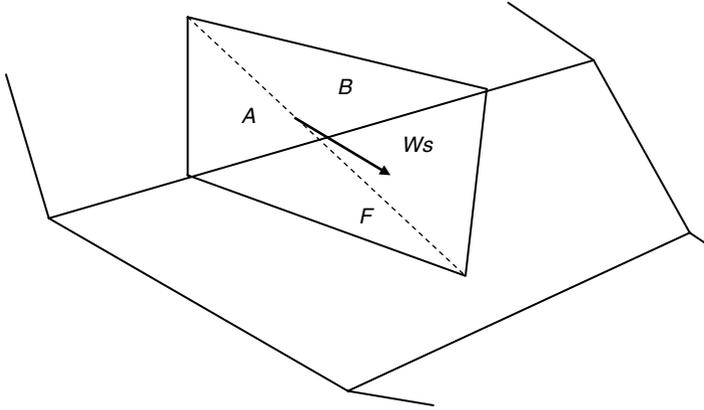
$$\vec{s} \cdot \vec{n}_F = \cos(\theta_{sF}).$$

The uphill bolt forces are then

$$T_A = F_A \cos(\theta_{sF}) \quad \text{and} \quad T_B = F_B \cos(\theta_{sF})$$

which completes the analysis.

42. Consider a potential wedge failure with seismic load Ws illustrated in the associated sketch. The seismic force is estimated as wedge weight W times a seismic coefficient a_s appropriate for the region. The seismic force acts horizontally through the center of the wedge and has an azimuth equal to the dip direction of the line of intersection. Modify a wedge safety factor formula to include such a seismic force.



Given: Given: wedge failure data (dip and dip direction, cohesion and friction angles, specific weight, upland, face and foreland dip and dip directions, water table) and seismic coefficient.

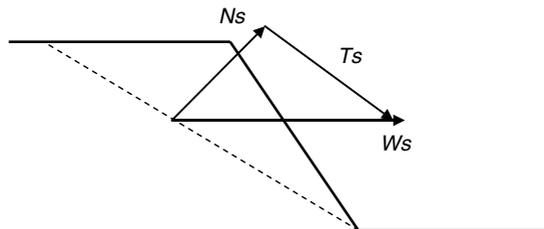
Find: Factor of safety formula.

Solution:

The seismic force (Ws) will decrease resistance by decreasing frictional resistance via normal force (N) components, but also will add to the driving force directly by downhill components in much the same way as in planar block slides. Thus,

$$FS_w = \frac{[R_A + (N_A - N_{sA}) \tan(\phi_A) + R_B + (N_B - N_{sB}) \tan(\phi_B)]}{D + T_{sA} + T_{sB}}$$

To be useful, details for calculating the seismic force components are needed. With reference to the sketch that shows the line of intersection in true dip, the normal and tangential components are seen to be similar to weight components, although the normal component of seismic force acts in the opposite direction of weight. Therefore, the formulas for normal weight components may be used to compute the normal components of seismic force.



Thus, from equations 2.31 in the text

$$N_{sA} = \frac{(Ns) \sin(\delta_b)}{\sin(\delta_a + \delta_b)}, \quad N_{sB} = \frac{(Ns) \sin(\delta_a)}{\sin(\delta_a + \delta_b)}, \quad Ws = Wa_s, \quad Ns = (Ws) \cos(\delta_s)$$

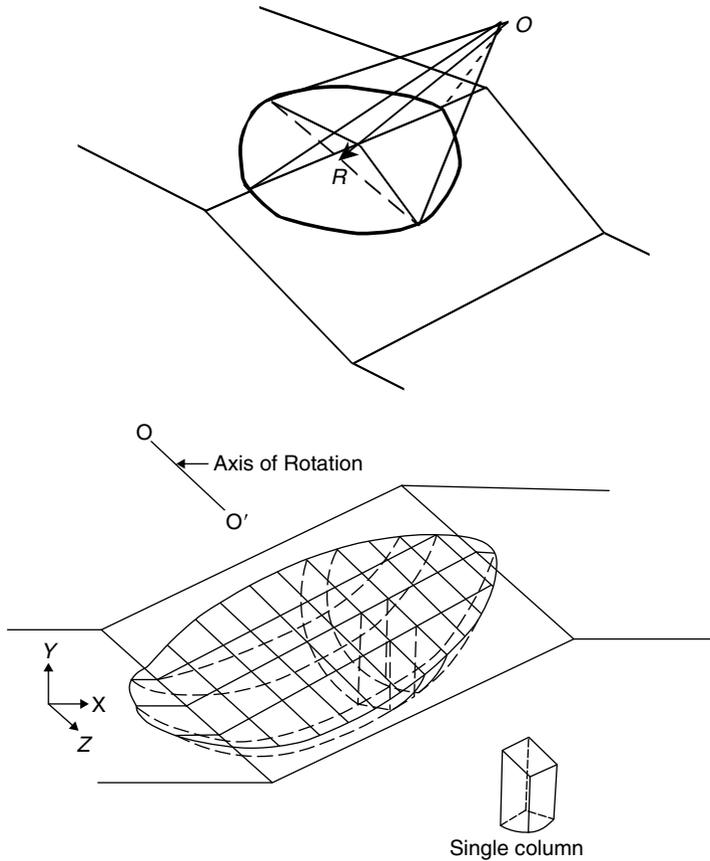
where Ws follows from the definition and Ns is simply the normal component of Ws .

The angle between the horizontal and the downhill direction is simply the inclination of the line of intersection δ_s . The downhill seismic forces are then

$$T_{sA} = Ws \cos(\delta_s) \quad \text{and} \quad T_{sB} = Ws \cos(\delta_s)$$

which completes the analysis.

43. Consider a rotational slide in the form of a spherical segment as shown in the sketches. Outline a “method of columns” analogous to the method of slices that accounts for three-dimensional effects not present in the two-dimensional analysis using the method of slices. (After Chen, 1981)

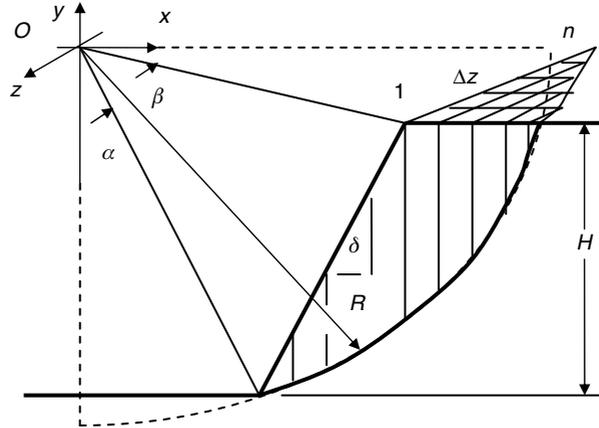


Given: Spherical slid mass with center of rotation at O with radius R in a slope of height H with angle δ from the horizontal.

Find: Method of columns similar to the method of slices.

Solution:

First fix a coordinate system with origin at the center of rotation, then partition the slide mass into slabs. Begin with a conventional slices partition of a central slab; proceed to the outer boundary of the spherical segment as shown in the sketch using a distance Δz for a total of n slabs. Let the angles from the toe of the central slab to the crest be α and β (sphere radius = R).



The mid-planes of a slab “ i ” have coordinates $-z(i) = (2i - 1)(\Delta z/2)$ from $i = 1$ to $i = n$ slabs. The radius $r(i)$ of the i th slab is $r(i) = R^2 - z(i)^2$. Each slab may now be treated in the same manner in the two-dimensional method of slices. Recall, moment equilibrium requires $\int_A RTdA = \int_V r\gamma dV$ that is approximated by $\sum R \sin(\alpha_j) W_j = \sum RT_j(\text{strength})/f_s$. The last is just equation 2.43 in the text. Assuming, as in the method of slices, that the global FS is equal to the local slice f_s , one has for the central slab with radius R

$$FS = \frac{\sum RT_j(\text{strength})}{\sum R \sin(\alpha_j) W_j}$$

The same analysis applies to the other slabs with radius $r(i)$. In consideration of all slabs (i) requiring summation over (i):

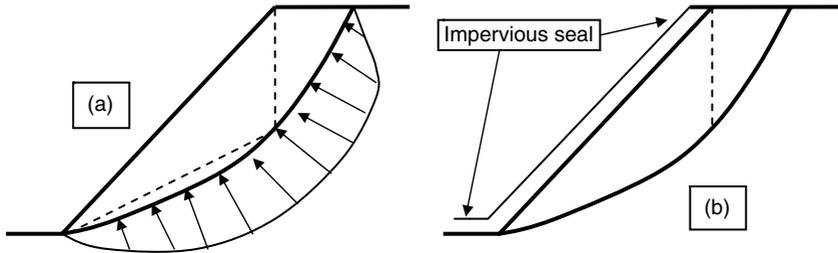
$$FS = \frac{\sum \left[\sum r(i) T_j(\text{strength}) \right]_i}{\sum \left[\sum r(i) \sin(\alpha_j) W_j \right]_i}$$

Geometric details could be computed systematically in a spread sheet for a site-specific spherical segment slip surface. Obviously, only the outline is given here. Two references that lead to more details about the method of columns are:

Chen, R. H. (1981) “Three-dimensional slope stability analysis.” Joint Highway Research Project, Engineering Experiment Station, Purdue University, Report JHRP-81-17.

Hovland, H. J. (1977) “Three-dimensional slope stability analysis method.” Journal of the Geotechnical Engineering Division, ASCE, Vol. 103, No. 9, pp. 971–986.

44. Consider a rotational failure as shown in the sketch with the water table at the crest. The water pressure on the circular slip surface is given by $p = \gamma_w z$, that is, by the product of water specific weight times water depth. The pressure distribution will show an increase from zero at the crest to a maximum and then a decrease to zero at the toe of the slope (a). An impervious seal is applied to the slope, but no drain holes are provided (b). Derive a formula that shows the effect of the seal on the slope safety factor



In this cases (a) and (b), the water pressure on the slip surface will increase steadily with depth below the water table to a point directly below the crest. At points further down the slip surface the water pressure in (a) depends on the vertical distance between the slip surface and the slope face. In case (b), the water pressure below the crest will be greater than in case (a) from the difference in water heads (distance below water tables). This difference will affect all slices below the slope face and is equal to the distance from the crest elevation to the slope face. Thus, the pressure difference between the two cases is $\Delta p = \gamma_w \Delta z$ and according to equation 2.45 in the text the *decrease* in slope safety factor is

$$\Delta FS = \frac{\sum_j (W_n - \Delta P) \tan(\phi)}{\sum_j W_s}$$

where $\Delta P = A \Delta p$ (A = area of slice base) and summation is over only the slices below the crest to the left of the dotted lines. The decrease in safety factor is the effect of the seal without drains.

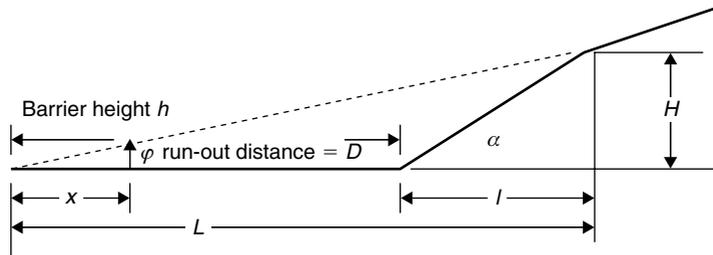
45. An unstable slope poses a threat to a housing development below. The height to the unstable mass is $H = 185$ ft and the slope angle $\alpha = 30$ degrees. The effective angle of sliding friction is ϕ . (a) Estimate the “run-out” distance the mass center moves from the slope toe before coming to rest. (b) Estimate a barrier height required to stop the slide mass before traveling the entire run-out distance.

Given: Unstable slope

Find: (a) run-out distance, (b) barrier height.

Solution:

First sketch the situation.



From the sketch:

$$D = L - l$$

$$D = \underline{\underline{[H/\cot(\phi)] - [H\cot(\alpha)]}}$$

The height of a barrier necessary to stop the mass center short of the full run-out distance is given by

$$\tan(\phi) = h/x$$

$$\therefore \underline{\underline{h = x \cot(\phi)}}$$