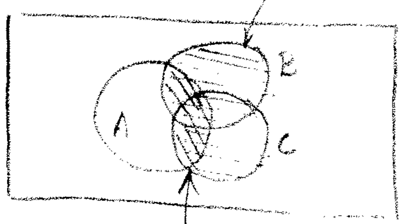
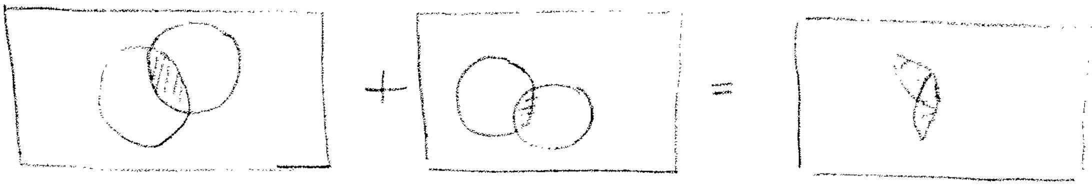
2.1

B+C

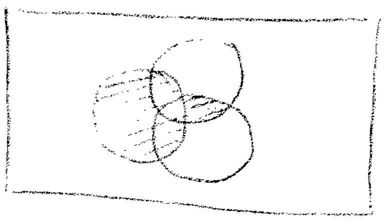


A · (B+C)

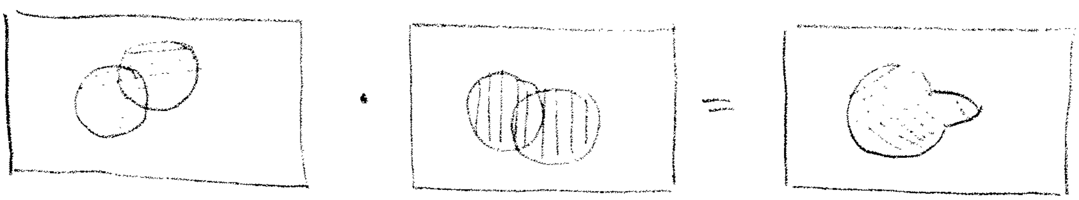


A · B   A · C   AB + AC

(b)



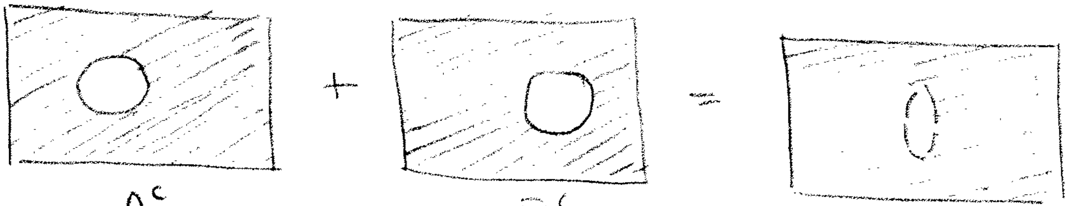
A + BC



A + B   A + C   (A+B)(B+C)

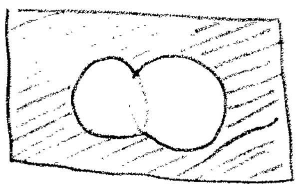
(c)

AB $\qquad$ $(AB)^c$



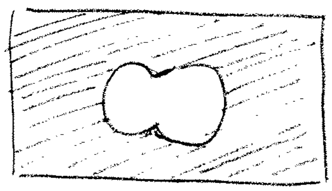
A^c $\qquad$ B^c $\qquad$ $A^c + B^c$

(d)



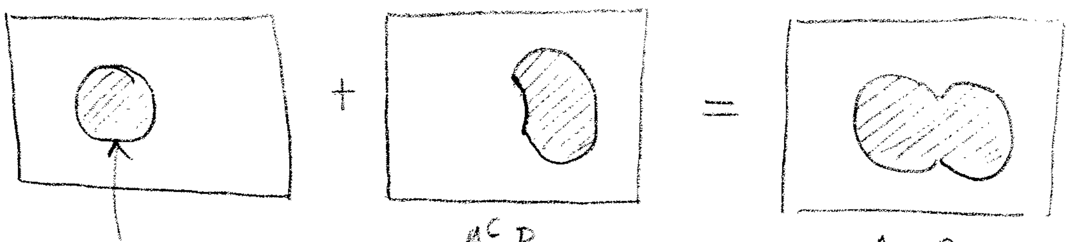
$(A+B)^c$

see above for diagrams of A^c and B^c $\Rightarrow$



$A^c \cdot B^c$

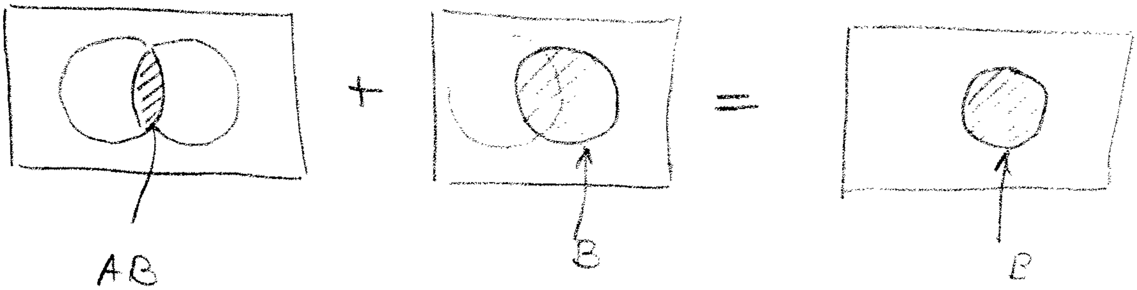
(e)



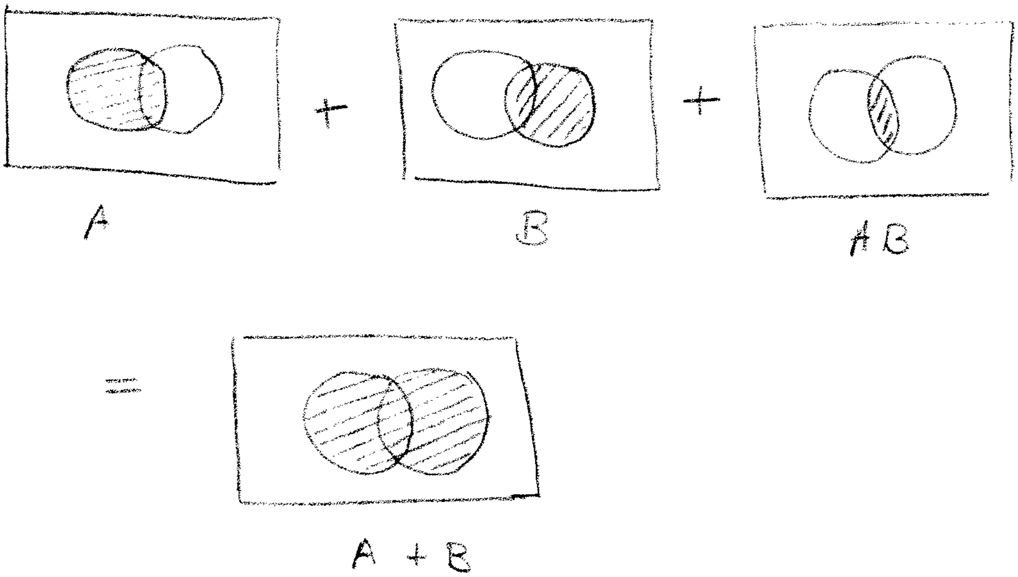
A $\qquad$ $A^c B$ $\qquad$ $A + B$

2.1 cont'd (2)

(f)



(g)



$\boxed{2.2}$

$$S = \{a_1, a_2, a_3, a_4, a_5, a_6\}$$

$$A_1 = \{a_1, a_2, a_4\}, \quad A_2 = \{a_2, a_3, a_6\}, \quad A_3 = \{a_1, a_3, a_5\}$$

(a)

(i) $A_1 + A_2 = \{a_1, a_2, a_3, a_4, a_6\}$

(ii) $A_1 A_2 = \{a_2\}$

(iii) $A_3^c = \{a_2, a_4, a_6\}$

$A_1 + A_3^c = \{a_1, a_2, a_4, a_6\}$

$(A_1 + A_3^c) A_2 = \{a_2, a_6\}$

(b)(i) $A_2 + A_3 = \{a_1, a_2, a_3, a_5, a_6\}$

$A_1(A_2 + A_3) = \{a_1, a_2\}$ (1)

$A_1 A_2 = \{a_2\}, \quad A_1 A_3 = \{a_1\}$

$A_1 A_2 + A_1 A_3 = \{a_1, a_2\}$ (2)

$\therefore$ (1) = (2)

$\boxed{2.2}$ Cont'd

(ii) $A_1 + A_2 A_3 = (A_1 + A_2)(A_1 + A_3)$

$A_2 A_3 = \{a_3\}$

$A_1 + A_2 A_3 = \{a_1, a_2, a_3, a_4\}$ ①

$A_1 + A_2 = \{a_1, a_2, a_3, a_4, a_6\}$

$A_1 + A_3 = \{a_1, a_2, a_3, a_4, a_5\}$

$(A_1 + A_2)(A_1 + A_3) = \{a_1, a_2, a_3, a_4\}$ ②

$\therefore$ ① = ②

(iii)

$A_1 + A_2 = \{a_1, a_2, a_3, a_4, a_6\}$

$(A_1 + A_2)^c = \{a_5\}$ ①

$A_1^c = \{a_3, a_5, a_6\}$, $A_2^c = \{a_1, a_4, a_5\}$

$A_1^c A_2^c = \{a_5\}$ ②

$\therefore$ ① = ②

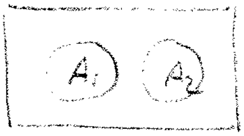
2.3

Axioms

(I) $\Pr[A] \geq 0$ (area always ≥ 0)

(II) $\Pr[S] = 1$ we normalize the area of the Venn diagram so $\Pr[S] = 1$

(III)



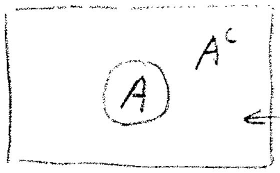
(no intersection)

The area of the union of two events is the sum of the areas

(IV) cannot be shown by Venn diagram

Corollaries (Table 2.4)

$\Pr[A^c] = 1 - \Pr[A]$

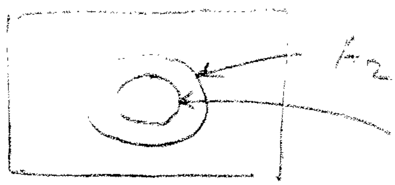


S has area 1

$0 \leq \Pr[A] \leq 1$

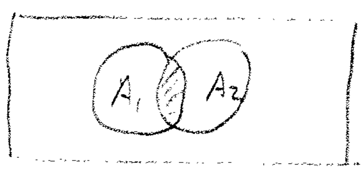
Area of A always ≥ 0
$A \subseteq S$ $\therefore$ area $\leq$ that of S.

If $A_1 \subseteq A_2$
then $\Pr[A_1] \leq \Pr[A_2]$



$A_1A_2 = \phi \Rightarrow \Pr[A_1A_2] = 0$ Null set has area zero.

$$\Pr[A_1 + A_2] = \Pr[A_1] + \Pr[A_2] - \Pr[A_1A_2]$$



Intersection is included twice if we sum the areas.

$\boxed{2.4}$

$$\Pr[A_1 + A_2 + A_3] = \Pr[A_1 + (A_2 + A_3]$$

$$= \Pr[A_1] + \Pr[A_2 + A_3] - \Pr[A_1(A_2 + A_3)]$$

$$= \Pr[A_1] + \Pr[A_2] + \Pr[A_3] - \Pr[A_2 A_3]$$

$$- \Pr\{A_1 A_2 + A_1 A_3\}$$

$$= \Pr[A_1] + \Pr[A_2] + \Pr[A_3] - \Pr[A_2 A_3]$$

$$-\left(\Pr[A_1 A_2] + \Pr[A_1 A_3] - \Pr[A_1 A_2 A_1 A_3]\right)$$

$$= \Pr[A_1] + \Pr[A_2] + \Pr(A_3] - \Pr[A_1 A_2] - \Pr\{A_2 A_3]$$

$$- \Pr\{A_1 A_3] + \Pr[A_1 A_2 A_3]$$

$\boxed{2.5}$

(ii) ~~(i)~~ $Pr[A_1 + A_2] = Pr[A_1] + Pr[A_2] - Pr[A_1 A_2]$

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{6} = \frac{5}{6}$

(i) ~~(ii)~~ $Pr[A_1 A_2] = \frac{1}{6}$

(iii) ~~(iii)~~ $Pr[(A_1 + A_3^c) A_2] = Pr[\{a_2, a_6\}]$

$= \frac{2}{6}$

2.6

(a) The sample space has 8 outcomes

000 001 010 011 100 101 110 111

$S = \{0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7\}$

(b) All outcomes are equally likely.

$A_1 = \{s > 5\} = \{6, 7\}$

$Pr[A_1] = \frac{2}{8} = 1/4$

(c)

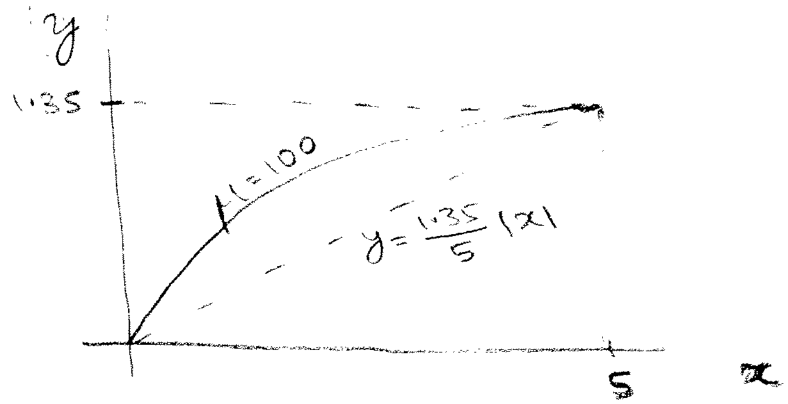
$A_2 = \{3 \leq s \leq 6\} = \{3, 4, 5, 6\}$

$Pr[A_2] = \frac{4}{8} = 1/2$

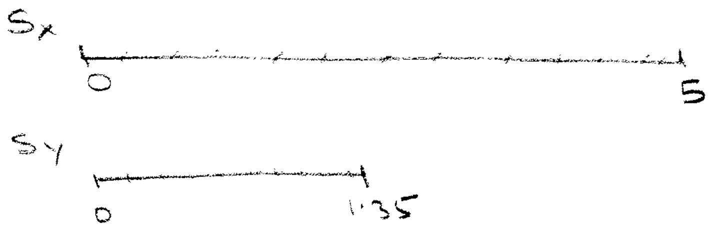
2.7

$$y = \frac{\log_e(1 + 100|x|)}{\log_e(1+100)} \quad \underline{\text{for } \mu = 100}$$

Input-output transfer characteristic



(a) Sample spaces



(b) $A_1 = \{ y < 1 \}$

$$\Pr[Y<1] = \Pr[X<1] = \frac{1}{5}$$

(c) $$\Pr\left[\tfrac{1}{2} < X \le 1\right] = \frac{1/2}{5} = 1/10$$

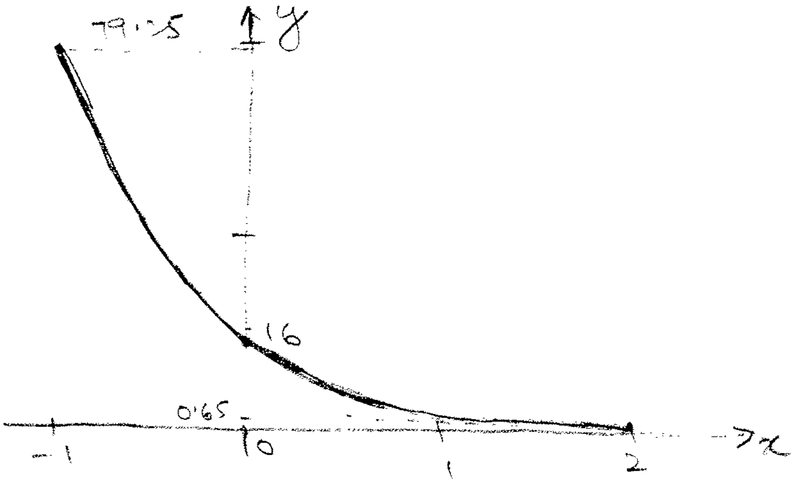
$$\Pr\left[\tfrac{1}{2} < Y \le 1\right] = \Pr[0.0905 < X \le 1] = \frac{0.9095}{5}$$
$$= 0.1819$$

(d) $$\Pr[1 < Y \le 1.25] = \Pr[1 < X \le 3.1919]$$
$$= \frac{2.1919}{5} = 0.4384$$

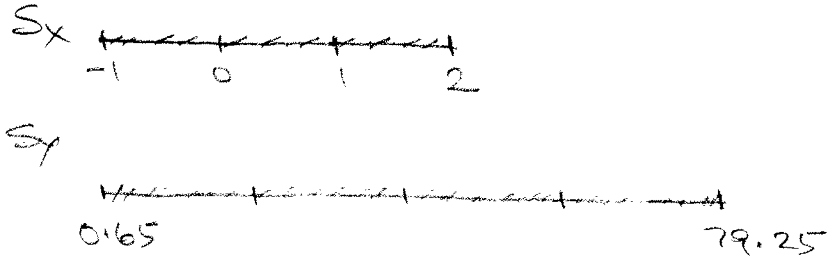
$\boxed{2.8}$

$$y = 16\,e^{-1.6x},$$

$$-1 \le x \le 2$$



(a) sample spaces (not to scale)



(b)

$$\Pr[Y \le 16] = \Pr[X > 0]$$
$$= \frac{2}{3}$$

$$\Pr[2 \le Y < 20] = \Pr[-0.1395 < X \le 1.2997]$$

$$= 0.4797$$

$\boxed{2.9}$

(a) $Pr[A+B] = Pr[A] + Pr[B] = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

(b)
$$Pr[A+B] = Pr[A] + Pr[B] - Pr[A] \cdot Pr[B]$$
$$= \frac{1}{4} + \frac{1}{3} - \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$$

(c) No. If it were possible then

$$P[A+B+C+D] = Pr[A] + Pr[B] + Pr[C] + Pr[D] > 1$$

$\therefore$ This is not possible.

2.10

Twelve-sided unfair die.

Events

$A = \{\text{odd numbered side}\}$

$B = \{4, 5, 6, 7, 8\}$

$Pr[1] = Pr[3] = Pr[5] = Pr[7] = Pr[9] = Pr[11]$

$Pr[2] = Pr[4] = Pr[6] = Pr[8] = Pr[10] = Pr[12]$

(a) $Pr[2] = 2\,Pr[1]$

$Pr[A] = \frac{6}{18} = 1/3$

(b) $Pr[B] = Pr[\{3 \text{ even}, 2 \text{ odd}\}]$

$= \frac{8}{18} = 4/9$

(c) $\{AB\} = \{1, 3, 5, 7, 9, 11\} \cap \{4, 5, 6, 7, 8\}$

$= \{5, 7\}$

$Pr[AB] = Pr[\{2 \text{ odd}\}]$

$= \frac{2}{18} = 1/9$

$\boxed{2.11}$

The sample space of the outcomes from this experiment:

$$S_I = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$$

$I \triangleq$ face value of 8-sided die − face value of 6-sided die

For different values of I, the number of arrangements are as follows:

−5	1−6						1
−4	1−5,	2−6					2
−3	1−4,	2−5,	3−6				3
−2	1−3,	2−4,	3−5,	4−6			4
−1	1−2,	2−3,	3−4,	4−5,	5−6		5
0	1−1,	2−2,	3−3,	4−4,	5-5,	6−6	6
1	2−1,	3−2,	4-3,	5−4,	6−5,	7−6	6
2	3−1,	4−2,	5−3,	6−4,	7−5,	8-6	6
3	4−1,	5−2,	6−3,	7−4,	8−5		5
4	5−1,	6−2,	7−3,	8−3			4
5	6−1,	7−2,	8−3				3
6	7−1,	8-2					2
7	8−1						1
						Total	48

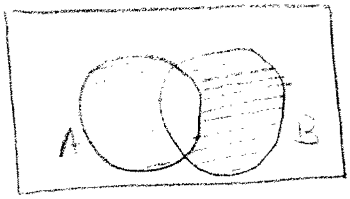
$$\Pr[\{I=-1\} \cup \{I=5\}] = \frac{5+3}{48}$$

$$= 1/6$$

2.12

(a) A = "hard drive fails" B = "memory fails"

We want the probability of the following event:



our event is:

$AB^c + A^cB$

$$A = A \cdot B + A \cdot B^c$$

$$Pr[A] = Pr[AB] + Pr[AB^c]$$

$$0.3 = 0.1 + Pr[AB^c] \Rightarrow Pr[AB^c] = 0.2$$

likewise

$$Pr[B] = Pr[AB] + Pr[A^cB]$$

$$0.2 = 0.1 + Pr[A^cB] \Rightarrow Pr[A^cB] = 0.1$$

Finally

$$Pr[AB^c + A^cB] = Pr[AB^c] + Pr[A^cB] = 0.2 + 0.1 = 0.3$$

(b) "No failures" $= A^cB^c$

Note that

$$A^cB^c = (A+B)^c$$

$$Pr[A+B] = Pr[A] + Pr[B] - Pr[AB]$$

$$= 0.3 + 0.2 - 0.1 = 0.4$$

$$Pr[\text{No failures}] = Pr[(A+B)^c] = 1 - Pr[A+B]$$

$$= 1 - 0.4 = 0.6$$

2-13

(a) $N = 26 \cdot 25 \cdot 24 \cdot 23$

$= 388,800$

$$\text{Prob} = \frac{1}{388,800} = 2.787 \times 10^{-6}$$

(b)

$$\Pr[\text{'S' in } 10^{th} \text{ position}] = \frac{1}{26}$$

$= 0.038$

$\boxed{2.14}$

(a) Pr[one or more good]

$$= \Pr[A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8]$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{7}{8}$$

Alternately,

$$\Pr[\text{one or more good}] = 1 - \Pr[\text{all bad}]$$

$$= 1 - \Pr[A_1] = 1 - \frac{1}{8} = \frac{7}{8}$$

(b) In this case it is easiest to use the alternate approach above;

$$\Pr[\text{one or more good}] = 1 - \Pr[A_1]$$

However, in this case the probability of a bad disk is $\frac{3}{8}$, so

$$\Pr[A_1] = \left(\frac{3}{8}\right)^3 = \frac{27}{512}$$

$$\Pr[\text{one or more good}] = 1 - \frac{27}{512} = \frac{485}{512}$$

$$\simeq 0.947$$

2.15

Draw the sample space and find the probabilities of each elementary event:

event	prob	
GGG	$(0.8)^3 =$	0.512
GGB	$(0.8)^2(0.2) =$	0.128
GBG	$(0.8)^2(0.2) =$	0.128
GBB	$(0.8)(0.2)^2 =$	0.032
BGG	$(0.8)^2(0.2) =$	0.128
BGB	$(0.8)(0.2)^2 =$	0.032
BBG	$(0.8)(0.2)^2 =$	0.032
BBB	$(0.2)^3 =$	0.008

(a) Event is: GGG probability = 0.512

(b) Event is: BBB probability = 0.008

(c) Event is:

GGB	GBG	BGG
0.128	0.128	0.128

probability = 0.128 + 0.128 + 0.128 = 0.384

(d) Event is:

GGB	GBG	BGG	GGG
0.128	0.128	0.128	0.512

probability = 0.128 + 0.128 + 0.128 + 0.512 = 0.896

$\boxed{2\text{-}15}$ (Alternate soln)

(a) $Pr\{GGG\} = \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{64}{125} = 0.512$

(b) $Pr[BBB] = \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{125} = 0.008$

(c) $Pr[GGB + GBG + BGG]$

$$= 3\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) = \frac{48}{125} = 0.384$$

(d) $Pr[GGB + GBG + BGG + GGG]$

$$= 3\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^3 = \frac{48}{125} + \frac{64}{125}$$

$$= \frac{112}{125} = 0.896$$

2-16

(a)

$$16\cdot16\cdot16\cdot16 = 16^4 = 65,536$$

(b)

$$16\cdot15\cdot14\cdot13 = \frac{16!}{12!} = 43,680$$

2.17

(a)

$$\binom{8}{2} = \frac{8!}{2!\,6!} = 28$$

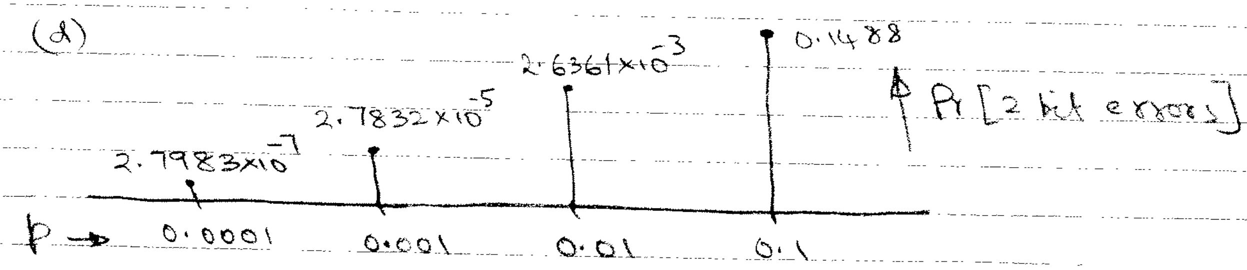
(b)

$$(1-p)\,p\,(1-p)(1-p)(1-p)\,p\,(1-p)(1-p) = p^2(1-p)^6$$

(c)

$$\Pr[\text{2 bit errors}] = \binom{8}{2} p^2 (1-p)^6 = 28\,p^2(1-p)^6$$

(d)



2-18

(a) $\Pr[\text{Retrans. in 3rd packet}] = (1-p)^2 p = (1-0.01)^2 (0.01) = 9.801 \times 1$

(b) $\Pr[\text{Retrans in } 10^{th} \text{ packet}] = (1-p)^9 p = 9.135 \times 10^{-3}$

(c)

$\Pr[\text{Retrans within the first 5 packets}]$

$$= \sum_{k=0}^{4} (1-p)^k p = (1-p)^0 p + (1-p)^1 p + (1-p)^2 p + (1-p)^3 p + (1-p)^4 p$$

$$= 0.049$$

(d)

$$\sum_{k=0}^{N-1} (1-p)^k p \geq 0.1$$

$$p \cdot \frac{1-(1-p)^N}{1-(1-p)} = 1-(1-p)^N \geq 0.1$$

$$-(1-p)^N \geq -0.9 \quad \text{or} \quad (1-p)^N \leq 0.9$$

$$N \log_{10} 0.99 \leq \log_{10} 0.9$$

$$-N(4.3648) \leq -0.04576$$

$$\text{or} \quad N \geq 10.4833 \quad \text{or} \quad \underline{\underline{N > 10}}$$

2-19:

(a)

$$9Q(1-Q)^8$$

(b) $\Pr[\text{Errors and not detected}] = \sum_{k \text{ even}} \Pr[k \text{ errors}]$

$$= \sum_{k \text{ even}} \binom{9}{k} Q^k (1-Q)^{9-k}$$

$\boxed{2.20}$

$$F = ACD + ACE + BCE + BCD$$

$$= C\,(AD + AE + BE + BD)$$

$$= C \cdot (A+B) \cdot (D+E)$$

$\boxed{2.21}$

$$F = AD + BE + ACE + BCD$$

$$= A\,(D + CE) + B\,(E + CD)$$

Another possible answer is as follows:

$$F^c = A^cB^c + A^cC^cE^c + D^cE^c + D^cB^cC^c$$

$$= A^c\,(B^c + C^cE^c) + D^c\,(E^c + B^cC^c)$$

$$F = \big(A + B \cdot (C+E)\big) \cdot \big(D + E \cdot (B+C)\big)$$

$\boxed{2.22}$

$$F = C \cdot (A+B) \cdot (D+E)$$

$$Pr[F] = Pr[C] \cdot Pr[A+B] \cdot Pr[D+E]$$

$$Pr[A+B] = Pr[A] + Pr[B] - Pr[AB]$$
$$= Pr[A] + Pr[B] - Pr[A] \cdot Pr[B]$$

$$Pr[A+B] = 0.1 + 0.1 - (0.1)(0.1) = 0.19$$

likewise $Pr[D+E] = 0.19$

$$Pr[F] = (0.1)(0.19)(0.19) = 0.00361$$

An alternate procedure

Go to the sample space and identify all the elementary events that result in failure:

$\boxed{2.23}$

		F	F^c
$A_1 A_2 A_3 A_4$	p^4	✓	
$A_1 A_2 A_3 A_4^c$	$p^3(1-p)$	✓	
$A_1 A_2 A_3^c A_4$	$p^3(1-p)$	✓	
$A_1 A_2 A_3^c A_4^c$	$p^2(1-p)^2$		✓
$A_1 A_2^c A_3 A_4$	$p^3(1-p)$	✓	
$A_1 A_2^c A_3 A_4^c$	$p^2(1-p)^2$	✓	
$A_1 A_2^c A_3^c A_4$	$p^2(1-p)^2$	✓	
$A_1 A_2^c A_3^c A_4^c$	$p(1-p)^3$		✓
$A_1^c A_2 A_3 A_4$	$p^3(1-p)$	✓	
$A_1^c A_2 A_3 A_4^c$	$p^2(1-p)^2$	✓	
$A_1^c A_2 A_3^c A_4$	$p^2(1-p)^2$	✓	
$A_1^c A_2 A_3^c A_4^c$	$p(1-p)^3$		✓
$A_1^c A_2^c A_3 A_4$	$p^2(1-p)^2$	✓	
$A_1^c A_2^c A_3 A_4^c$	$p(1-p)^3$		✓
$A_1^c A_2^c A_3^c A_4$	$p(1-p)^3$	✓	
$A_1^c A_2^c A_3^c A_4^c$	$(1-p)^4$		✓

Then:

$$\begin{aligned}
\Pr[F] = {} & p^4 + (p^3 - p^4) + (p^3 - p^4) \\
& + (p^3 - p^4) + (p^2 - 2p^3 + p^4) \\
& + (p^2 - 2p^3 + p^4) + (p^3 - p^4) \\
& + (p^2 - 2p^3 + p^4) + (p^2 - 2p^3 + p^4) \\
& + (p^2 - 2p^3 + p^4) + (p - 3p^2 + 3p^3 - p^4)
\end{aligned}$$

$$= p + 2p^2 - 3p^3 + p^4$$

$\boxed{2.23}$ con'd.

A slightly less tedius method is to compute

$$\Pr[F^c] = (p^2 - 2p^3 + p^4) + (p - 3p^2 + 3p^3 - p^4)$$
$$+ (p - 3p^2 + 3p^3 - p^4) + (p - 3p^2 + 3p^3 - p^4)$$
$$+ (1 - 4p + 6p^2 - 4p^3 + p^4)$$

$$= 1 - p - 2p^2 + 3p^3 - p^4$$

then

$$\Pr[F] = 1 - \Pr[F^c] = p + 2p^2 - 3p^3 + p^4$$

$\boxed{2.24}$

(a) $F = A \cdot (B + C) = AB + AC$

(b) $F = A + BC$

(c) <u>1st network</u> *

$P_1 = Pr[F] = Pr[AB + AC] = Pr[AB] + Pr[AC] - Pr[ABC]$

$= 2p^2 - p^3$

<u>2nd network</u>

$P_2 = Pr[F] = Pr[A + BC] = Pr[A] + Pr[BC] - Pr[ABC]$

$= p + p^2 - p^3$

Comparing the expressions:

$$2p^2 - p^3 = p^2 + p^2 - p^3 < p + p^2 - p^3$$

so $P_1 < P_2$ $\quad (P_1 = 0.019 \ , \ P_2 = 0.109)$

1st net has lower probability

---

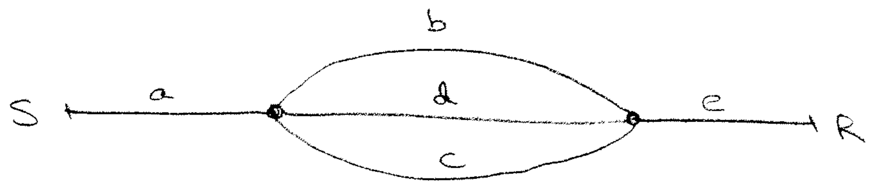
* Alternate computation for 1st net

$Pr[F] = Pr[A \cdot (B+C)] = Pr[A] \cdot Pr[B+C]$

$= Pr[A]Pr[B] + Pr[A] \cdot Pr[C] - Pr[A]Pr[BC]$

$= p^2 + p^2 - p^3$

2-25



Let A denote the event "Link a is OK," etc.

(a) The failure expression

$$F = A + BCD + E$$

(b)

$$\Pr[F] = \Pr[A+E] + \Pr[BCD] - \Pr[(A+E)BCD]$$

$$= p^3 + \Pr[A] + \Pr[E] - \Pr[AE] - \Pr[ABCD + BCDE]$$

$$= p^3 + p + p - p^2 - \Pr[ABCD] - \Pr[BCDE] + \Pr[ABCDE]$$

$$= 2p - p^2 + p^3 - 2p^4 + p^5$$

$$\Pr[\text{Path Established}] = \Pr[F^c] = 1 - \Pr[F]$$

$$\Pr[F^c] = 1 - 2p + p^2 - p^3 + 2p^4 - p^5$$

$$= 0.980099 0199$$

$\boxed{2.2b}$ (2.21)

(a)

$$\text{Comm} = \big(A \cap (C \cup D)\big) \cup B$$

(b)

$$\Pr[C \cup D] = \Pr[C] + \Pr[D] - \Pr[C] \cdot \Pr[D]$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4} = 0{,}75$$

(c)

Let's define the event $T = A \cap (C \cup D)$

then $\Pr[T] = \Pr[A] \cdot \Pr[C \cup D]$ (since independent)

$$= \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8} = .375$$

$$\text{Comm} = T \cup B$$

$$\Pr[\text{Comm}] = \Pr[T] + \Pr[B] - \Pr[T] \cdot \Pr[B]$$

$$= \frac{3}{8} + \frac{1}{2} - \frac{3}{8} \cdot \frac{1}{2} = \frac{11}{16} = 0.6875$$

2.2b cont'd. (1)

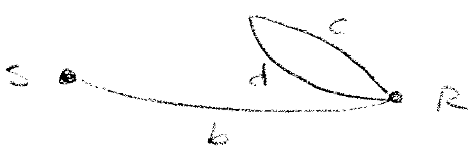
(d)

Let F be the event "S fails to communicate with R"

From part (c) we know that $Pr[F] = 1 - \frac{11}{16} = \frac{5}{16}$

We need to compute probability of failure of each link conditioned on this knowledge.

$$Pr[A^c|F] = \frac{Pr[F|A^c] \cdot Pr[A^c]}{Pr[F]}$$



Note that $Pr[F|A^c] = Pr[B^c] = \frac{1}{2}$

so $Pr[A^c|F] = \dfrac{\frac{1}{2} \cdot \frac{1}{2}}{5/16} = \dfrac{4}{5}$

$$Pr[B^c|F] = \frac{Pr[F|B^c] \cdot Pr[B^c]}{Pr[F]}$$

$Pr[F|B^c] = 1 - Pr[T] = \frac{5}{8}$

So $Pr[B^c|F] = \dfrac{\frac{5}{8} \cdot \frac{1}{2}}{5/16} = 1$

$$Pr[C^c|F] = \frac{Pr[F|C^c] \cdot Pr[C^c]}{Pr[F]}$$

$Pr[F|C^c] = Pr[(B^c \cap A^c) \cup (B^c \cap D^c)]$

$= Pr[B^c \cap A^c] + Pr[B^c \cap D^c] - Pr[B^c \cap A^c \cap C^c]$

$= \frac{1}{4} + \frac{1}{4} - \frac{1}{8} = \frac{3}{8}$

So $Pr[C^c|F] = \dfrac{\frac{3}{8} \cdot \frac{1}{2}}{5/16} = \dfrac{3}{5}$

$\boxed{2.2b}$ cont'd (2)

An identical calulation shows $\Pr[D^c \mid F] = \frac{3}{5}$

So the order in which the links should be tested is

b, a, c or d

Actually b does not <u>need</u> to be <u>tested</u> because given a failure of communication we <u>know</u> that b failed $\Pr[B^c \mid F] = 1$

2.27

From Example 2.7, the probability of failure is given by

$$\Pr[F] = p^4 - 3p^3 + 2p^2 + p$$

(a) Let $\Pr[F] < 0.001$, i.e.,

$$p^4 - 3p^3 + 2p^2 + p - 0.001 < 0$$

Solving for p using Matlab yields these roots:

$$1.7547\, e^{\pm j0.3261}, \quad -0.3254, \quad 0.0010$$

By ruling out the complex and negative values the desired p is

$$p < 0.001$$

(b) For $\Pr[F] < 0.05$, we solve

$$p^4 - 3p^3 + 2p^2 + p - 0.05 < 0$$

The roots of the polynomial using Matlab are

$$1.7449\, e^{\pm j0.3219}, \quad -0.3566, \quad 0.0460$$

The desired p is

$$p < 0.046$$

.28

(a) $X = \{$ failure to establish a connection between LA & Boston $\}$

$$X = (C+D)(A+B+E)$$

(b) $Y = \{$ Establish a Connection $\} = X^c$

$$\Pr[Y] = 1 - \Pr[X]$$

$$= 1 - \Pr[(C+D)(A+B+E)]$$

$$= 1 - \Pr[C+D]\ \Pr[A+B+E]$$

$$= 1 - \left(\Pr[C] + \Pr[D] - \Pr[CD]\right)$$
$$\left(\Pr[A] + \Pr[B+E] - \Pr[A(B+E)]\right)$$

$$= 1 - (p + p - p^2)\left(p + \Pr[B] + \Pr[E] - \Pr[BE]\right.$$
$$\left. - \Pr[A]\Pr[B+E]\right)$$

$$= 1 - (p + p - p^2)\left(p + p + p - p^2 - p(p + p - p^2)\right)$$

$$= 1 - (2p - p^2)(3p - p^2 - 2p^2 + p^3)$$

$$= 1 - (2p - p^2)(3p - 3p^2 + p^3)$$

(c) $p = 0.01$, $\quad P[Y] = 0.9994$

2.29

(a) $A = \{\text{first bit is a } 1\} = \{4, 5, 6, 7\}$

$B = \{\text{value} > 5\} = \{6, 7\}$

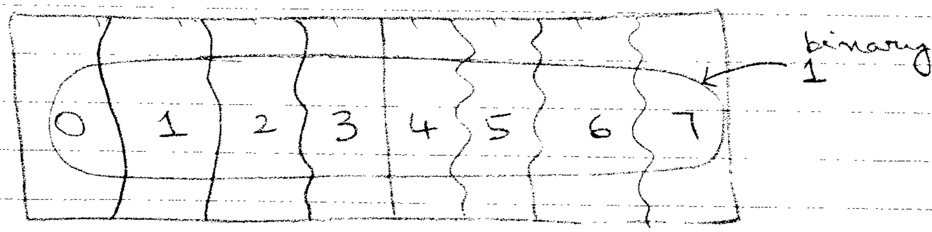
$$Pr[B/A] = \frac{Pr[AB]}{Pr[A]} = \frac{Pr[\{6,7\}]}{Pr[\{4,5,6,7\}]}$$

$$= \frac{\frac{1}{16} + \frac{1}{16}}{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}} = \frac{1/8}{1/2} = 1/4$$

(b)
$$Pr[A|B] = \frac{Pr[B|A]\, Pr[A]}{Pr[B]} \quad \text{by Bayes' rule}$$

$$= \frac{\frac{1}{4} \cdot \frac{1}{2}}{\frac{1}{16} + \frac{1}{16}} = \frac{1/8}{1/8} = 1$$

(c) To determine the prob. of a binary 1 or 1_2, consider that the three bits per level have equal probabilities



$$Pr[1] = Pr[1/0_D]\, P[0_D] + Pr[1|1_D]\, Pr[1_D] + Pr[1|2]\, Pr[2]$$
$$+ Pr[1|3]\, Pr[3] + Pr[1|4]\, Pr[4] + Pr[1|5]\, Pr[5]$$
$$+ Pr[1|6]\, Pr[6] + Pr[1|7]\, Pr[7]$$
$$= 0.5$$

(d) Similarly,

$$Pr[0] = 0.5$$

or

$$Pr[0] = 1 - Pr[1] = 0.5$$

2.30

M = memory failure

D = hard disk failure

(a)

$$Pr[M] \cdot Pr[D] = (0.01)(0.02) = 0.0002$$

$$Pr[MD] = 0.0014$$

$\therefore$ Not independent

(b) $$Pr[M|D] = \frac{Pr[MD]}{Pr[D]} = \frac{0.0014}{0.02} = .07$$

2.31

(a) $$Pr[M] \cdot Pr[D] = (0.02)(0.015) = 0.0003$$

$$Pr[MD] = 0.0003 \quad \therefore \text{independent}$$

(b) $$Pr[M|D] = \frac{Pr[MD]}{Pr[D]} = \frac{0.0003}{0.015} = 0.02$$

2-32

M = Memory failure

D = Hard disk failure

(a) $Pr[M] \cdot Pr[D] = (0.02)(0.015)$

$= 0.0003$ Yes

(b)

$$Pr[M|D] = \frac{Pr[MD]}{Pr[D]}$$

$$= \frac{0.0003}{0.015}$$

$$= 0.015$$

or

$Pr[M|D] = Pr[M]$ since M & D are independent

$\boxed{2.33}$ Let $p = Pr[\text{1st bit error}]$

then $Pr[\text{2nd bit error} \mid \text{1st bit error}] = 2p$

$Pr[\text{two consecutive errors}] = Pr[\text{1st bit error}, \text{2nd bit error}]$

$= Pr[\text{2nd bit error} \mid \text{1st bit error}] \cdot Pr[\text{1st bit error}$

$0.0002 = 2p \cdot p = 2p^2$

$\Rightarrow p = 0.01$

$\boxed{2.34}$ Let H = hardware failure

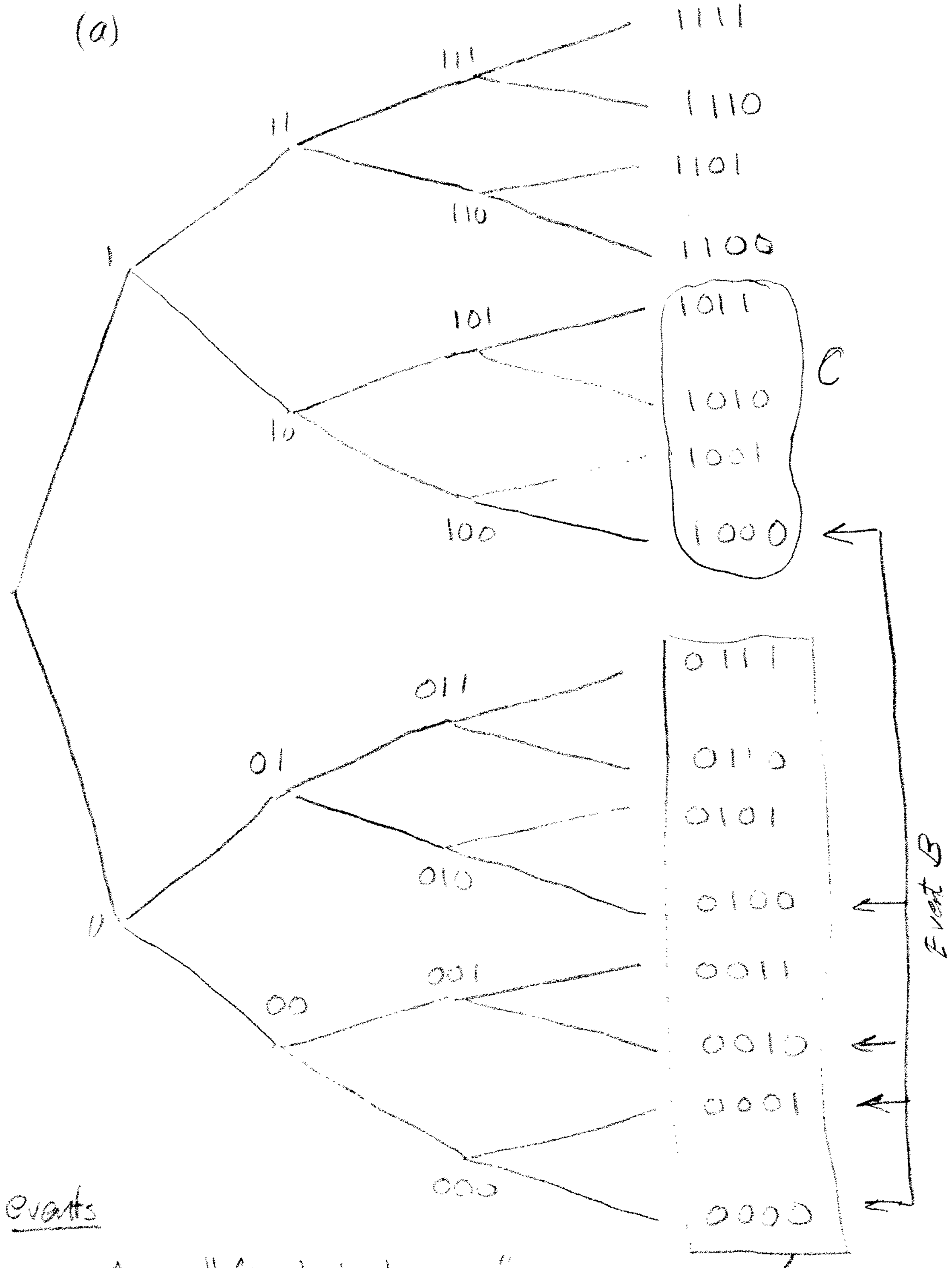
S = software failure

$$Pr[H \mid S] = \frac{Pr[S/H] \cdot Pr[H]}{P[S]}$$

$$= \frac{(0.02)(.001)}{0.005} = 0.004$$

2.35

(a)



events

A = "first bit zero"

B = "more zeros than ones"

C = "first two bits are 10"

2.35 cont'd.

(b) $$\Pr[B|A] = \frac{\Pr[B \cdot A]}{\Pr[A]} = \frac{4/16}{8/16} = \frac{1}{2}$$

(c) $$P[B|C] = \frac{\Pr[B \cdot C]}{\Pr[C]} = \frac{1/16}{4/16} = \frac{1}{4}$$

(d) $$\Pr[A|B] = \frac{\Pr[A \cdot B]}{\Pr[B]} = \frac{4/16}{5/16} = \frac{4}{5}$$

2.36

L event Beatle is late

B jeep breaks down

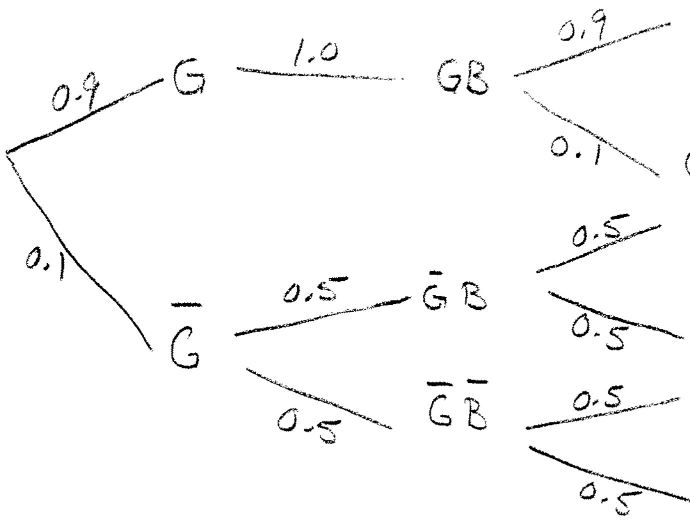
$$\Pr[L] = \Pr[L|B]\Pr[B] + \Pr[L|B^c]\Pr[B^c]$$

$$= (.9)(.4) + (.2)(.6)$$

$$= .36 + .12 = 0.48$$

2.37

(a)



	Prob.	Mission Fails	Mission Succeeds
GBF	0.81		✓
$GB\bar{F}$	0.09		✓
$\bar{G}BF$	0.025		✓
$\bar{G}B\bar{F}$	0.025	✓	
$\bar{G}\bar{B}F$	0.025	✓	
$\bar{G}\bar{B}\bar{F}$	0.025	✓	

b) (i) 0.075

(ii) 0.025

c) Pr[more than one system failed] = Pr[mission failed] = 0.075

(i) $0/0.75 = 0$

(ii) $0.050 / 0.075 = 2/3$

(iii) $0.050/0.075 = 2/3$

d) Pr{beer cooler failed} = 0.05

Pr{mission suceeded | beer cooler failed} = $0/0.05 = 0$

$\boxed{2.38}$

$A = \{ \text{IE supports a given feature} \}$

$B = \{ \text{NN supports a given feature} \}$

$Pr[A] = 1 - \epsilon, \quad Pr[B] = 1 - \delta$

(a)

$$Pr[\text{Browser fails to access}] = \underbrace{Pr[\text{Fails} \mid IE]}_{Pr[A^c]} Pr[IE] + \underbrace{Pr[\text{Fails} \mid NN]}_{Pr[B^c]} Pr[NN]$$

$$= \epsilon \times \frac{1}{2} + \delta \times \frac{1}{2} = \frac{1}{2}(\epsilon + \delta)$$

(b)

$$Pr[IE \mid \text{fails to access}] = \frac{Pr[\text{fails to access} \mid IE]\; Pr[IE]}{Pr[\text{fails to access}]}$$

$$= \frac{\epsilon \times 1/2}{\frac{1}{2}(\epsilon + \delta)} = \frac{\epsilon}{\epsilon + \delta}$$

2-39

16 ICs: 3 Defective + 13 Good

Sample space: S = {Combinations of r = 2 chosen from n = 16}

(a) Event A = {At least 1 of 2 selected ICs is Good}

The # of outcomes in S is given by

$$N_S = \binom{16}{2} = \frac{16!}{2!\,14!} = 120$$

Consider the complement of A:

A^c = {None of 2 selected ICs are Good}

$$N_{A^c} = \binom{3}{2}\binom{13}{0} = \frac{3!\ 13!}{2!\,1!\ \ 13!} = 3$$

$$\Pr[A] = 1 - \Pr[A^c]$$

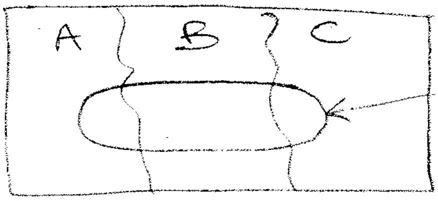
$$= 1 - \frac{3}{120} = \frac{117}{120} = 0.975$$

(b)

Let B = {Randomly Select an IC from the box}

$$\Pr[B] = \frac{13}{16} = 0.8125$$

2NB3



D ≜ {Selected component is defective}

A ≜ {Component from manufacture A}

(a) By the principle of total probability, we have

$$\Pr[D] = \Pr[DA] + \Pr[DB] + \Pr[DC]$$

$$= \Pr[D|A]\Pr[A] + \Pr[D|B]\Pr[B] + \Pr[D|C]\Pr[C]$$

$$= 0.10\left(\tfrac{1}{3}\right) + 0.05\left(\tfrac{1}{3}\right) + 0.20\left(\tfrac{1}{3}\right)$$

$$= 0.1167$$

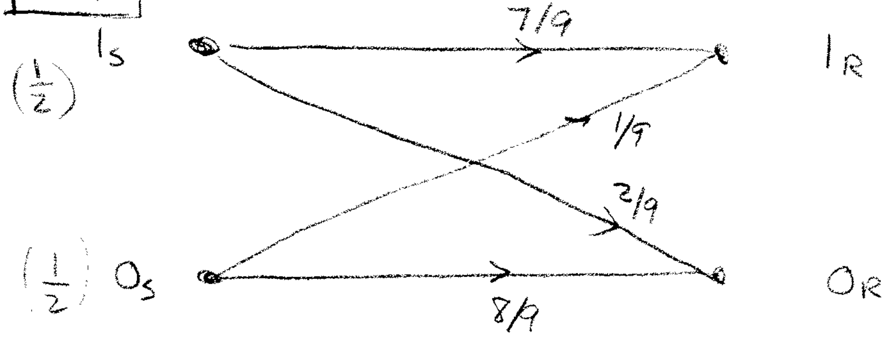
(b) By Bayes' rule, we have

$$\Pr[A|D] = \frac{\Pr[D|A]\Pr[A]}{\Pr[D]}$$

$$= \frac{(0.10)\ 1/3}{0.1167}$$

$$= \frac{2}{7} = 0.2857$$

$\boxed{2.40}$



(a) $Pr[1_R] = Pr[1_R | 1_s] \cdot Pr[1_s] + Pr[1_R|0_s]\, Pr[0_s]$

$$= \frac{7}{9} \cdot \frac{1}{2} + \frac{1}{9} \cdot \frac{1}{2} = \frac{8}{18} = \frac{4}{9}$$

(b) $Pr[\text{error}] = Pr[\text{error}| 1_s]\; Pr[1_s] + Pr[\text{error}| 0_s]\, Pr[0_s]$

$$= \frac{2}{9}\frac{1}{2} + \frac{1}{9}\frac{1}{2} = \frac{3}{18} = \frac{1}{6}$$

(c) $Pr[1_s | 1_R] = \dfrac{Pr[1_R | 1_s] \cdot Pr[1_s]}{Pr[1_R]}$

$$= \frac{\frac{7}{9} \cdot \frac{1}{2}}{4/9} = \frac{7}{8}$$

2.41

(a)

$$\Pr[1_R | 0_S] = 0.4$$

(b)

$$\Pr[1_R] = \Pr[1_R | 0_S]\Pr[0_S] + \Pr[1_R | 1_S]\Pr[1_S]$$

$$= (0.4)\tfrac{1}{3} + (0.75)\tfrac{2}{3}$$

$$= \frac{19}{30} = 0.6333$$

(c)

$$\Pr[\text{error}] = \Pr[\text{error} | 0_S]\Pr[0_S] + \Pr[\text{error} | 1_S]\Pr[1_S]$$

$$= (0.4)\tfrac{1}{3} + (0.25)\tfrac{2}{3}$$

$$= \frac{9}{30} = 0.3$$

(d)

$$\Pr[0_S | 1_R] = \frac{\Pr[1_R | 0_S]\ \Pr[0_S]}{\Pr[1_R]}$$

$$= \frac{(0.4)\tfrac{1}{3}}{0.6333}$$

$$= \frac{4}{19} = 0.211$$

$\boxed{2.42}$

(a) $Pr[\text{Error}] = Pr[\text{Error} \mid 0_s]\, Pr[0_s] + Pr[\text{Error} \mid 1_s]\, Pr[1_s]$

$$= \rho \cdot \tfrac{1}{2} + \rho \cdot \tfrac{1}{2} = 0.001$$

$$\Rightarrow \rho = 0.001$$

(b) Similarly

$$Pr[\text{Error}] = \rho\,(0.2) + \rho\,(0.8) = 0.001$$

$$\Rightarrow \rho = 0.001$$

(c)

$$Pr[0_s \mid 0_R] = \frac{Pr[0_R \mid 0_s]\, Pr[0_s]}{Pr[0_R]}$$

$$= \frac{Pr[0_R \mid 0_s]\, Pr[0_s]}{Pr[0_R \mid 0_s]\, Pr[0_s] + Pr[0_R \mid 1_s]\, Pr[1_s]} \quad \text{(Bayes' Rule)}$$

$$= \frac{(1-\rho) P_0}{(1-\rho) P_0 + \rho\, P_1} = \frac{0.999\,(\frac{1}{2})}{1/2} = 0.999$$

2-43

(a)

S	R	Prob
0	0	$\frac{1}{2}(1-\varepsilon_0)$
0	1	$\frac{1}{2}\varepsilon_0$
1	0	$\frac{1}{2}\varepsilon_1$
1	1	$\frac{1}{2}(1-\varepsilon_1)$

(b) "Error" = "01" $\cup$ "10"

$$Pr[\text{Error}] = \frac{1}{2}\varepsilon_0 + \frac{1}{2}\varepsilon_1 = \frac{1}{2}(\varepsilon_0 + \varepsilon_1)$$

(c)
$$Pr[1\text{ sent} \mid \text{Error}] = \frac{Pr[1\text{ sent} \cap \text{Error}]}{Pr[\text{Error}]}$$

$$= \frac{\frac{1}{2}\varepsilon_1}{\frac{1}{2}(\varepsilon_0+\varepsilon_1)} = \frac{\varepsilon_1}{\varepsilon_0+\varepsilon_1}$$

(d)
$$Pr[1\text{ sent} / 1\text{ rcvd}] = \frac{Pr[1\text{ rcvd} \mid 1\text{ sent}] \cdot Pr[1\text{ sent}]}{Pr[1\text{ rcvd}]}$$

$$= \frac{\frac{1}{2}(1-\varepsilon_1)}{\frac{1}{2}\varepsilon_0 + \frac{1}{2}(1-\varepsilon_1)} = \frac{1-\varepsilon_1}{1-\varepsilon_1+\varepsilon_0}$$

$\boxed{2.44}$

(a)

$$Pr[A_R] = Pr[A_R|A_S]\,Pr[A_S] + Pr[A_R|B_S]\,Pr[B_S] + Pr[A_R|D_S]\,Pr[D_S]$$

$$= 0.9\times\frac{1}{4} + 0.05\times\frac{1}{4} + 0.05\times\frac{1}{4} = 0.25$$

(b)

$$Pr[\text{error}] = Pr[A_R|B_S]\,Pr[B_S] + Pr[A_R|D_S]\,Pr[D_S]$$
$$+ Pr[B_R|A_S]\,Pr[A_S] + Pr[B_R|C_S]\,Pr[C_S]$$
$$+ Pr[C_R|B_S]\,Pr[B_S] + Pr[C_R|D_S]\,Pr[D_S]$$
$$+ Pr[D_R|A_S]\,Pr[A_S] + Pr[D_R|C_S]\,Pr[C_S]$$
$$= 8\left(0.05\times\frac{1}{4}\right) = 0.1$$

(c) From (a), $Pr[D_R] = 0.25$.

(i) $$Pr[A_S|D_R] = \frac{Pr[D_R|A_S]\,Pr[A_S]}{Pr[D_R]} = 0.05$$

(ii) $$Pr[B_S|D_R] = \frac{Pr[D_R|B_S]\,Pr[B_S]}{Pr[D_R]} = 0$$

(iii) $$Pr[C_S|D_R] = \frac{Pr[D_R|C_S]\,Pr[C_S]}{Pr[D_R]} = 0.05$$

(iv) $$Pr[D_S|D_R] = \frac{Pr[D_R|D_S]\,Pr[D_S]}{Pr[D_R]}$$

$$= \frac{0.9\times 1/4}{1/4} = 0.9$$

$\boxed{2.45}$

(a)

$$Pr[A_R] = Pr[A_R|A_S]\,Pr[A_S] + Pr[A_R|B_S]\,Pr[B_S]$$

$$= 0.97 \times 3/16 + 0.05 \times \tfrac{1}{4} = 0.1944$$

$$Pr[D_R] = Pr[D_R|D_S]\,Pr[D_S] + Pr[D_R|C_S]\,Pr[C_S]$$

$$= 0.95 \times \tfrac{1}{4} + 0.03 \times 5/16 = 0.2469$$

(b)

$$Pr[\text{error}] = Pr[A_R|B_S]\,Pr[B_S] + Pr[B_R|A_S]\,Pr[A_S]$$

$$+ Pr[C_R|D_S]\,Pr[D_S] + Pr[D_R|C_S]\,Pr[C_S]$$

$$= 0.04$$

(c)

(i)

$$Pr[A_S|D_R] = \frac{Pr[D_R|A_S]\;Pr[A_S]}{Pr[D_R]} = 0$$

(ii)

$$Pr[B_S|D_R] = \frac{Pr[D_R|B_S]\;Pr[B_S]}{Pr[D_R]} = 0$$

Because $Pr[D_R|B_S] = 0$

(iii)

$$Pr[C_S|D_R] = \frac{Pr[D_R|C_S]\;Pr[C_S]}{Pr[D_R]}$$

$$= \frac{0.03 \times 5/16}{0.2469} = 0.0386$$

(iv)

$$Pr[D_S|D_R] = \frac{Pr[D_R|D_S]\;Pr[D_S]}{Pr[D_R]}$$

$$= \frac{0.95 \times 1/4}{0.2469} = 0.9619$$

$\boxed{2\text{-}46}$

(a) $Pr[err|0_S] = Pr[1_R|0_S] + Pr[2_R|0_S]$

$= 0.15 + 0.05 = 0.20$

(b) $Pr[err|1_S] = Pr[0_R|1_S] + Pr[2_R|1_S]$

$= 0.15 + 0.05 = 0.20$

$Pr[err|2_S] = Pr[0_R|2_S] + Pr[1_R|2_S]$

$= 0.15 + 0.05 = 0.20$

$$Pr[err] = Pr[err|0_S]Pr[0_S] + Pr[err|1_S]Pr[1_S] + Pr[err|2_S]Pr[2_S]$$

$$= (0.20)\tfrac{2}{5} + (0.20)\tfrac{2}{5} + (0.20)\tfrac{1}{5} = 0.20$$

(c) $$Pr[1_S|1_R] = \frac{Pr[1_R|1_S]Pr[1_S]}{Pr[1_R]}$$

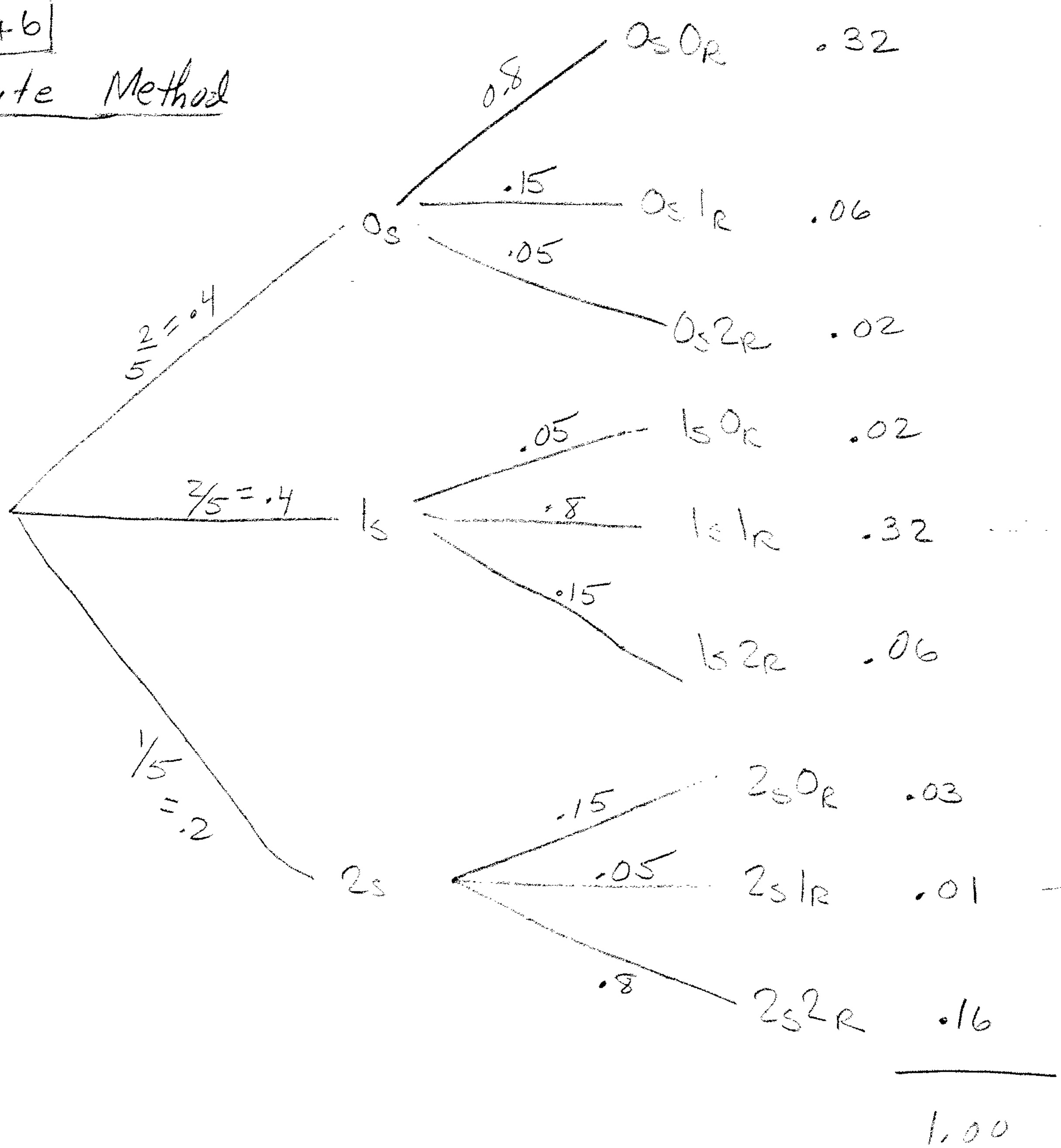
$$= \frac{Pr[1_R|1_S]Pr[1_S]}{Pr[1_R|1_S]Pr[1_S] + Pr[1_R|0_S]Pr[0_S] + Pr[1_R|2_S]Pr[2_S]}$$

$$= \frac{(0.8)(0.4)}{(0.8)(0.4) + (0.15)(0.4) + (0.05)(0.2)} = \frac{32}{39} = 0.821$$

(d) $$Pr[err|1_R] = 1 - Pr[1_S|1_R] = \frac{7}{39} = 0.179$$

2.46

Alternate Method



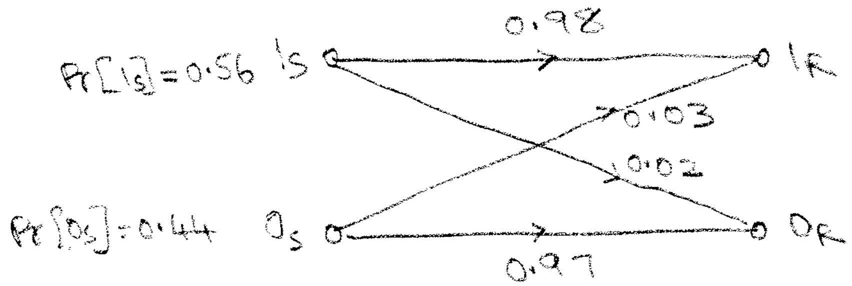
(a) $\dfrac{.06 + .02}{.06 + .02 + .32} = \dfrac{.08}{.40} = \dfrac{1}{5} = .2$

(b) $.06 + .02 + .02 + .06 + .03 + .01 = 0.2$

(c) $\dfrac{.32}{.06 + .32 + .01} = \dfrac{.32}{.39} = \dfrac{32}{39} = 0.821$

(d) $\dfrac{.06 + .01}{.06 + .32 + .01} = \dfrac{.07}{.39} = \dfrac{7}{39} = 0.179$

$\boxed{2\text{-}47}$



Probability of a bit error in this channel is

$$Pr[\text{error}] = p = Pr[1R|0s]\,Pr[0s] + Pr[0R|1s]\,Pr[1s]$$

$$= (0.03)\,0.44 + (0.02)\,0.56$$

$$= 0.0244$$

$$Pr[<3\text{ errors}] = Pr[0\text{ errors}] + Pr[1\text{ error}] + Pr[2\text{ errors}]$$

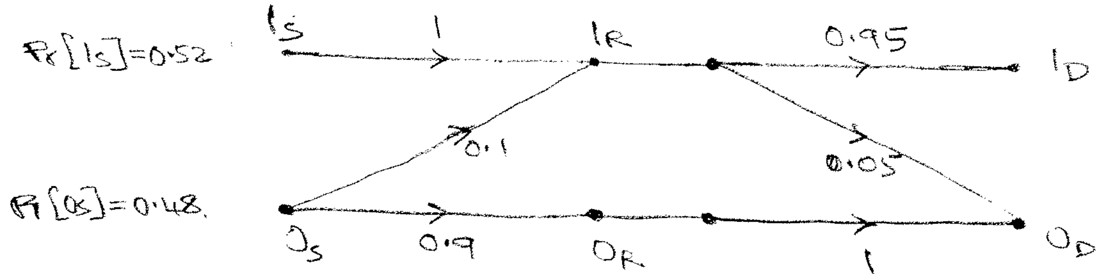
$$= \binom{25}{0} p^0 (1-p)^{25} + \binom{25}{1} p^1 (1-p)^{24} + \binom{25}{2} p^2 (1-p)^{23}$$

$$= 0.5393 + 0.3372 + 0.1012$$

$$= 0.9777$$

2-48

(a)



(b)

$$\Pr[1_R] = \Pr[1_R|1_S]\Pr[1_S] + \Pr[1_R|0_S]\Pr[0_S]$$

$$= 1(0.52) + 0.1(0.48)$$

$$= 0.568$$

(c)

$$\Pr[0_D] = \Pr[0_D|0_R]\Pr[0_R] + \Pr[0_D|1_R]\Pr[1_R]$$

$$= 1(0.432) + 0.05(0.568)$$

$$= 0.4604$$

$$\Pr[0_R|0_D] = \frac{\Pr[0_D|0_R]\Pr[0_R]}{\Pr[0_D]}$$ (Bayes' rule)

$$= \frac{1(0.432)}{0.4604}$$

$$= 0.9383$$

$\boxed{2.49}$

(a) $$Pr[\alpha] = Pr[\alpha, A] + Pr[\alpha, B]$$

$$= Pr[\alpha|A]\cdot Pr[A] + Pr[\alpha|B]\cdot Pr[B]$$

$$= (0.8)(0.6) + (0.1)(0.4) = 0.52$$

(b) for α received, compare:

$$Pr[A|\alpha] \overset{A}{\underset{B}{\gtrless}} Pr[B|\alpha]$$

$$\frac{(.8)(.6)}{Pr[\alpha]} \overset{A}{\underset{B}{\gtrless}} \frac{(.1)(.4)}{Pr[\alpha]} \implies \text{choose } A$$

for β received compare:

$$(.2)(.6) \overset{A}{\underset{B}{\gtrless}} (.9)(.4) \implies \text{choose } B$$

(c) for α received:

$$(.8)(.6) \overset{A}{\underset{B}{\gtrless}} (.9)(.4) \implies \text{choose } A$$

for β received:

$$(.2)(.6) \overset{A}{\underset{B}{\gtrless}} (.1)(.4) \implies \text{choose } A$$

$$Pr[\text{error}] = Pr[B] = 0.4$$

2-50

FORMULA FOR SUCCESS ← 19 letters

Letters = { Space, A, C, E, F, L, M, O, R, S, U }

(a) $Pr[\text{Space}] = \frac{2}{19}$, $Pr[A] = \frac{1}{19}$, $Pr[C] = \frac{2}{19}$

$Pr[E] = \frac{1}{19}$, $Pr[F] = \frac{2}{19}$, $Pr[L] = \frac{1}{19}$

$Pr[M] = \frac{1}{19}$, $Pr[O] = \frac{2}{19}$, $Pr[R] = \frac{2}{19}$

$Pr[S] = \frac{3}{19}$, $Pr[U] = \frac{2}{19}$

(b)

$$H = -\sum_i Pr[\text{Letter } i] \log_2 Pr[\text{Letter } i]$$

$$= -4\left(\frac{1}{19}\log_2\frac{1}{19}\right) - 6\left(\frac{2}{19}\log_2\frac{2}{19}\right) - \frac{3}{19}\log_2\frac{3}{19}$$

$= 3.3661$ bits/Letter

2-51

(a) Lets label the intermediate nodes 0 & 1.

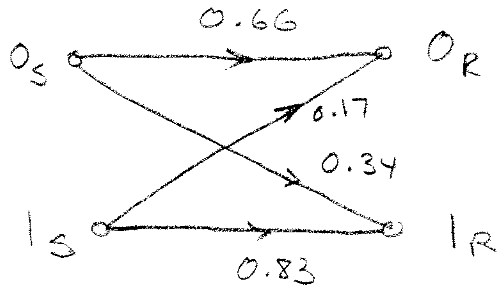
$$Pr[0_R|0_S] = Pr[0_R|0_S\,0]\,Pr[0|0_S] + Pr[0_R|0_S\,1]\,Pr[1|0_S]$$

$$= (0.8)(0.8) + (0.1)(0.2) = 0.66$$

Likewise

$$Pr[1_R|1_S] = (0.9)(0.9) + (0.2)(0.1) = 0.83$$

eq. channel:



(b) $$Pr[\text{error}] = Pr[0_R|1_S]\,Pr[1_S] + Pr[1_R|0_S]\,Pr[0_S]$$

$$= (0.17)(0.5) + (0.34)(.5) = 0.255$$

(c) $$Pr[0_R|1_S] = (0.9)(0.1) + (0.2)(0.9) = 0.27$$

$$Pr[1_R|0_S] = (0.1)(0.8) + (0.8)(0.2) = 0.24$$

$$Pr[\text{error}] = (0.27)(0.5) + (0.24)(0.5) = 0.255$$

No

-52

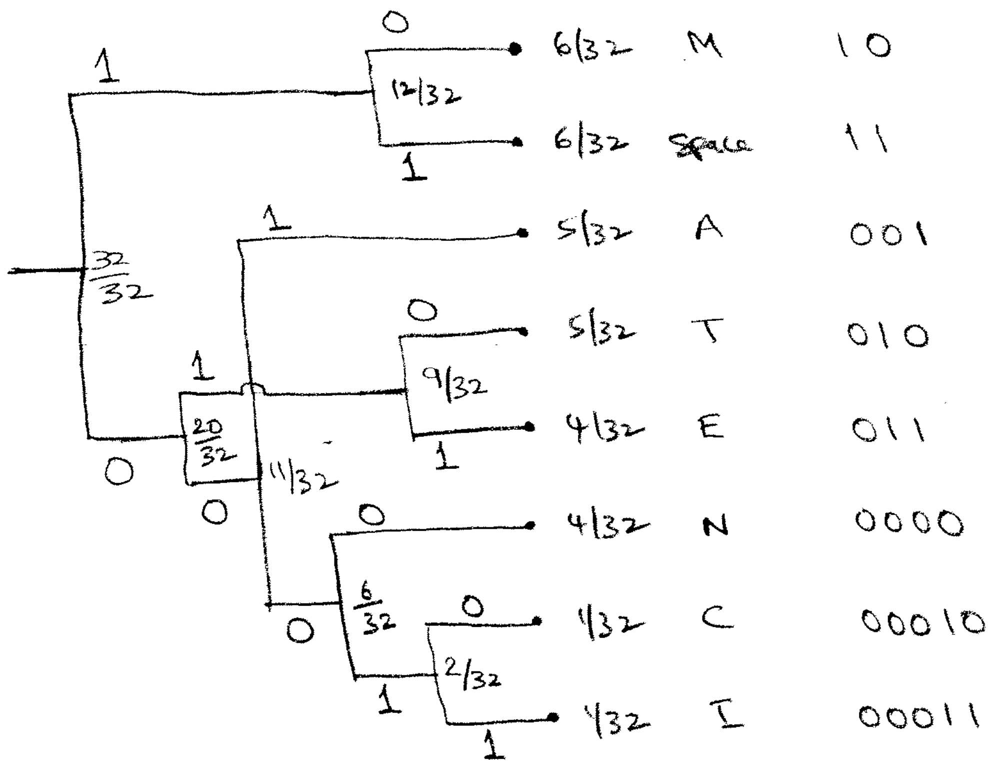
(a) Symbol Set, $\{S_i\} = \{A, C, E, I, M, N, T, \text{Space}\}$

$Pr[A] = Pr[T] = 5/32$ $\quad$ $Pr[C] = Pr[I] = 1/32$

$Pr[E] = Pr[N] = 4/32$ $\quad$ $Pr[M] = Pr[\text{Space}] = 6/32$

(b) $H = -\sum_{i=1}^{8} Pr[S_i] \log_2 Pr[S_i] = 2.8050$ bits

(c)



(d) Average length

$$\bar{l} = \sum_{i=1}^{8} l_i \, Pr[S_i]$$

$$= 2 \times \frac{6}{32} + 2 \times \frac{6}{32} + 3 \times \frac{5}{32} + 3 \times \frac{5}{32} + 3 \times \frac{4}{32} + 4 \times \frac{4}{32}$$

$$+ 5 \times 1/32 + 5 \times 1/32$$

$= 2.875$ bits