CHAPTER 2:

Problem 1

(a) A slab optical waveguide has a symmetrical step refractive index profile based on pure silica as the cladding material with a refractive index of 1.480 at 1550 nm free-space wavelength.

(i) The scalar wave equation for the z-propagating transverse electric monochromatic lightwaves is given by

$$\frac{d^2E_y}{dx^2} + (k^2 n_j^2 - \beta^2)E_y = 0$$

where n_j ($j = 1$ *or* 2) represents the refractive index in either the core or the cladding regions, $k = 2\pi/\lambda$ is the free space wave number, λ is the free space wavelength, β is the propagation constant of the lightwaves along the z-direction.

Write down the wave solutions for both the even and odd TE guided modes in the core and cladding regions. The corresponding two eigenvalue equations of these guided modes are given by

$v = u \tan u$ for even TE guided modes

$v = -\dfrac{u}{\tan u}$ for odd guided TE modes

where u and v are defined by

$$u^2 = a^2(k^2 n_1^2 - \beta^2)$$
$$v^2 = a^2(-k^2 n_2^2 + \beta^2)$$

You are required to prove just one of the above eigenvalue equations (either one of your choice).

(ii) Obtain an expression for the normalised frequency V-parameter and thence design a slab optical waveguide so that it can guide two TE optical guided modes at an operating free-space wavelength of 1550 nm. It is recommended that the following parameters of the optical waveguide should be specified: the slab core thickness, the cladding thickness, the relative refractive index difference between the core and cladding regions and the cut-off wavelength of TE modes of order higher then the two TE modes.

(iii) Sketch the electric field and intensity distribution of the fundamental guided TE mode in the transverse plane of the designed waveguide in Part (ii).

(b) Describe briefly the attenuation or loss of silica planar waveguide layer and the dispersion curve of such planar waveguide whose refractive index difference is smaller than 1%.

Solution:

(a)

(i) The wave equation $\frac{d^2E_y}{dx^2} + (k^2 n_j^2 - \beta^2)E_y = 0$ can be derived by referring to eq. (2.1) and the equations given in the Appendix as follows

The planar optical waveguide is a non-conducting medium without free charge, thus the Maxwell's equations are:

$$\nabla x E = -\frac{\partial B}{\partial t}$$

$$\nabla x H = +\frac{\partial D}{\partial t} \qquad (1.3b)$$

$$\nabla . D = 0$$

$$\nabla . B = 0$$

The flux densities B and D are the corresponding electric and magnetic flux densities and are given by

$$\mathbf{D} = \varepsilon_r \varepsilon_0 \mathbf{E} + \mathbf{P} \qquad (1.3c)$$

$$\mathbf{B} = \mu_o \mathbf{H} \qquad (1.3d)$$

where μ_O is permeability in vacuum, P is the electric polarization that requires a microscopic quantum mechanical approach. Assuming that there is no pollination effect acting on the optical fields then by taking the curl of (1.3b) we have:

$$\nabla x \nabla x E = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \qquad (1.4a)$$

thus expanding the curl(curl) operator we have:

$$\nabla x \nabla x \underset{\sim}{E} = \nabla(\nabla . \underset{\sim}{E}) - \nabla^2 E$$

then using (1.3c) and $\nabla(\nabla . \underset{\sim}{E}) = 0$ we have

$$\nabla^2 E + \frac{1}{c^2}\frac{\partial^2 E}{\partial \ t^2} = 0 \qquad (1.4b)$$

With the time dependent of the electric field as in eqn. (1.3a) we have $d^2/dt^2 = -\omega^2$ and $d^2/dz^2 = \beta^2$ and substituting into equation (1.4a) and using $\omega^2/c^2 = n^2(\omega)k_o^2$ we have the wave equation:

$$\boxed{\nabla_t^2 \ E + (\beta^2 - n^2(\omega)k_0^2)E = 0} \qquad (1.4c)$$

where $$\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad (1.4d)$$

Therefore for a planar optical waveguide with an infinite extension in the y direction we have d/dy = 0 and for TE modes only E_y is significant, the wave equation becomes:

$$\frac{d^2 E_y}{dx^2} + (\beta^2 - \omega^2 \mu\varepsilon)E_y = 0 \qquad (1.5a)$$

where μ and ε are the permeability and permittivity of medium n_1 or n_2. [$\mu = \mu_o$ and $\varepsilon = \varepsilon_r \ \varepsilon_o$], non magnetic and (glass) dielectric material. Similarly a wave equation involved H_y is given by

$$\frac{d^2 H_y}{dx^2} + (\beta^2 - \omega^2 \mu\varepsilon)H_y = 0 \qquad (1.5b)$$

Equations (1.5a) and (1.5b) can be rewritten using $k=\omega/c$ and $c=(\mu_0\varepsilon_0)^{-1/2}$ is the light velocity in vacuum and k is the wave number in vacuum, as

$$\frac{d^2 E_y}{dx^2} + (\beta^2 - k^2 n_j^2)E_y = 0 \qquad (1.5c)$$

$$\frac{d^2 H_y}{dx^2} + (\beta^2 - k^2 n_j^2)H_y = 0 \qquad (1.5d)$$

where $n_j = n_1$ or n_2 depending whether the equations are applied in the core or cladding regions; $n_1 = (\varepsilon_{r1})^{1/2}, n_2 = (\varepsilon_{r2})^{1/2}$ with ε_{r1} and ε_{r2} are the relative permittivities of the core and cladding respectively. From (1.5c) and (1.5d) we observe that the variation of the fields along Ox as:

- *sinusoidal* behavior when $k^2 n_j^2 > \beta^2$ or guided waves inside the core, and

- *exponential* (decay) behavior when $k^2 n_j^2 < \beta^2$ ie no radiation in cladding region

In another word for a properly designed optical waveguide the optical field is oscillating in regions where the longitudinal propagation constant is smaller than the plane-wave propagation constant and "evanescent" with an exponential-like behaviour elsewhere Thus the lightwaves are "trapped" in the core region and guided through the waveguide length. In the next section we analyse the wave equation so that conditions for guiding lightwaves are established.

(ii) Optical waves are guided along the waveguide when their EM fields are oscillatory in the slab waveguide region and exponentially decay in the cladding region, that is

$$kn_2 \le \beta \le kn_1 \tag{1.6}$$

We now define a transverse propagation constant **u/a** and transverse decay constant **v/a** as

$$\frac{u^2}{a^2} = k^2 n_1^2 - \beta^2 \tag{1.7a}$$

$$\frac{v^2}{a^2} = -k^2 n_2^2 + \beta^2 \tag{1.7b}$$

Adding the equations (1.7a) and (1.7b) gives:

$$\frac{u^2}{a^2} + \frac{v^2}{a^2} = k^2 (n_1^2 - n_2^2) \tag{1.8}$$

or alternatively

$$\boxed{V^2 = u^2 + v^2 = k^2 a^2 (n_1^2 - n_2^2)} \tag{1.9}$$

We observe that the equations (1.7a) and (1.7b) represent the propagation constant in the transverse direction as illustrated in Figure 1 below.

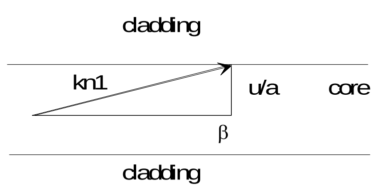

Figure 1 : Representation of the propagation constant along the "ray" direction, the propagation direction z and the transverse direction.

In order for the lightwaves to be guided or effectively oscillating in the transverse direction we can see that the transverse propagation constant u/a must be positive in the core region and negative in the cladding region.

The parameter V is defined as the normalised frequency (V) which is dependent only on the guide thickness and light frequency, (i.e. wavelength) and the refractive index difference between the core and cladding regions (the slab and superstrate or substrate regions for planar optical waveguides) .

Eqns (1.5a) and (1.5b) show that the field E_y for TE modes and H_y for TM mode are a linear combination of cos(ux/a) and sin(ux/a) inside the core layer (i.e when $|x| \leq a$) and exponentially decay form on outside the core or in the cladding layer (that is when $|x|>a$) with exp(-vx/a) and exp(+vx/a) in the superstrate or substrate. We therefore have a continuum of optical guided modes depending whether the solution function follows a symmetrical or anti-symmetric pattern (e.g. cosine or sine functions).

Mathematically the general solution of the wave equations given in (1.5c) and (1.5d) would be a combination of the sine and cosine or even and odd functions respectively. In the following sections we split the solution into two parts the even and odd modes corresponding with the even and odd functions. We can write a combination of these solutions as we have seen usually done in mathematics of differential equations.

Even TE modes (for modes with solution function cosine):

- For $|x| \leq a$, that is inside the core region the only significant component for TE mode is E_y and H_x

$$E_y(x) = A\cos\frac{ux}{a} \quad (1.10a) \qquad H_y = 0 \quad \text{and} \quad H_z = A\sin\frac{ux}{a} \quad (1.10b)$$

- For $|x| > a$, that is the field portion of the lightwaves are in the cladding region

$$E_y = Ce^{-\frac{v}{a}(x-a)} \quad (1.10c)$$

The arbitrary constants A and C can be found by applying the boundary conditions as follows:

- *we have the value of E_y at $x = a^+$ and $x = a^-$ must be equal; using eqns (1.10a) and (1.10b):*

  $A\cos u = Ce^{-\frac{v}{a}(x-a)}$ *evaluated at x = a this equality becomes*

  $$C = A\cos u \quad (1.10d)$$

- *The coefficient A (thus C) can then be found by using H_z and one of Maxwell's equation (1.4c) as*

  H_z (at core $x = a^+$) = H_z (at cladding $x = a^-$)

at x = a$^+$ *in core we have* $$H_z = \frac{1}{j\omega\mu_0}\frac{dE_y}{dz} = \frac{1}{j\omega\mu_0}(-\frac{u}{a}A\sin\frac{u}{a}x)$$ *(1.10e)*

(1.1

and in cladding at x = a$^-$ $$H_z = \frac{1}{j\omega\mu_0}(-C\frac{v}{a}e^{-\frac{v}{a}(x-a)})$$ (1.10f)

Therefore eqauting these this boundary condition (1.10e) and (1.10f) we obtain

$$C = A\frac{u}{v}\sin u$$ (1.10g)

The eigenvalue equation for the even modes can be achieved by equating (1.10d) and (1.10g):

$$v = u \tan u$$ (1.10k)

this equation is called the eigenvalue equation which can be solved to find the propagation constant β along the z direction. The number of guided modes that can be supported by the slab optical waveguide can then be easily determined. The number of possible values of β gives the number of guided even TE modes, thus whether the waveguide is a single mode or multimode pending on the number of odd modes possibly supported by the waveguide. This is investigated in the next part.

Odd TE modes

Similarly for odd TE modes, we have the solution function follows a sine function. Writing the solution for Ey in the core region and the evanescent field in the cladding regions then applying the boundary conditions at the core-cladding interface we obtain the eigenvalue equation for odd modes as:

$$v = -\frac{u}{\tan u}$$ (1.11d)

Graphical Solutions

Combining equations (1.9), (1.10) and (1.12) we observe that the waveguides can support only <u>discrete modes</u> and the propagation constant β related to u and v parameters can be found by solving graphically the <u>intersection</u> between <u>circles of V</u> and <u>curves representing eqns (1.11d) or (1.12d).</u> These solutions are illustrated in Figure 2.

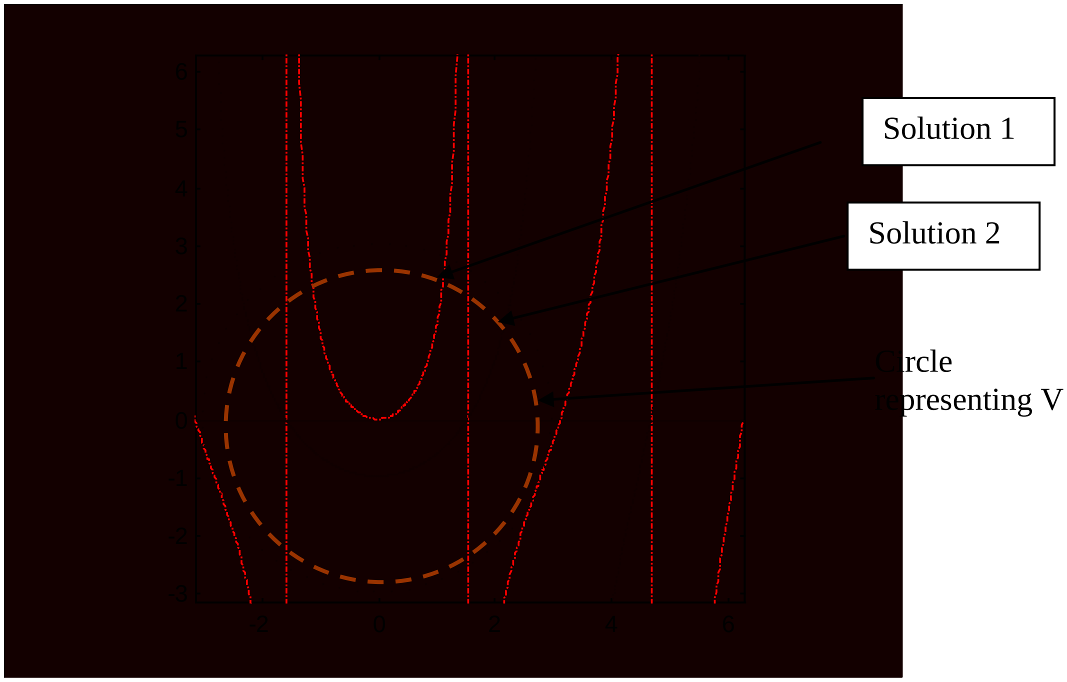

Figure 4 Graphical solution of eqns (1-9), (1.10d) and (1.12)

________v = utan u, __ _ __v = -u/tan u, and -------.$V^2 = u^2 + v^2$

Cut-off properties

From Figure 4 we observe that:

- V = 0, ie zero optical frequency or λ is zero, that is there exist no lightwaves. Thus we observe that we always have at least one guided (even) mode, TE_o
- $V < \pi/2$, there exists only one guided mode TE_{e0} (fundamental even mode)
- $V > \pi/2$, odd TE_0 mode appears (second mode – fundamental odd mode)
- $V = \pi$, third mode (TE_{e1}, first order even mode)

That is, each time V reaches a multiple integer of $\pi/2$, a new TE mode reaches its cut-off. The fundamental TE even mode is cut off only when V = 0, that is when there is no waveguide.

(iii) The field distribution of the guided mode would follow sin or cos distribution inside the core and exponentially decay in the cladding region. A rough sketch can be seen in Figure 3.

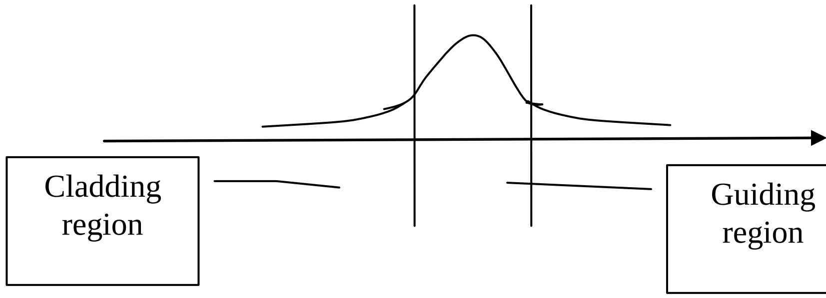

Figure 3: Sketch of the fundamental mode amplitude distribution in the waveguide region and cladding region. If higher order odd mode then the distribution follows a sinusoidal shape instead of the cosine.

Problem 2:

A slab optical waveguide has a symmetrical refractive index based on pure silica as the substrate material. The core refractive index is 1.50 and its thickness is 4.00 μm. The cladding thickness is 20 μm. The refractive index difference is 0.09. The operating wavelength is 1300.0 nm.

(a) Is the operating wavelength in the UV, visible, near infra-red, infrared or far infra-red region?

(b) Is the silica material less lossy at 1300 nm than at 1550 nm? Give reasons.

(c) Find the normalised frequency parameter V for the planar optical waveguide. Thence find the number of odd and even guided modes. If possible write a procedure in MATLAB to calculate the propagation constants of these guided modes.

(d) If the cladding refractive index is to 1.515, would the optical waveguide support any guided mode at 1300 nm wavelength? If it does, find the number of guided modes for this structure. Sketch the field and intensity distribution of these modes across the waveguide cross section.

(e) Using the refractive index profile as in Part (d), design the geometrical structure of the waveguide so that it can support only one TE-even mode.

Solution Hints:

Using the graphical solution, determine the value of V nand hence the number of guided modes. Determine the odd and even mode. Check the ref index difference to see whether the waveguide is a weakly guiding type, if it is then pay attention to this fact and then calculate the propagation constant of the guided mode and then the field distribution, even means symmetric and odd means asymmetric distribution.

Problem 3

Assuming that the refractive indices at 1550 nm wavelength are the same, repeat Problem 2 with an operating optical wavelength of 1550 nm.

Problem 4

The structure of an optical planar waveguide splitting junction Y. Sketch this structure with the y angle of 2 degrees of arc. The requirement is that the output optical fields at the output ports of the Y junction must be that of a single even TE-mode. Using silica as the substrate material of a refractive index of 1.500 at 1530 nm wavelength in vacuum, design the planar optical waveguide sections of the Y junction. Designers should assume that the splitting tilted junction area would support the same guided modes as that of the output straight branches.