Solutions Manual to Electroacoustics

Problem 3.1

The outgoing wave is written

$$\underline{p}(r) = \frac{A}{r} e^{-jkr}$$

using

$$\rho \frac{\partial \underline{u}}{\partial t} = -\frac{1}{r} \frac{\partial \underline{p}}{\partial r}$$

we find that

$$\underline{u}(r) = \frac{A}{r\rho c} e^{-jkr} \left[1 - \frac{j}{kr} \right]$$

Here

$$\rho c = 1.2 \cdot 343 \approx 412 \quad \left[\frac{\text{Ns}}{\text{m}^3} \right]$$

Since we were given that the acoustic pressure amplitude at 1 m from the point source is 0.1 Pa, we have

$$\left| \underline{p}(1) \right| = 0.1 = \frac{A}{1} \quad [\text{Pa}]$$

so

$$A = 0.1 \quad [\text{Pa} \cdot \text{m}]$$

The phase difference between sound pressure and particle velocity is

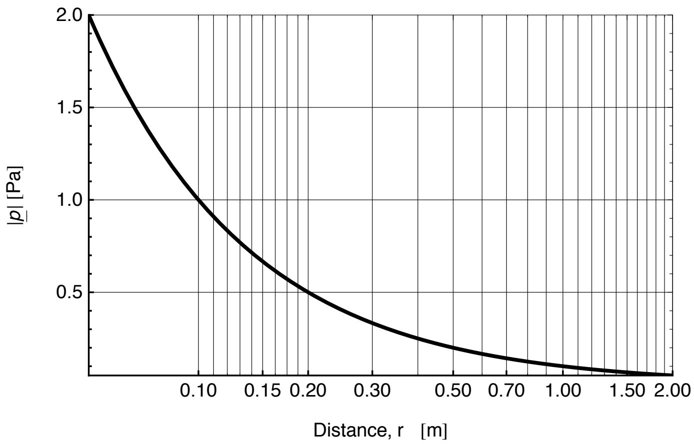
$$\theta = \arg\left(\frac{\underline{p}}{\underline{u}} \right) = \arg\left(\frac{\rho c \left[1 - \frac{1}{jkr} \right]}{1 + \left(\frac{1}{kr} \right)^2} \right) = \tan^{-1}\left(\frac{\text{Im}}{\text{Re}} \right) = \tan^{-1}\left(\frac{1}{kr} \right)$$

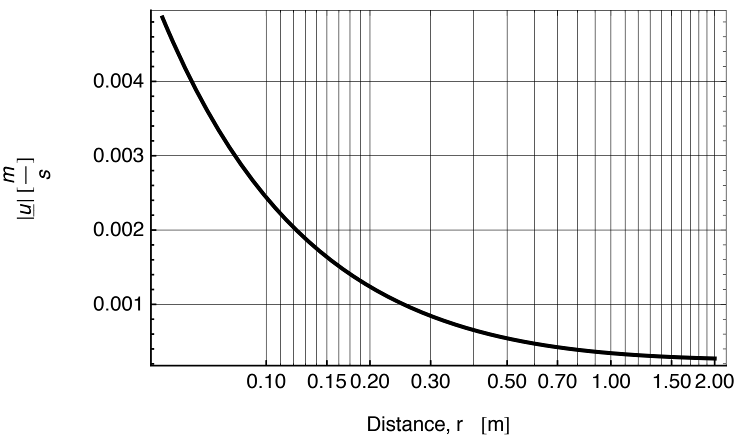
In summary:

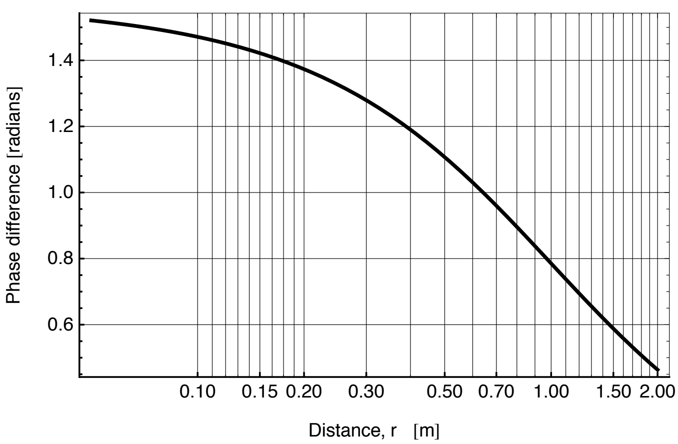
$$\left| \underline{p}(r) \right| = \frac{A}{r} \quad [\text{Pa}]$$

$$|\underline{u}(r,k)| = \frac{A}{r\rho c}\sqrt{1+\frac{1}{(kr)^2}} \quad \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

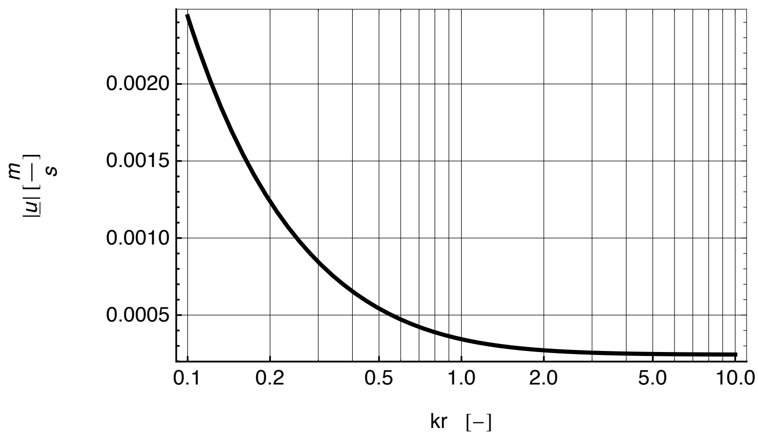
This gives us as functions of r:

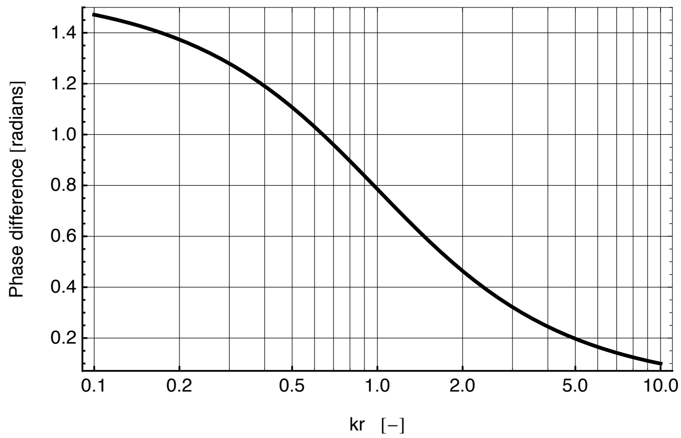






and as functions of kr:





Problem 3.2

The sound pressure and particle velocity due to the sphere vibration is

$$\underline{p}(r) = \frac{A}{r} e^{-jkr}$$

$$\underline{u}(r) = \frac{A}{r\rho c} e^{-jkr} \left[1 - \frac{j}{kr} \right]$$

which gives us the sound field impedance

$$Z(r,k) = \frac{\rho c}{1 + \frac{1}{jkr}}$$

For a spherical wave the intensity is

$$I(r,k) = \tilde{u}^2 \operatorname{Re}\left[\underline{Z}(r,k) \right] = \tilde{u}^2 \frac{\rho c}{1 + \frac{1}{(kr)^2}}$$

We know the particle velocity at the sphere surface (r=r0) so the power there is

$$P = 4\pi r_0^2 I(r_0,k) = \tilde{u}^2(r_0)\frac{4\pi r_0^2 \rho c}{1+\frac{1}{(kr_0)^2}}$$

The velocity is related to the displacement as

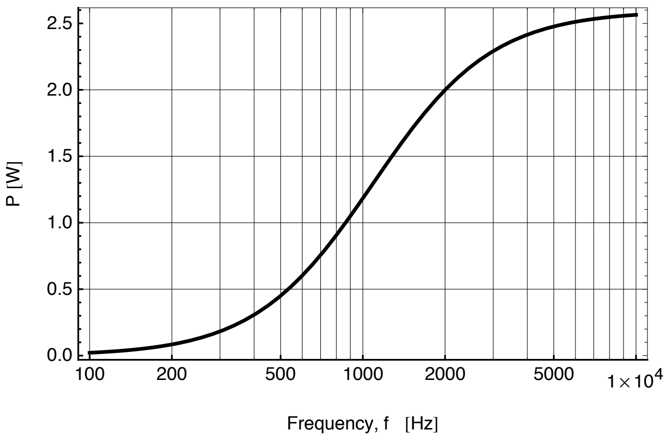
$$\underline{u} = j\omega \underline{x}$$

so we find that

$$\tilde{u}^2 = (\omega\tilde{x})^2 = \left(\omega\frac{\hat{x}}{\sqrt{2}}\right)^2 = 0.2 \quad \left[\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2\right]$$

The turnover point in frequency is determined by $kr_0=1$

$$0.05k = 0.05\cdot\frac{2\pi f}{343} = 1$$



Problem 3.3

The sound pressure in front of the rigid wall is determined by the incident and reflected waves. or the free wave we find

$$\underline{p}(x)_{free} = \hat{p}e^{-jkx}$$

$$\tilde{p}(x)_{free} = \frac{\hat{p}}{\sqrt{2}}$$

The reflection coefficient is $\underline{r}$=1 since the wall is rigid so the pressure as a function of position relative the wall at x=0 is

$$\underline{p}(x)_{with} = \hat{p}\left[e^{-jkx} + e^{jkx}\right]$$

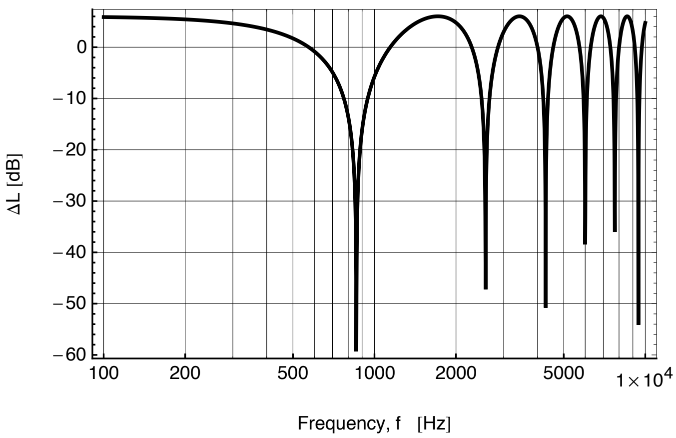
$$\tilde{p}(x)_{with} = \frac{\hat{p}}{\sqrt{2}}\left[2\cos(kx)\right]$$

The sound pressure ratio is

$$\frac{\tilde{p}(x)_{with}}{\tilde{p}(x)_{free}} = 2\cos(kx)$$

Using the level concept we find that

$$\Delta L = 10\log\left[\left(2\cos\left(\frac{0.1\cdot 2\pi f}{343}\right)\right)^2\right] \quad [\text{dB}]$$



Problem 3.4

Since the loudspeakers radiate in phase the total sound pressure at the measurement is the sum of the partial pressures at this point

a)

$$\tilde{p} = 0.63 + 0.11 + 0.20 = 0.94 \quad [\text{Pa}]$$

$$L_p = 20\log\left(\frac{0.94}{0.00002}\right) = 93.4 \quad [\text{dB}]$$

b)

$$\tilde{p} = \frac{0.63}{2} + 0.11 + 0.20 = 0.625 \quad [\text{Pa}]$$

$$L_p = 20\log\left(\frac{0.625}{0.00002}\right) = 89.9 \quad [\text{dB}]$$

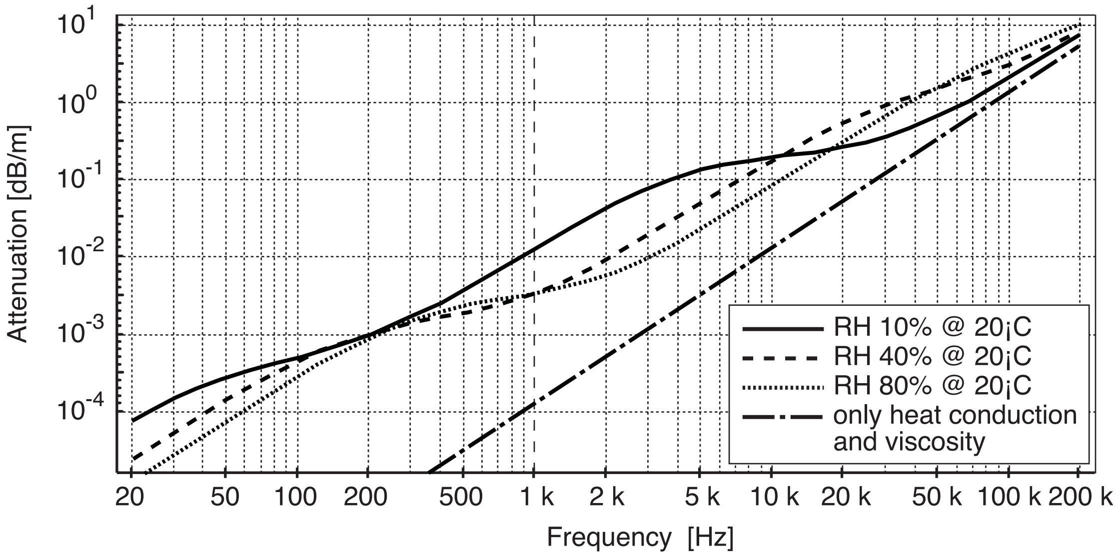
c)

$$\tilde{p} = 2\cdot 0.63 + 0.11 + 0.20 = 1.57 \quad [\text{Pa}]$$

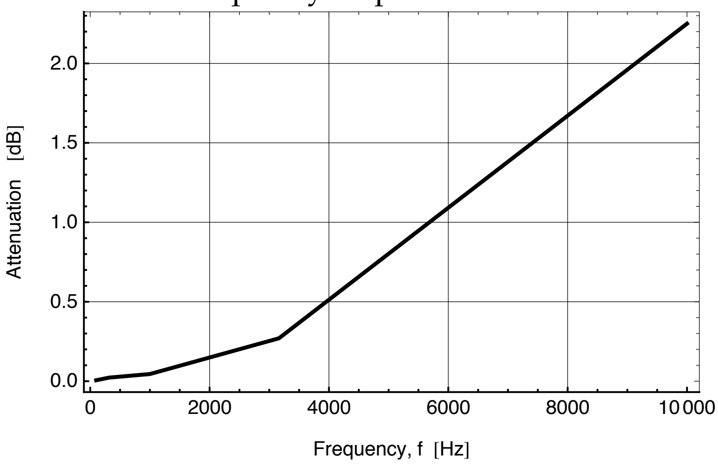
$$L_p = 20\log\left(\frac{1.57}{0.00002}\right) = 97.9 \quad [\text{dB}]$$

Problem 3.5

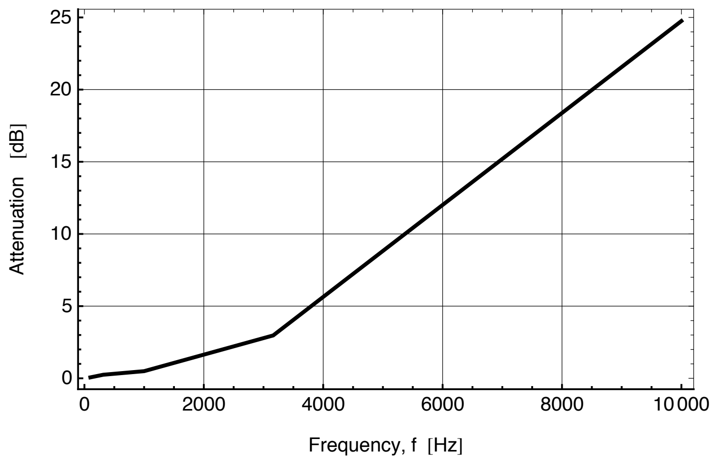
The attenuation due to the sound absorption by air over a distance x is $\Delta L \approx 4.3\, mx$. The attenuation coefficient is found from



Attenuation frequency response over 9 m



Attenuation frequency response over 99 m



Problem 3.6

The reflection coefficient of a wave hitting an impedance change is given by Equation 3.47.

$$\underline{r} = \frac{\hat{p}_r}{\hat{p}_i} = \frac{\underline{Z}_2 - \underline{Z}_1}{\underline{Z}_2 + \underline{Z}_1}$$

which applies also if the impedances are complex. Assume that the foil is very large and mounted so that it is perpendicular to the plane wave hitting the foil. If the mass per unit area of the foil is m″ we have in this case

$$\underline{Z}_1 = \rho c$$

$$\underline{Z}_2 = j\omega m'' + \rho c$$

The reflection coefficient is then

$$\underline{r} = \frac{j\omega m''}{j\omega m'' + 2\rho c}$$

and sound pressure in front of the foil is

$$\underline{p}(x) = \hat{p}\left[e^{-jkx} + \underline{r}e^{jkx}\right]$$

$$\tilde{p}(x) = \frac{\hat{p}}{\sqrt{2}}\left|e^{-jkx} + \underline{r}e^{jkx}\right|$$

so $|\underline{r}|$ and m″ can be determined from the sound pressure pattern.

An alternative but slightly less exact method is to send a short "tone burst" of high frequency sound (say 100 cycles) towards the foil, measure the amplitude of the reflected burst, and calculate $|\underline{r}|$ and then m″.

Problem 3.7

Assume that we use the tone burst method suggested in the previous problem and that we use very high frequencies and that the distances l_1 and l_2 are sufficiently large for the bursts not to interfere in their respective media.
The intensity transmission is

$$I_1 = I_{in}\left(1-\tau_{12}\right)$$

$$I_3 = I_{in}\tau_{12}\left(1-\tau_{21}\right)\tau_{23}$$

The intensity transmission coefficients are

$$\tau_{12} = 1-\left|\underline{r}_{12}\right|^2 = 1-\left|\frac{\underline{Z}_1-\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}\right|^2 = \tau$$

$$\tau_{21} = 1-\left|\underline{r}_{21}\right|^2 = 1-\left|\frac{\underline{Z}_2-\underline{Z}_1}{\underline{Z}_2+\underline{Z}_1}\right|^2 = \tau$$

$$\tau_{23} = 1-\left|\underline{r}_{23}\right|^2 = 1-\left|\frac{\underline{Z}_1-\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}\right|^2 = \tau$$

The impedances of the media are real since we are dealing with the gases directly and are not concerned with interference effects. The incident and transmitted pulse amplitudes A_1 and A_3 will be

$$I_1 = I_{in}\left(1-\tau\right)$$

$$I_3 = I_{in}\left(1-\tau\right)\tau^2$$

$$\frac{A_3}{A_1} = \sqrt{\frac{I_3}{I_1}} = \tau$$

This leads to the equation

$$\frac{A_3}{A_1} = \frac{4Z_1Z_2}{\left(Z_1+Z_2\right)^2}$$

Solving for Z_2 we find

$$Z_2 = -\left[1-2\frac{A_1}{A_3}\right]Z_1 \pm \sqrt{\left(\left[1-2\frac{A_1}{A_3}\right]Z_1\right)^2 - Z_1^2}$$

$$Z_2 = Z_1\left[2\frac{A_1}{A_3}-1+\sqrt{\frac{A_1}{A_3}\left(\frac{A_1}{A_3}-1\right)}\right]$$

Now we use the time between bounces in medium 2 to determine the speed of sound there

$$c_2 = \frac{2l_2}{\Delta t}$$

and we can then finally find the density of medium 2

$$\rho_2 = \frac{Z_2}{c_2}$$

Problem 3.8

The sound pressure amplitude at 1 m distance is

$$\hat{p}_1 = \tilde{p}_1\sqrt{2} = 0.89 \quad [\text{Pa}]$$

The distance to the measurement point is found from

$$r_{10} = \sqrt{10^2 + 0.5^2} \approx 10.01 \quad [\text{m}]$$

The sum pressure is then at 50 Hz

$$k_{50} = \frac{2\pi 50}{343} \approx 0.92 \quad \left[\frac{\text{radians}}{\text{m}\cdot\text{s}}\right]$$

$$\hat{p}_{sum,50} = \frac{2\hat{p}_1}{r_{10}} e^{-jk_{50}r_{10}} = \frac{2\cdot 0.89}{10.01} e^{-jk_{50}r_{10}} \approx 0.18 e^{-j9.2} \quad [\text{Pa}]$$

and at 1 kHz

$$k_{1k} = \frac{2\pi 1000}{343} \approx 18.3 \quad \left[\frac{\text{radians}}{\text{m}\cdot\text{s}}\right]$$

$$\hat{p}_{sum,1k} = \frac{2\hat{p}_1}{r_{10}} e^{-jk_{1k}r_{10}} = \frac{2\cdot 0.89}{10.01} e^{-jk_{1k}r_{10}} \approx 0.18 e^{-j183} \quad [\text{Pa}]$$

The particle velocity will be the vector sum of the particle velocity contributions by the two sources. Assume the sources ±0.5 on the y-axis and the observation point 10 m away on the x-axis. The contribution in the y-direction will cancel out and for the x-direction we find

$$\hat{u}_{sum,50} = \frac{\hat{p}_{sum,50}}{\rho c} \frac{10}{\sqrt{10^2 + 0.5^2}} \approx 1.6\cdot 10^{-5} e^{-j9.2} \quad \left[\frac{\text{m}}{\text{s}}\right]$$

$$\hat{u}_{sum,1k} = \frac{\hat{p}_{sum,1k}}{\rho c} \frac{10}{\sqrt{10^2 + 0.5^2}} \approx 1.6\cdot 10^{-5} e^{-j183} \quad \left[\frac{\text{m}}{\text{s}}\right]$$

Problem 3.9

The sound pressures will now cancel out at the point of observation. Since the particle velocity is the vector sum of the particle velocity contributions the particle velocity in the x-direction is zero. The particle velocity in the y-direction is found from

$$\hat{u}_{sum,50} = \frac{\hat{p}_{sum,50}}{\rho c} \frac{0.5}{\sqrt{10^2 + 0.5^2}} \approx 6.3 \cdot 10^{-7} e^{-j9.2} \quad \left[\frac{\text{m}}{\text{s}}\right]$$

$$\hat{u}_{sum,1k} = \frac{\hat{p}_{sum,50}}{\rho c} \frac{0.5}{\sqrt{10^2 + 0.5^2}} \approx 6.3 \cdot 10^{-7} e^{-j183} \quad \left[\frac{\text{m}}{\text{s}}\right]$$

Note that there will be particle velocity but no sound pressure.

Problem 3.10

The rms sound pressure for one loudspeaker at 1 m distance is found from

$$90 = 20\log\left(\frac{\tilde{p}}{2 \cdot 10^{-5}}\right).$$

$$\tilde{p} = 2 \cdot 10^{-5} \cdot 10^{4.5} \approx 0.63 \quad [\text{Pa}]$$

The sound field impedance is

$$\underline{Z}(r) = \rho c \frac{jkr}{1 + jkr}$$

$$\text{Re}\left[\underline{Z}(r)\right] = \rho c \,\text{Re}\left[\frac{jkr}{1 + jkr}\right]$$

The sound power radiated by one of the loudspeakers in free field is

$$P_0 = 4\pi r^2 I(r) = 4\pi r^2 \frac{\tilde{p}^2}{(\rho c)^2} \text{Re}\left[\underline{Z}(r)\right]$$

$$kr = \frac{2\pi 50}{343} \quad \left[\frac{\text{radians}}{\text{m} \cdot \text{s}}\right]$$

$$P_0 = 4\pi \frac{(0.63)^2}{412} \frac{kr}{\sqrt{1 + (kr)^2}} \approx 8.2 \cdot 10^{-3} \quad [\text{W}]$$

When the loudspeakers are placed next to one another the sound pressure will increase to twice its previous value at the observation point so the radiated power will be four times as large.

$$P_{double} \approx 33 \cdot 10^{-3} \quad [\text{W}]$$

Problem 3.11

The loudspeakers at distance d now make a dipole source. The sound pressure from the dipole is given by equation 3.38.

$$\frac{\tilde{p}_d}{\tilde{p}_0} = \sqrt{(kd)^2 + \left(\frac{d}{r}\right)^2} \cos(\theta)$$

At far distance and high frequency the sound pressure is

$$\frac{\tilde{p}_d}{\tilde{p}_0} \approx kd\cos(\theta)$$

Integration over the full sphere then gives the power ratio

$$\frac{P_d}{P_0} \approx \frac{(kd)^2}{3} \approx \frac{\left(\frac{2\pi 50}{343}\cdot 0.03\right)^2}{3} \approx 2.5\cdot 10^{-4}$$

$$P_d \approx 2.1\cdot 10^{-6} \quad [W]$$