Chapter 4

4-1

Let f(s) be the p.d.f. of project completion time.

Probability that the project complete between time *x* and *y* is $\int_x^y f(x)dx$.

And the probability that the project completes in exactly x time unit is $\int_x^x f(x)dx = 0$

4-2

G has the lowest crash ratio, crash by 1 day, cost = 2775+25=2800

E has the next lowest crash ration, crash by 2 days, cost = 2800+200=3000

C has the next lowest crash ration, crash by 1 day, cost = 3000+250=3250

Project duration = 11-4=7 days

4-3

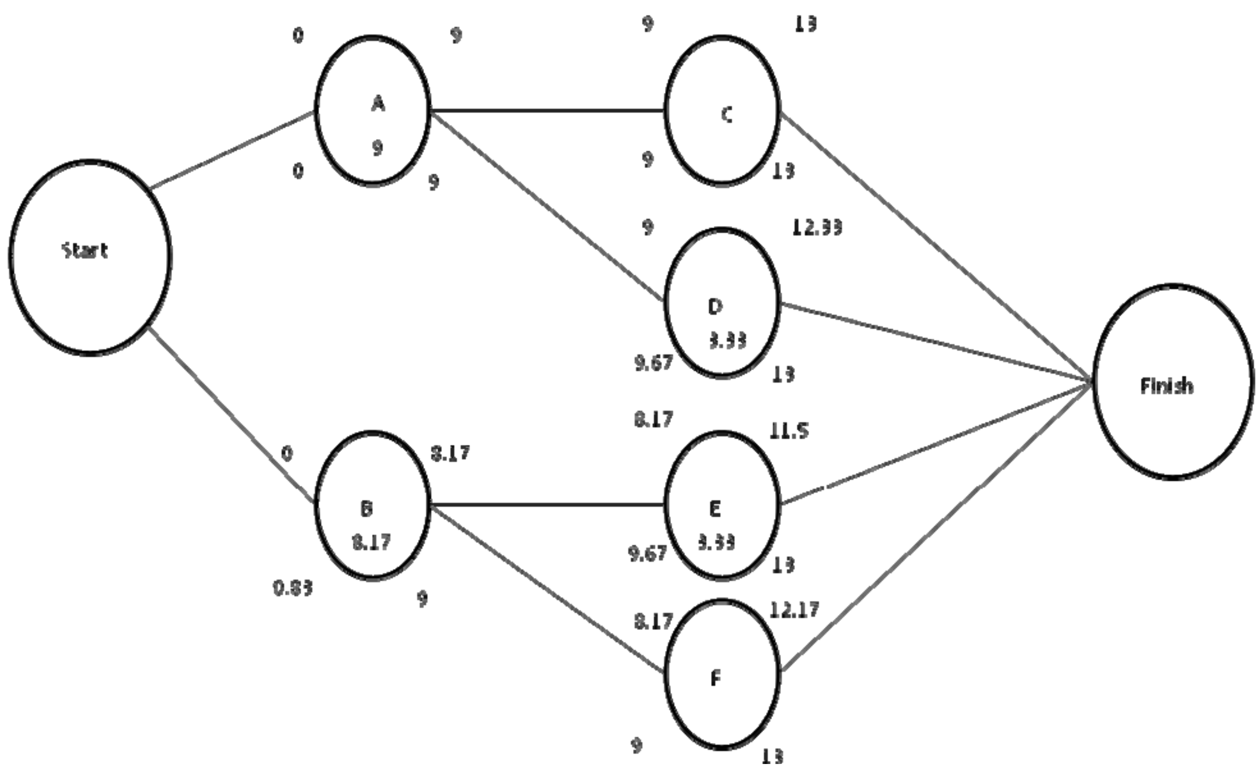
Using the methods presented in the text,

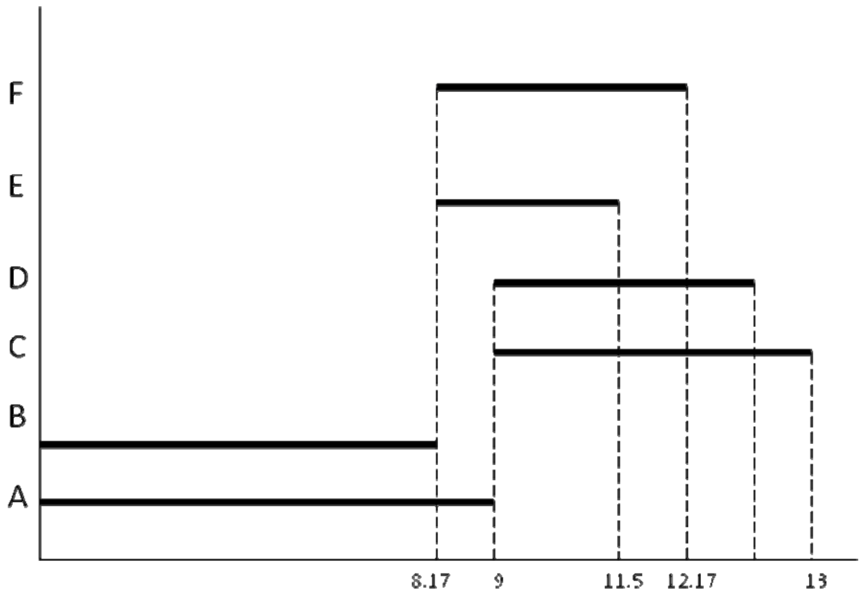
$$\bar{t} = \frac{480}{20} = 24, R = 42 - 8 = 34, k = \frac{3}{d_2} = \frac{3}{3.735} = 0.8032$$
$$kR = 27.31$$

Since $kR > \bar{t}$, then $a = 0, b = 2\bar{t} = 48, m = t = 24$

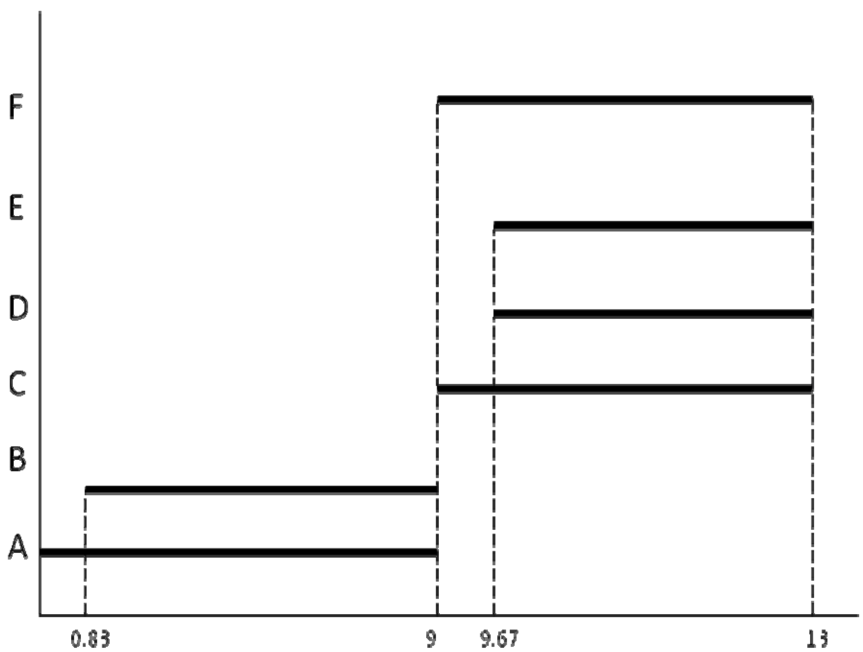
4-4

Activity	Precedent	a	m	b	t_e	s^2
A	-	7	9	11	9	0.44
B	-	7	8	10	8.17	0.25
C	A	2	4	6	4	0.44
D	A	1	3	7	3.33	1
E	B	1	3	7	3.33	1
F	B	2	4	6	4	0.44



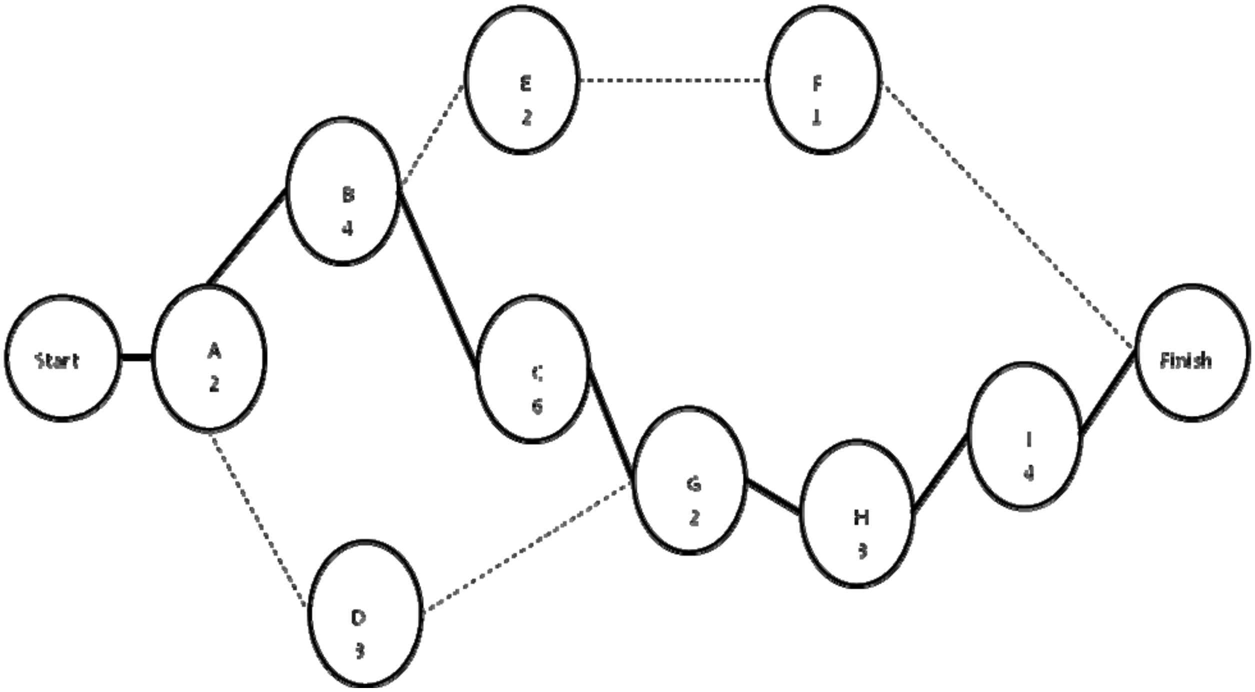


Gantt Chart for ES Schedule



Gantt Chart for LS Schedule

4-5



Activity	ES	EC	LS	LC	TS
A	5	7	14	16	9
B	7	11	16	20	9
C	11	17	20	26	9
D	7	10	23	26	16
E	11	13	32	34	21
F	13	14	34	35	21
G	17	19	26	28	9
H	19	22	28	31	9
I	22	26	31	35	9

4-6

$$FS(j) = \underset{i \in S(j)}{Min}\{ES(i)'s - EC(j)\}$$
$$= \underset{i \in S(j)}{Min}\{ES(i)'s\} - EC(j)$$
$$TS(j) = LC(j) - EC(j)$$

Comparing both and canceling like terms.

Since the earliest starting time of all successors of *j* cannot be greater than the latest completion time of the activity,

i.e. $LC(j) \geq \underset{i \in S(j)}{Min}\{ES(j)'s\} \Rightarrow TS(j) \geq FS(j)$

4-7

From 4-6, since $TS(j) \geq FS(j)$, it follows that FS may or may not be greater than zero.

4-8

TS=2 indicates that only 2 time units can be delayed on the critical path activities with the project still meeting the deadline. Since 2 critical activities are delayed by 2 time units each, the project will be delayed by 2 time units.

4-10

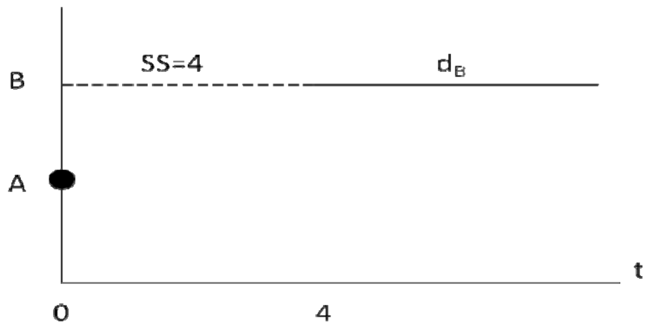
a). Since SS = 4, we must have $t \geq 4$

Since FF = 3, we must have

$$(t + d_B) - d_A \geq 3$$
$$\text{i.e. } d_B \geq (3 - t) + d_A$$
$$\Rightarrow d_B \geq 3 + d_A - t$$

$$\text{But } t \geq 4$$
$$\therefore d_B \geq 3 + d_A - 4$$
$$\Rightarrow d_B \geq d_A - t$$

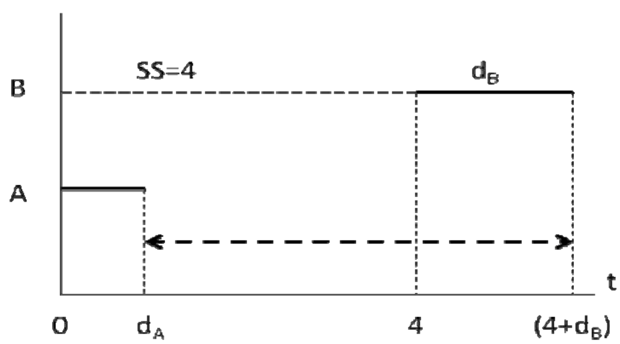
b). <u>Case 1:</u> $d_A = 0$



$d_B \geq 0-1$
$\Rightarrow d_B \geq -1$

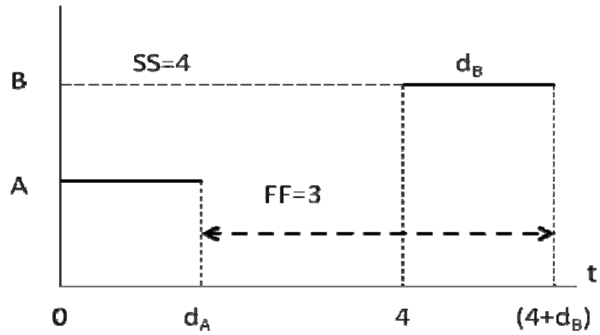
Since duration cannot be negative, we have $d_B \geq 0$

<u>Case 2</u>: $0 \leq d_A \leq 1$

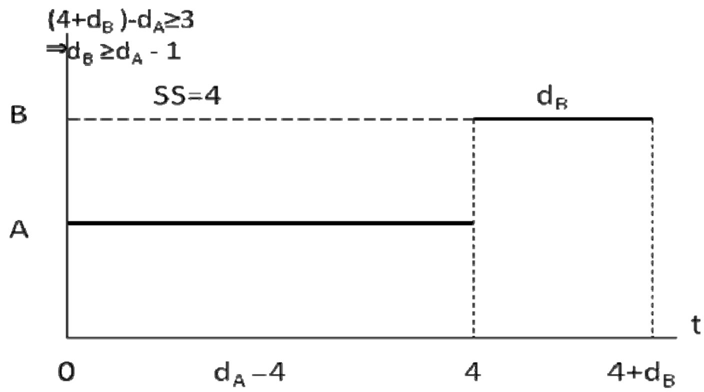


$(4+d_B)-d_A \geq 3$
i.e. $d_B \geq d_A + 3-4$
$\Rightarrow d_B \geq d_A - 1$

Case 3: $1 \leq d_A \leq 4$

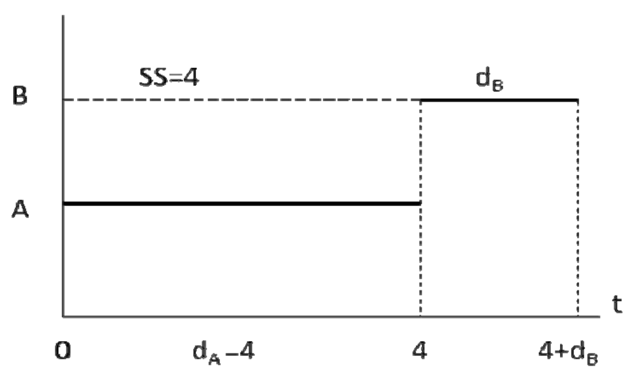


Case 4: $d_A = 4$



$(4+d_B)-d_A \geq 3$
$\Rightarrow d_B \geq d_A - 3$ which satisfies $d_B \geq d_A - 1$

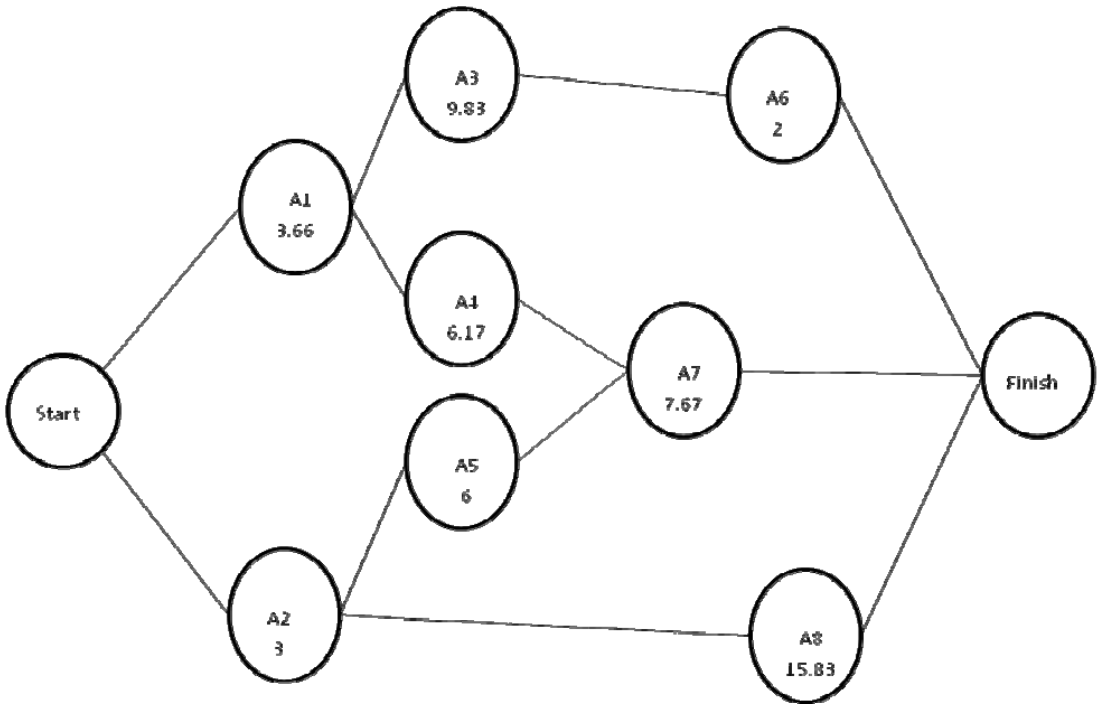
Case 5: $d_A > 4$



$(d_A + d_B) - d_A \geq 3$
i.e. $d_B \geq d_A - 3$ which satisfies $d_B \geq d_A - 1$

Note: It is assumed that jobs start at the earliest possible times.

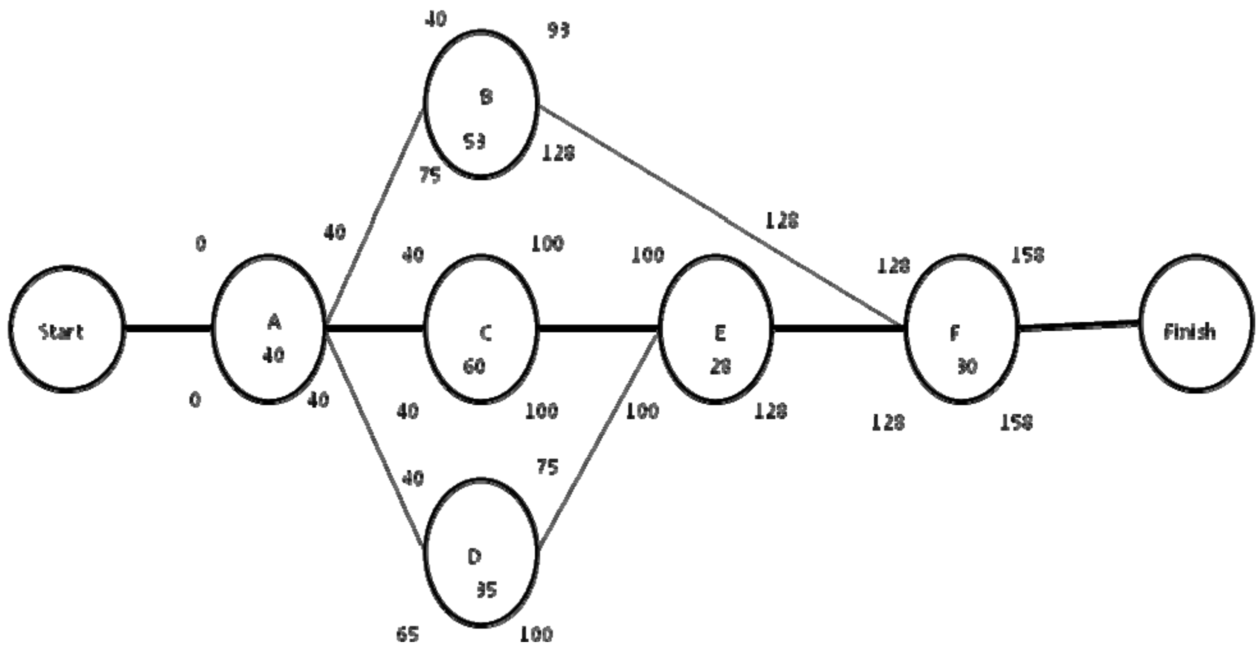
4-11



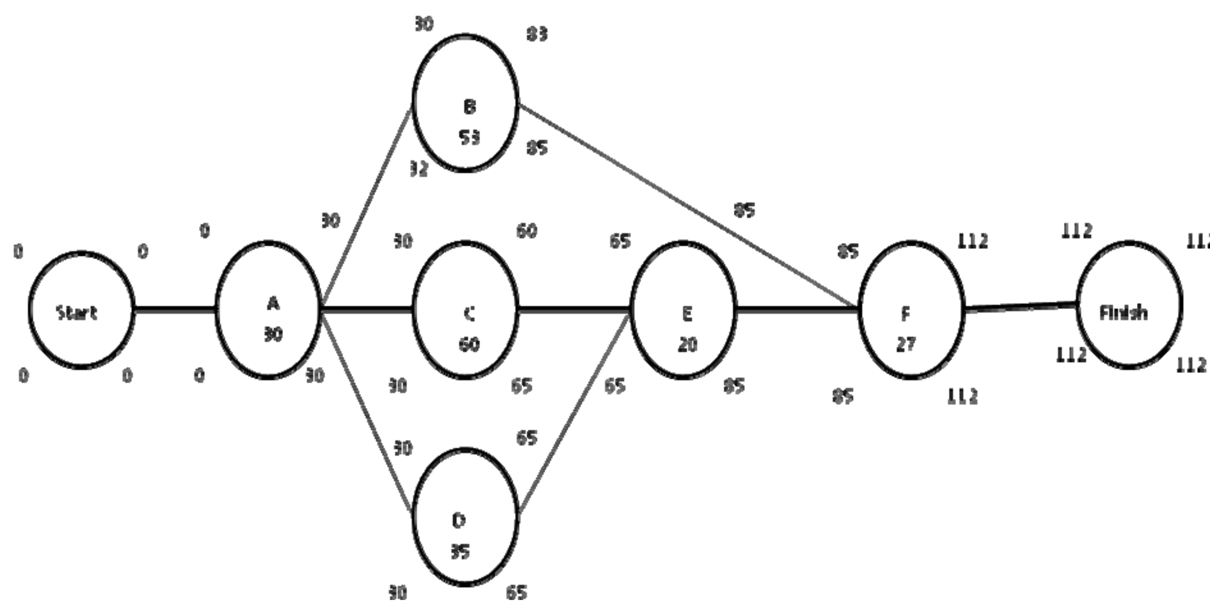
Activity	t_e	s^2	ES	EC	LS	LC
A1	3.66	0.44	0	3.66	1.33	4.99
A2	3	0.11	0	3	0	3
A3	9.83	0.11	3.66	13.49	7	16.83
A4	6.17	0.03	3.66	9.83	4.99	11.16
A5	2	0	3	5	9.16	11.16
A6	2	0	13.49	15.49	16.83	18.83
A7	7.67	0.44	9.83	17.5	11.16	18.83
A8	15.83	1.36	3	18.83	3	18.83

4-12

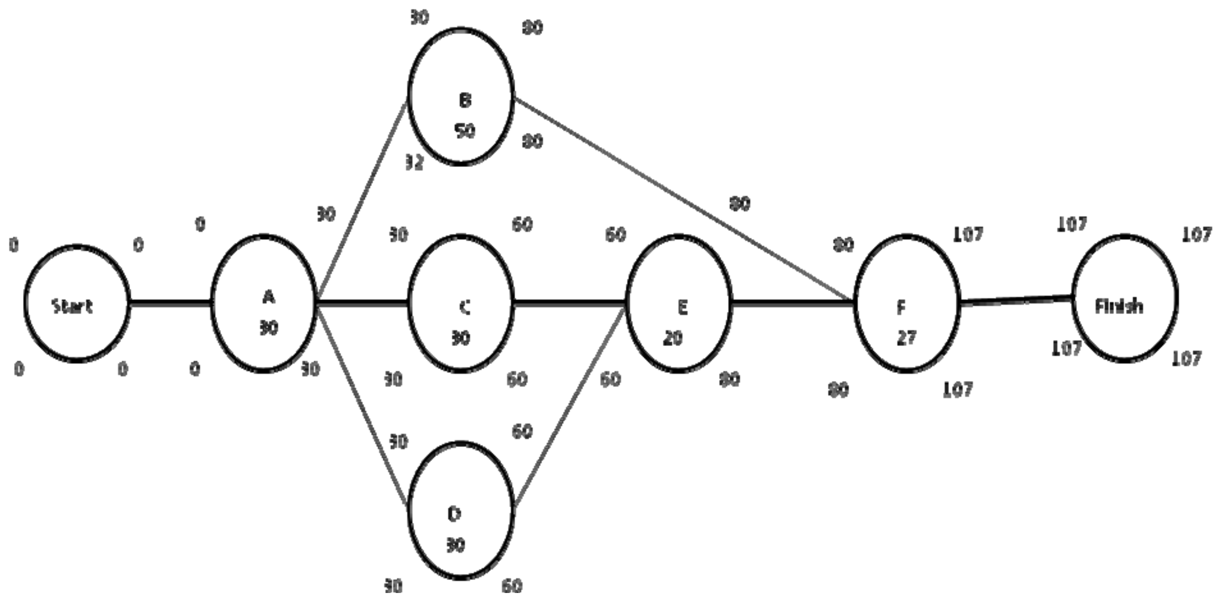
a)



Total Cost with normal time = $69,500



Activities Crashed: A,C,E,F



Total Cost = \$77,300

b)

$$Te = 158$$
$$S^2 = V[A] + V[C] + V[E] + V[F]$$
$$= 10 + 600 + 40 + 30 = 680$$
$$s = 26.07$$

$$P(T < Td) = P(T < 159)$$
$$= P\left(z < \frac{159 - 158}{26.07}\right)$$
$$= P(z < 0.038)$$
$$= 0.516$$

c) Penalty of \$3,000 or less implies Td < 160

$$P(T < Td) = P(T < 160)$$
$$= 0.5306$$

d) Probability of penalty of more than \$3,000 = 1 – Prob(\$3000 or less)

= 1 – 0.5306

= 0.4694

e) Penalty of less than $3000 implies Td < 161

$$
\begin{aligned}
P(T < Td) &= P(T <= 161) \\
&= P(Z <= 0.115) \\
&= 0.5458
\end{aligned}
$$

4-13

$$
\begin{aligned}
P(T \le 40) &= 1 - P(T \ge 40) \\
&= 1 - \int_{40}^{b} \frac{2(b-t)}{(b-m)(b-a)} dt \\
&= 1 - \int_{40}^{46} \frac{2(46-t)}{(46-36)(46-24)} dt \\
&= 1 - 0.163 \\
&= 0.8363
\end{aligned}
$$

4-14

$$P(x \le 40) = \frac{40-24}{46-24} = 0.7272$$

4-15

	Duration	**a**	**m**	**b**	t_e	s^2
A	6	4.2	6	8.4495	6.1082	0.5016
B	5	3.5	5	7.2361	5.1227	0.3877
C	2	1.4	2	3.4142	2.1357	0.1127
D	4	2.8	4	6	4.1333	0.2844
E	5	3.5	5	7.2361	5.1227	0.3877
F	6	4.2	6	8.4495	6.1082	0.5016
G	1	0.7	1	2	1.1167	0.0469
H	7	4.9	7	9.6458	7.0910	0.6256

$s^2 = \sigma_B^2 + \sigma_E^2 + \sigma_H^2 = 1.4010$

$\therefore s = 1.1836$

$T_e = 17$

$S^2 = 2.1871$

$= 1.4789$

$P(T \le T_d) = P\left(z \le \frac{T_d - T_e}{S} \right) = 0.85$

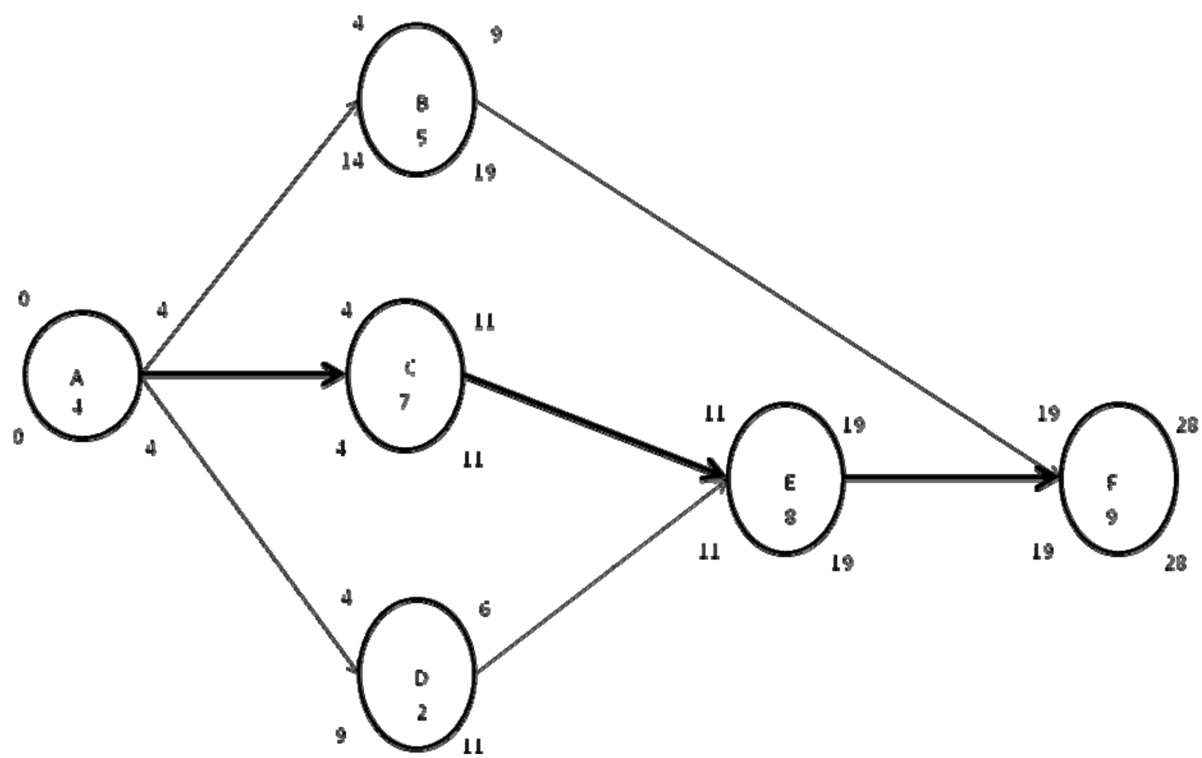
From Normal Distribution Table $z = 1.0364$

$\frac{T_d - 17}{1.4789} = 1.0364$

$Td = 18.53 \cong 19$ days

4-16

Expected value = $\sum_j x_j (P(x_j))$



T_e=28

Critical Path: A-C-E-F

Variance of the project duration = 0.5+0.25+0.375+0.4375=1.5625

Standard Deviation of the project duration = 1.25

$$P(\text{Penalty} = \$0) = P(T \le 28) = P(Z < \frac{28-28}{1.25}) = 0.5$$

$$P(\text{Penalty} = \$3000) = P(28 < T \le 29) = P\left(Z < \frac{29-28}{1.25}\right) - P\left(Z < \frac{28-28}{1.25}\right)$$

$$= P\left(Z < 0.8\right) - P\left(Z < 0\right)$$

$$= 0.2881$$

$$P(\text{Penalty} = \$6000) = P(29 < T \le 30) = 0.1571$$

$$P(\text{Penalty} = \$9000) = P(30 < T \le 31) = 0.0466$$

$$P(\text{Penalty} = \$12000) = P(31 < T \le 32) = 0.0075$$

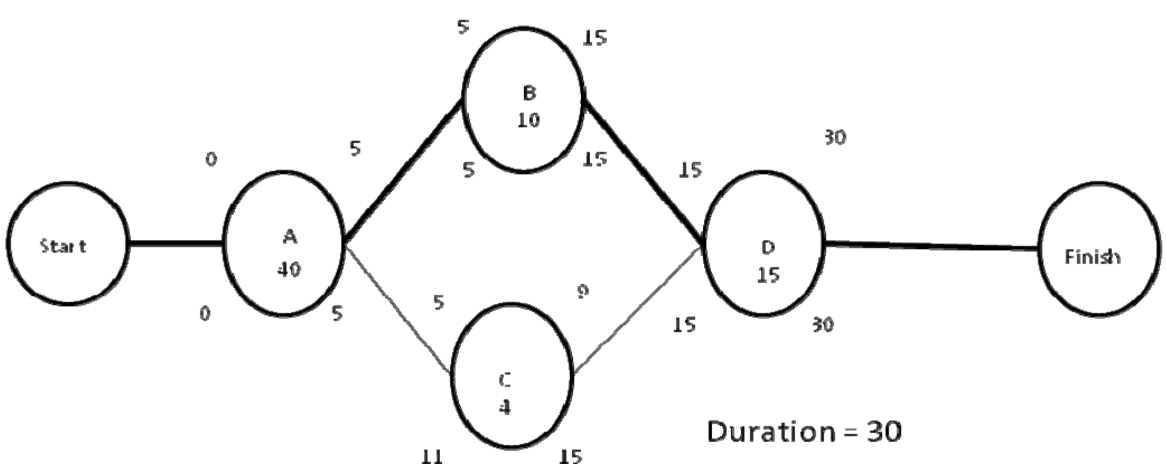
$$P(\text{Penalty} = \$15000) = P(32 < T \le 33) = 0.0007$$

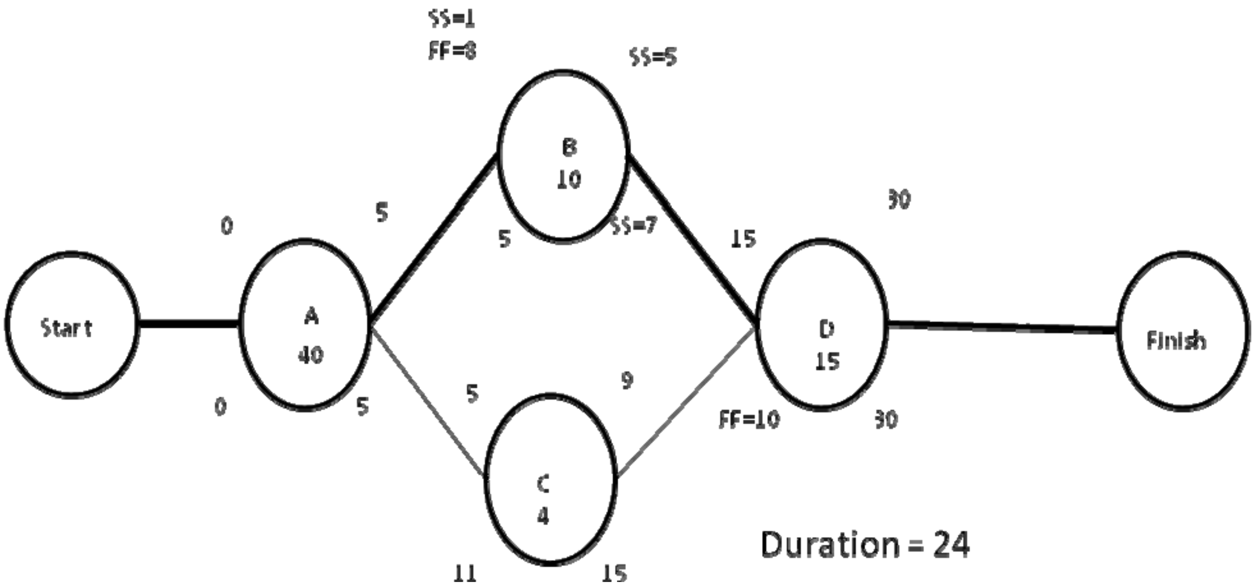
$$P(\text{Penalty} = \$18000) = P(33 < T \le 34) = 0.0000$$

$$\text{Expected Penalty } = \$0(0.5) + \$3000(0.2881) + \$6000(0.1571) + \$9000(0.0466) + \$12000(0.0075) + \$15000(0.0007)$$

$$\text{Expected Penalty } = \$2326.8$$

4-17

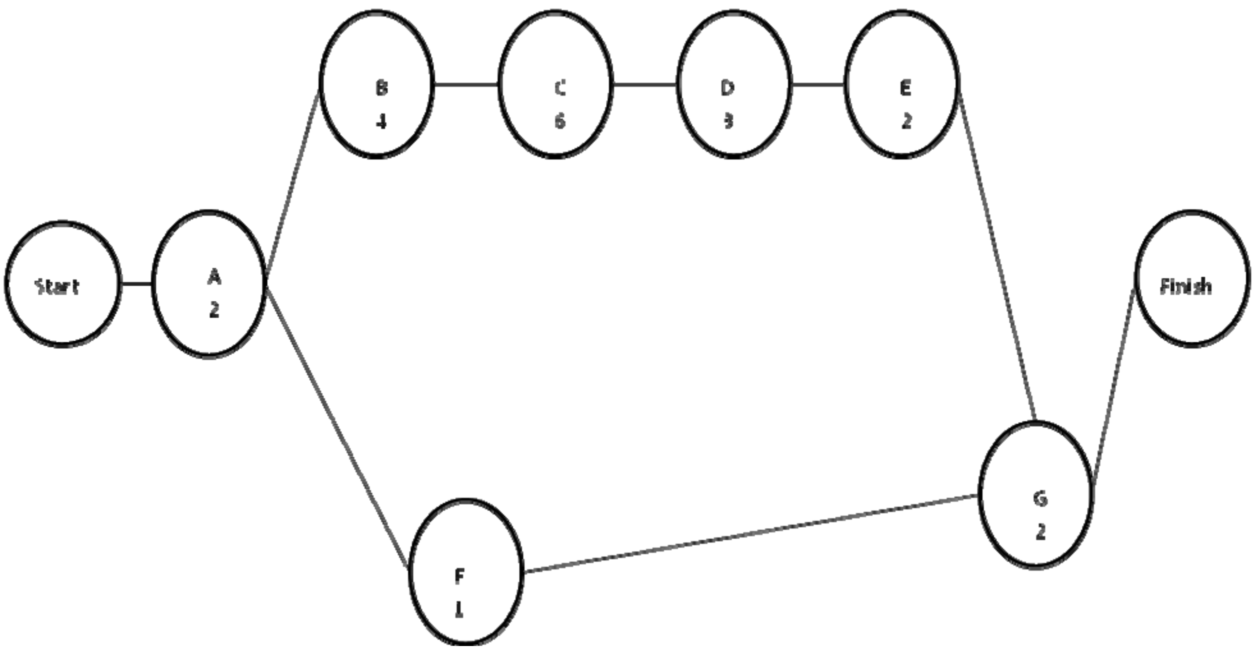




4-18

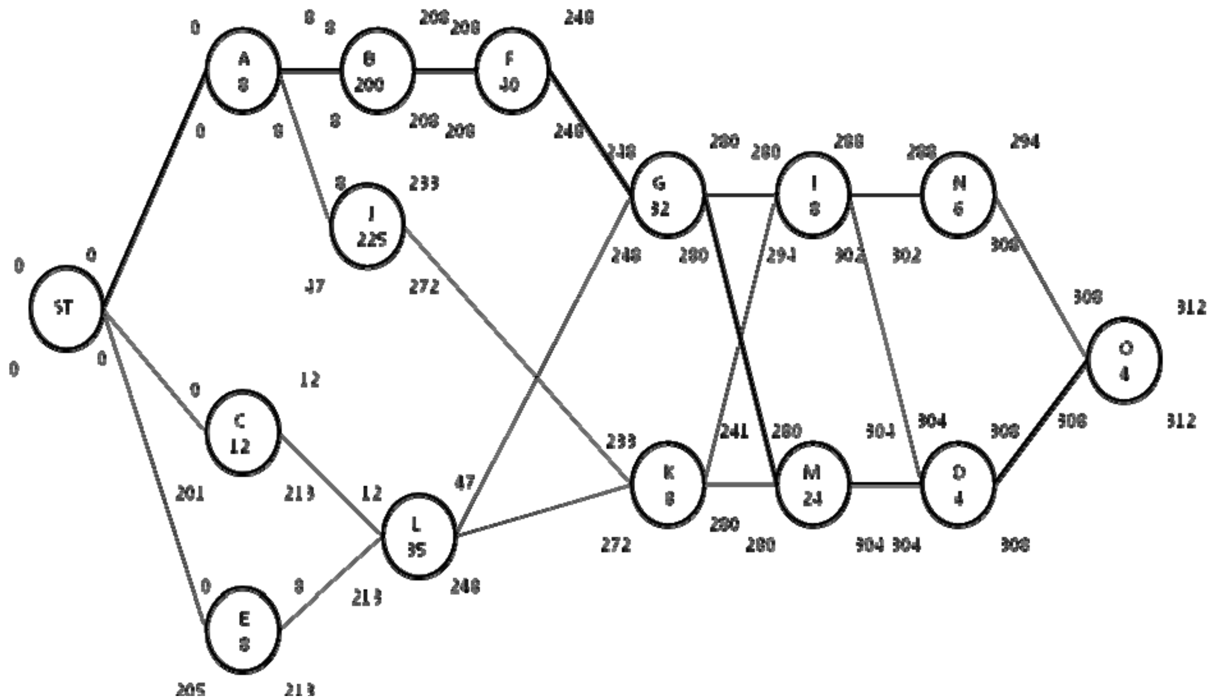
a) Crashing of noncritical activity has no effect on the critical path and the project duration.

b)



This critical path is A-B-C-D-E-F-G and duration = 6. Crashing the noncritical activity F from 2 to 1 has no effect on the critical path and project duration.

4-19

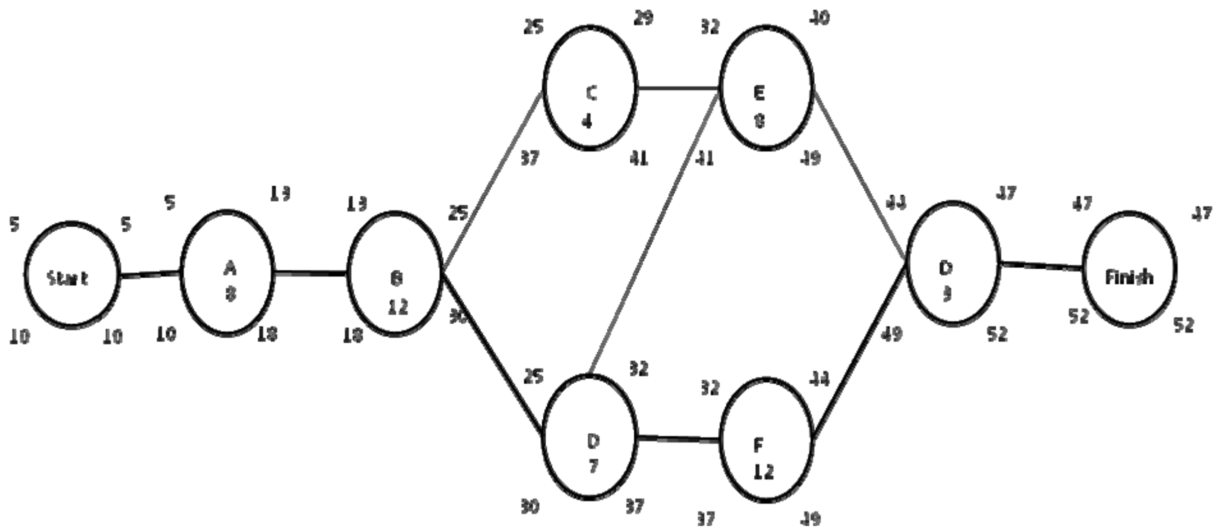


c) Paths in order of decreasing criticality:

k	Activities in Path	ΣTS on Path (S_k)	Path Criticality Index (C_k%)	Remark
1	ABFGMDO	0	100	Most Critical
2	ABFGIDO	14	97	
3	ABFGINO	28	94.1	
4	AJKMDO	78	83.5	
5	AJKIDO	92	80.5	
6	AJKINO	106	77.6	
7	CLGMDO	402	15	
8	ELGMDO	406	14.2	
9	CLGIDO	416	12	
10	ELGIDO	420	11.2	
11	CLGINO	430	9.1	
12	ELGINO	434	8.2	
13	CLKMDO	441	6.8	
14	ELKMDO	445	5.9	
15	CLKIDO	455	3.8	
16	ELKIDO	459	2.6	

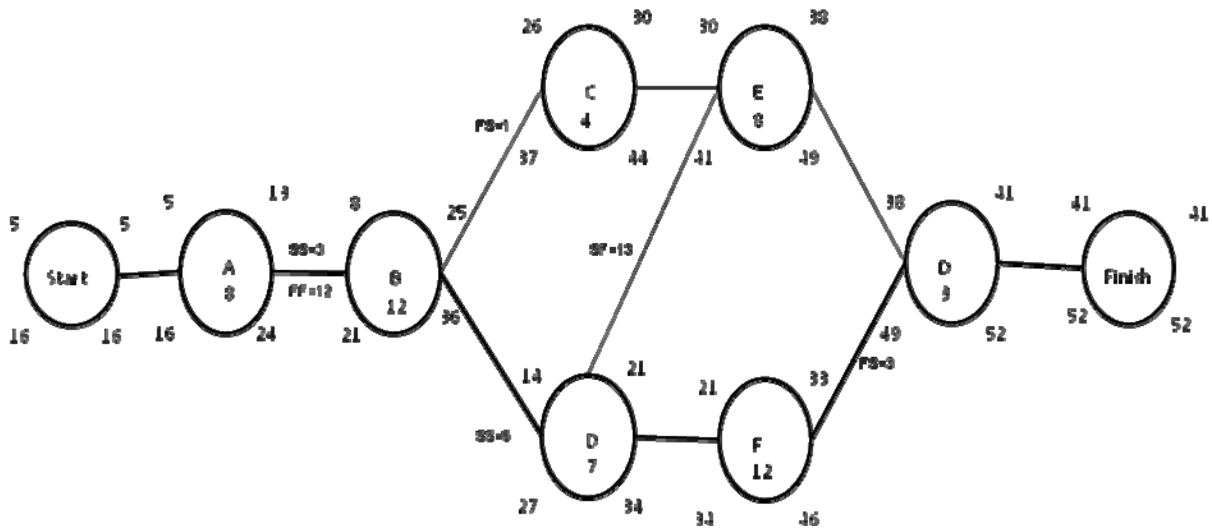
17	CLKINO	469	0.8	
18	ELKINO	473	0	Least Critical

4-20

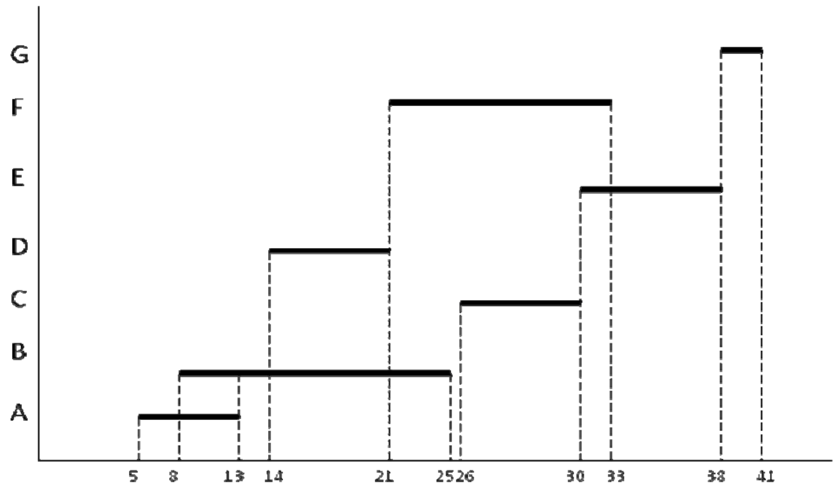


Activity	ES	EC	LS	LC	TS	FS
A	5	13	10	18	5	0
B	13	25	18	30	5	0
C	25	29	37	41	12	3
D	25	32	30	37	5	0
E	32	40	41	49	9	4
F	32	44	37	49	5	0
G	44	47	49	52	5	0

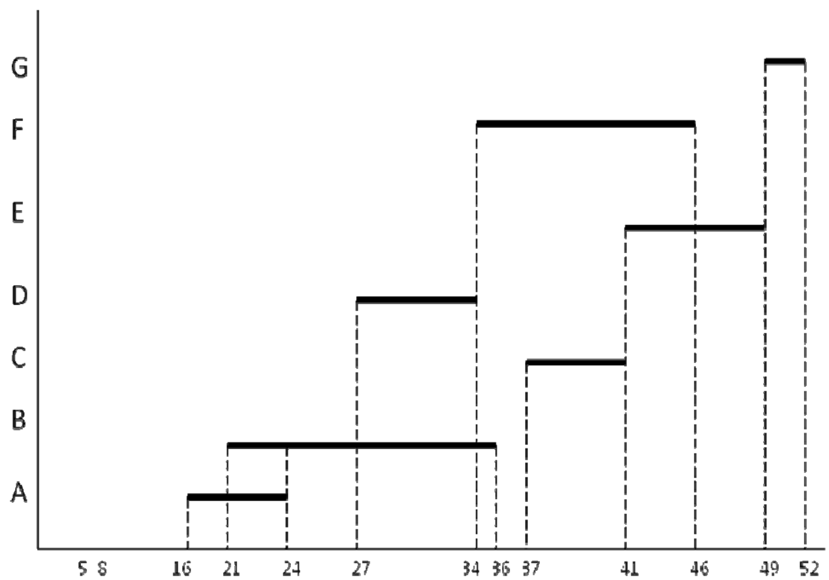
4-21



Activity	ES	EC	LS	LC	TS	FS
A	5	13	16	24	11	-5
B	8	25	21	36	13	-11
C	26	30	37	41	11	0
D	14	21	27	34	13	0
E	30	38	41	49	11	0
F	21	33	34	46	13	5
G	38	41	49	52	11	0

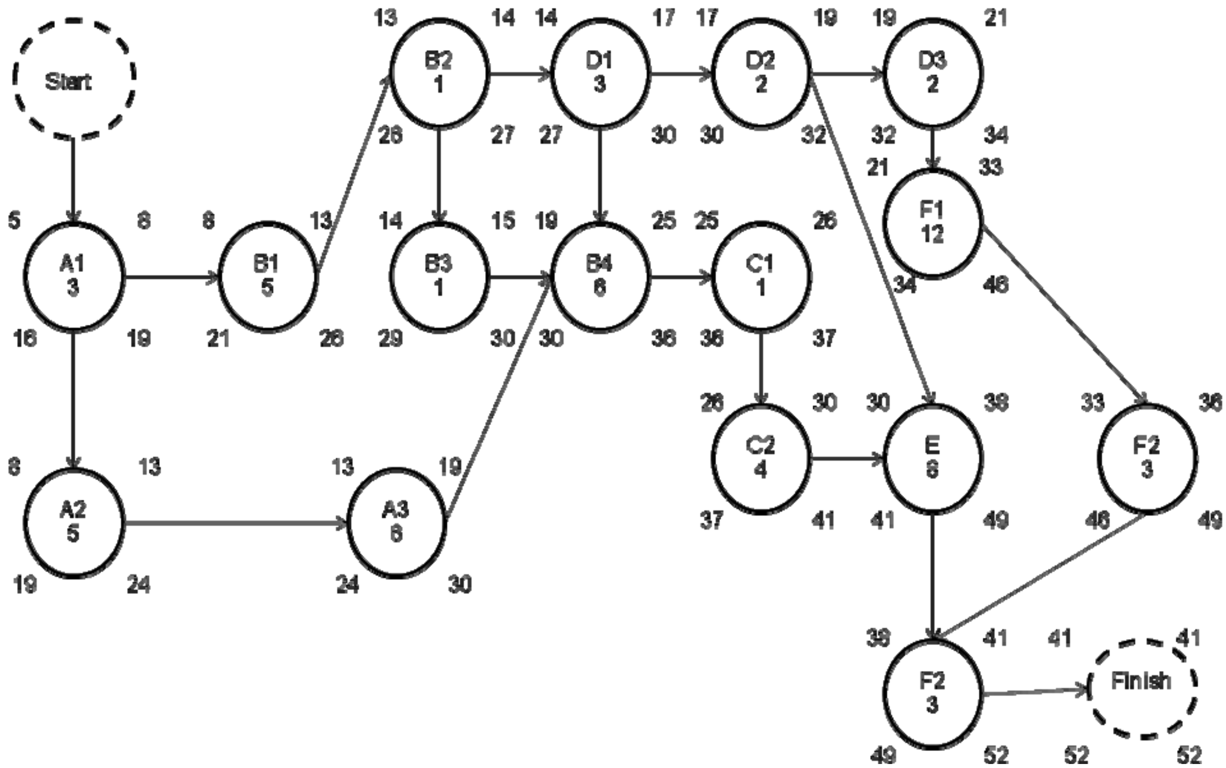


Gantt Chart based on ES



Gantt Chart based on LS

4-22



4-23

MBE because of the focus on the critical path. That is, attention is directed at the problem area

4-24

Activity precedence constraint – Precedence has to be satisfied regardless of resource availability.

4-25

Critical Path: A-B-D-F-G

Te = 42

$$S_e = \frac{(b-a)^2}{36} = \frac{\left((m+1)-(m-1)\right)^2}{36} = \frac{1}{9}$$

$$S^2 = V(A)+V(B)+V(D)+V(F)+V(G) = 0.5555$$

$$S = 0.7454$$

$$P\left(T \le 51\right) = P\left(z \le \left(51-42\right)/0.7454\right)$$

$$\approx 1.00$$

4-26

Since the mode is closer to *b* than *a*, m_d is between *a* and *c*. For the range of $a \le x \le c$, the cdf triangular distribution is

$$F(x) = \int_a^x \frac{2(t-a)}{(b-a)(c-a)} dt$$

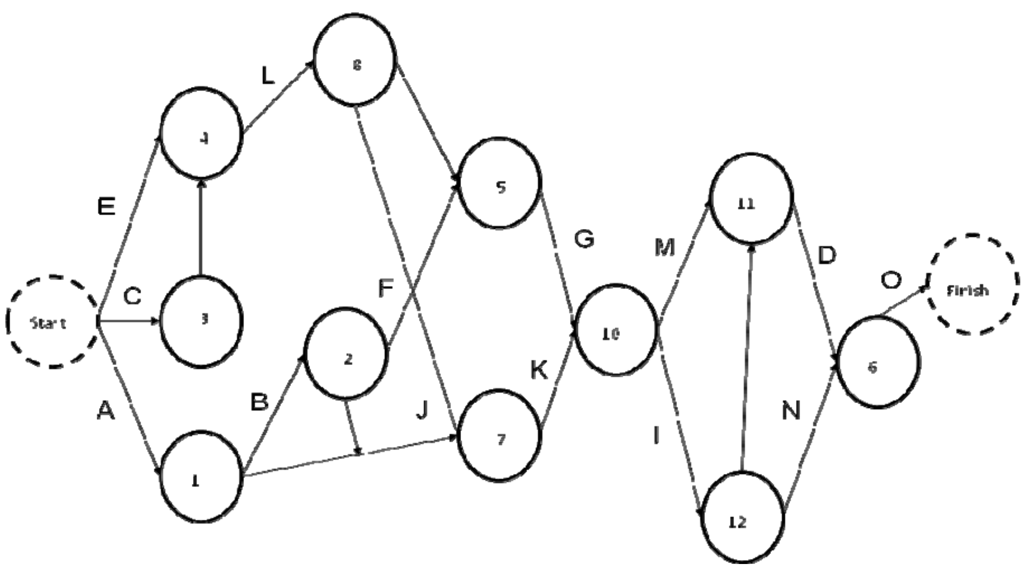
Given

$$F(m_d) = \int_a^{m_d} \frac{2(t-a)}{(b-a)(c-a)} dt = 0.5$$

$$\frac{1}{(b-a)(c-a)}\left[\frac{t^2}{2} - at\right]_a^{m_d} = 1$$

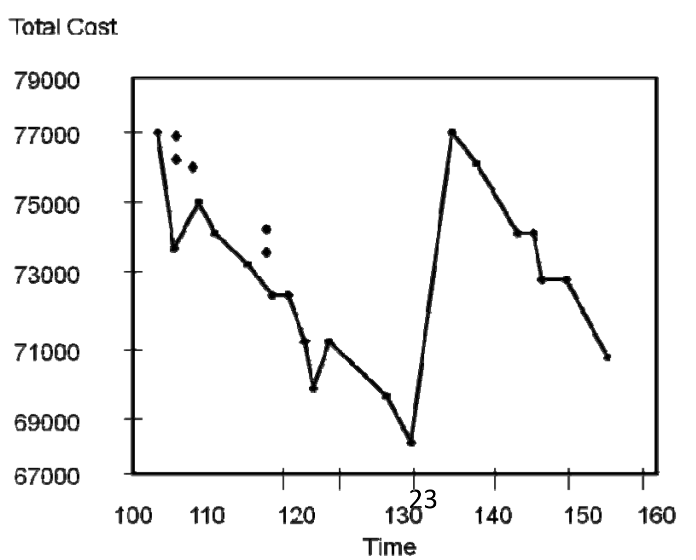
$$...m_d = a + \sqrt{\frac{(b-a)(c-a)}{2}} \quad \text{for } a \le x \le c$$

4-27



4-28 Compression Strategies

Proj. Duration	Strategy	Activity Crashed	Total Cost
155	S1	F	$70,500
150	S1	E	$72,500
148	S1	A	$72,500
147	S1	E, F	$73,500
145	S1	A, F	$73,500
140	S1	A, E	$75,500
137	S1	A, E, F	$76,500
133	S1	C	$68,000
130	S1	C, F	$69,000
125	S1	C, D, F	$71,000
	S2	C, E	$71,000
123	S1	A, C	$71,000
123	S2	C, D	$70,000
122	S1	C, E, F	$72,000
120	S1	A, C, D, F	$74,000
	S2	C, D, E, F	$74,000
	S3	A, C, D, F	$74,000
	S4	B, C, D, E	$73,300
	S5	A, C, F	$72,000
118	S1	A, C, D	$73,000
115	S1	A, C, E	$74,000
113	S1	A, C, D, E	$76,000
112	S1	A, C, E, F	$75,000
110	S1	A, C, D, E, F	$77,000
	S2	A, B, C, D, E	$76,300
	S3		$74,000
107	S1	A, B, C, D, E, F	$77,300



Time-Cost Trade off Curve