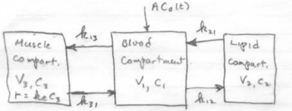
R. Hw, et. al

2 ND Edition

Solutions - Chapter 2 Problems - 2nd Ed.

2.1



a) Mass Balances for Each Compartment

rate of Accumulation of Mass in Compartment i = rate of Mass into Compartment i - rate of Mass out of Comp. i - rate of Mass metabolized in Comp. i

$i = 1, 2, 3$

Compartment 1:

$$\frac{d(V_1C_1)}{dt} = A\,C_0(t) + V_2C_2k_{21} + V_3C_3k_{31} - (V_1C_1k_{12} + V_1C_1k_{13})$$

Expanding LHS

$$V_1\frac{dC_1}{dt} + C_1\frac{dV_1}{dt} = AC_0(t) + V_2C_2k_{21} + V_3C_3k_{31} - V_1C_1(k_{12} + k_{13})$$

If Volumes are Constant, $\frac{dV_1}{dt} = \frac{dV_2}{dt} = \frac{dV_3}{dt} = 0$

So: $$V_1\frac{dC_1}{dt} = AC_0(t) + V_2C_2k_{21} + V_3C_3k_{31} - V_1C_1(k_{12} + k_{13}) \quad (1)$$

Compartment 2:

$$V_2\frac{dC_2}{dt} = k_{12}V_1C_1 - k_{21}V_2C_2 \quad (2)$$

Compartment 3:

$$V_3\frac{dC_3}{dt} = V_1C_1k_{13} - V_3C_3k_{31} - k_eV_3C_3 \quad (3)$$

IC's at $t=0$, $C_1(0) = C_2(0) = C_3(0)$.

b) (see next Sheet)

c) (See Simulink outputs for injection: $AC_0(t) = A\delta(t)$
where $\delta(t)$ = unit impulse function - injection
$AC_0(t) = AU(t)$! where $U(t)$ = unit step function (i.v. drip)

Simulink EQuations

-Compartment 1- $\frac{dC_1}{dt} = \left(\frac{A}{V_1}\right) C_o(t) + \left(\frac{V2k_{2,1}}{V_1}\right) C_2 + \left(\frac{V3k_{3,1}}{V_1}\right) C_3 - \left[\frac{V1C_1}{V_1}\left(\text{[redacted]}\ K_{1,2} + k_{1,3}\right)\right]$

-Compartment 2 $\frac{dC_2}{dt} = \left(\frac{V_1k_{1,2}}{V_2}\right) C_1 - k_{2,1}C_2$

-Compartment 3- $\frac{dC_3}{dt} = \left(\frac{V_1k_{1,3}}{V_3}\right) C_1 - k_{3,1}C_3 - k_eC_3$

The Simulink file created to model the three compartments and the equations governing them is seen on the next page. The model was then run in two of the initial condition scenarios; the injection, and the IV drip.

[redacted]

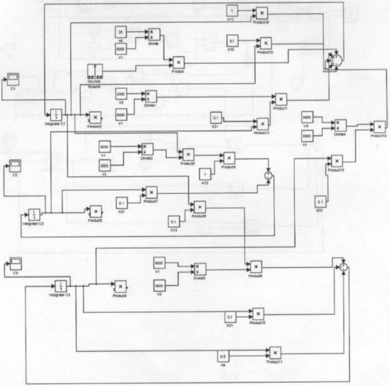


Figure 1; Simulink Block Diagram

Results:

Figure 2: Injection Simulation

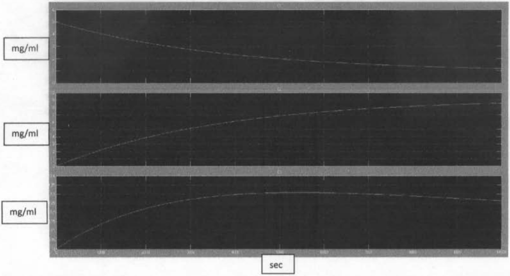
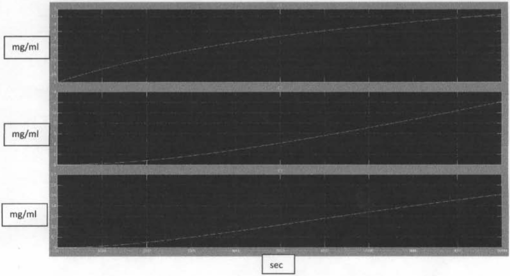


Figure 3: I.V. Simulation



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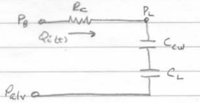
Solutions to Problems – Ch. 2: 2.2 – 2.3

2.2

a)



P_L = Lung Pressure
P_B = Barometric Pressure
Q_i = Flow During inspiration & Exp.
P_{alv} = Alveolar Pressure
R_c = Airway Resistance to Flow

Remember:

Ⓐ $C = \frac{dV}{dP}$

$dV = C\,dP$

Ⓑ $dV = \int Q_i\,dt$

Ⓒ $dP = P_L - P_{alv} = \frac{1}{C_{eq}} \int Q_i(t)\,dt$

Ⓓ $C_{eq} = \left(\frac{1}{C_{cw}} + \frac{1}{C_L}\right)^{-1}$ Compliances in Series

Ohms Law:

$$P_B - P_{alv} = \underbrace{R_c Q_i}_{P_B - P_L} + \underbrace{\left(\frac{1}{C_{cw}} + \frac{1}{C_L}\right)\int Q_i\,dt}_{P_L - P_{alv}} \quad (1)$$

$P_B - P_L + P_L - P_{alv} = P_B - P_{alv}$

Differentiating (1) wrt time

$$\frac{d(P_B - P_{alv})}{dt} = R_c \frac{dQ_i}{dt} + \left(\frac{1}{C_{cw}} + \frac{1}{C_L}\right) Q_i \quad (2)$$

Since P_B = Constant, $\frac{dP_B}{dt} = 0$

b)

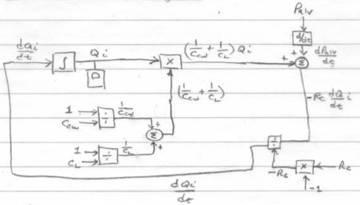
$$-\frac{dP_{alv}}{dt} = R_c \frac{dQ_i}{dt} + \left(\frac{1}{C_{cw}} + \frac{1}{C_L}\right) Q_i \quad (3)$$

c) Asthma: Increase R_c (airway resistance)

asbestos lung Disease: decrease C_L (stiffer Lung)

Scoliosis: decrease C_{cw} – (Deformity)

d) Simulink Block Diagram

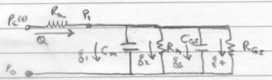


From Eq'n (3)

$$\frac{dQ_i}{dt} = -\frac{1}{R_c}\left[\left(\frac{1}{C_{cw}}+\frac{1}{C_L}\right)Q_i + \frac{P_{aiv}}{dt}\right] \Longrightarrow \text{Simulink Block Diagram}$$

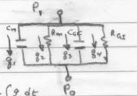
2.3 Solutions to Problems in Ch. 2: 2.3 - 2ND ED.

a)



$$Q = \frac{P_a - P_1}{R_a}$$

$$\boxed{P_1 = P_a - R_a Q} \quad (1)$$



$$P_1 - P_0 = R_m q_2 = R_{GI} q_4 = \frac{1}{C_m}\int q_1\,dt = \frac{1}{C_{GI}}\int q_3\,dt$$

$P_0 = atm = 0$

$$\boxed{P_1 = R_m q_2 = R_{GI} q_4 = \frac{1}{C_m}\int q_1\,dt = \frac{1}{C_{GI}}\int q_3\,dt} \quad (2)$$

$$\boxed{q_1 + q_2 + q_3 + q_4 = Q.} \quad (3)$$

From (2)

$$\boxed{\frac{dP_1}{dt} = R_m \frac{dq_2}{dt} = R_{GI}\frac{dq_4}{dt} = \frac{1}{C_m} q_1 = \frac{1}{C_{GI}} q_3} \quad (4)$$

From (1)

$$\boxed{\frac{dP_1}{dt} = \frac{dP_a}{dt} - R_a \frac{dQ}{dt}} \quad (5)$$

Equating (4) and (5)

$$\boxed{\frac{dP_a}{dt} - R_a\frac{dQ}{dt} = R_m \frac{dq_2}{dt} = R_{GI}\frac{dq_4}{dt} = \frac{1}{C_m} q_1 = \frac{1}{C_{GI}} q_3} \quad (6)$$

From (3)

$$\boxed{\frac{dQ}{dt} = \frac{dq_1}{dt} + \frac{dq_2}{dt} + \frac{dq_3}{dt} + \frac{dq_4}{dt}} \quad (7)$$

From (4)

$$q_1 = C_m \frac{dP_1}{dt} \Rightarrow \frac{dq_1}{dt} = C_m \frac{d^2P_1}{dt^2} \quad (8) \Rightarrow \boxed{q_1 = C_m \frac{dP_1}{dt}} \quad (A)$$

$$q_3 = C_{GI} \frac{dP_1}{dt} \Rightarrow \frac{dq_3}{dt} = C_{GI} \frac{d^2P_1}{dt^2} \quad (9) \Rightarrow \boxed{q_3 = C_{GI} \frac{dP_1}{dt}} \quad (B)$$

$$\frac{dq_2}{dt} = \frac{1}{R_m} \frac{dP_1}{dt} \quad (10) \Rightarrow \boxed{q_2 = \frac{1}{R_m} P_1} \Rightarrow (C)$$

$$\frac{dq_4}{dt} = \frac{1}{R_{GI}} \frac{dP_1}{dt} \quad (11) \Rightarrow \boxed{q_4 = \frac{1}{R_{GI}} P_1} \Rightarrow (D)$$

Substituting (8), (9), (10) and (11) into (7)

$$\frac{dQ}{dt} = C_m \frac{d^2P_1}{dt^2} + C_{GI} \frac{d^2P_1}{dt^2} + \frac{1}{R_m} \frac{dP_1}{dt} + \frac{1}{R_{GI}} \frac{dP_1}{dt} \qquad \boxed{P_1 = P_a - R_a Q} \quad (E)$$

$$\boxed{\frac{dQ}{dt} = (C_m + C_{GI}) \frac{d^2P_1}{dt^2} + \left(\frac{1}{R_m} + \frac{1}{R_{GI}}\right) \frac{dP_1}{dt}} \quad (12) \qquad \boxed{\frac{dP_1}{dt} = \frac{dP_a}{dt} - R_a \frac{dQ}{dt}} \quad (F)$$

using (5)

$$\boxed{\frac{dP_1}{dt} = \frac{dP_a}{dt} - R_a \frac{dQ}{dt} \quad (5)}$$

Then: $$\boxed{\frac{d^2P_1}{dt^2} = \frac{d^2P_a}{dt} - R_a \frac{d^2Q}{dt^2} \quad (13)}$$

using (5) and (13) in (12) gives:

$$\frac{dQ}{dt} = (C_a + C_{GI})\left[\frac{d^2P_a}{dt^2} - R_a \frac{d^2Q}{dt^2}\right] + \left[\frac{1}{R_m} + \frac{1}{R_{GI}}\right]\left[\frac{dP_a}{dt} - R_a \frac{dQ}{dt}\right]$$

Collecting Terms

$$\frac{dQ}{dt} = (C_a + C_{GI}) \frac{d^2P_a}{dt^2} - R_a (C_a + C_{GI}) \frac{d^2Q}{dt^2} + \left[\frac{1}{R_m} + \frac{1}{R_{GI}}\right] \frac{dP_a}{dt} - R_a \left[\frac{1}{R_m} + \frac{1}{R_{GI}}\right] \frac{dQ}{dt}$$

$$(C_a + C_{GI}) \frac{d^2P_a}{dt^2} + \left[\frac{1}{R_m} + \frac{1}{R_{GI}}\right] \frac{dP_a}{dt} = R_a (C_a + C_{GI}) \frac{d^2Q}{dt^2} + R_a \left[\frac{1}{R_m} + \frac{1}{R_{GI}} + 1\right] \frac{dQ}{dt}$$

Integrating Both Sides once with respect to t

$$\boxed{(C_a + C_{GI}) \frac{dP_a}{dt} + \left[\frac{1}{R_m} + \frac{1}{R_{GI}}\right] P_a = R_a (C_a + C_{GI}) \frac{dQ}{dt} + R_a \left[\frac{1}{R_m} + \frac{1}{R_{GI}} + 1\right] Q} \quad (14)$$

Setting up the Simulink Block Diagram:

1st Solve for the highest derivative of the dependent variable, $Q(t)$

– From (14)

$$R_a(C_m+C_{GI})\frac{dQ}{dt} = (C_m+C_{GI})\frac{dP_a}{dt} + \left[\frac{1}{R_m}+\frac{1}{R_{GI}}\right]P_a - R_a\left[\frac{1}{R_m}+\frac{1}{R_{GI}}+1\right]Q$$

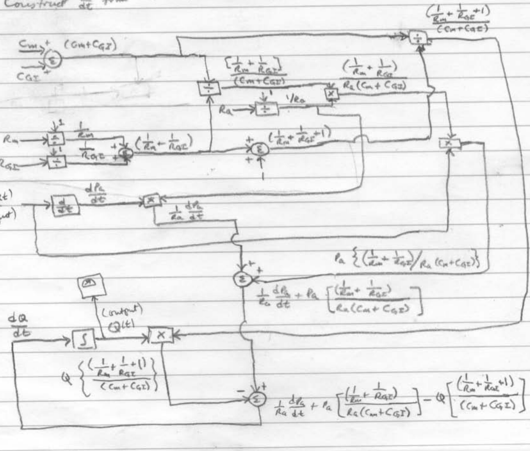
$$\frac{dQ}{dt} = \frac{\cancel{(C_m+C_{GI})}}{R_a\cancel{(C_m+C_{GI})}}\frac{dP_a}{dt} + \frac{\left[\frac{1}{R_m}+\frac{1}{R_{GI}}\right]}{R_a[C_m+C_{GI}]}P_a - \frac{\cancel{R_a}\left[\frac{1}{R_m}+\frac{1}{R_{GI}}+1\right]}{\cancel{R_a}(C_m+C_{GI})}Q$$

$$\frac{dQ}{dt} = \left(\frac{1}{R_a}\right)\frac{dP_a}{dt} + \frac{\left[\frac{1}{R_m}+\frac{1}{R_{GI}}\right]}{R_a(C_m+C_{GI})}P_a - \frac{\left[\frac{1}{R_m}+\frac{1}{R_{GI}}+1\right]}{(C_m+C_{GI})}Q \quad (15)$$

$P_a = P_a(t)$ – specified input.

To use Simulink to solve for $Q(t)$, first assume that you know $\frac{dQ}{dt}$, then construct $\frac{dQ}{dt}$ from the terms on the RHS of (15) and feed it back

b)



once $Q(t)$ is known,

$P_1(t) = P_a(t) - R_a Q(t)$ from ①

once $P_1(t)$ is known,

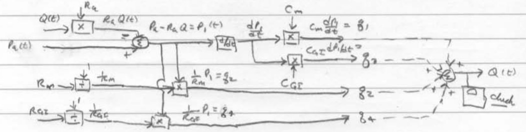
$q_1 = C_m \frac{dP_1}{dt}$ from (A)

$q_3 = C_{GI} \frac{dP_1}{dt}$ from (B)

$q_2 = \frac{1}{R_m} P_1$ from (C)

$q_4 = \frac{1}{R_{GI}} P_1$ from (D)

The Extension of the Simulink Block Diagram is Then.



c) Let $P_a = P_{a0} + A\,U(t)$

where P_{a0} is the initial mean arterial Pressure (90-100 mm Hg)

and $A = 60 - 80$ mm Hg.

d) Let R_{GI} decrease to 0.5 times original value. At same time let R_m double.

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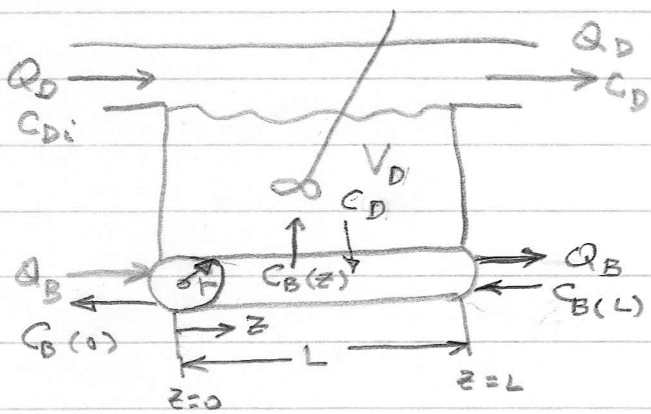
Ch. 1

Chapter 2 Problems – Solutions (#4-11)

2.4



→ Co-current flow

← Countercurrent flow

Steady-state mass Balances

1. over Dialysate Compartment

rate of Accum. of Mass in Volume D = rate of Mass in − rate of mass out

→ 0 steady state

rate of Mass into V_D : $Q_D C_{Di} + \int_0^L P_D S\,[C_B(z) - C_D]\,dz$

rate of Mass out of V_D ; $Q_D C_D$

Then: from Mass Balance ⇒ $0 = Q_D[C_{Di} - C_D] + \int_0^L P_D S\,[C_B(z) - C_D]\,dz$ (1)

2. over Blood compartment: Co-Current flow

using the B-K-R-C-P model (eq'n 2.70 – 2.71)

$$C_B(L) = C_{B0}\,e^{-(2SP_D/V_z r)L} + C_D\left[1 - e^{-(2SP_D/V_z r)L}\right]$$

where $V_z = \dfrac{Q_B}{A_c} = \dfrac{Q_B}{\pi r^2 L}$ and $S = 2\pi r L$, the surface area available for transport.

$$C_B(z) = C_{B0}\,e^{-(2SP_D/V_z r)z} + C_D\left[1 - e^{-(2SP_D/V_z r)z}\right] \quad (2)$$

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We can show (after some Algebra) and using equation (2) that:

$$\int_0^L P_D S\,[C_B(z) - C_D]\,dz = \frac{V_z r (C_{B0} - C_D)}{2}\left[1 - e^{-(2SP_D/V_z r)L}\right]$$

after some more Algebra, we can show that

$$C_D = \frac{\dfrac{V_z r\, C_{B0}}{2Q_D}\left[1 - e^{-(2SP_D/V_z r)L}\right]}{1 + \dfrac{V_z r}{2Q_D}\left(1 - e^{-(2SP_D/V_z r)L}\right)} \qquad (3)$$

where $C_B(z)$ is given by equation (2)

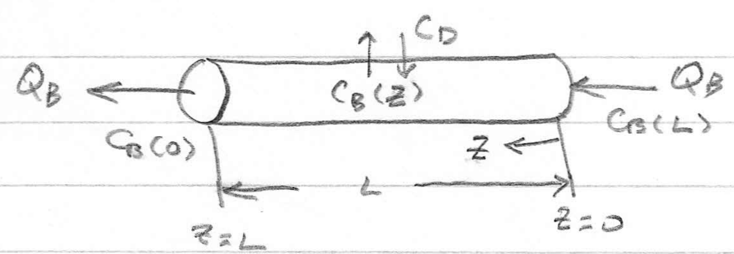
b. For the Countercurrent Blood Flow Model
Eq'n (1) remains the same.
Eq'n (2) becomes:

$$C_B(z) = C_B(L)\, e^{-(2SP_D/V_z r)z} + C_D\left[1 - e^{-(2SP_D/V_z r)z}\right]$$

where C_D is still given by Eq'n (3) above.
The B-K-R-C-P model then becomes:

$$C_B(0) = C_B(L)\, e^{-(2SP_D/V_z r)L} + C_D\left[1 - e^{-(2SP_D/V_z r)L}\right]$$

where now, the Blood Flow Portion is renamed as follows:



2.5

a) at S.S. 1 mole of Hb (64,485 g Hb/g-mole) takes up 8 moles of O (16 g/g-m)

To compare:

$$\frac{1\ g\ Hb}{64{,}485\ g/g\text{-mole}} = 1.55\times10^{-5}\ \text{moles of Hb}$$

1.55×10^{-5} Mols of Hb Carries $8(1.55\times10^{-5}) = 1.24\times10^{-4}$ Mols of O

or 0.62×10^{-4} moles of O_2

By the ideal gas Law, 1 mole of a gas occupies 22.4 L at STP (e.g. 1 Atm and 0 °C).

Assume: Avg. Arterial BP = 100 mm Hg = 760 + 100 = 860 mm Hg absolute

Avg. Body Temp = 37 °C = 273 + 37 = 310 °K

b) Volume of O_2 Carried by 1 g Hb (at Body T & Mean Arterial P) is then

$$22.4\,L \times 0.62\times10^{-4}\ \text{mols}\ O_2 \times \frac{760}{860} \times \frac{310}{273} = 1.394\times10^{-3}\,L = \boxed{1.394\ \frac{ml\ O_2}{g\ Hb}}$$

This is very close to the published value.

a) Oxygen Carrying Capacity of 100 ml blood.

$$\frac{15\ g\ Hb}{100\ ml\ blood} \times 1.394\ \frac{ml\ O_2}{g\text{-}Hb} = 20.1\ \frac{ml\ O_2}{100\ ml\ blood}$$

2.6

a)
$$300\ \frac{ml\ O_2}{min} = \underbrace{\left(C.O\ \frac{100\ ml\ blood}{min}\right) \times \overset{Hct}{0.45}}_{\text{Plasma Flow}} \times \overset{\text{Dissolved } O_2}{\frac{0.003\ ml\ O_2}{(100\ ml\ plasma \cdot p_{O_2})}} \times \overset{Pa_{O_2}}{100\ mm\ Hg\ p_{O_2}}$$

$$300 = C.O\ (0.135)$$

$$\frac{C.O}{\left(\frac{100\ ml\ blood}{min}\right)} = \frac{300}{0.135} = 2222.2\ dL/min = \boxed{222.2\ L/min}$$

Normal C.O. is 5 L/min.

b) At Steady-State, Oxygen utilization rate is the rate of transport of O_2 at the Capillaries. This is the difference between the rate of O_2 delivered by arterial blood and the rate of O_2 leaving in venous blood.

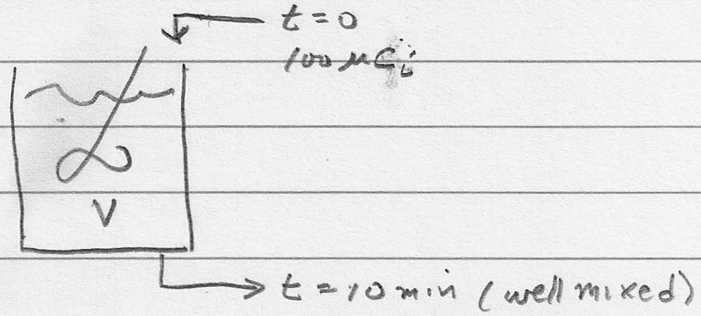
Neglecting dissolved O_2, this mass balance becomes.
Assume that the Hb in arterial blood is 98% saturated.

$$300\ \frac{ml\,O_2}{min} = \underbrace{\left(5000\ \frac{ml\ blood}{min}\right)}_{C.O.}\left(\frac{15\ g\,Hb}{100\ ml\ blood}\right) \times \frac{ml\,O_2}{g\,Hb}\left[0.98 - 0.75\right]$$

$$x = \frac{300}{750(0.23)} = 1.74\ \frac{ml\,O_2}{g\,Hb}.$$

This is about 20% higher than the reported value of 1.39 ml O_2/g Hb.

2.7)



Expected Concentration in well mixed Sample is $\frac{100\ \mu Ci}{V\,ml}$

Reported Concentration: $0.45\ \frac{\mu Ci}{ml}$

$$\frac{100}{V} = 0.45, \quad V = \frac{100}{0.45} = \boxed{222.2\ ml}$$

Since the lable was carried by red cells, this only measures the red cell volume. Since blood is plasma + red cells, you have to calculate the plasma volume.

$Hct = 0.48$ = fraction of whole blood that accounts for red cells

So total blood volume, x is:

$$0.48x = 222.2$$

$$x = 462.9\ ml$$

and Plasma volume is: $462.9 - 222.2 = \boxed{240.7\ ml}$

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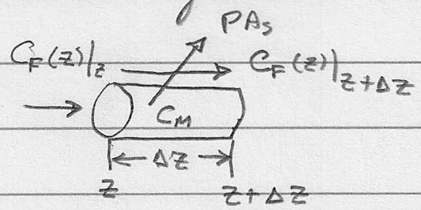
Ch. 2

12.8)

a. Co-Current Flow

A mass Balance on The Maternal side at Steady-State for a solute which is being transported between Maternal and Fetal Systems is: Assuming well-mixed Maternal Side (See Pg. 29, Fig. 1.13b)



Differential control volume, Δz

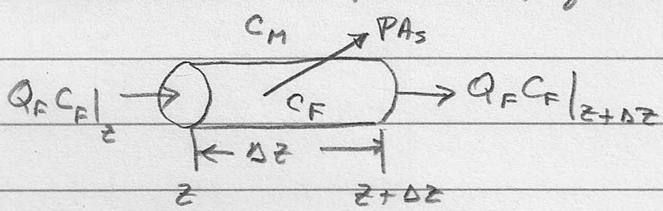
} The Exchange of Material takes Place through The surface. For a cylinder, The Surface area is: $S = 2\pi r L$. For an arbitrary Surface, $S = A_s L$, where A_s is The Surface area/Length

rate of Mass in − rate of Mass out − rate of Mass transported = 0

$$Q_M(C_{MA} - C_M) - \int_0^L PA_s\left[C_M - C_F(z)\right]dz = 0 \qquad (1)$$

Since $C_M \neq f(z)$ if The maternal side is well-mixed.
However, $C_F = f(z)$ so we have to sum up all The maternal-fetal transport over all z, with C_M Constant and $C_F = C_F(z)$

A Mass Balance on The Fetal Side over a differential slice of exchange volume at steady-State, gives:



$$Q_FC_F|_z - Q_FC_F|_{z+\Delta z} - PA_s\left[C_F - C_M\right]\Delta z = 0$$

Divide by Δz and take limit as $\Delta z \to 0$

$$-\frac{d}{dz}(Q_FC_F) - PA_s\left[C_F - C_M\right] = 0$$

For Q_F = Constant

$$Q_F\frac{dC_F}{dz} = -PA_s\left[C_F - C_M\right] \qquad (2)$$

rearranging (2).

$$\frac{dC_F}{C_F - C_M} = -\frac{PA_s}{Q_F}\,dz$$

Since $C_M \neq f(z)$, we can integrate this to get

$$\int_0^L \frac{dC_F}{C_F - C_M} = -\frac{PA_s}{Q_F}\int_0^L dz$$

$S = A_s L$ = total surface area available for exchange

Integrating

$$\ln\left[C_F - C_M\right]\Big|_0^L = -\frac{PA_s z}{Q_F}\Big|_0^L = -\frac{PA_s L}{Q_F} = -\frac{PS}{Q_F} \qquad (3)$$

$\Longrightarrow$ we can also write the integral as:

$$\ln\left[C_F - C_M\right] = -\frac{PA_s z}{Q_F}$$

or $C_F - C_M = e^{-\frac{PA_s z}{Q_F}}$

So that:

$$\boxed{C_F(z) = C_M + e^{-\frac{PA_s z}{Q_F}}} \qquad (4)$$

$$\ln\left[C_{FV} - C_M\right] - \ln\left[C_{FA} - C_M\right] = -\frac{PS}{Q_F}$$

$$\ln\left[\frac{C_{FV} - C_M}{C_{FA} - C_M}\right] = -\frac{PS}{Q_F}$$

$$\boxed{\frac{C_{FV} - C_M}{C_{FA} - C_M} = e^{-\frac{PS}{Q_F}}}$$

From (3)

$$C_F - C_M = e^{-\frac{PA_s z}{Q_F}}, \quad C_F = C_M + e^{-\frac{PA_s z}{Q_F}} \qquad (5)$$

$$-\int_0^L PA_s\left[C_M - C_F(z)\right]dz = +\int_0^L PA_s\left[C_F(z) - C_M\right]dz$$

From (5)

$$PA_s\int_0^L \left[C_F(z) - C_M\right]dz = PA_s\int_0^L e^{-\frac{PA_s z}{Q_F}}\,dz = -\frac{PA_s}{\frac{PA_s}{Q_F}}\,e^{-\frac{PA_s z}{Q_F}}\Big|_0^L = -Q_F\left[e^{-\frac{PA_s L}{Q_F}} - 1\right]$$

$$PA_s\int_0^L \left[C_F(z) - C_M\right]dz = Q_F\left[1 - e^{-\frac{PA_s L}{Q_F}}\right] = Q_F\left[1 - e^{-\frac{PS}{Q_F}}\right] \qquad (6)$$

Substituting (6) back into (1) lets us solve for C_M

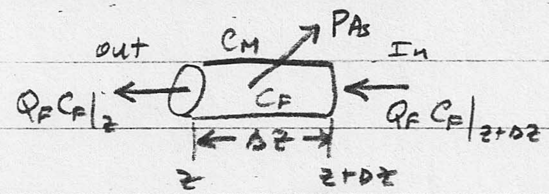
$$Q_M(C_{MA} - C_M) = -Q_F\left[1 - e^{-PS/Q_F}\right]$$

$$\boxed{C_M = C_{MA} - \left(\frac{Q_F}{Q_M}\right)\left[1 - e^{-PS/Q_F}\right]} \qquad (7)$$

b. Countercurrent Flow

The Maternal Mass Balance is the same (eq'n. 1).

The Fetal Mass Balance becomes:



$$Q_F C_F|_{z+\Delta z} - Q_F C_F|_z - PA_s[C_F - C_M]\Delta z = 0$$

Divide by Δz and take limit as $\Delta z \to 0$

$$\frac{d}{dz}(Q_F C_F) - PA_s[C_F - C_M] = 0$$

For Q_F = constant, Eq'n. 2, now becomes

$$Q_F \frac{dC_F}{dz} = PA_s[C_F - C_M] \qquad (2A)$$

Rearranging:

$$\frac{dC_F}{C_F - C_M} = \frac{PA_s}{Q_F}\,dz$$

Integrating

$$\ln[C_F - C_M] = e^{\frac{PA_s}{Q_F}z}$$

$$\boxed{C_F - C_M = e^{\frac{PA_s}{Q_F}z}} \qquad \boxed{C_F = C_M + e^{\frac{PA_s}{Q_F}z}} \qquad (5A)$$

also:

$$\int_0^L \frac{dC_F}{[C_F - C_M]} = \int_0^L \frac{PA_s}{Q_F}\,dz$$

$$\boxed{S = A_s L}$$

$$\ln[C_F - C_M]\Big|_0^L = \frac{PA_s z}{Q_F}\Big|_0^L$$

$$\ln[C_{FV} - C_M] - \ln[C_{FA} - C_M] = \frac{PA_s L}{Q_F} = \frac{PS}{Q_F}$$

$$\ln\left[\frac{C_{FV} - C_M}{C_{FA} - C_M}\right] = \frac{PS}{Q_F}$$

or $\left[\frac{C_{FV} - C_M}{C_{FA} - C_M}\right] = e^{PS/Q_F}$

From (5A)

$$P A_s \int_0^L [C_M - C_F]\, dz = -P A_s \int_0^L [C_F - C_M]\, dz = -P A_s \int_0^L e^{\frac{P A_s}{Q_F} z}\, dz$$

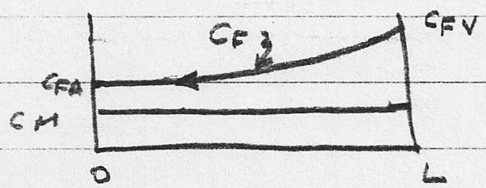
$\boxed{S = A_s L}$

$$= \frac{-P A_s}{\frac{P A_s}{Q_F}} e^{\frac{P A_s}{Q_F} z}\Big|_0^L = -Q_F \left[e^{\frac{P A_s}{Q_F} L} - 1 \right] = Q_F \left[1 - e^{\frac{P S}{Q_F}} \right]$$

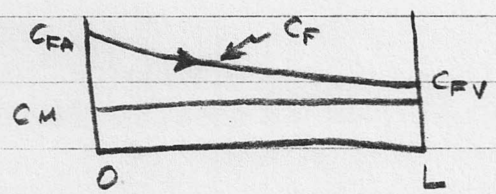
Substituting this back into (1) allows us to solve for C_M

$$Q_M (C_{MA} - C_M) - Q_F \left[1 - e^{P S / Q_F} \right] = 0$$

$$\boxed{C_M = C_{MA} - \left(\frac{Q_F}{Q_M} \right) \left[1 - e^{P S / Q_F} \right]} \qquad (7A)$$



countercurrent flow [From (5A)]



cocurrent flow [From (5)]

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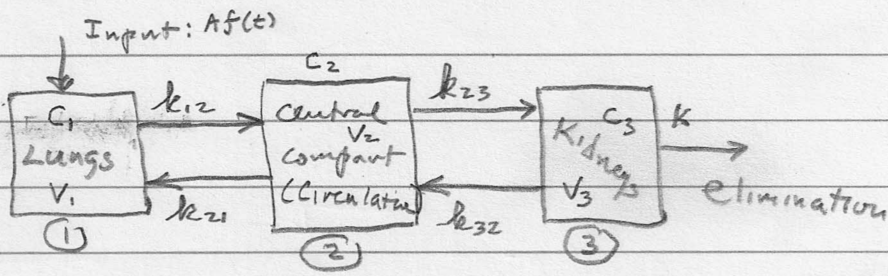
Ch. 2

2.9]

This problem is similar to problem 2.1 and can be approached in a similar manner.

(See last Pages)

a)



See: Figure E2.8: 3 - compartment model.

(In this problem, elimination is in kidney compartment)

b) using unsteady - state mass Balances: See eq'ns (2.34 - 2.36)

Lungs: Compartment 1: Assume $V_1 = V_2 = V_3$ = Constant Volume

rate of Accumulation = rate in - rate out

(1) $$V_1 \frac{dc_1}{dt} = k_{12} C_1 + A\delta(t) - k_{21} C_2$$ where $A\delta(t)$ = Bolus input of size A

$\delta(t)$ = unit Impulse function

Circulation: Compartment 2:

(2) $$V_2 \frac{dc_2}{dt} = k_{12} C_1 + k_{32} C_3 - [k_{21} + k_{23}] C_2$$

Kidneys: Compartment 3:

(3) $$V_3 \frac{dc_3}{dt} = k_{23} C_2 - [k_{32} + K] C_3$$

Initial Conditions: at $t = 0$, $C_1 = C_2 = C_3 = 0$

Parameters: $V_1, V_2, V_3, k_{12}, k_{21}, k_{23}, k_{32}, K, A$

Boundary Conditions: Input = $Af(t)$ in lung compartment

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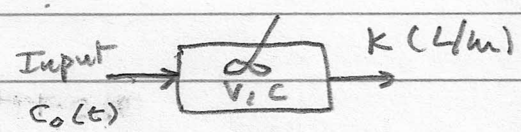
2.10)



a. rate of acc = rate in - rate out

$C_o(t)$ = Rate of Drug Delivery

$$V\frac{dc}{dt} = C_o(t) - KC$$

$$V\frac{dc}{dt} + KC = C_o(t)$$

b. $C_o(t)$, V and possibly C-vs-t Data, depending on the Complexity of $C_o(t)$

c. $C_o(t) = A\,\delta(t)$

$$V\frac{dc}{dt} + KC = A\,\delta(t)$$

Divide by K

$$\left(\frac{V}{K}\right)\frac{dc}{dt} + c = \left(\frac{A}{K}\right)\delta(t)$$

Let $\tau = \frac{V}{K}$ and $A' = \frac{A}{K}$

$$\tau\frac{dc}{dt} + c = A'\delta(t)$$

This is a 1^{ST} order linear ordinary Differential Eq'n. It's Solution for an impulse input is well known:

$$C(t) = \frac{A'}{\tau}e^{-t/\tau} \qquad (1)$$

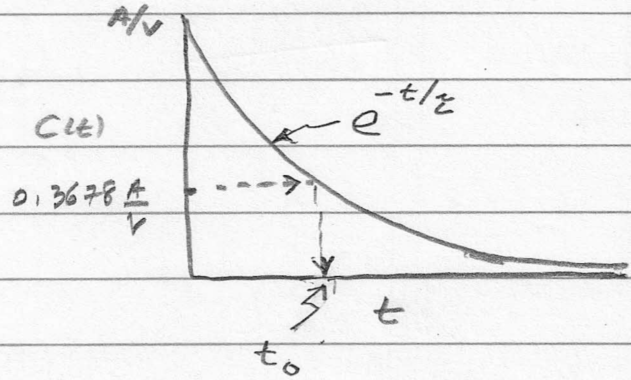
at $t = 0$, $C(0) = \frac{A'}{\tau} = \frac{\frac{A}{K}}{\frac{V}{K}} = \frac{A}{V}$

Collect experimental data on $C(t)$-vs-t

From 1:

at $t = \tau$

$C(t=\tau) = \frac{A}{V} e^{-1.0} = 0.3678 \frac{A}{V}$, A is amount injected
V = Volume of Compartment



$t_0 = \frac{t}{\tau} = 1.0$

$\therefore \; t = \tau = t_0$

So That $t_0 = \tau = \frac{V}{K}$

$\boxed{K = \frac{V}{t_0}}$ Knowing V, Calculate K

d. Can Estimate an average K, using a total mass Balance

1. Know how much drug is administered at t = 0, That is? A grams injected.

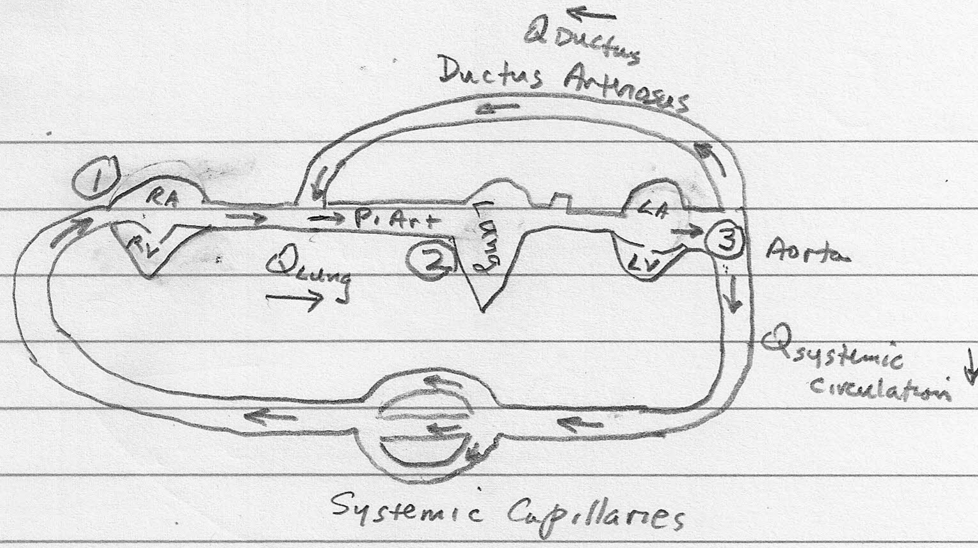
2. In 24 hours can measure volume of urine collected and Concentration of Drug (average) in that volume. If Drug is not metabolized (assumption in model) then total amount of drug recovered is $C_u V_u$, where V_u = total volume of Urine collected in 24 hours and C_u is concentration of drug in the urine.

3. The average Flow of urine is F_u (L/hr)

4. Then The average rate of excretion of drug is $\frac{C_u V_u}{24}$ (g/hr) = R_u

5. Then: K(L/hr) = R_u(g/hr) / C_u (g/L)

2.11



Sampling Site	O_2 Content (C_{O_2}) (ml O_2/100 ml blood)
1. Right Atrium	13
2. Pulmonary Artery	16
3. Aorta	19

Fick Principle:

$$Q = \frac{\dot{V}_{O_2}}{C_{aO_2} - C_{vO_2}}$$

(Fick Principle is just a mass Balance.)

$\dot{V}_{O_2}$ = Oxygen Consumption: 240 ml/min

a. Systemic Circulation a-V O_2 Difference is 3 - 1

$$Q_{systemic} = \frac{240\ (ml\ O_2/min)}{(0.19 - 0.13)\left(\frac{ml\ O_2}{ml\ blood}\right)} = 4000\ \frac{ml\ blood}{min} = \boxed{4\ L/min}$$

b. Lung Circulation a-V O_2 Difference is 2 - 1

$$Q_{Lung} = \frac{240}{(0.16 - 0.13)} = 8000\ \frac{ml\ blood}{min} = \boxed{8\ L/min}$$

c. By a mass Balance, The Flow Through The Ductus Arteriosus is just The flow Through The Lung - Flow Through The Systemic Circ.

$$Q_{Ductus} = 8\ L/min - 4\ L/min = \boxed{4\ L/min.}$$