

## Chapter 2: Principles of Mathematics Education

### Chapter Summary

Chapter 2 is intended to expose the reader to several components that constitute an effective mathematics program. The 2014 NCTM document *Principles to Actions: Ensuring Mathematical Success for All* delineates six principles – teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism. Each of these elements is ultimately the responsibility of a mathematics teacher. The chapter describes the principles and provides the reader with examples of why it is important to consider each of them in the preparation of a mathematics teacher.

In addition to discussing the principles, the chapter also helps the reader to think about how the components are connected together. For example, if a state institutes a high-stakes assessment, such as a graduation test, then school districts must reconsider the content of their program (curriculum) and the strategies that teachers are using (teaching) in the classroom in light of the design of the test. Consequently, the assessment, curriculum, and teaching assessments are interconnected with one another.

Each of the topics in this chapter is elaborated upon in subsequent chapters. For example, Chapter 3 addresses the topic of teaching and learning from a psychological standpoint; Unit 2 (Chapters 4 and 5) is devoted to curriculum; Unit 3 (Chapters 6, 7, 8, and 9) focuses on teaching, and Unit 4 (Chapters 10 and 11) provides specifics on assessment. Access and equity are major topics of Unit 5 (Chapters 12 and 13), and the appropriate use of tools technology is discussed in Chapter 7, as well as being woven throughout the textbook. While different mathematics education programs may call for using the chapters of the book in a variety of sequences, it is strongly recommended that reading and discussion of Chapters 1 and 2 precede the study of any subsequent chapters of the book, as these two chapters provide the foundation and overview that underlie the content of the rest of the text.

### Learning Outcomes

After reading Chapter 2, the student should be able to answer the following questions:

- What are the six principles of mathematics education, as outlined by the NCTM in *Principles to Actions: Ensuring Mathematical Success for All*, and what does each mean?
  - \*\* The 2014 NCTM document *Principles to Actions: Ensuring Mathematical Success for All* delineates six principles – teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism. A mathematics teacher should be aware of these components and how their interplay determines the level of success of a mathematics program.

- How can the principles be used to help mathematics teachers define an ideal mathematics education program?
  - \*\* Teachers need to be able to think about the “big picture” of mathematics education when functioning in their jobs. Acknowledgement of the important roles of factors such as addressing access and equity and using tools and technology will ultimately drive the decisions that are made by educators.
- How does a traditional mathematics teaching model compare to one that embraces the six principles?
  - \*\* A lecture-based classroom that emphasizes the acquisition of skills over the development of conceptual understanding has dominated the teaching of mathematics for many years. When the needs of the learner are taken into account, then other considerations, such as how to appropriately use technology and how to best measure student progress begin to transform classroom teaching.
- What is the role of professional development in the improvement of classroom teaching practices?
  - \*\* The teaching profession is constantly evolving and in a state of change. It is the responsibility of each teacher to seek out the latest in theory and practice to improve one’s teaching skills throughout a teaching career.
- How do the six principles interact with one another – specifically, what is the relationship among curriculum, teaching, and assessment?
  - \*\* The six principles do not exist in isolation. Instead, each of these elements of the mathematics teaching profession often interacts in a complex manner. For example, one way to address concerns of access and equity is through the use of tools and technology. Teachers need to recognize the relationships between these principles.

### **Sample Classroom Activities**

1. Divide the class into six “expert” groups and assign one of the principles (teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism) to each group. Have each group discuss the role of its principle in the design of a mathematics program. Groups may refer to the source document from NCTM that describes the principles. Then, re-assemble the students into groups of six, in which each group includes one person from each of the original principles groups. Each person should share the highlights of their expert group’s discussion, and the group should conclude by discussing how the principles interact with one another.
2. Ask each teacher candidate or in-service teacher to spend one day in a school, observing more than one mathematics teacher in a variety of courses or grade levels. In the methods classroom, have each person report on what was observed in the schools. Were the observed lessons more like that of Mr. Dussan, as described in the beginning of Chapter 2, or did teachers appear to be addressing the principles, as illustrated by using authentic assessments, technology, and other means described in Chapter 2?

3. Algebra I, as described in Chapter 2, is sometimes referred to as a “gatekeeper” course because students often cannot move on to the study of geometry, trigonometry, and pre-calculus topics until first mastering basic algebra. Provide each small group of students with a current Algebra I book and an older version of the same book. Have the group examine the contents of the book and discuss how textbook authors address Algebra I content differently today than they did in the past. Look for ways that equity is being addressed, as well as activities that engage the learner in the use of hands-on materials or technology.
4. Have teacher candidates or in-service teachers explore some of the web-based professional development tools that are available on the Internet. Examples include Annenberg Media’s Learner.org site, as well as NCTM’s Illuminations ([illuminations.nctm.org](http://illuminations.nctm.org)) website. Discuss some of the benefits and potential drawbacks to using the web as a resource for professional development.
5. Divide the methods class into small groups and assign a mathematics software package, a graphing calculator, or an interactive website to each group of students. For example, groups may explore Geometer’s Sketchpad, Geogebra, Fathom, Microsoft Excel, a graphing calculator, or a website such as the National Library of Virtual Manipulatives ([nlvm.usu.edu](http://nlvm.usu.edu)). Each group should explore its technological tool and report back to the rest of the class on how the technology might be used in the classroom and the possible advantages and disadvantages in the teaching of mathematics.

### **Masters for PowerPoint Slides**

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# The NCTM Principles, 2014

**Teaching and Learning.** An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically (p. 7).

**Access and Equity.** An excellent mathematics program requires that all students have access to a high quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential (p. 59).

**Curriculum.** An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world (p. 70).

**Tools and Technology.** An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking (p. 78).

**Assessment.** An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement (p. 89).

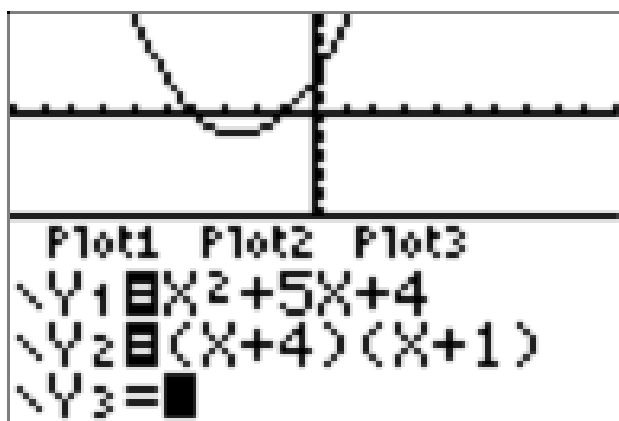
**Professionalism.** In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for personal and collective professional growth toward effective teaching and learning of mathematics (p. 99).

# Factoring a Trinomial

## Alternative Methods to Check

$$x^2 + 5x + 4 = (x + 4)(x + 1)$$

### #1: Examine the Graphs

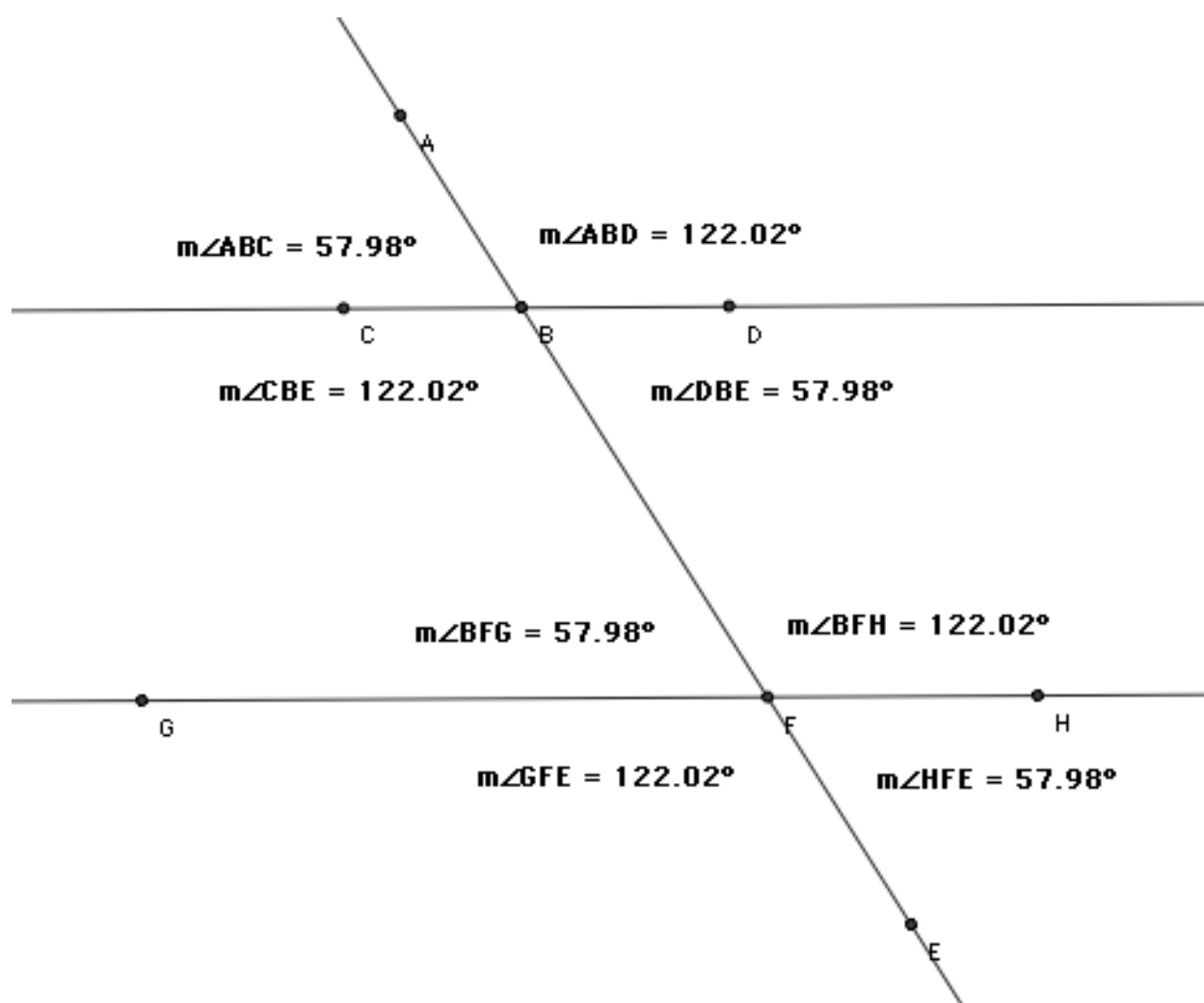


### #2: Examine the Table

X	Y <sub>1</sub>	Y <sub>2</sub>
0	4	4
1	10	10
2	18	18
3	28	28
4	40	40
5	54	54
6	70	70
X=0		

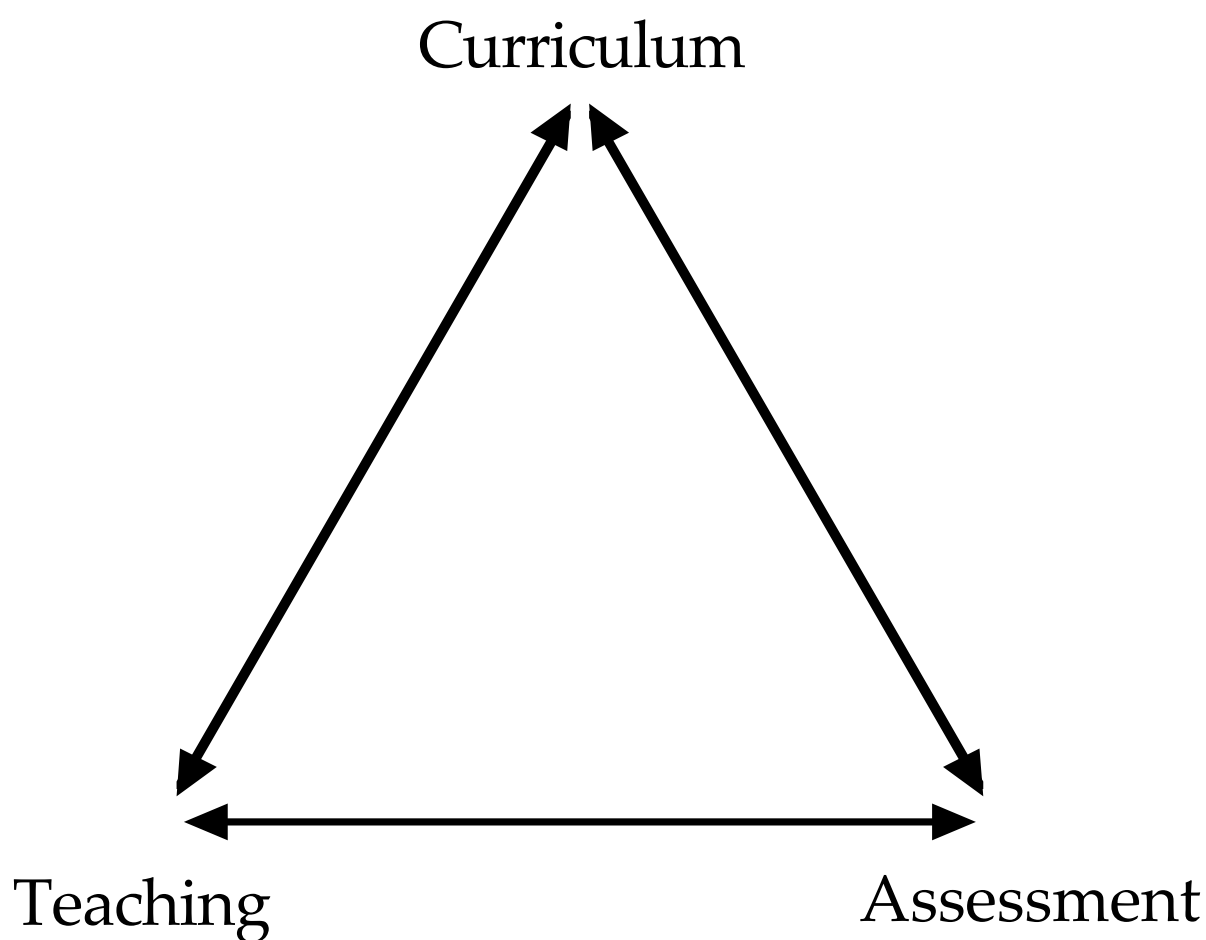
# Parallel Lines with Dynamic Geometry Software

Students explore parallel lines cut by a transversal using dynamic geometry software, such as Geometer's Sketchpad:



## Connecting the Principles

The six NCTM principles interact with and depend on one another:



## Test Bank Items

### ***MULTIPLE CHOICE***

1. Which of the following is a false assumption that is commonly held by new teachers of mathematics?
  - a) Technology is helpful to learning mathematics.
  - b) Students will benefit from exposure to the same learning environment that was experienced by the teacher.
  - c) Mathematics is perceived as being very difficult by many of their students.
  - d) Teaching methods can and do make a difference in student success.
  - e) All students are capable of learning mathematics.
2. In which year did the National Council of Teachers of Mathematics (NCTM) publish *Principles to Actions: Ensuring Mathematical Success for All*, from which the 6 principles were described?
  - a) 1980.
  - b) 1989.
  - c) 2000.
  - d) 2010.
  - e) 2014.
3. Which of the following refers to the essential elements that are needed to create an exemplary mathematics program for students?
  - a) Principles.
  - b) Indicators.
  - c) Standards.
  - d) Goals.
  - e) Focal Points.
4. Which of the following is NOT one of the six NCTM principles?
  - a) Teaching and Learning.
  - b) Tools and Technology.
  - c) Assessment.
  - d) Environment.
  - e) Professionalism.
5. Which of the following principles refers to “high expectations and strong support for all students”?
  - a) Curriculum.
  - b) Access and Equity.
  - c) Teaching and Learning.
  - d) Assessment.
  - e) Tools and Technology.



6. Which of the following principles states that students should be engaged in “meaningful learning through individual and collaborative experiences”?
  - a) Tools and Technology.
  - b) Curriculum.
  - c) Teaching and Learning.
  - d) Access and Equity.
  - e) Assessment.
7. The analogy of learning to play guitar, as explained in Chapter 2, illustrates which of the following?
  - a) Music, like mathematics, can be difficult to master.
  - b) A student might be skilled at playing guitar but not the piano, just as one student might feel comfortable with the use of technology while another does not.
  - c) The focus in a mathematics classroom should go beyond skill-based procedures.
  - d) Students need to learn the basics in mathematics, just as they do in music.
  - e) Mathematics lessons, like music lessons, must be challenging.
8. Problem solving can best be described as which of the following?
  - a) The opportunity to put skills to use in a real-life situation.
  - b) The best way to wrap up a mathematics unit on a given topic.
  - c) The opportunity to practice routine skills.
  - d) Showing students an approach that can be used anytime a similar situation occurs.
  - e) Assigning homework from the book that is clearly marked as “problems.”
9. Which of these can be described as the “flexible use of procedures in the problem-solving process”?
  - a) Productive Disposition.
  - b) Fluency.
  - c) Conceptual Understanding.
  - d) Strategic Competence.
  - e) Adaptive Reasoning.
10. An example of relating procedural knowledge to conceptual ideas that is presented in Chapter 2 involves which of the following?
  - a) Proving the congruence of triangles.
  - b) Using the distributive property.
  - c) Finding the percentage of a number.
  - d) Combining similar terms.
  - e) Determining the volume of a solid.
11. The inter-relating of mathematical ideas is referred to as which of the following?
  - a) Applying mathematics.
  - b) Reasoning mathematically.
  - c) Making connections.
  - d) Using procedures.
  - e) Finding solutions.

12. Which of the following is the most important factor in the learning of mathematics?
  - a) Textbooks used.
  - b) Technology used.
  - c) Skill of the teacher.
  - d) Strength of the curriculum.
  - e) Support of the community.
13. Which of the following is NOT necessarily associated with effective mathematics teaching?
  - a) High level of competence in mathematics.
  - b) Strict adherence to lesson plans and textbooks.
  - c) A well designed set of teaching strategies.
  - d) Knowledge about which teaching approaches work best in a given situation.
  - e) Understanding of how students learn.
14. Which of the following is a decision that a teacher must make almost every day?
  - a) Selection of content and processes to emphasize.
  - b) Determining how much time to spend on a given topic.
  - c) Selecting appropriate problems to solve.
  - d) Determining how to measure student understanding.
  - e) All of these.
15. Which of the following is NOT considered a high-leverage teaching practice?
  - a) Establish mathematics goals to focus learning.
  - b) Pose purposeful questions.
  - c) Build procedural fluency from conceptual understanding.
  - d) Research and use an extensive variety of web resources.
  - e) Facilitate meaningful mathematical discourse.
16. The Mathematical Teaching Practices are for teachers as which of the follow is for students?
  - a) Standards for Mathematical Practice.
  - b) Principles of Assessment.
  - c) Disposition Checklist.
  - d) Professional Teaching Standards.
  - e) None of these.
17. Which course is often referred to as a “gatekeeper” for the study of higher mathematics?
  - a) Geometry.
  - b) Pre-Algebra.
  - c) Algebra I.
  - d) Trigonometry.
  - e) Algebra II.

18. Which is the determining factor on which teachers are ordinarily given the assignment to teach the higher-level mathematics courses in secondary and middle schools?
- a) Mathematics the teacher studied in college.
  - b) Years of teaching experience in the district.
  - c) Preference of the teacher.
  - d) Requests made by potential students.
  - e) Reputation of the teacher.
19. Which of the following most accurately defines *equity* in mathematics education?
- a) All students have a right to equally access all areas of the curriculum.
  - b) All students should be taught the same.
  - c) All students should obtain high grades in mathematics.
  - d) All students should be exposed to both the best and the weakest teachers in their schooling experience.
  - e) All students should use the same textbooks in their mathematics classes.
20. Which of the following can contribute significantly to helping every student learn mathematics?
- a) Cooperative group work.
  - b) Hands-on teaching strategies.
  - c) Use of computers.
  - d) Individual tutoring.
  - e) All of these.
21. Which of the following is often used interchangeably with the word “curriculum” in the community?
- a) Lesson plans.
  - b) Activities.
  - c) Units.
  - d) Textbooks.
  - e) Principles.
22. Which of the following terms refers to “all of the experiences that a student has throughout his or her educational career”?
- a) Mathematics education.
  - b) Student learning.
  - c) Curriculum.
  - d) Standards.
  - e) Assessment.
23. What is the name of the educational theorist who developed the term *implicit curriculum*?
- a) Gardner.
  - b) Eisner.
  - c) Dussan.
  - d) Vygotsky.
  - e) Annenberg.

24. Which of the following is NOT recognized as part of the implicit curriculum?
- a) The seating arrangement in the classroom.
  - b) The psychological environment established by the teacher.
  - c) The courses taken by a student.
  - d) The way that lessons are conducted.
  - e) The manner in which technology is incorporated in teaching.
25. Which of the following is a way that a student can verify that a trinomial has been factored properly?
- a) Examine the graphs of the trinomial and product of the binomials.
  - b) Substitute a value for the variable and simplify the trinomial and the product of the binomials.
  - c) Examine a table of values for the trinomial and for the product of the binomials.
  - d) Re-multiply the binomial solution to compare the product to the original trinomial.
  - e) All of these.
26. What is the most important reason for why a teacher should consider using a graphing calculator in graphing and tabular modes to verify the solutions to polynomial simplifying problems?
- a) The strategy promotes equity.
  - b) The strategy is always enjoyable to students.
  - c) The strategy is easier than using pencil and paper approaches.
  - d) The strategy prepares the students for real-life problems involving polynomials.
  - e) The strategy requires the use of less class time.
27. Which of the following is LEAST associated with curriculum?
- a) What material is taught.
  - b) How the material is taught.
  - c) The sequence in which the material is taught.
  - d) Which concepts are emphasized.
  - e) Why the material is included in a course.
28. Sequencing mathematical topics in a reasonable order is most likely to result in which of the following benefits?
- a) Covering topics more quickly and studying more mathematics.
  - b) Avoiding duplication of topics and building on student knowledge.
  - c) Using technology and providing more visuals for students.
  - d) Appealing to the interests of students and increasing achievement.
  - e) Increasing course-taking and raising awareness of the usefulness of mathematics.
29. Which of the following describes a *transversal*?
- a) A line that intersects two parallel lines.
  - b) A ray that bisects an angle.
  - c) A segment that joins the endpoint of a longer segment to its midpoint.
  - d) A line that serves as the base for a triangle.
  - e) A line that is perpendicular to another given line.

30. According to the author in Chapter 2, the use of technology to explore a problem, rather than simply drawing a picture with pencil-and-paper, is more likely to produce which result?
- a) Accuracy of the figure drawn.
  - b) Success in teaching the concept.
  - c) Motivation of the students.
  - d) Attention of the class.
  - e) Acceptable behavior by the students.
31. A common misuse of technology in teaching mathematics is which of the following?
- a) Using technology in all units throughout a school year.
  - b) Using graphing calculators with students before they are ready.
  - c) Restricting the use of technology to checking answers.
  - d) Playing mathematical games.
  - e) Solving problems that should have been worked by hand.
32. Which of the following is an important technological tool that is appropriate for use in secondary and middle school mathematics classrooms?
- a) Graphing calculators.
  - b) Computers.
  - c) Personal Electronic Devices.
  - d) Interactive whiteboards.
  - e) All of these.
33. Which groups of students often benefit the most from the use of technology when learning mathematics, according to the author in Chapter 2?
- a) Middle school students.
  - b) Students in high-level mathematics courses.
  - c) Students who are frequently absent from school.
  - d) Students with learning disabilities.
  - e) Students with negative attitudes toward mathematics.
34. Which statement most accurately characterizes the NCTM's vision of mathematical assessment?
- a) Gathering information about students.
  - b) Determining student grades.
  - c) Evaluating student progress.
  - d) Giving tests to students.
  - e) Viewing student work at the end of a unit.

35. Which of the following would be considered a formal means of assessing student progress?
- a) Observing students working on a team project.
  - b) Observing students as they make a presentation to the class.
  - c) Reading student journal entries.
  - d) Interviewing a student while doing seatwork.
  - e) Administering a test.
36. Which statement best describes the ideal role of assessment in mathematics?
- a) End-of-unit measurement of progress.
  - b) Somewhat of an interruption to the teaching process.
  - c) Something that is done more “to” the student than “for” the student.
  - d) An ongoing process.
  - e) The best measure of a teacher’s effectiveness.
37. Which of the following is considered an “essential component” of effective assessment in a mathematics classroom?
- a) Frequent testing or quizzing of student knowledge.
  - b) Timely feedback on assessments given to students.
  - c) “Curving” of grades to promote positive student attitudes.
  - d) Ensuring that assessments are always objective.
  - e) Fairness, promoted by ensuring that all students are assessed the same way.
38. Assessment should serve to improve which of the following?
- a) The teaching and learning of mathematics.
  - b) The design of the curriculum.
  - c) The accuracy of final grades.
  - d) The evaluation of a teacher.
  - e) The use of tools and technology.
39. Which of the following most accurately characterizes the process of becoming a teacher?
- a) An undergraduate education degree program.
  - b) A graduate education degree program.
  - c) Ten years of teaching experience.
  - d) A lifelong endeavor.
  - e) An impossible goal to achieve.
40. A learning community of teachers often focuses on which of the following?
- a) Examining lesson plans.
  - b) Examining teaching practices.
  - c) Examining assessments.
  - d) Working together for mutual improvement.
  - e) All of these.

41. Which of the following professional development activities best lends itself to addressing self-directed plans for improvement?
- a) Exploring websites.
  - b) Attending workshops.
  - c) Taking college courses.
  - d) Attending conferences.
  - e) Participating in teacher meetings.
42. Learner.org is a popular professional development site because it provides the teacher with hundreds of which of the following?
- a) Examples of student work.
  - b) Lesson plans.
  - c) Classroom activities.
  - d) Streaming video clips.
  - e) Problems to pose to students.
43. According to the author in Chapter 2, perhaps the most important task of the teacher is to develop his or her skills to do which of the following?
- a) To make mathematics fun for students.
  - b) To challenge the students to perform to their potential.
  - c) To use technology in the classroom.
  - d) To write effective lesson plans.
  - e) To motivate and engage the learners.
44. Which of the following most accurately characterizes the six NCTM principles?
- a) Each principle should be considered separate from the others, as they represent very different aspects of the mathematics teaching profession.
  - b) The most important is the learning principle, since the ultimate measure of success is what the students have learned.
  - c) The equity and technology principles are the most closely related, as one promotes the other, yet both are different from the other four principles.
  - d) The six principles are all interwoven, as each connects in some way with the others.
  - e) Three of the principles relate primarily to the teacher, while the other three are directed at the students.
45. Which of the following can be considered the major components of the mathematics teaching profession?
- a) Equity, teaching, and learning.
  - b) Curriculum, assessment, and teaching and learning.
  - c) Technology, teaching and learning, and equity.
  - d) Teaching, learning, and assessment.
  - e) Assessment, technology, and equity.

46. Curricular changes in a school district are most likely to bring about changes in which of the following?
- a) Equity.
  - b) Teaching.
  - c) Learning.
  - d) Technology.
  - e) Problem solving.
47. When a teacher decides to assign a quarterly project in mathematics, which of the principles is being directly addressed?
- a) Assessment.
  - b) Teaching.
  - c) Learning.
  - d) Equity.
  - e) Curriculum.
48. Which of the following does research suggest may be one of the best ways to bring about reform in mathematics teaching practices in a school district?
- a) Rewriting the curriculum.
  - b) Purchasing interactive whiteboards for each classroom.
  - c) Adoption of different assessment strategies.
  - d) Purchase of new textbooks.
  - e) Involvement of the community in decision-making.
49. The NCTM *Standards* describes teaching mathematics in which of the following ways?
- a) A profession that lends itself to “recipes” for success.
  - b) A profession in which it is fairly easy to help teachers become effective.
  - c) A complex profession with no universal approaches that work for all.
  - d) A profession with enough research to know what works with virtually all students, most of the time.
  - e) A profession with little valid research to drive day-to-day decisions.
50. Which of the following is NOT a characteristic of the teaching profession?
- a) Making the best of current research.
  - b) Putting research to practice in the classroom.
  - c) Believing that every student can learn.
  - d) Reflecting on practice to improve over time.
  - e) Recognizing that teachers who are relatively new to the profession are the least likely to be successful.



51. Becoming an effective teacher of mathematics can be best described by which of the following?
- a) Success in the field depends on a high level of success in college.
  - b) The development of a teacher is a career-long process.
  - c) Most teachers are not successful until they have at least 10 years of experience.
  - d) It is more difficult to teach mathematics than any other subject area.
  - e) Most teachers of mathematics will never feel truly successful in their entire career.

### ***ESSAY***

1. Discuss how a teacher's personal success in doing mathematics, together with his or her experiences as a student, influence the way that the teacher runs his or her own classroom.
2. List and provide a brief description of each of the six NCTM principles of an effective mathematics program.
3. Explain how a teacher's view on the question, "Do you believe that *all* students are capable of learning mathematics?" will affect his or her teaching practices.
4. A broad use of the term "curriculum" implies more than simply the textbook used in teaching mathematics. Explain a more comprehensive view of the meaning of curriculum, as described in the NCTM principles.
5. Give an example of a mathematics lesson that illustrates the teaching and learning principle at work. (You can either discuss the example presented in Chapter 2 or describe a situation of your own.)
6. Explain how the three major components of mathematics education – curriculum, teaching and learning, and assessment – connect and influence one another.

### ***PERFORMANCE TASKS***

1. Identify a personal challenge or weakness in your own mathematics or education background, such as discomfort with teaching geometry or lack of knowledge about the use of inquiry in teaching. Search to find a website, such as [learner.org](http://learner.org), that provides professional development information on your topic of choice. Investigate what is available on the web to provide you with support and make a short presentation to the class that includes demonstrating the website that you have identified.
2. Interview two mathematics teachers – one who is relatively new to the profession and one who has been teaching for 20 years or more. Ask them which of the six principles they believe is most important, most difficult to address, and so forth. Compare the responses in a reflective paper, describing how experience in the field may influence the manner in which teachers view the principles.

3. Obtain a copy of the NCTM principles from the source document, *Principles to Actions: Ensuring Mathematical Success for All*. Select any two of the principles, read about them in the original document, and create a checklist of specific actions that you can take as a teacher to address each of the principles. Support the items on your checklist with short comments or quotes from the source document.

***MULTIPLE CHOICE TEST BANK SOLUTIONS:***

- |       |       |       |       |
|-------|-------|-------|-------|
| 1. B  | 14. E | 27. E | 40. E |
| 2. E  | 15. D | 28. B | 41. A |
| 3. A  | 16. A | 29. A | 42. D |
| 4. D  | 17. C | 30. C | 43. E |
| 5. B  | 18. B | 31. C | 44. D |
| 6. C  | 19. A | 32. E | 45. B |
| 7. C  | 20. E | 33. D | 46. B |
| 8. A  | 21. D | 34. A | 47. A |
| 9. B  | 22. C | 35. E | 48. C |
| 10. B | 23. B | 36. D | 49. C |
| 11. C | 24. C | 37. B | 50. E |
| 12. C | 25. E | 38. A | 51. B |
| 13. B | 26. A | 39. D |       |