Chapter 2

Planning Production in Supply Chains

Solutions to Numerical Exercises only

2.6)

(a) Last Value method

$F_{13}=F_{14}=F_{15} = D_{12} = 41$

(b) Averaging method

$$F_{13}=F_{14}=F_{15} = \frac{34+33+42+34+36+43+34+33+43+31+35+41}{12}$$

$= 36.583 \approx 37$

(c) Three-month moving average method

$$F_{13}=F_{14}=F_{15} = \frac{31+35+41}{3}$$

$= 35.67 \approx 36$

(d) Exponential smoothing with $\alpha = 0.25$

Assume F_1 as the average of the 12 months demand = 36.58. The forecasts are shown in Table 1.

Table 1. Forecasts (Exponential smoothing)

Month	**Sales**	**Forecast (F_t)**
1	34.00	36.58
2	33.00	35.94
3	42.00	35.20
4	34.00	36.90
5	36.00	36.18
6	43.00	36.13
7	34.00	37.85

8	33.00	36.89
9	43.00	35.92
10	31.00	37.69
11	35.00	36.01
12	41.00	35.76

F_{13}=F_{14}=F_{15} = (0.25 x 41) + (0.75 x 35.76) = 37.07 ≈ 37

(e) Holt's method with α = 0.4 and β = 0.5

For illustration, L_1 is assumed to be equal to D_1 and T_1 is 1. Hence, L_1 = 34 and T_1 = 1. The forecasts are shown in Table 2.

Table 2. Forecasts (Holts Method)

Month	Sales	Estimate of Level (L_t)	Estimate of Trend (T_t)	Forecast (F_t)
1	34.00	34.00	1.00	35.00
2	33.00	34.60	0.80	35.40
3	42.00	34.44	0.32	34.76
4	34.00	37.66	1.77	39.42
5	36.00	37.25	0.68	37.94
6	43.00	37.16	0.30	37.46
7	34.00	39.67	1.40	41.08
8	33.00	38.25	-0.01	38.24
9	43.00	36.14	-1.06	35.08
10	31.00	38.25	0.52	38.77
11	35.00	35.66	-1.03	34.63
12	41.00	34.78	-0.96	33.82

For Month 2,

$L_2 = \alpha D_1 + (1-\alpha)F_1 =$ (0.4) 34 + (0.6) 35 = 34.60

$T_2 = \beta(L_2 - L_1) + (1-\beta)T_1 =$ (0.5) (34.60 – 34) + (0.7) 1 = 0.80

F_2 = L_2 + T_2 = 35.40

L_{13} = (0.4 x 41) + (0.6 x 33.82) = 36.69

T_{13} = 0.5 x (34.78 – 35.66) + (0.5 x -0.96) = 0.48

F_{13}= 36.69 + (1 x 0.48) = 37.17 ≈ 37

F_{14} = 36.69 + (2 x 0.48) = 37.65 ≈ 38

F_{15} = 36.69+ (3 x 0.48) = 38.13 ≈ 38

(f) Since the demands are higher during the third month of each quarter, there is a definite seasonality pattern present in the demands. Hence, all the methods should be modified to include seasonality.

2.7)

Let D_t and F_t be the demand and forecast for period t respectively. Then the forecast error for period t is given by

$$e_t = F_t - D_t$$

Further, let the four weights for the four period weighted moving average method be w_1 (latest period for which demand data is available, period t), w_2 (period t-1), and w_3 (period t-2). Joe Kool wishes to minimize the sum of the *absolute* values of errors (which is a non-linear objective). To linearize this, let

$$e_t = e_t^+ - e_t^-, \quad \text{where } e_t^+, e_t^- \geq 0, \forall t$$
$$\Rightarrow |e_t| = e_t^+ + e_t^-$$

The resulting linear program is given as follows:

Objective: $\min Z = \sum_{t=4}^{12} (e_t^+ + e_t^-)$

subject to the constraints:

$5200w_1+5405w_2+5325w_3-5510 = e_4^+ - e_4^-$
$5510w_1+5200w_2+5405w_3-5765 = e_5^+ - e_5^-$
$5765w_1+5510w_2+5200w_3-5210 = e_6^+ - e_6^-$
$5210w_1+5765w_2+5510w_3-5375 = e_7^+ - e_7^-$
$5375w_1+5210w_2+5765w_3-5585 = e_8^+ - e_8^-$
$5585w_1+5375w_2+5210w_3-5460 = e_9^+ - e_9^-$

$$5460w_1+5585w_2+5375w_3-4905 = e_{10}^{+}-e_{10}^{-}$$
$$4905w_1+5460w_2+5585w_3-5755 = e_{11}^{+}-e_{11}^{-}$$
$$5755w_1+4905w_2+5460w_3-6320 = e_{12}^{+}-e_{12}^{-}$$
$$w_1+w_2+w_3=1$$
$$w_1 \geq w_2 \geq w_3 \geq 0$$
$$e_t^{+}, e_t^{-} \geq 0 \quad \forall t=4,5,6,...,12$$

The problem is solved using EXCEL solver. Optimal weights are w_1=0.55; w_2=0; w_3=0.45

Forecast for month 13 = (0.55 x 6320) + (0 x 5755) + (0.45 x 4905) =5683.25 ≈ 5684

2.8)

Table 3 gives the forecast and their error for the various methods in Exercise 2.6.

Table 3. Forecast errors for different methods

Month	Sales	Last Value	Averaging Method	Three-month Moving average	Exponential Smoothing	Holts method	E_t (Last Value)	E_t (Averaging)	E_t (Three-month Moving average)	E_t (Exponential Smoothing)	E_t (Holts method)
1	34.00	-	-	-	36.58	35	-	-	-	-	-
2	33.00	34.00	34.00	-	35.94	35.4	-	-	-	-	-
3	42.00	33.00	33.50	-	35.2	34.76	-	-	-	-	-
4	34.00	42.00	36.33	36.33	36.9	39.42	8.00	2.33	2.33	2.90	5.42
5	36.00	34.00	35.75	36.33	36.18	37.94	-2.00	-0.25	0.33	0.18	1.94
6	43.00	36.00	35.80	37.33	36.13	37.46	2.00	-7.20	-5.67	-6.87	-5.54
7	34.00	43.00	37.00	37.67	37.85	41.08	9.00	3.00	3.67	3.85	7.08
8	33.00	34.00	36.57	37.67	36.89	38.24	0.00	3.57	4.67	3.89	5.24
9	43.00	33.00	36.13	36.67	35.92	35.08	-1.00	-6.88	-6.33	-7.08	-7.92
10	31.00	43.00	36.89	36.67	37.69	38.77	9.00	5.89	5.67	6.69	7.77
11	35.00	31.00	36.30	35.67	36.01	34.63	-3.00	1.30	0.67	1.01	-0.37
12	41.00	35.00	36.18	36.33	35.76	33.82	1.00	-4.82	-4.67	-5.24	-7.18

BIAS and MAD values for the methods are given in Table 4.

Table 4. BIAS and MAD for different methods

Method	BIAS	MAD
Last Value	25.00	3.67
Averaging	20.95	2.33
Three-month Moving average	24.67	2.74
Exponential Smoothing	23.33	2.59
Holts method	30.44	3.42

Three-month moving average method has the lowest BIAS and MAD values (0.67 and 3.78). So, the three-month moving average method is recommended for forecasting.

2.9)

Comparison of forecasting methods

Month	E_t (Method 1)	E_t (Method 2)
1	-26	-37
2	51	48
3	-56	-30
4	-82	-90
5	60	40
6	-20	-35

(a) Compute MAD, MSE, BIAS

MAD = $\frac{1}{n}\sum_{t=1}^{n}|e_t|$

MSE = $\frac{1}{n}\sum_{t=1}^{n}e_t^2$

BIAS = $\sum_{t=1}^{n}e_t$

Method 1:
MAD = (26+51+56+82+60+20)/6= 49.17
MSE = $(26^2+51^2+56^2+82^2+60^2+20^2)/6$= 2856.17
BIAS=-26+51-56-82+60-20= -73

Method 2:
MAD = (37+48+30+90+40+35)/6= 46.67
MSE = $(37^2+48^2+30^2+90^2+40^2+35^2)/6=2583.00$
BIAS=-37+48-30-90+40-35= -104

(b) Compute tracking signal

$TS_t = BIAS_t/MAD_t$
$TS_1 = e_1/ \mid e_1\mid$
$TS_2 = (e_1 + e_2)/ \{|e_1|+ |e_2|\}/2$
.
.
$TS_6 = (e_1 + e_2 +\ldots+ e_5+ e_6)/ \{|e_1|+|e_2| +\ldots+|e_6|\}/6$

	Tracking Signal	
Month	**Method 1**	**Method 2**
1	-1.000	-1.000
2	0.649	0.259
3	-0.699	-0.496
4	-2.102	-2.127
5	-0.964	-1.408
6	-1.485	-2.229

(c) Both methods tend to under forecast since the bias is negative.

Method 2 appears to be more biased than Method 1.

But since:

$(\text{MAD})_{\text{Method 2}} < (\text{MAD})_{\text{Method 1}}$

$(\text{MSE})_{\text{Method 2}} < (\text{MSE})_{\text{Method 1}}$

Method 2 is recommended for forecasting purposes.

2.10)

Decision Variables:

x_1 Normal production in week 1 for use in week j for j = 1, 2, 3, 4

x_2 Normal production in week 2 for use in week j for j = 2, 3, 4

x_3 Overtime production in week 2 for use in week j for j = 2, 3, 4

x_4 Normal production in week 3 for use in week j for j = 3, 4

x_5 Overtime production in week 3 for use in week j for j = 3, 4

x_6 Normal production in week 4 for use in week 4

I_j Inventory at the end of week j, j = 1, 2, 3, 4

b_j Backorder at the end of week j, j = 1, 2, 3, 4

Objective: To minimize the sum of production, inventory and backorder costs.

$$\text{Minimize } Z = 10x_1 + 10x_2 + 15x_3 + 15x_4 + 15x_5 + 15x_6 + 3\sum_{j=1}^{4} I_j + 4\sum_{j=1}^{4} b_j$$

Subject to,

Inventory balance constraints: The left hand side of the equation is the sum of inventory at the end of week t-1 and the production during week t minus the back order at the end of week t-1. If the sum is less than the demand, backorder (b_j) exists. Else, inventory (I_j) exists.

$$I_0 + x_1 - b_0 = 300 - b_1 + I_1$$

$$I_1 + x_2 + x_3 - b_1 = 700 - b_2 + I_2$$

$$I_2 + x_4 + x_5 - b_2 = 900 - b_3 + I_3$$

$$I_3 + x_6 - b_3 = 800 - b_4 + I_4$$

$$I_0 = 0$$

Initial Inventory constraint: Initial inventory at the beginning of week 1 (end of week 0) is set to be 0.

$I_0 = 0$

Initial and Final back order level constraint: Initial back order is assumed to be 0 and the final back order at the end of week 4 is 0, assuming that all the back orders must be filled by the end of the fourth week.

$b_0 = 0$

$b_4 = 0$

Regular time production capacity constraints: Regular production capacity in each week is limited to 700.

$x_1 \leq 700$

$x_2 \leq 700$

$x_4 \leq 700$

$x_6 \leq 700$

Over time production capacity constraints: Over time production capacity in week 2 and 3 is limited to 200.

$x_3 \leq 200$

$x_5 \leq 200$

Non-negativity constraints

$x_1, x_2, x_3, x_4, x_5, x_6, b_1, b_2, b_3, b_4, I_1, I_2, I_3, I_4 \geq 0$

2.11)

	Week1	Week2	Week3	Week4	Dummy	
Week1	10	13	16	19	0	700
Week2 (Normal)	14	10	13	16	0	700
Week2 (OT)	23	15	18	21	0	200
Week3 (Normal)	27	19	15	18	0	700
Week3 (OT)	36	28	20	23	0	200
Week4	35	27	19	15	0	700
	300	700	900	800	500	

Variables:

x_{1j} Normal production in week 1 for use in week j for $j = 1, 2, 3, 4$

x_{2j} Normal production in week 2 for use in week j for $j = 1, 2, 3, 4$

x_{3j} Overtime production in week 2 for use in week j for $j = 1, 2, 3, 4$

x_{4j} Normal production in week 3 for use in week j for $j = 1, 2, 3, 4$

x_{5j} Overtime production in week 3 for use in week j for $j = 1, 2, 3, 4$

x_{6j} Normal production in week 4 for use in week j for $j = 1, 2, 3, 4$

Note: x_{21}, x_{31}, x_{41}, x_{51}, x_{61}, x_{42}, x_{52}, x_{62}, x_{63}, are production to fill the backorders

2.12)

	Customer A			Customer B				
	June	July	August	June	July	August	Dummy	
June RT	100	110	120	100	110	120	0	40
June OT	120	130	140	120	130	140	0	10
July RT	105	100	110	M	100	110	0	40
July OT	125	120	130	M	120	130	0	10
Aug RT	110	105	100	M	M	100	0	40
Aug OT	125	125	120	M	M	120	0	10
	30	20	15	20	20	10	35	150

2.13)

(a) $L_t \leq (0.1)W_t \quad$ for $t = 1, 2, ..., 6$

(b) $R_t \leq 50 \quad$ for $t = 1, 2, ..., 6$

(c) $|W_t - W_{t-1}| \leq 10 \quad$ for t=1,2,....,6

This can be linearized as follows:

$W_t - W_{t-1} \leq 10$
$W_{t-1} - W_t \leq 10$

Note: W_1 = 20

(d) $I_t \leq 120 \quad$ for $t = 1, 2, ..., 6$

2.14)

(a) Let index i =1,...,6 represent the six months.

x_i	Production in month i
y_i^+	Increase in production in month i
y_i^-	Decrease in production in month i
I_i	Inventory at the end of month i
d_i	Demand for month i

(b) & (c)

The production planning formulation may be written as follows.

Objective:

$$\text{Minimize } Z = 5\sum_{i=1}^{6} y_i^+ + 3\sum_{i=1}^{6} y_i^- + 3\sum_{i=1}^{6} I_i$$

Subject to,

Demand constraints: The demand in month i should be met. In general, this may be expressed as: $I_{i-1} + x_i = d_i + I_i$.

$$\begin{aligned} 1000+x_1 &= 2500+I_1 \\ I_1+x_2 &= 5000+I_2 \\ I_2+x_3 &= 7500+I_3 \\ I_3+x_4 &= 10000+I_4 \\ I_4+x_5 &= 9000+I_5 \\ I_5+x_6 &= 6000+I_6 \end{aligned}$$

Inventory constrains: Capacity limits on inventory as well as ending inventory in June.

$$\begin{aligned} I_i &\leq 7000 \quad \forall\, i=1,\dots,6 \\ I_6 &\geq 3000 \end{aligned}$$

Production balance: Production between two successive months is related by the increase or decrease in production.

$$x_1 = 2000 + y_1^+ - y_1^-$$

$$x_2 = x_1 + y_2^+ - y_2^-$$

$$x_3 = x_2 + y_3^+ - y_3^-$$

$$x_4 = x_3 + y_4^+ - y_4^-$$

$$x_5 = x_4 + y_5^+ - y_5^-$$

$$x_6 = x_5 + y_6^+ - y_6^-$$

Non-negativity constraints:

$$x_i, y_i^+, y_i^-, I_i \geq 0 \qquad \forall i=1,\ldots,6$$

2.15 (Forecasting Case Study)

Using the data for the years 2007 – 2010, prepare the initial estimates of the seasonal factors for each quarter

$$\text{Seasonality Index (SI)} = \frac{\text{Average demand during that period}}{\text{Overall averageof demand for all periods}}$$

- The overall average of quarterly demand using the demand values for 16 quarters

Overall Average = (800+750+600+1500+1700+1100+680+2000+2100+2200+1300+3100+2400+3060+1800+4000) / 16 = 1818.125

- The quarterly average using the four demand values for each quarter is as follows:

Quarter 1 average = (800+1700+2100+2400)/4 = 1750
Quarter 2 average = (750+1100+2200+3060)/4 = 1777.5
Quarter 3 average = (600+680+1300+1800)/4 = 1095
Quarter 4 average = (1500+2000+3100+4000)/4 = 2650

- The seasonality factor for each quarter can then be calculated using the seasonality index formula:

  Seasonality Index for Quarter 1 = 1750/ 1818.125 = 0. 96253
  Seasonality Index for Quarter 2 = 1777.5 / 1818.125 = 0. 977656
  Seasonality Index for Quarter 3 = 1095 / 1818.125 = 0. 602269
  Seasonality Index for Quarter 4 = 2650/ 1818.125 = 1. 457546

For the six different smoothing constant levels of (α, β) values, forecasts are determined using Holt's method for the years (2007-2010). BIAS and STD error measures are calculated and listed in Table 5.

Table 5. BIAS and STD values for different (α, β) values

(Alpha, Beta) Level	(Alpha, Beta) values	BIAS	STD
1	(0.1, 0.1)	-6673.39	778.51
2	(0.1, 0.2)	-5466.121	763.86
3	(0.1, 0.3)	-4463.673	**762.05**
4	(0.2, 0.2)	-2808.392	768.64
5	(0.2, 0.3)	-1969.469	781.91
6	(0.3, 0.3)	**-1035.496**	798.29

Based on the two error measures: BIAS and STD, there is no dominant (ALPHA, BETA) value which could be recommended (see Table 5). BIAS is the least for (0.3, 0.3) = *-1035.496*. But, STD (*798.29*) is the worst value for this smoothing constant level. STD is comparably good for (0.1, 0.2), (0.1, 0.3) and (0.2, 0.2) with 763.86, 762.05 and 768.64 correspondingly.

In order to recommend a best smoothing constant value (ALPHA, BETA), we use a weighted sum approach. The following steps are done:

1) Sum the values of the columns corresponding to BIAS and STD (Table 6). This sum is used to normalize the BIAS and STD values.

Table 6. Sum of columns

(Alpha, Beta) Level	(Alpha, Beta) values	BIAS	STD
1	(0.1, 0.1)	-6673.39	778.51057
2	(0.1, 0.2)	-5466.121	763.86054
3	(0.1, 0.3)	-4463.673	**762.05034**
4	(0.2, 0.2)	-2808.392	768.63743
5	(0.2, 0.3)	-1969.469	781.91101
6	(0.3, 0.3)	**-1035.496**	798.2958
	Total	-22416.541	4653.265

2) Calculate the normalized values by dividing each value in column by its corresponding column total (Table 7):

Table 7. Normalized values

(Alpha, Beta) Level	(Alpha, Beta) values	BIAS	STD
1	(0.1, 0.1)	0.2976994	0.1673041
2	(0.1, 0.2)	0.2438432	0.1641558
3	(0.1, 0.3)	0.1991241	0.1637668
4	(0.2, 0.2)	0.1252821	0.1651824
5	(0.2, 0.3)	0.0878578	0.1680349
6	(0.3, 0.3)	0.0461934	0.171556

3) Set weights for the error measures. We assume equal weights for both BIAS and STD *i.e.*, 0.5 for STD and 0.5 for BIAS. Table 8 shows the weighted sum.

Table 8. Weighted Sum

(Alpha, Beta) Level	(Alpha, Beta) values	Weighted Sum
1	(0.1, 0.1)	0.232502
2	(0.1, 0.2)	0.203999
3	(0.1, 0.3)	0.181445
4	(0.2, 0.2)	0.145232
5	(0.2, 0.3)	0.127946
6	(0.3, 0.3)	0.108875

The minimum weighted sum is selected as the best (ALPHA, BETA) levels. (0.3, 0.3) has the minimum weighted sum of 0.108875. So, we recommend using this level for the values of smoothing constants.

c) (0.3, 0.3)

F_{Qi} is the forecast for quarter i for the year 2011
L_{Q1} is the level forecast for quarter 1 for the year 2011
T_{Q1} is the Trend forecast for quarter 1 for the year 2011
S_{Qi} is the seasonality index for quarter i

L_{Q1} = 3062.89; T_{Q1} = 171.96

$$F_{Q1} = (L_{Q1} + T_{Q1})\, SI_{Q1} = 3113.64$$
$$F_{Q2} = (L_{Q1} + 2.T_{Q1})\, SI_{Q2} = 3330.69$$
$$F_{Q3} = (L_{Q1} + 3.T_{Q1})\, SI_{Q3} = 2155.39$$
$$F_{Q4} = (L_{Q1} + 4.T_{Q1})\, SI_{Q4} = 5466.88$$

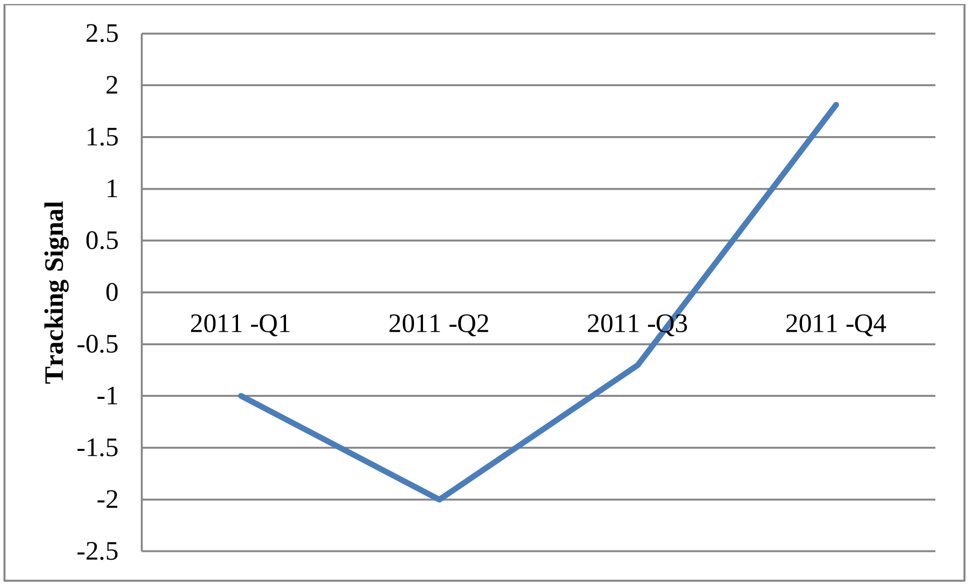
Period	Actual demand	Actual forecast	Errors (e_t)	e_t^2	Tracking signal (TS_k)
2011 -Q1	3600	3113.64	-486.36	236545.1	-1
2011 -Q2	3900	3330.69	-569.31	324112.8	-2
2011 -Q3	1500	2155.39	655.39	429531.4	-0.702
2011 -Q4	3320	5466.88	2146.88	4609081	1.811
Total			1746.60	5599270.21	
			MSE	1399817.55	
			STD	1183.14	

BIAS: The method was under forecasting in the years 2007- 2010. However, in 2011 the method is over forecasting.

STD: Increased over the 2007-2010 data.

Tracking Signal:

Figure 1. Tracking signal values for Holt's method



TS_k values are within the limits.

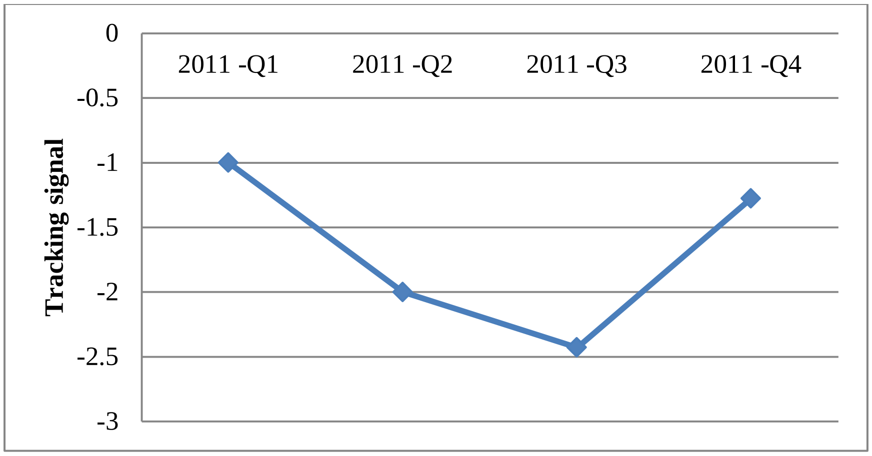
(d) $\overline{F_{Q1}} = \overline{F_{Q2}} = \overline{F_{Q3}} = \overline{F_{Q4}} = \dfrac{2493.429 + 3129.937 + 2988.699 + 2744.34}{4} = 2839.101$

Seasonalized forecast (F_{Qi}) = $S_{Qi} \times \overline{F_{Qi}}$

F_{Q1} = 2732.72; F_{Q2} = 2775.66; F_{Q3} = 1709.90; F_{Q4} =4138.12;

Period	Actual demand	Actual forecast	Errors (e_t)	e_t^2	Tracking signal (TS_k)
2011 -Q1	3600	2732.72	-867.28	752174.6	-1
2011 -Q2	3900	2775.66	-1124.34	1264134.00	-2
2011 -Q3	1500	1709.90	209.90	44058.82	-2.42794
2011 -Q4	3320	4138.12	818.12	669318.5	-1.27644
Total			-963.596	2729686	
			MSE	682421.5	
			STD	826.0881	

Figure 2. Tracking signal values for moving average



Tables 9 and 10 show the comparison results of the two methods. Figure 3 shows the tracking signal plot for the methods.

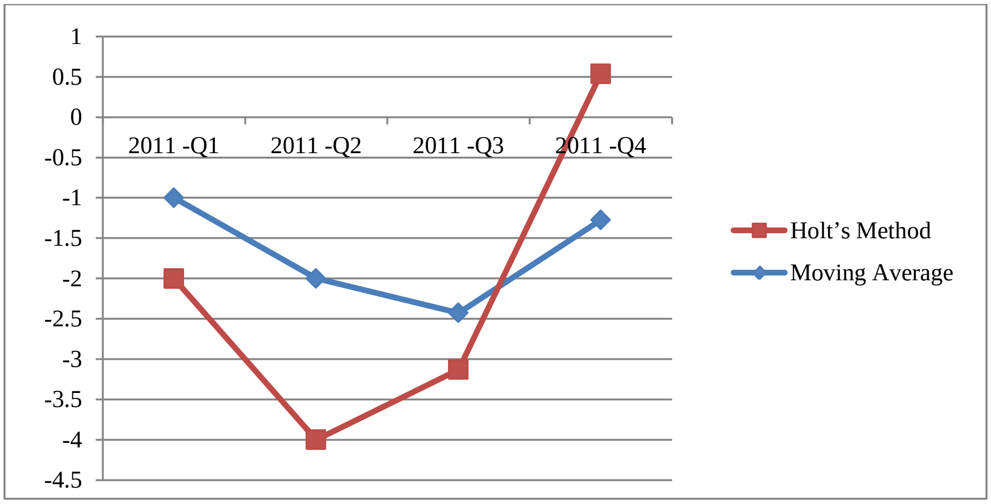
Table 9. BIAS and STD for the moving average and Holt's

Method	BIAS	STD
Moving Average	-963.596	826.0881
Holt's Method	1746.60	1183.14

Table 10. Tracking Signal for the moving average and Holt's

	Moving Average	Holt's Method
TS_1	-1.000	-1.000
TS_2	-2.000	-2.000
TS_3	-2.427	-0.702
TS_4	-1.276	1.811

Figure 3. Tracking signal values – Moving Average vs. Holt's method



The following are the conclusions from the comparison:

1) Moving average is better than the Holt's method. From the sales data, it can be observed that the sales for the year 2011 have fluctuated in almost all the quarters. In quarter 1, it has gone up considerably than the trend. In quarter 3 and 4, sales has dropped from its trend till 2010. Moving average method gives a good forecast because of its ability to identify short-term fluctuations quicker than the Holt's method.
   Even with a considerably high value of smoothing constant values in the Holt's method, which can react to the changes in data quickly, resulting forecast errors and variance is larger. It is evident from the results in Table 9.
2) Tracking signal values show that both the forecast methods are within the acceptable value.

2.16 (Aggregate Planning case study)

(a) Given below is an example plan for Chase strategy

Table 11. Chase Strategy for Exercise 2.16

Month	Workers	RT production
1	35	350
2	60	600
3	60	600
4	80	800
5	130	1300
6	200	2000

Month	Workers	RT production
7	250	2500
8	300	3000
9	240	2400
10	180	1800
11	150	1500
12	150	1500

Note: The above plan (Table 11) assumes no overtime use. There are several other plans possible under the chase strategy.

Table 12. Chase strategy production plan for Exercise 2.16

Month	Regular time production	Over time production	Demand	Cumulative Inventory at the end of the month *t*
1	350	0	500	0
2	600	0	600	0
3	600	0	600	0
4	800	0	800	0
5	1300	0	1300	0
6	2000	0	2000	0
7	2500	0	2500	0
8	3000	0	3000	0
9	2400	0	2400	0
10	1800	0	1800	0
11	1500	0	1500	0
12	1500	0	1200	300

Table 13. Chase strategy work force analysis for Exercise 2.16

Month	Total workforce	Regular time production	Over time production	Hired	Fired
1	35	35	0	0	65
2	60	60	0	25	0
3	60	60	0	0	0
4	80	80	0	20	0
5	130	130	0	50	0
6	200	200	0	70	0
7	250	250	0	50	0
8	300	300	0	50	0
9	240	240	0	0	60
10	180	180	0	0	60
11	150	150	0	0	30
12	150	150	0	0	0

Final Inventory = 300; Final workforce = 150;
Cost of the chase strategy = $4,455,000

(b) Given below is an example plan for Level strategy

Average demand / month = 1517. Hence, use a level work force of 150 workers and over time.

Table 14. Level Strategy for Exercise 2.16

Month	Initial Inventory	Production		Demand
		RT	OT	
1	150	1500	0	500
2	1150	1500	0	600
3	2050	1500	0	600
4	2950	1500	0	800
5	3650	1500	0	1300
6	3850	1500	0	2000
7	3350	1500	0	2500
8	2350	1500	0	3000
9	850	1500	100	2400
10	50	1500	250	1800
11	0	1500	0	1500
12	0	1500	0	1200

Final inventory = 300; Final workers = 150
Note: There are several other plans possible under level strategy also.

Table 15. Level strategy production plan for Exercise 2.16

Month	Regular time production	Over time production	Demand	Cumulative Inventory at the end of the month *t*
1	1500	0	500	1150
2	1500	0	600	2050
3	1500	0	600	2950
4	1500	0	800	3650
5	1500	0	1300	3850
6	1500	0	2000	3350
7	1500	0	2500	2350
8	1500	0	3000	850
9	1500	100	2400	50
10	1500	250	1800	0
11	1500	0	1500	0
12	1500	0	1200	300

Table 16. Level strategy work force analysis for Exercise 2.16

Month	Total workforce	Regular time production	Over time production	Hired	Fired
1	150	150	0	50	0
2	150	150	0	0	0
3	150	150	0	0	0
4	150	150	0	0	0
5	150	150	0	0	0
6	150	150	0	0	0
7	150	150	0	0	0
8	150	150	0	0	0
9	150	150	50	0	0
10	150	150	125	0	0
11	150	150	0	0	0
12	150	150	0	0	0

Final Inventory = 300; Final workforce = 150;
Cost of the Level strategy= $4,243,750

(c) **LP Model to determine the optimal production plan for 2012**

Decision Variables

W_t Total workers available beginning of month *t*, after hiring and firing

RP_t Workers assigned to regular time production in month *t*

OP_t Workers assigned to over time production in month *t*

F_t Number of workers fired at the beginning of month *t*

H_t Number of workers hired at the beginning of month *t*

I_t Cumulative inventory at the end of month *t*

X_t Number of units produced during regular time production in month *t*

Y_t Number of units produced during over time production in month *t*

Input Variable

D_t Demand at time period *t*

Objective Function: The objective is to minimize the sum of regular time production, over time production, hiring, firing, and average inventory holding costs

$$\text{Minimize } Z = 200\sum_{t=1}^{12} X_t + 300\sum_{t=1}^{12} Y_t + 500\sum_{t=1}^{12} H_t + 3000\sum_{t=1}^{12} F_t + 50\sum_{t=1}^{12} \frac{I_t}{2}$$

Size of the workforce: Total workforce available at the beginning of month *t* is equal to the total workforce available at the beginning of month *t*-1 plus the number hired at the beginning month *t*, minus the number fired at the beginning of month *t*.

$$W_t = W_{t-1} + H_t - F_t \qquad \text{for } t = 1, 2, \ldots, 12$$

Expected number of workers at the beginning: Total workforce available at the beginning of month 0 is equal to 100.

$$W_0 = 100$$

Desired work force at the end of month 12: Total desired workforce during month 12 is equal to 150.

$$W_{12} \geq 150$$

Regular and overtime production capacity constraints: Regular production at month *t* is equal to the product of regular production capacity of a worker and the number of workers assigned to regular time production in month *t*

$$X_t = 10RP_t \qquad \text{for } t = 1,2 \ldots, 12$$

Overtime production capacity constraints: Overtime production at month *t* is equal to the product of overtime production capacity of a worker and the number of workers assigned to overtime production in month *t*

$$Y_t = 2\ OP_t \qquad \text{for } t = 1,2 \ldots, 12$$

Demand/Inventory Balance: The left hand side of the equation is the sum of the current regular production (X_t), over time production (Y_t), and the inventory carried over (I_{t-1}). The sum is the total amount available to meet demand in month t. If it exceeds demand (D_t) then we will have an inventory of I_t at the end of month t. No shortages are allowed.

$X_t + Y_t + I_{t-1} = D_t + I_t$ for $t =$ 1,2 …., 12

Initial inventory at the beginning of month 1: Inventory available at the end of time 0 (or beginning of time 1) is 150.

$I_0 = 150$

Desired inventory at the end of month 12: Desired inventory at the end of month 12 is greater than or equal to 100

$I_{12} \geq 100$

Workforce assignment constraints: The number of workers assigned to regular production in month t is equal to the total workforce available at the beginning of month t after hiring and firing.

$W_t = RP_t$ for $t =$ 1,2 …., 12

Overtime production constraints: The number of workers assigned to over time production in month t should be less than the total workforce available at the beginning of month t.

$OP_t \leq W_t$ for $t =$ 1,2 …., 12

Non-negativity constraints

$W_t,\ RP_t, OP_t,\ H_t,\ F_t, I_t,\ X_t \geq 0$ for all $t = 1,\ 2,\ 3…..,12$

(d) The LP model is solved using LINGO and the results are shown in Tables 17 and 18:

Table 17. LP Optimal Production Plan for Exercise 2.16

Month	Regular time production	Over time production	Demand	Cumulative Inventory at the end of the month *t*
1	1000	0	500	650
2	1000	0	600	1050
3	1000	0	600	1450
4	1000	0	800	1650
5	1836.5	0	1300	2186.5
6	1836.5	0	2000	2023.1
7	1836.5	0	2500	1359.6
8	1836.5	0	3000	196.2
9	1836.5	367.3	2400	0
10	1800	0	1800	0
11	1500	0	1500	0
12	1500	0	1200	300

Table 18. Workforce Analysis for Exercise 2.16

Month	Total workforce	Regular time production	Over time production	Hired	Fired
1	100	100	0	0	0
2	100	100	0	0	0
3	100	100	0	0	0
4	100	100	0	0	0
5	183.7	183.7	0	83.7	0
6	183.7	183.7	0	0	0
7	183.7	183.7	0	0	0
8	183.7	183.7	0	0	0
9	183.7	183.7	183.7	0	0
10	180	180	0	0	3.7
11	150	150	0	0	30
12	150	150	0	0	0

Final Inventory = 300; Final workforce =150;

Optimal cost of the LP model = $4,121,154

(e)

(i) Monthly Inventory levels: Chase strategy has the least monthly inventory levels. The optimal inventory level given by the LP model is higher than that of the chase strategy. The level strategy has the highest monthly inventory levels among the three. Table 19 shows the monthly inventory levels under the three production plans.

Table 19. Monthly inventory levels under Chase, Level, and LP model (Exercise 2.16)

Month	Chase strategy	Level strategy	LP model
1	0	1150	650
2	0	2050	1050
3	0	2950	1450
4	0	3650	1650
5	0	3850	2186.5
6	0	3350	2023.1
7	0	2350	1359.6
8	0	850	196.2
9	0	50	0
10	0	0	0
11	0	0	0
12	300	300	300

(ii) Hiring and Firing: Level strategy has the lowest number of hiring and firing. In this problem, under the level strategy, there was hiring of 50 workers in month 1 and no firing. The optimal hiring and firing plan by the LP model had a hiring of 84 workers and firing of 34 workers. The chase strategy had the highest number of hiring and firing (hiring of 265 workers and firing of 215 workers in total). Hiring and firing levels for the three production plans are shown in Table 20.

Table 20. Hiring and Firing under Chase, Level, and LP model (Exercise 2.16)

Month	Chase strategy		Level strategy		LP model	
	Hiring	Firing	Hiring	Firing	Hiring	Firing
1	0	65	50	0	0	0
2	25	0	0	0	0	0
3	0	0	0	0	0	0
4	20	0	0	0	0	0
5	50	0	0	0	83.7	0
6	70	0	0	0	0	0
7	50	0	0	0	0	0
8	50	0	0	0	0	0
9	0	60	0	0	0	0
10	0	60	0	0	0	3.7
11	0	30	0	0	0	30
12	0	0	0	0	0	0

(iii) Over time use: Under chase strategy, there was 0 over time use. Level strategy used 175 workers in over time. The LP model had the highest over time use with 184 workers. Table 21 shows the over time use for the three production plans.

Table 21. Over time use under Chase, Level, and LP model (Exercise 2.16)

Month	Chase strategy	Level strategy	LP model
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0
9	0	50	183.7
10	0	125	0
11	0	0	0
12	0	0	0

(iv) <u>Total cost</u>: LP model yields a production plan with the lowest cost, followed by the level strategy. The production plan by the chase strategy has the highest total cost. Table 22 shows the total cost for the three production plans.

Table 22. Total cost under Chase, Level, and LP model (Exercise 2.16)

	Chase strategy	**Level strategy**	**LP model**
Total cost	4455000	4243750	4121154

Chapter 3

Inventory Management Methods and Models

Solutions to Numerical Exercises only

3.8) Note that the data is already sorted by decreasing annual sales (in $). Therefore, to determine reasonable cut-off points for the A, B, and C categories we only need to supplement the sorted sales data with the cumulative percentage of overall sales and the cumulative percentage of overall items. The table below includes this data, along with suggested categorizations of A, B, and C items.

Stock-keeping unit (SKU)	Annual Sales ($)	Cumulative % of SKUs	Cumulative % of Sales	Class
J-625	$904,366	4.00%	15.77%	A
Z-454	$838,481	8.00%	30.39%	A
W-681	$757,060	12.00%	43.59%	A
J-909	$635,764	16.00%	54.68%	A
T-988	$596,075	20.00%	65.08%	A
B-570	$482,492	24.00%	73.49%	A
M-117	$390,553	28.00%	80.30%	B
H-033	$262,363	32.00%	84.88%	B
W-998	$212,713	36.00%	88.58%	B
C-841	$151,413	40.00%	91.23%	B
R-596	$125,234	44.00%	93.41%	B
Y-764	$81,392	48.00%	94.83%	B
F-496	$48,131	52.00%	95.67%	C
M-154	$37,409	56.00%	96.32%	C
M-615	$30,011	60.00%	96.84%	C
A-620	$29,830	64.00%	97.36%	C
K-388	$27,592	68.00%	97.84%	C
B-237	$24,633	72.00%	98.27%	C
K-778	$22,551	76.00%	98.67%	C
Y-319	$19,836	80.00%	99.01%	C
T-670	$16,058	84.00%	99.29%	C
S-802	$14,996	88.00%	99.56%	C
T-172	$10,106	92.00%	99.73%	C
G-676	$8,783	96.00%	99.88%	C
M-687	$6,624	100.00%	100.00%	C