$$\eta = \left[\frac{V_{out} \cdot I_{load}}{V_{in} \cdot \left(I_{load} + I_s \right)} \right] \Rightarrow \eta = \left(\frac{2.5}{6.6} \right) \Rightarrow \eta = 37.8\%$$

From the circuit:

$$V_{in} - I_{load} \cdot R_s - I_s \cdot R_s - V_{out} = 0 \Rightarrow$$

$$R_s = \left(\frac{(V_{in} - V_{out})}{(I_{load} + I_s)} \right) \Rightarrow R_s = \left(\frac{(12-5)}{(0.5 \cdot 1.1)} \right) \Rightarrow R_s = 12.72\Omega$$

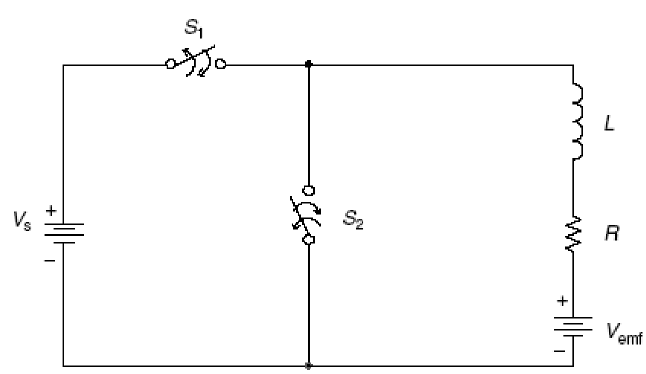
The power dissipation across R_s is:

$$P_{R_S} = \left(I_{load} + I_s \right)^2 \cdot R_s$$

If R_s is greater than 12.72Ω, then the regulator is less efficient because the power dissipated is greater. Thus, the calculated R_s is the minimum value.

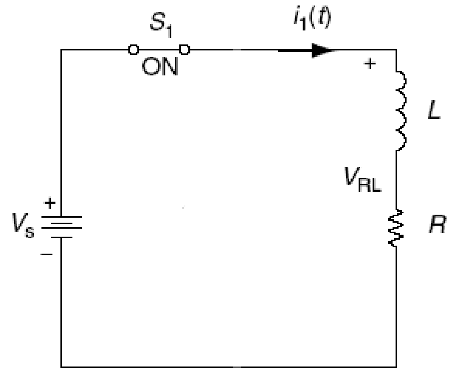
Problem 2

The fundamental switching converter shown in Figure 1.7 has a resistance value of 1Ω and an inductance of 1 H, V_s = 3.72 V, and a V_{emf} = 0V. During the interval of $0 < t \le 1$ sec, S_1 is on and S_2 is off. S_1 is off and S_2 is on during the interval of $1 \le t < 2$ sec. The current flowing through the inductor at the beginning of the switching cycle is 1 A. Determine the peak-to-peak ripple current in the inductor during the first switching cycle. Draw and label the waveforms for the current flowing through the inductor and the voltage across the resistor.



$$T = 2\sec$$
$$D = 0.5$$
$$I_0 = 1A$$
$$V_{emf} = 0$$

Mode 1: S_1 on, S_2 off, $0 < t \leq 1$.



Using Kirchoff's voltage law, we have:

$$V_s(t) = i_1(t)R + L\frac{di_1(t)}{dt}$$

Using Laplace transformation, we can write:

$$\frac{V_s}{S} = I_1(S)\cdot R + L\cdot[S\cdot I_1(S) - I_1(0)]$$

Grouping terms:

$$\frac{V_s}{S} = I_1(S)\cdot(R + S\cdot L) - L\cdot I_1(0)$$

Dividing both sides by R:

$$(V_s/R)\cdot(1/S) = I_1(S)\cdot\left[1 + S\cdot(L/R)\right] - (L/R)\cdot I_1(0)$$

Solving for I_1:

$$I_1(S) = \frac{(V_s/R)\cdot(1/S)}{(L/R)\cdot\left[S + (R/L)\right]} + \frac{(L/R)\cdot I_1(0)}{(L/R)\cdot\left[S + (R/L)\right]}$$

$$I_1(S) = (V_s/R)\cdot\frac{(R/L)}{S\cdot[S + (R/L)]} + \frac{I_1(0)}{[S + (R/L)]}$$

$$I_1(S) = \frac{V_s}{R} \cdot \left(\frac{1}{S} - \frac{1}{S + R/L} \right) + I_1(0) \cdot \left(\frac{1}{S + R/L} \right)$$

Taking the inverse Laplace transform, we can obtain the time domain equation for $i_1(t)$

$$i_1(t) = \frac{V_s}{R} \cdot (1 - e^{-t \cdot (R/L)}) + I_1(0) \cdot e^{-t \cdot (R/L)}$$

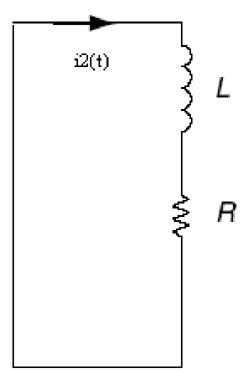
Replacing with the values yields:

$$i_1(t) = 3.72 \cdot (1 - e^{-t}) + e^{-t}$$

The peak to peak current ripple is the value for $i_1(t)$ calculated at end of mode 1 minus the initial value $i_1(0) = I_0$. Therefore,

$$I_{1pp} = 3.72 \cdot \left(1 - e^{-1}\right) + e^{-1} - 1 \Rightarrow I_{1pp} = 1.719A$$

Mode 2: S_1 off, S_2 on, $1 \le t < 2$. The equivalent circuit for mode 2 is



From the Kirchoff's voltage law:

$$0 = R \cdot i_2(t) + L \cdot \frac{di_2(t)}{dt}$$

Using Laplace transform:

$$0 = R \cdot I_2(S) + L \cdot [S \cdot I_2(S) - I_2(0)]$$

Grouping terms:

$$I_2(S) \cdot (R + S \cdot L) = I_2(0) \cdot L$$

Dividing both sides by R:

$$I_2(S)\cdot[1+S\cdot(L/R)] = I_2(0)\cdot(L/R)$$

Solving for $I_2(S)$:

$$I_2(S) = I_2(0)\cdot\frac{(L/R)}{1+S\cdot(L/R)}$$

$$I_2(S) = I_2(0)\cdot\frac{1}{S+(R/L)}$$

Using the inverse Laplace transform:

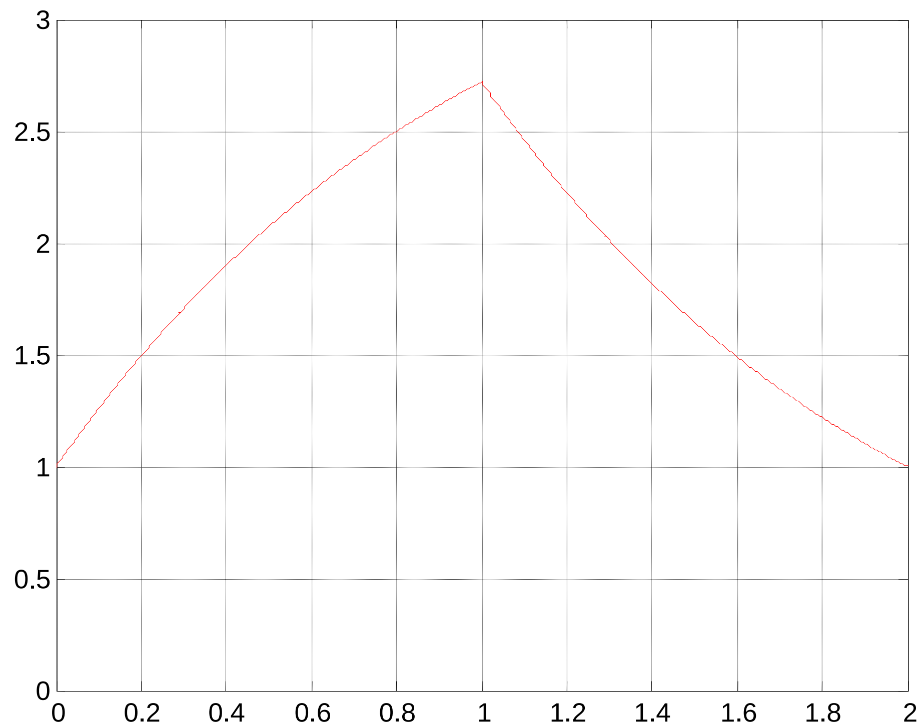
$$i_2(t) = i_2(0)\cdot e^{-t(R/L)}$$

The initial current for $i_2(t)$ is $i_1(1)$. Calculating from the above equation for t=2 gives

$$i_2(2) = i_1(1)\cdot e^{-1} \Rightarrow i_2(2) = 1.719A\cdot e^{-1} = 1A$$

Which coincides with the current flowing through the inductor at the beginning of the switching cycle.

The waveforms for the current flowing through the inductor and for the voltage across the resistor are shown in the figure below. Note that the voltage drop across the resistor is $V_R(t) = i_L(t).R$. For $R = 1\,\Omega$, it is the same as the current waveform.

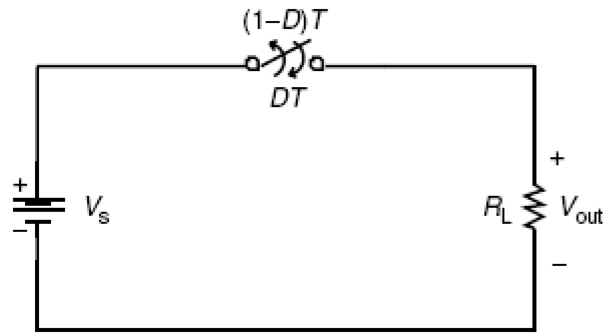


Matlab script to generate the waveforms:

```
clear variables
clc
R=1       %Resistance
L=1       %Inductance
Vs=3.72   %Source Voltage
Vemf=0    %EMF Voltage
D=0.5     %Duty Cycle
Io=1      %Initial Current
T=2       %Switching Period
%Mode 1
t1=0:0.001:D*T
i1=((Vs-Vemf)/R)*(1-exp(-t1*(R/L)))+Io*exp(-t1*(R/L))
%Initial current for mode 2
I2=((Vs-Vemf)/R)*(1-exp(-(D*T)*(R/L)))+Io*exp(-(D*T)*(R/L))
%Mode 2
t2=D*T:0.001:T
i2=I2*exp(-(t2-(D*T))*(R/L))
plot(t1,i1,'r',t2,i2,'r')
axis([0 T 0 3])
grid on
```

Problem 3

The fundamental switching converter of Figure 1.5 has a resistive load of 10 Ω and an input voltage of 48 V. The switching frequency is 1 kHz with a duty cycle of 50%. The semiconductor switch has a voltage drop of 1V in its on-state.
(a) Determine the average output voltage and the rms output voltage of this switching converter.
(b) Using Fourier series, express the output voltage in terms of their fundamental component and the next two higher harmonics. Find the rms value of the fundamental component of the output voltage.



$$V_S = 48V$$
$$R_L = 10\Omega$$
$$f_S = 1kHz$$
$$D = 0.5$$
$$V_{SW_{on}} = 1V$$

(a) From equation 1.12, the average output voltage is

$$V_a = \frac{1}{T}\int_0^{DT} (V_s - V_{SW_{on}})dt = D\cdot(V_s - V_{SW_{on}}) \Rightarrow V_a = 23.5V$$

From Equation 1.14, the rms value for the output voltage is

$$V_{out,rms} = \sqrt{\frac{1}{T}\int_0^{DT} \left(V_s - V_{SW_{on}}\right)^2 dt} = \left(V_s - V_{SW_{on}}\right)\cdot\sqrt{D} \Rightarrow V_{out,rms} = 33.234V$$

(b) The output voltage expression using Fourier series up to the third harmonic may be derived from Equation1.21 because D=0.5. V_{out} is an odd function given by

$$V_{out}(t) = \left(V_s - V_{SW_{on}}\right)\left\{D + \left[\frac{(1-\cos\pi)}{\pi}\right]\sin\left(2\pi f_s t\right) + \left[\frac{(1-\cos 3\pi)}{3\pi}\right]\sin\left(6\pi f_s t\right) + \left[\frac{(1-\cos 5\pi)}{5\pi}\right]\sin\left(10\pi f_s t\right)\right\}$$

The rms value of the fundamental component of the output voltage is calculated as