Chapter 2 Basic Elements and Components

Solution 2.1

(a) Force, current, fluid flow rate, heat transfer rate.

(b) Velocity, voltage, pressure, temperature

(c)-(g) No

Solution 2.2

Consider a cylindrical surface at *r*adius r and length *l*. The heat transfer rate in the radial direction is

$$Q = -2\pi rlk\frac{dT}{dr}$$

or,

$$\frac{Q}{r} = -2\pi lk\frac{dT}{dr}$$

Integrate wrt *r* from d_i to d_o, noting that *Q* is independent of *r*. We get

$$Q\ln\frac{d_o}{d_i} = -2\pi lk[T_o - T_i]$$

or,

$$T_i - T_o = \ln\frac{d_o}{d_i}/(2\pi lk)Q$$

Hence,

$$R_k = \frac{\ln d_o/d_i}{2\pi lk}$$

Solution 2.3

Under these conditions (see text), the efficiency of the lead screw will be less than zero. Hence the unit will not be functional and will be in a locked condition.

Solution 2.4

Preloading increases friction and hence reduces the lead-screw efficiency. Also, preloading introduces extra stresses, which will increase noise and wear, and will reduce the operating life of the unit.

Solution 2.5

The equations of motion (from Newton's second law) are

For the rotor: $T - T_R = J\alpha$ (i)

For the table: $F - F_R = ma$ (ii)

where T_R = resistance torque from the lead screw
α = angular acceleration of the motor rotor (and lead screw)
F = driving force from the lead screw
F_R = external resistance force on the table.

Assuming a rigid lead screw without backlash, the motion compatibility condition is written as

$$a = r\alpha \tag{iii}$$

The load transmission equation for the lead screw is:

$$F = \frac{e}{r}T_R \tag{iv}$$

Finally, equations (i) through (iv) can be combined to give

$$T = \left(J + \frac{mr^2}{e}\right)\frac{a}{r} + \frac{r}{e}F_R \tag{v}$$

Note that $F_R = +mg$ when the load is driven up and $F_R = -mg$ when the load is driven down. Hence,

$$T = \left(J + \frac{mr^2}{e}\right)\frac{a}{r} \pm \frac{r}{e}mg \tag{vi}$$

For the given lead, $r = \dfrac{5\times10^{-3}}{2\pi}$ m/rad

For starting conditions:

$a = 0$ and $e = 0.5$

Substitute $m = 500$ kg and the above numerical values in (vi) to obtain the starting torque

$$T_s = \frac{5\times10^{-3}}{2\pi\times0.5}\times500\times9.81 \text{ N/m} = 7.8 \text{ N.m}$$

For moving up:
Substitute numerical values in (vi) with $F_R = +mg$, e = 0.65, J = 0.25 kg.m^2, and a = 3.0 m/s^2. The dynamic torque is

$$T_d = 0.25\times\frac{3.0\times2\pi}{5\times10^{-3}}+500\times(3.0+9.81)\times\frac{5\times10^{-3}}{2\pi\times0.65} \text{ N.m} = 942.0 + 7.8 \text{ N.m} \quad \text{(vii)}$$

or, T_d = 949.8 N.m

For moving down:
Substitute numerical values in (vi) with $F_R = +mg$ and the rest as in the previous case.

$$T_d = 0.25\times\frac{3.0\times2\pi}{5\times10^{-3}}+500\times(3.0-9.81)\times\frac{5\times10^{-3}}{2\pi\times0.65} \text{ N.m} = 942.0 - 4.2 \text{ N.m} \quad \text{(viii)}$$

or, T_d = 937.8 N.m

Note that the first term on the RHS of (vii) or (viii) is much larger than the second term. Hence, the torque required to accelerate the rotor and lead screw is much larger than that used to drive the load.

Solution 2.6

Step 1: First suppose that the outer gear is free to rotate but the connecting arm is fixed.

Step 2: Give a clockwise (cw) angular velocity ω to the pinion

Step 3: Angular velocity of the planetary gear = $\omega\frac{r_p}{r_g}$ (counter-clockwise or ccw)

Angular velocity of the outer gear = $\omega\frac{r_p}{r_g}\times\frac{r_g}{(r_p+2r_g)} = \omega\frac{r_p}{(r_p+2r_g)}$ (ccw)

Step 4: To fix the outer gear (as required in the problem) give a cw rotation of $\omega\frac{r_p}{(r_p+2r_g)}$ to the entire unit.

Angular velocity of pinion = $\omega_i = \omega+\omega\frac{r_p}{(r_p+2r_g)} = \omega\frac{2(r_p+r_g)}{(r_p+2r_g)}$

Angular velocity of the planetary gear = $\omega\frac{r_p}{r_g} - \omega\frac{r_p}{(r_p+2r_g)}$ ccw $= \omega_i\frac{r_p}{2r_g}$ ccw

Note: This result could have been obtained directly in view of the fact that the mesh point between the planetary gear and the outer gear has zero velocity (since the outer gear is fixed).

$$\text{Angular velocity of the arm} = \omega \frac{r_p}{(r_p + 2r_g)} \text{ ccw } = \omega_i \frac{r_p}{2(r_p + r_g)}$$

Solution 2.7

Advantages:

1. Provides a convenient means of torque amplification as well as speed reduction.
2. Widely available in a variety of types and designs.
3. Cost effective

Disadvantages:

1. Backlash due to clearance between tooth width and tooth spacing, which is necessary for proper meshing. Sophisticated tooth profiles, spring loading of the gears, and feedback control can reduce/eliminate backlash.
2. Friction due to sliding motion between drive teeth and driven teeth at the meshing points, and also at the bearings. Lubrication and surface treatment of the gear teeth, and the use of low-friction bearings can reduce the frictional loss.
3. Added weight and inertia. Use light-weight material.

Solution 2.8

See the free-body diagram shown in Figure S2.8, in the case of a loss-free (i.e., 100% efficient) gear transmission.

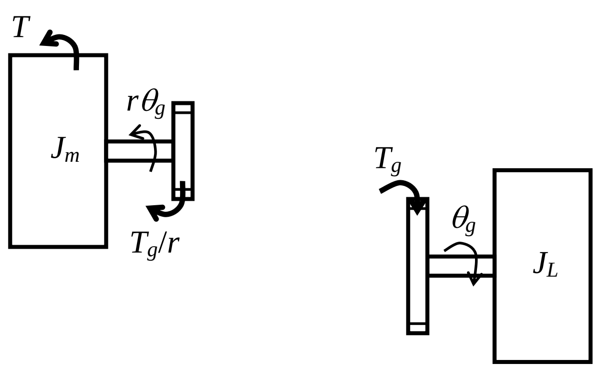

Figure S2.8 Free-body diagram.

Newton's 2nd law gives

$$J_m r \ddot{\theta}_g = T - \frac{T_g}{r} \qquad \text{(i)}$$

and

$$J_L \ddot{\theta}_g = T_g \qquad \text{(ii)}$$

where T_g = gear torque on the load inertia. Eliminate T_g in (i) and (ii). We get

$$\ddot{\theta}_g = \frac{rT}{(r^2 J_m + J_L)}$$

Solution 2.9

Repeatability may be poor (typically higher than 0.05°). Instability problems may occur due to nonlinearities (limit cycles, etc.). Vibrations and wear can be unsatisfactory. Positioning accuracy can be poor for high-precision applications (e.g., errors greater than 0.1°). Preloading reduces stiffness of the drive (and may be crucial in high-speed and high-torque applications). Also, preloading will increase tooth wear problems. Other conventional techniques do not completely eliminate backlash but will increase the overall cost of the drive system.

Harmonic drive, as discussed in the text, is a backlash-free transmission that can provide high speed ratios (and torque ratios).

Solution 2.10

Advantages of indirect-drive mechanisms: Reduces joint weight
$\Rightarrow$ increases bandwidth (speed of operation) and reduces torque requirements (due to lower mass and lower inertia loading)

Disadvantages of indirect-drives: Increased friction, backlash, increased time constants due to transmission dynamics (conflicting with reduced inertia loading).

Solution 2.11

Case	Rigid Spline	Wave Generator	Flexispline	Speed Ratio
0	Fixed	Input	Output	r
1	Fixed	Output	Input	$1/r$
2	Output	Input	Fixed	$r + 1$
3	Input	Output	Fixed	$1/(r + 1)$
4	Output	Fixed	Input	$(r + 1)/r$
5	Input	Fixed	Output	$r/(r + 1)$

The direction of the drive motion depends on which of the tooth pitch values is smaller. Suppose that the flexispline has a larger pitch angle (i.e., fewer teeth) than the rigid spline has. In this case the direction of operation is listed below.

Case	Input/Output Direction
0	Opposite
1	Opposite
2	Same
3	Same
4	Same
5	Same

Also, the cases 1 and 3 may not be feasible (i.e., the harmonic drive may not be back-drivable). Furthermore, the cases 4 and 5 (where the transmission ratio is almost 1) are not particularly useful in practice.

Solution 2.12

If the flexible shaft is rigidly coupled to the motor, there can be a sharp bend at the coupled location. The associated stresses can be high and may result in premature failure of the joint. The flexible coupling relaxes much of the stress by providing a smoother and flexible joint.

A flexible coupling may be used to link a rigid shaft to a motor. This will enable to offset minor misalignments between the motor axis and the shaft axis. But, it cannot practically provide a large change of direction in the drive axis. If a large change in direction is needed, the use of a flexible shaft would be a suitable choice.

Solution 2.13

Backlash is a high-frequency phenomenon. Direct sensing will be needed to detect it. But, the backlash state will change before the control system has time to compensate for it through feedback control. In fact, the feedback approach, being slower, would be more inappropriate here.

Backlash may be eliminated by using direct-drive actuators (which do not use gear transmissions) or by using backlash-free transmissions such as harmonic drives or traction drives.

Solution 2.14

Straightforward application of Newton's second law (torque = rate of change of angular momentum; which is valid about a fixed point or centriod), about the fixed point of rotation (joint) gives

$$\tau = I\ddot{\theta} + b\dot{\theta} + c\sin\dot{\theta} + mgl\cos\theta$$

Solution 2.15

Write equations of motion separately for the motor side and the load side of the system. Then combine them using the motion compatibility condition (no backlash) and the power transfer relation (with efficiency *e*) at the gear interface. We get

$$\tau = \left(J_m + \frac{J_l}{er^2}\right)\ddot{\theta}_m + \left(b_m + \frac{b_l}{er^2}\right)\dot{\theta}_m$$

where

$$J_{eq} = J_m + \frac{J_l}{er^2} \text{ at motor}$$

$$b_{eq} = b_m + \frac{b_l}{er^2} \text{ at motor}$$

Solution 2.16

Wire diameter = 1 mil = 1×10^{-3} in = $1\times10^{-3}\times2.54\times10^{-2}$ m
Wire length = 12 in = $12\times2.54\times10^{-2}$ m
Wire resistance $R = \rho\times\dfrac{12\times2.54\times10^{-2}}{\dfrac{\pi}{4}(1\times10^{-3}\times2.54\times10^{-2})^2}\,\Omega$

or, $R = \rho\times6.0\times10^{8}\,\Omega.$

Hence, multiply ρ by 6.0×10^8 to obtain the resistivity in Ω.cmil/ft.

Solution 2.17

The two resistances are:

$R_1 = 21\times10^2\ \Omega\ \pm\ 5\%$

$R_2 = 20\times10^1\ \Omega\ \pm\ 5\%.$

They correspond to the resistance ranges:

19.95×10^2 to $19.95\times10^2\,\Omega$ and

19.0×10^1 to $21.0\times10^1\,\Omega$.

Consider the two extreme cases in the two intervals.
The parallel connection of 19.95×10^2 and 19.0×10^1 gives

$$\frac{1}{R} = \frac{1}{19.95\times10^2} + \frac{1}{19.0\times10^1}$$

or,

$R = 17.35 \times 10^1\ \Omega$.

Similarly, the parallel connection of 22.05×10^2 and 21.0×10^1 gives

$$\frac{1}{R} = \frac{1}{22.05\times10^2} + \frac{1}{21.0\times10^1}$$

or,

$R = 19.17 \times 10^1\ \Omega$.

This is equivalent to

$R = 18.2\times10^1\ \Omega\ \pm\ 5\%$.

The corresponding color code is: brown, gray, brown, gold.

Solution 2.18

For self-compensation, we must have the same output after the temperature changes through ΔT. Hence, we must have

$$\frac{R_o}{(R_o + R_c)} S_{so} = \frac{R_o(1+\alpha.\Delta T)}{\left[R_o(1+\alpha.\Delta T)+R_c\right]} S_{so}(1+\beta.\Delta T)$$

where the subscript *o* denotes values before the temperature change. Cancellation of the common terms and cross-multiplication gives

$$R_o\beta + R_c(\alpha+\beta) = (R_o + R_c)\alpha\beta\Delta T$$

Now, since both $\alpha.\Delta T$ and $\beta.\Delta T$ are usually much smaller than unity, we may neglect the right-hand-side (second-order) term in the preceding equation. This gives the following expression for the compensating resistance:

$$R_c = -\left[\frac{\beta}{\alpha+\beta}\right] R_o$$

Note that this type of compensation is feasible if the temperature coefficient of the strain gage sensitivity (β) is negative, which typically possible. Also, the material of the compensating resistor must have a negligible temperature coefficient of resistance. Alternatively, we should locate R_c in a separate, temperature-controlled environment.

Solution 2.19

Using the given values, we have $S_{q26} = \varepsilon S_{v26} =$ 460.0 pC/N and $\varepsilon_r = \varepsilon / \varepsilon_0 = 500.0$.

Also, $\varepsilon_0 = 8.85\times10^{-12}$ F/m.

Hence, $S_{v26} = \dfrac{460.0\times10^{-12}}{500.0\times8.85\times10^{-12}}$ V.m/N = 0.104 V.m/N.

Now use $S_v = \dfrac{1}{d}\dfrac{\partial v}{\partial p}$ with $d = 1.0\times10^{-3}$ m.

The voltage drop (for a unity applied stress) is $0.104\times1.0\times10^{-3}$ V = 1.04×10^{-4} V.

Solution 2.20

Reluctance can be interpreted as the resistance to the flow of magnetic flux through a magnetic circuit element. Electrical resistance is the resistance to the flow of current through an electric circuit element. This analogy is illustrated in Table S2.20.

Table S2.20 The analogy between magnetic reluctance and electric resistance.

	Electric Circuit	**Magnetic Circuit**
Effort Variable	Voltage v	Magnetomotive force (mmf) $F = ni$
Flow Variable	Current i	Magnetic flux ϕ
Element Parameter	Resistance R	Reluctance $\Re$
Material Parameter	Resistivity ρ	Permeability μ
Constitutive Equation	$v = R\,i$	$F = \Re\phi$
Element Equation	$R = \dfrac{\rho L}{A}$	$\Re = \dfrac{L}{\mu A}$

Consider Figure P2.20. The continuity of magnetic flux gives:

$$\phi = \phi_1 + \phi_2$$

Divide throughout by the mmf F of the magnetic source ($F = ni$):

$$\frac{\phi}{F} = \frac{\phi_1}{F} + \frac{\phi_2}{F}$$

which, by definition of reluctance, gives

$$\frac{1}{\Re} = \frac{1}{\Re_1} + \frac{1}{\Re_2}$$

Solution 2.21

$$\delta v_o = \pm\frac{10}{10+200}\times 5.0 \text{ V} = \pm 0.24 \text{ V}$$

This is approximately a fluctuation of $\pm\dfrac{0.24}{20}\times100\% = \pm1.2\%$.

Solution 2.22

The temperature-voltage characteristics are highly nonlinear in both cases. Also, parameter values may be incompletely known and may drift with time.

For a bipolar junction transistor we have

$$v_{eb} = \frac{kT}{q} \ln \frac{i_c}{I_s}$$

with

$$I_s = aT^3 \exp(-\frac{qV_g}{kT})$$

Substitution gives

$$v_{eb} = \frac{kT}{q} \ln \frac{i_c}{aT^3} + V_g$$

Differentiate with respect to T, to obtain the sensitivity:

$$\frac{\partial v_{eb}}{\partial T} = \frac{k}{q} \ln \frac{i_c}{aT^3} - \frac{3k}{q}$$

Solution 2.23

When the base lead is in position 1, the emitter to base voltage v_{be} = 1.0 V which is greater than 0.7 V. Hence the transistor is forward biased, and will conduct. Then the output voltage v_{ce}, which is equal to v_o, will be very small (about 0.2 V). When the base lead is in position 2, v_{be} = 0.0 V and the transistor will not conduct, providing a very large output impedance (open-circuit condition). Then, $v_o = v_{ref}$. It follows that the output v_o can switch between 0.2V and v_{ref} using this circuit.

Solution 2.24

$$v_{average} = dv_{ref}$$

Solution 2.25

A suitable circuit is shown in Figure S4.10. The base voltage of the *npn* transistor is triggered to 5 V, the resulting forward bias provides a low-resistance path with a small voltage drop (e.g., 0.2 V) between C and E. The corresponding current through the LED is

$$\frac{10.0-0.2}{200.0+10} = 0.038 \text{ A} = 38.0 \text{ mA}.$$

This current is sufficient to turn on the LED with the required brightness. When the base voltage is triggered to 0 V, the transistor will not conduct due to the resulting high impedance between C and E. The LED will be turned off as a result.

This device may be used as a warning indicator for an automobile engine (e.g., for overheating).

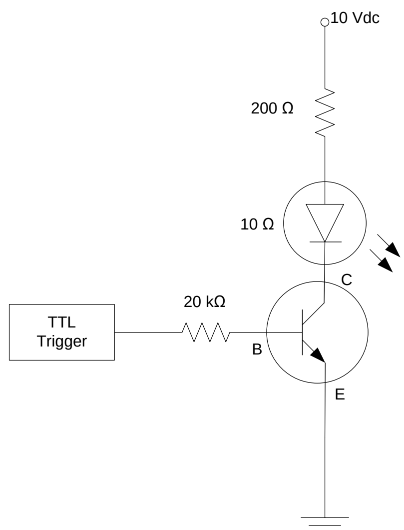

Figure S2.25 A digitally-triggered switching circuit for LED.