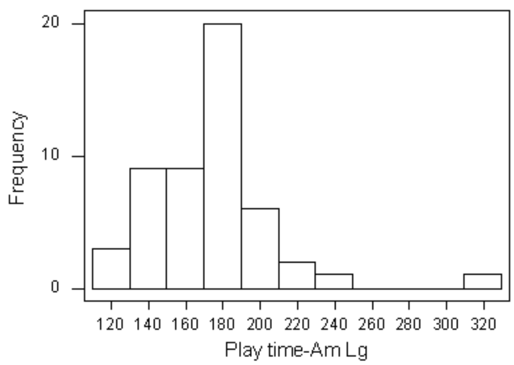
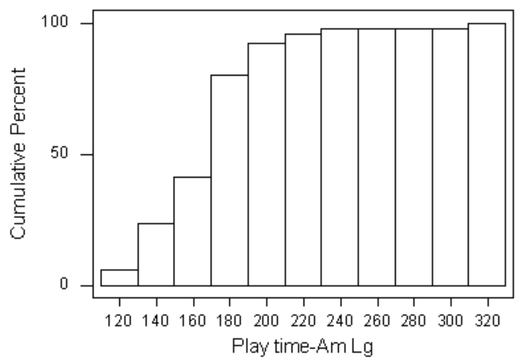
Chapter 2 Solutions – Part 1

Exercise in Empirical Methods

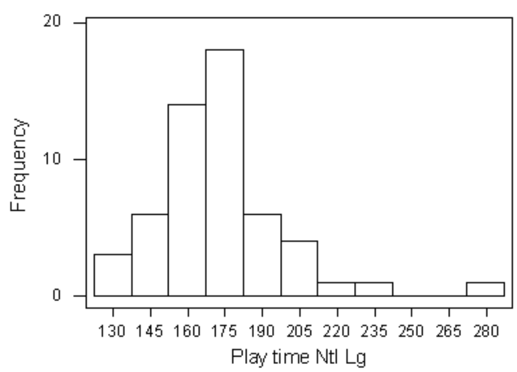
2-1. Game Times in Baseball games

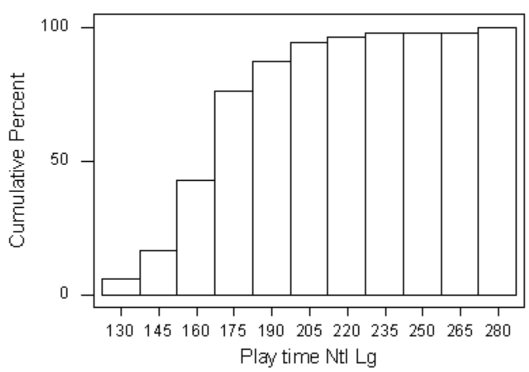
Histogram and Cumulative frequency distribution of playtime in Am. League





Histogram and Cumulative frequency distribution of playtime in Ntl. League





The chance the game will end before 180 minutes in Am Lg.: 80%, in Ntl Lg.: 85%
The time before which 95% of the games will end - in Am Lg.: 200 minutes, in Ntl Lg.: 200 min.
These results have been read out of the cum. freq. Distribution graph as approximate answers.
The exact numbers can be obtained by tallying the observations in a frequency distribution table.

2-2. Whiteness of PVC Resin:

By comparing the averages (see the descriptive statistics) and observing the box plot (see below) of the data obtained before and after the project, it seems that the whiteness has improved because of the improvement project.

```
DESCRIPTIVE STATISTICS:
```

Before Improvement

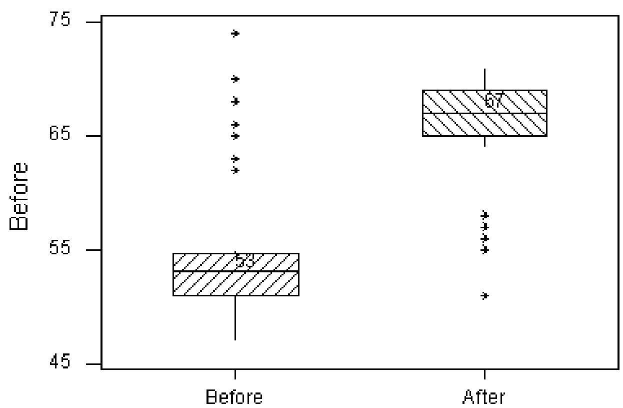
Variable	N	Mean	Median	TrMean	StDev	SE Mean
Before	72	53.847	53.000	53.203	5.383	0.634

Variable	Minimum	Maximum	Q1	Q3
Before	47.000	74.000	51.000	54.750

After Improvement

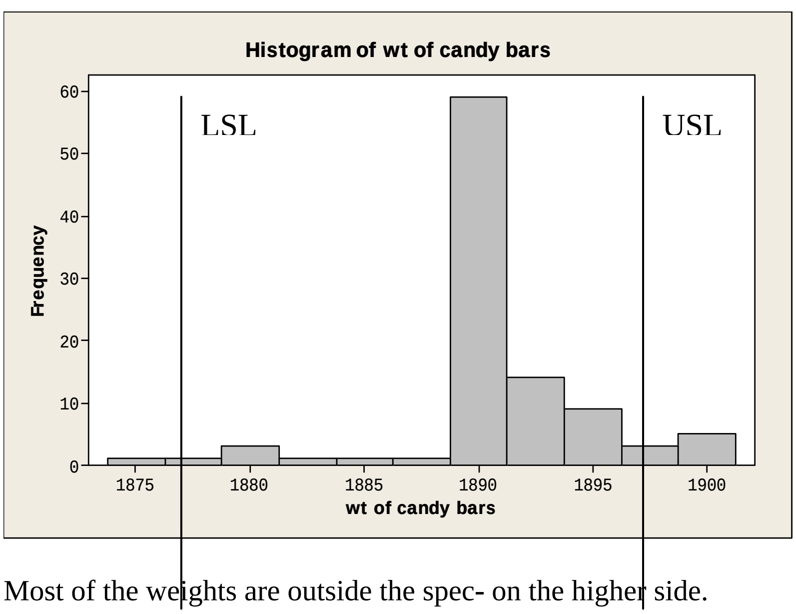
Variable	N	Mean	Median	TrMean	StDev	SE Mean
After	72	65.917	67.000	66.375	4.158	0.490

Variable	Minimum	Maximum	Q1	Q3
After	51.000	71.000	65.000	69.000



2-3. Weight of Candybars

Histogram of the candy bar weights:



Most of the weights are outside the spec- on the higher side.

Stem & Leaf Diagram of the candy bar weights

```
 1    187  6
 2    187  8
 5    188  011
 6    188  3
 6    188
 7    188  6
 20   188  8999999999999
(47)  189  00000000000000000000000111111111111111111111111
 31   189  22222222233333
 17   189  4445555
 10   189  6677
 6    189  89
 4    190  0011
```

```
This is not a pretty Stem & Leaf diagram possibly because there are too many
repeated values in the stem of 189. But the data was real-world data and
sometimes the pictures don’t look pretty.  Yet, the S&L diagram can be
interpreted to convey some meaningful information about the data.
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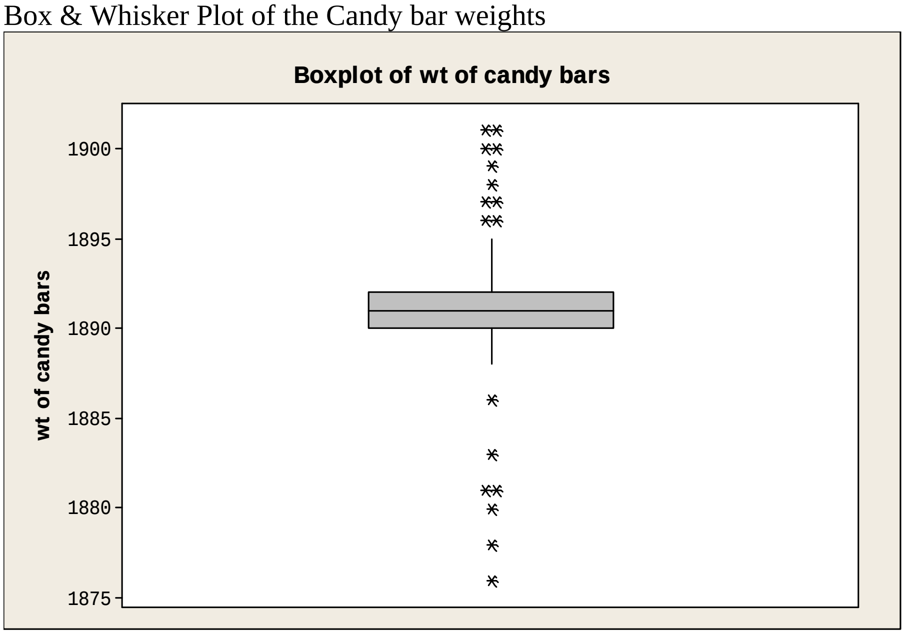
Box & Whisker Plot of the Candy bar weights



There are many outliers in this data as shown by stars printed by Minitab at the outlying values. Minitab identifies an outlier as a value that lies outside of 1.5 (standard deviations) from the quartile box. Thus, for this data, any value outside of 1890 – 1.5(4.06) = 1883.9 and 1892 + 1.5(4.06) = 1898.09 is an outlier.

Descriptive Statistics: wt of candy bars

Variable	N	Mean	SE Mean	StDev	Minimum	Q1	Median
wt of candy bars	98	1891.0	0.410	4.06	1876.0	1890.0	1891.0

Variable	Q3	Maximum
wt of candy bars	1892.0	1901.0

Mode: 1891
(We see in the Stem & Leaf diagram, the most number of observations, 24 of them, occur with a value of 891)

Range: 1901 – 1876 = 25
IQR = 1892 – 1890 = 2

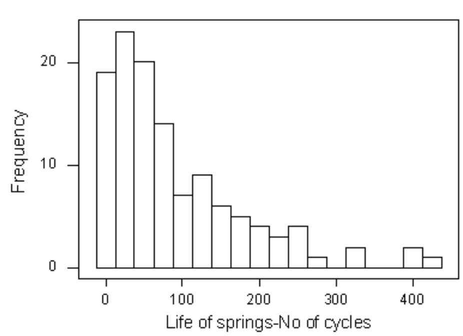
2-4 Breaking Time of Mold Springs

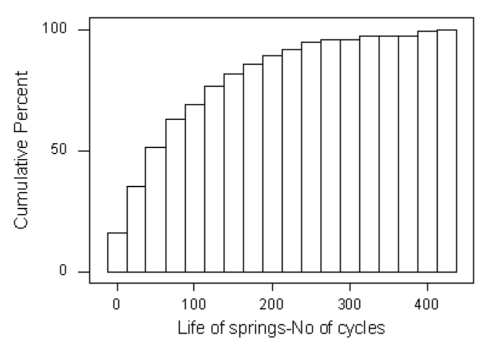
Variable	N	Mean	Median	TrMean	StDev	SE Mean
Life of	120	90.59	60.00	80.76	90.31	8.24

Variable	Minimum	Maximum	Q1	Q3
Life of	2.00	427.00	25.50	131.75

Note that the mean and standard deviation of the data are almost equal. Also note the wide variability in the data as shown by the wide range between the minimum and the maximum values. The percentage failing before the mean of 90 cycles can be read from the Cumulative Percent graph (shown below) as: about 65%.

Note the non-symmetric nature of the histogram. The data do not come from a normal population. The shape and other characteristics indicate that the data possibly come from a population which has a distribution called the exponential distribution.

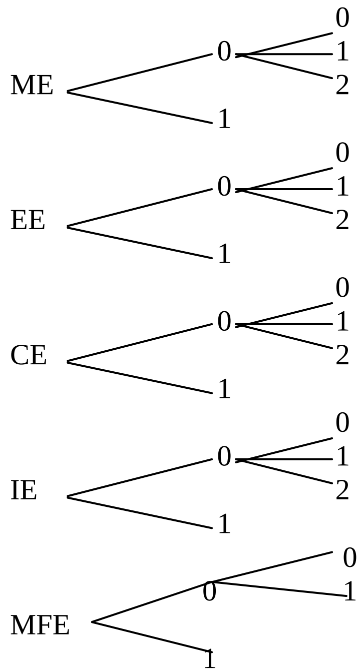




CHAPTER 2 SOLUTIONS – Part 2

Exercises in Probability

2.5



$$S = \left\{ \begin{array}{l} \text{ME1, ME00, ME01, ME02} \\ \text{EE1, EE00, EE01, EE02} \\ \text{CE1, CE00, CE01, CE02} \\ \text{IE1, IE00, IE01, IE02} \\ \text{MfE1, MfE00, MfE01, MfE02} \end{array} \right\}$$

(b) A = {ME01, EE01, CE01, MfE01}

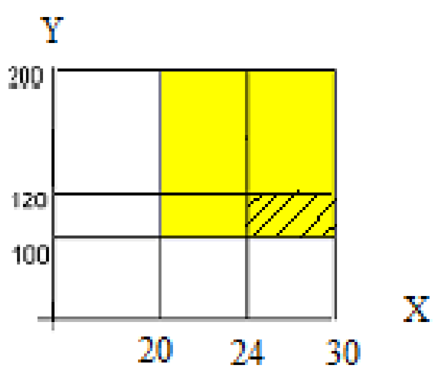
2.6 E

$$S = \left\{\begin{array}{cccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}\right\}$$

E

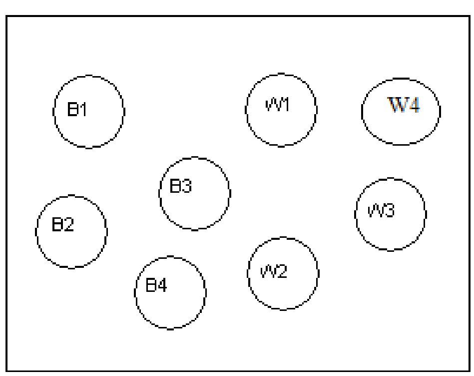
Elements of E : {5.1, 6.1, 6.2, 1.5, 1.6, 2.6}

2.7 S (shown shaded) =



Event A: $\{X > 24, Y < 120\}$ is shown in hatch

2.8



$$S = \left\{ \begin{array}{l} B_1B_1,\ B_1B_2,\ B_1B_3,\ B_1B_4,\ B_1W_1,\ B_1W_2,\ B_1W_3,\ B_1W_4 \\ B_2B_1,\ B_2B_2,\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots B_2W_3, B_2W_4 \\ B_3B_1,\ B_3B_2,\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots B_3W_3, B_3W_4 \\ B_4B_1,\ B_4B_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots B_4W_3, B_4W_4 \\ W_1B_1,\ W_1B_2,\ W_1B_3,\ W_1B_4,\ W_1W_1,\ W_1W_2,\ W_1W_3, W_1W_4 \\ W_2B_1,\ W_2B_2,\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots W_2W_3, W_2W_4 \\ W_3B_1,\ W_3B_2,\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots W_3W_3, W_3W_4 \\ W_4B_1, W_4B_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots W_4W_3, W_4W_4 \end{array} \right\}$$

64 elements in sample space *S*

$$\text{A (both black)} = \left\{ \begin{array}{l} B_1B_1, B_1B_2, B_1B_3, B_1B_4 \\ B_2B_1, B_2B_2, B_2B_3, B_2B_4 \\ B_3B_1, B_3B_2, B_3B_3, B_3B_4 \\ B_4B_1, B_4B_2, B_4B_3, B_4B_4 \end{array} \right\}$$

16 elements in event *E*

2.9 *S* = {same as in the previous problem except omit the eight elements B_1B_1, B_2B_2, B_3B_3, B_4B_4,

W_1W_1, W_2W_2, W_3W_3, W_4W_4}

56 elements in *S*

A= {same as in the previous problem except omit the four elements B_1B_1, B_2B_2, B_3B_3, B_4B_4}

12 elements in *E*

2.10 *P*(Red)= 3/6 *P*(Yellow)=2/6 *P*(Blue)= 1/6

P(Not Red) = [1 – *P*(Red)] = [1 – (3/6)] = (3/6) = **(1/2)**

2.11 For two balls picked with replacement, from problem 2.8, *S* has 64 equally likely elements. The event {Both Black} has 16 equally likely elements. Therefore, P(Both Black) = **16/64 = 1/4**

2.12 For two balls picked without replacement, from problem 2.8, *S* has 56 equally likely elements. The event {Both Black} has 12 equally likely elements. Therefore, *P*(Both Black) = 12/56 = **3/14**

2.13

S:

$$\left\{\begin{matrix} 1.1 & 1.2 & 1.3 & 1.4 & 1.5 & 1.6 \\ 2.1 & 2.2 & 2.3 & 2.4 & 2.5 & 2.6 \\ 3.1 & 3.2 & 3.3 & 3.4 & 3.5 & 3.6 \\ 4.1 & 4.2 & 4.3 & 4.4 & 4.5 & 4.6 \\ 5.1 & 5.2 & 5.3 & 5.4 & 5.5 & 5.6 \\ 6.1 & 6.2 & 6.3 & 6.4 & 6.5 & 6.6 \end{matrix}\right\}$$

B(total is >7) *A*(first die is a 4)

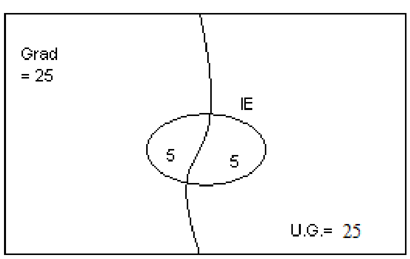
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 15/36 + 6/36 – 3/36 = **(18/36)**

2.14 $S = \left\{\begin{matrix} 1.1 & 1.2 & 1.3 & 1.4 & 1.5 & 1.6 \\ 2.1 & 2.2 & 2.3 & 2.4 & 2.5 & 2.6 \\ 3.1 & 3.2 & 3.3 & 3.4 & 3.5 & 3.6 \\ 4.1 & 4.2 & 4.3 & 4.4 & 4.5 & 4.6 \\ 5.1 & 5.2 & 5.3 & 5.4 & 5.5 & 5.6 \\ 6.1 & 6.2 & 6.3 & 6.4 & 6.5 & 6.6 \end{matrix}\right\}$

A: The difference in two numbers is at least 2

$P(A)$ = 20/36 = **(5/9)**

2.15 S :

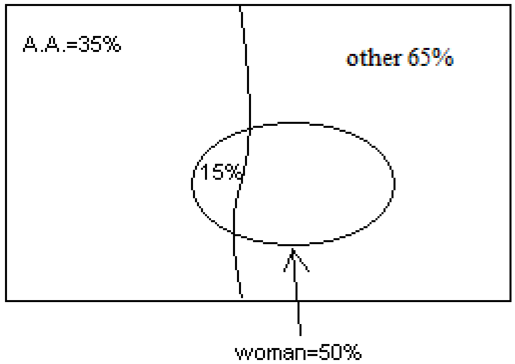


Total of 50 elements in the sample space S

$P(Grad)$ = 25/50 $P(IE)$ = 10/50 $P(Grad \cap IE)$ = 5/50

$P(Grad \cup IE)$ = 25/50 + 10/50 – 5/50 = 30/50 = **(3/5)**

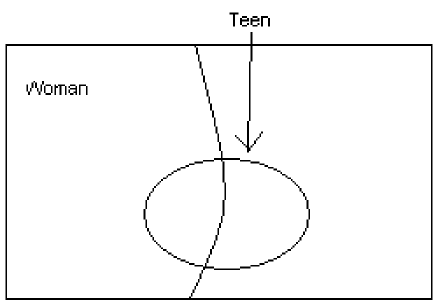
2.16 *S*:



$P(AA \cup W) = P(AA) + P(W) - P(AA \cap W) = 0.35 + 0.5 - 0.15 = 0.7$

$P(\overline{AA \cup Woman}) = [\,1 - P(AA \cup W)\,] = 1 - 0.7 = \mathbf{0.3}$

2.17 *S* :



(a) $P(Teen) = 0.2 \quad P(Woman \mid Teen) = 0.5$

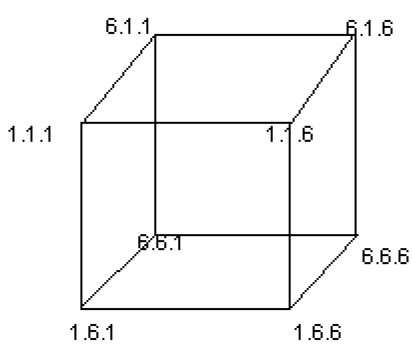
$P(Woman \cap Teen) = (0.5) * (0.2) = \mathbf{0.1}$

(b) $P(Teen \mid Woman) = 0.25$

$P(Teen \mid Woman) = [\,P(Teen \cap Woman)\,]/[P(Woman)]$

$P(Woman) = (0.1/0.25) = \mathbf{0.4}$

2.18 *S* =



216 elements in S
The event {All show the same face} = {1.1.1, 2.2.2, 3.3.3, 4.4.4, 5.5.5, 6.6.6}

P(*All will show the same face*) = 6/216
P(*All will not show the same face*) = [1 – P(*All will show same face*)] = **210/216**

2.19 a) S = {HH, HT, TH, TT}
Let A ={exactly one head}, then A = {HT, TH}
P{HT} = (0.2)*(0.4) = 0.08 because the tosses are independent
Similarly P{TH} = 0.08
Hence P(A) = P(TH OR HT) = P(TH) + P(HT) because they are mutually exclusive.
= 0.16 + 0.16 **= 0.32**

b) Let's pick one of the outcomes with 2 heads out of 5 tosses: {HHTTT}.
The probability of this occurring is (0.2)*(0.2)*(0.8)*(0.8)*(0.8) = $(0.2)^2\,(0.4)^3$ because the tosses are independent.
When a coin is tossed five times, there are $\binom{5}{2}$ = 10 outcomes with 2 heads and 3 tails.
Therefore, the probability of getting 2 heads when the coin is tossed 5 times
= $10(.2)^2(.4)^3$ = **0.2048**
.

2.20 (a) Since each question can be answered in two ways, the number of possible ways of answering the 8 questions simultaneously = 2^8 = 256
(b) Since only one of these is a correct set, the probability a random set is correct = 1/256 = **.0039**

2.21 Look at the operation like filling the boxes below:
Start filling the boxes from the rightmost box, which can be filled in 4 ways.

\# of ways of filling the boxes:

B	G	B	G	B	G	B

1 * 1 * 2 * 2 * 3 * 3 * 4 = **144 ways**

2.22 We did a similar problem earlier using the first principles by first writing the sample space. We cannot use that method here because there are too many elements in S. Now we will use the multiplication theorem and independence.

Let A be the event the first pick is green. $P(A)$ = 15/27
Let B be the event the second pick is green. $P(B)$ = 15/27 (due to replacement)

P(*Both green*) = $P(A \cap B)$ = 15/27 * 15/27 = **0.309** (because of independence)

2.23 Let A be the event the first pick is green. $P(A)$ = 15/27 = 0.519.

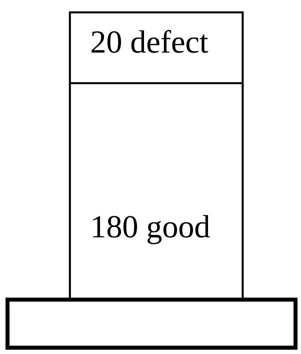
Let B be the event the second pick is green. Because the first ball is not replaced, B is not independent of A.
So we use the conditional probability $P(B|A)$, which is = 14/26.
$P(Both\ green) = P(B \mid A) * P(A)$ = 14/26 * 15/27 = **0.299**

2.24 (a) $P(0\ defective) = \dfrac{\binom{27}{4}\binom{3}{0}}{\binom{30}{4}} = \dfrac{77550}{27405} = 0.64$

(b) $P(2\ defectives) = \dfrac{\binom{27}{2}\binom{3}{2}}{\binom{30}{4}} = 0.077$

(c) $P(2\ or\ less) = P(2) + P(1) + P(0) = 0.077 + \dfrac{\binom{27}{4}\binom{3}{1}}{\binom{30}{4}} = 0.107 + .64 =$ **0.824**

2.25



There are 10 defective units out of 200
$P(Reject\ Tower) = P\ (More\ than\ 2\ defective\ units\ ) =$
$1 - P(2\ or\ less\ defective) = 1 - \{P(0) + P(1) + P(2)\} =$

$$1 - \frac{\binom{190}{8}\binom{10}{0} + \binom{190}{7}\binom{10}{1} + \binom{190}{6}\binom{10}{2}}{\binom{200}{8}} = 1 - 0.6957 = \mathbf{0.0043}$$

2.26 $P(Defective) = P(Def \mid A)*P(A) + P(Def \mid B)*P(B) + P(Def \mid C)*P(C) +$
$P(Def \mid D)*P(D)$ = (0.02)*(0.28) + (0.015)*(0.32) + (0.025)*(0.18) + (0.01)*(0.22) =
0.0171

2.27 $P(Vote) = P(Vote \mid D)*P(D) + P(Vote \mid R)*P(R) + P(Vote \mid I)*\mathrm{P}(I) =$
(0.75)*(0.44) + (0.05)*(0.36) + (0.60)*(0.2) = **0.468**

2.28 $S =$

$$\left\{\begin{matrix} 1.1 & 1.2 & 1.3 & 1.4 & 1.5 & 1.6 \\ 2.1 & 2.2 & 2.3 & 2.4 & 2.5 & 2.6 \\ 3.1 & 3.2 & 3.3 & 3.4 & 3.5 & 3.6 \\ 4.1 & 4.2 & 4.3 & 4.4 & 4.5 & 4.6 \\ 5.1 & 5.2 & 5.3 & 5.4 & 5.5 & 5.6 \\ 6.1 & 6.2 & 6.3 & 6.4 & 6.5 & 6.6 \end{matrix}\right\}$$

Y = Difference between the two numbers.

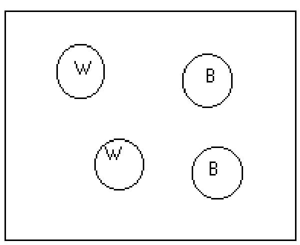
$R_Y = \{0, 1, 2, 3, 4, 5\}$
p.m.f. of Y=

Y	0	1	2	3	4	5
P(Y)	6/36	10/36	8/36	6/36	4/36	2/36

$$\mu_y = (0)(6/36) + \ldots + (5)(2/36) = \frac{(0+10+16+18+16+10)}{36} = 70/36 = \mathbf{1.944}$$

$$\sigma^2_y = (0 - 1.94)^2 (6/36) + \ldots + (5 - 1.95)^2 (2/36) = \mathbf{2.05}$$

2.29



$R_x : \{1, 2\}$

$$p(1) : \frac{\binom{2}{2}\binom{2}{1}}{\binom{4}{3}} = \frac{(1*2)}{4} = 0.5 \qquad p(2) : 1 - P(1) = 0.5$$

p.m.f. of X :

x	1	2
p(x)	.5	.5

$$\mu_x = 1 * .5 + 2 * .5 = \mathbf{1.5}$$
$$\sigma^2_x = (1 - 1.5)^2 * (0.5 + (2 - 1.5)^2 * (0.5) = \mathbf{0.25}$$

2.30 *P(X<4)* = $\int_2^4 \frac{2(1+x)}{27}dx$ = $\frac{2}{27}\int_2^4 (1+x)dx$ = 16/27

$\mu_x = \frac{2}{27}\int_2^5 x(1+x)dx = \frac{2}{27}\left[\left(\frac{x^2}{2}\right)\Big|_2^5 + \left(\frac{x^3}{3}\right)\Big|_2^5\right]$ = **3.66**

2.31 $\int f(x)dx = 1 \Rightarrow \int_0^1 ax^3 dx = 1 \Rightarrow a\left(\frac{x^4}{4}\right)\Big|_0^1 = 1 \Rightarrow \frac{a}{4} = 1 \Rightarrow a = 4$

F(x) = $\int_0^x 4t^3 dt = 4\left(\frac{t^4}{4}\right)\Big|_0^x = x^4, 0 < x < 1$

F(1/2) = 1/16, F(3/4) = 81/256, *P(1/2 < X < 3/4)* = F(3/4) **-** F(1/2) = **65/256**

$\mu_x = \int_0^1 x4x^3 dx =$ **4/5** $\sigma^2_x = \int_0^1 (x-4/5)^2 * 4x^3 dx =$ **4/150**

2.32 Let *X* be the number of engines failing during a flight:

X~Bi(4, 0.3)$\Rightarrow p(x) = \binom{4}{x}(.3)^x(.7)^{4-x}, x = 0, 1, 2, 3$

$P(X \le 2) = p(0) + p(1) + p(2) = \binom{4}{0}(.3)^0(.7)^4 + \binom{4}{1}(.3)^1(.7)^3 + \binom{4}{2}(.3^2(.7)^2 =$

$0.7^4 + (4)(0.3)(0.7)^3 + (6)(0.3)^2(0.7)^2$ = **0.9163**

2.33 *P(Accept) = P(# of defectives in the sample = 0)*
If *X* is # of defectives in the sample of 8, then *X~Bi*(8, 0.02)

$\Rightarrow p(x) = \binom{8}{x}(.02)^x(.98)^{8-x}$, x = 0, 1, 2, ... , 8

$p(0) = \binom{8}{0}(.02)^0(.98)^8$ = **0.85**

2.34 *X* = # of accidents per month, so *X~Po*$(\lambda = 3)$

$\Rightarrow p(x) = \frac{e^{-3}3^x}{x!}$, x = 0, 1, 2, ...

$P(X \le 2) = P(0) + P(1) + P(2) = e^{-3}\left[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right] = e^{-3}\left[1 + 3 + \frac{9}{2}\right]$ = **0.423**

2.35 X = # of salvage holes per casing $\Rightarrow X\sim Po(\lambda = 3) \Rightarrow p(x) = \dfrac{e^{-3}3^x}{x!}$, x = 0, 1, 2, ...

$$P(X > 6) = 1 - P(X \le 6) = 1 - [p(0) + p(1) + p(2) + p(3) + p(4) + p(5) + p(6)] =$$

$$1 - e^{-3}\left[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!} + \frac{3^6}{6!}\right] = 0.0335$$

Number of castings expected to be rejected out of 200 = 200(0.0335) = 6.7 $\simeq$ **7**.

2.36 $P(X \le 15) = P\left(z \le \dfrac{15-10}{5}\right) = \Phi(1) = \mathbf{0.8413}$

$$P(X \ge 12) = 1 - P(X < 12) = 1 - P\left(z < \frac{12-10}{5}\right) = 1 - \Phi(0.4) = 1 - .6554 = \mathbf{0.3446}$$

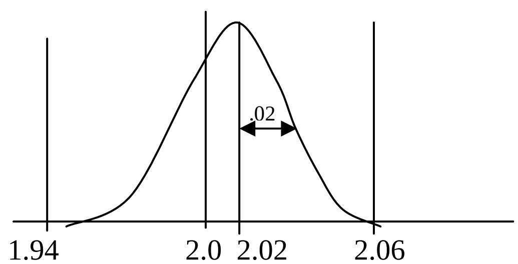
$$P(9 \le X \le 20) = \Phi\left(\frac{20-15}{5}\right) - \Phi\left(\frac{9-10}{5}\right) = \Phi(2) - \Phi(-0.2) = .9772 - .4207 =$$

0.5565

2.37 (a) $D\sim N(2.02, .02^2) \Rightarrow P(D < 1.94) = \Phi\left(\dfrac{1.94-2.02}{.02}\right) = \Phi(-4) = \mathbf{0}$

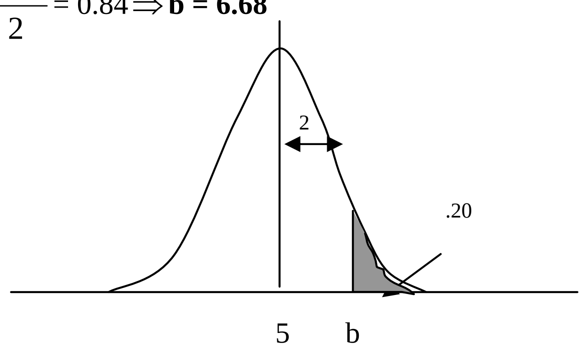
(b) $P(D > 2.06) = 1 - \Phi\left(\dfrac{2.06-2.02}{.02}\right) = 1 - \Phi(2) = 1 - .09772 = \mathbf{0.0228}$

(c) $P(1.94 < D < 2.06) = \mathbf{0.9772}$



2.38 We want b such that $P(X > b) = .2 \Rightarrow P(X \le b) = 0.8$ i.e. $P\left(z \le \dfrac{b-5}{2}\right) = .8$

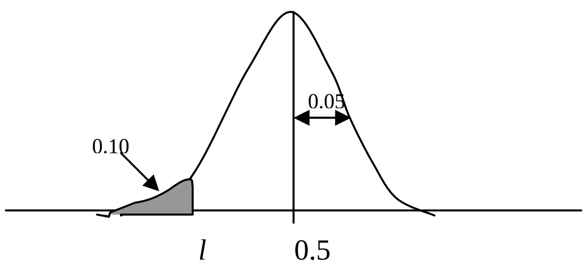
From table $\dfrac{b-5}{2} = 0.84 \Rightarrow$ **b = 6.68**



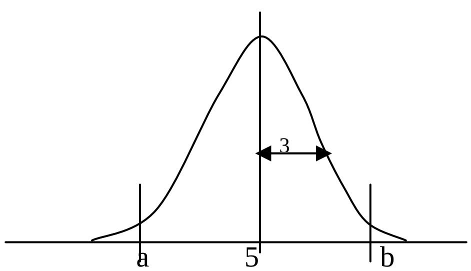
2.39 X= Thickness of blanks $X \sim N(0.5, 0.05^2)$

$$P(X < l) = 0.10 \Rightarrow P\left(z \le \frac{l - 0.5}{0.05}\right) = 0.10$$

$$\frac{l - 0.5}{0.05} = -1.28 \Rightarrow l = \mathbf{0.436}$$



2.40



$P(a \le X \le b) = 0.8$

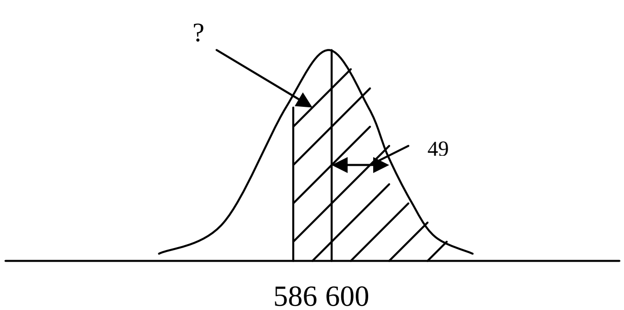
$$P\left(\frac{a-5}{3} \le z \le \frac{b-5}{3}\right) = 0.8 \Rightarrow P\left(z \le \frac{a-5}{3}\right) = 0.1 \text{ because of symmetry}$$

$$\Rightarrow \frac{a-5}{3} = -1.28 \Rightarrow$$

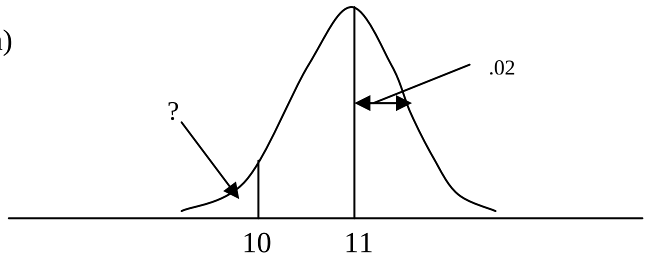
***a* = 1.16, *b* = 8.84**

2.41 X = Life of the batteries $X \sim N(600, 49^2)$

Need $P(X>586) = 1 - P\left(z \le \frac{586-600}{49}\right) = 1 - \Phi(-0.29) = 1 - 0.3859 = \mathbf{0.614}$



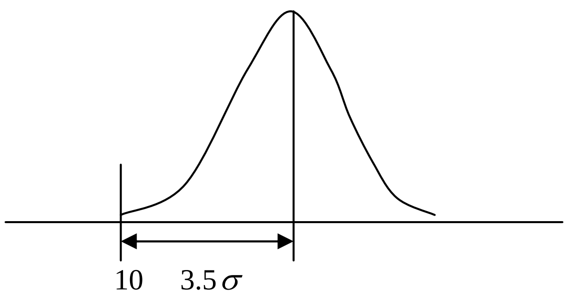
2.42 (a)



X = Amount of cooking oil in the cans, X ~ N (11, 0.2^2)

Need $P(X < 10) = P\left(z < \dfrac{10-11}{0.2}\right) = \Phi(-5) = \mathbf{0.00}$

(b)



Assuming $\Phi(-3.5) = 0$, the average can be lowered to 10 + (3.5)(0.2) = **10.70**.

2.43 99% C.I. for mean strength $\mu = \left[\bar{x} \pm z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}\right]$

$(1-\alpha) = 0.99 \Rightarrow \alpha = 0.01 \Rightarrow \alpha/2 = 0.005 \Rightarrow z_{\alpha/2} = 2.575$

99% C.I. for $\mu = \left[180 \pm (2.575)\dfrac{15}{\sqrt{12}}\right] = [180 \pm 11.15] =$ **[168.85, 191.15]**

2.44 $\bar{x} = 1389.9$, $\alpha/2 = 0.025$, $z_{\alpha/2} = 1.96$

95% CI. for the average life of bulbs $\mu = \left[1389.9 \pm 1.96\dfrac{100}{\sqrt{9}}\right] =$

$[1389.9 \pm 65.33]$ = **[1324.57, 1455.23] hrs.**

2.45 This is a case where the population std. deviation σ is not known. We use the sample standard deviation *s* as a measure of variability, and use the *t*-distribution

$(1-\alpha)100\%$ C.I. for the pop. mean $\mu = \left[\bar{x} \pm t_{\alpha/2,n-1}\dfrac{s}{\sqrt{n}}\right]$

$(1-\alpha) = 0.99 \Rightarrow \alpha/2 = 0.005 \Rightarrow t_{.005,19} = 2.861$

99% C.I. for $\mu = \left[1.15 \pm 2.861\dfrac{0.25}{\sqrt{20}}\right] = [1.15 \pm 0.16]$ = **[0.99, 1.31]**

2.46 From sample, $\bar{x} = 1389.9, s = 87$ Need 90% C.I. for μ

$\alpha = 0.10 \Rightarrow \alpha/2 = 0.05 \Rightarrow t_{0.05,8} = 1.86$

90% C.I. for $\mu = \left[1389.9 \pm 1.86 \frac{87}{\sqrt{9}}\right] = [1389.9 \pm 53.94]$ = **[1335.96, 1443.84]**

2.47 $\bar{x} = 6.234$, s = 0.025

99% C.I. for mean $\mu = \left[\bar{x} \pm t_{0.005,14} \frac{s}{\sqrt{n}}\right] = \left[6.234 \pm 2.977 \frac{0.025}{\sqrt{15}}\right]$ **= [6.215, 6.253]**

99% C.I. for variance $\sigma^2 = \left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-2}}\right] = \left[\frac{14(0.025)^2}{\chi^2_{0.005,14}}, \frac{14(0.025)^2}{\chi^2_{0.995,14}}\right]$

(=31.32) (=4.07)

= **[0.000273, 0.00215]**

99% C.I. for std deviation $\sigma = \left[\sqrt{\mathbf{0.000273}}, \sqrt{0.00215}\right] =$ **[0.0165, 0.0464]**

2.48 n=15 $\bar{x} = 70.61$ s=13.73

95% C.I. for mean $\mu = \left[\bar{x} \pm t_{0.025,14} \frac{13.73}{\sqrt{15}}\right] = \left[70.61 \pm 2.145 \frac{13.73}{\sqrt{15}}\right] = \left[70.61 \pm 7.604\right]$

= **[63.006, 78.214]**

To get the 95% C.I. for σ, we first obtain the 95% C.I. for σ^2

95% C.I. for $\sigma^2 = \left[\frac{14(13.73)^2}{\chi^2_{0.025,14}}, \frac{14(13.73)^2}{\chi^2_{0.975,14}}\right] = [100.96, 468.77]$

(=26.14) (=5.63)

95% C.I. for $\sigma = \left[\sqrt{100.96.45}, \sqrt{468.77}\right] =$ **[10.05, 21.65]**

2.49 H_0: $\mu = 90\%$ H_1: $\mu < 90\%$

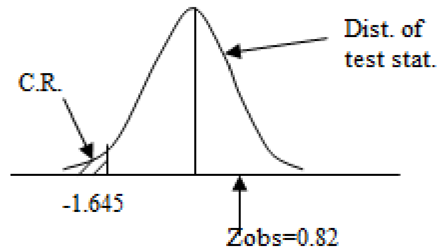
$\alpha = 0.05$ $\bar{x} = 90.82$ $\sigma = \sqrt{5}$ n=5

Test stat: $\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim$ z

C.R.: All z obs $\leq -z_\alpha = -z_{0.05} = -1.645$

$z_{obs} = \frac{90.82 - 90}{\sqrt{5}/\sqrt{5}} = 0.82$

z_{obs} is not in the C.R. $\Rightarrow$ do not reject H_o. There is no reason to believe the yield is less than 90%



2.50 H_0: $\mu = 125$

H_1: $\mu > 125$

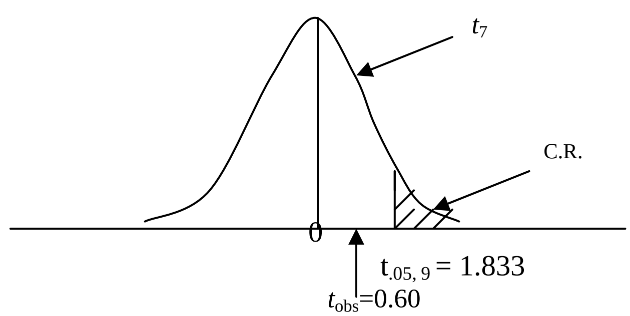
$\alpha = 0.05$ $\bar{x} = 128.8$ $s = 19.98$ n=10

Test stat: $\dfrac{\bar{x} - 125}{s / \sqrt{n}} \sim t_9$

C.R.: Reject H_0 if $t_{obs} > t_{crit.} = t_{0.05,9} = 1.833$

$t_{obs} = \dfrac{128.8 - 125}{19.98 / \sqrt{10}} = 0.60 \Rightarrow t_{obs}$ is not in C.R. $\Rightarrow$ do not reject H_0 (see sketch below)

Shelf life in not greater than 125 days.



2.51 H_0: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 \neq 0$

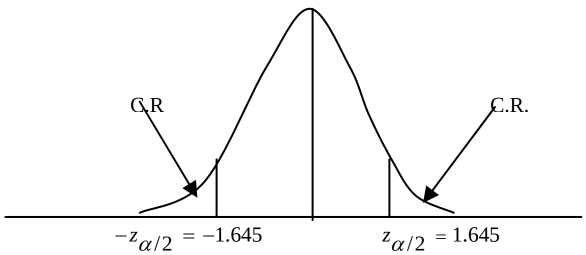
$\alpha = .10$ n_1=10 $\bar{x}_1 = 16.01$ n_2=10 $\bar{x}_2 = 16.021$

Test stat.: $\dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}} \sim z$

C.R.: $z_{obs} < -z_{\alpha/2} = -1.645$ or $z_{obs} > z_{\alpha/2} = 1.645$

$$z_{obs} = \frac{-0.011}{\sqrt{\frac{.015^2}{10} + \frac{.018^2}{10}}} = \frac{-.0.11}{0.0074} = -1.485$$

The z_{obs} is not in the C.R. $\Rightarrow$ Do not reject H_0. Both machines are filling equal volumes.



2.52

H_0: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 \neq 0$

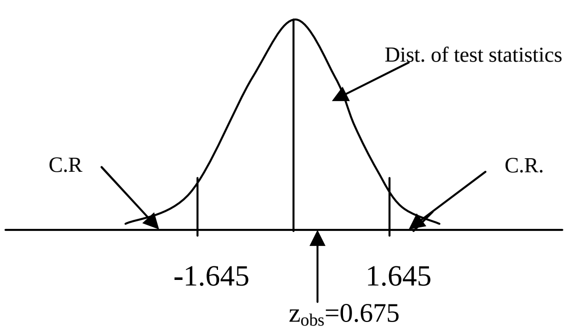
$\alpha = .10$ $n_1 = 8$ $\bar{x}_1 = 68575$ $n_2 = 8$ $\bar{x}_2 = 64525$

Test stat.: $\frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}} \sim z$

C.R.: $z_{obs} < -z_{\alpha/2}$ or $z_{obs} > z_{\alpha/2}$

$$z_{obs} = \frac{68575 - 64525}{\sqrt{\frac{12000^2}{8} + \frac{12000^2}{8}}} = \frac{4050}{6000} = 0675$$

z_{obs} is not in the C.R. $\Rightarrow$ Do not reject H_0. There seems to be no difference in salaries.



2.53 n = 98, X_{min}= 876, X_{max} = 901, R = 25, # of cells = 3.3 log (98) = 6.6
Make cell width = 5, # of cells = 6

Cell limits	Actual number a_i	Expected number e_i	Adjusted e_i	$(e_i-a_i)^2$
876 – 880	3	0.5	8.8	7.84
881 – 885	3	8.3		
886 – 890	37	33.4	33.4	12.96
891 – 895	45	40.4	40.4	21.16
896 – 900	8	12.3	13.3	10.89
901 – 905	2	1		
	98			**52.85**

H_o: Data came from $N\,(\mu = 891, \sigma^2 = 4.1^2)$
H_1: H_0 not true

To calculate the expected # of observations in each cell:

$$P(875.5 \le X \le 880.5) = \Phi\left(\frac{880.5-891}{4.1}\right) - \Phi\left(\frac{875.5-891}{4.1}\right) = 0.0052 - 0 = 0.0052 \ \ (*100)$$

$$P(880.5 \le X \le 885.5) = \Phi(-1.34) - \Phi(-2.56) = 0.0901 - 0.0052 = 0.085(*100)$$
$$P(885.5 \le X \le 890.5) = \Phi(-0.12) - \Phi(-1.34) = 0.4522 - 0.1112 = 0.341(*100)$$
$$P(890.5 \le X \le 895.5) = \Phi(1.10) - \Phi(-0.12) = 0.8643 - 0.4522 = 0.412(*100)$$
$$P(895.5 \le X \le 900.5) = \Phi(2.31) - \Phi(1.10) = 0.9896 - 0.8643 = 0.125(*100)$$
$$P(900.5 \le X \le 905.5) = \Phi(3.54) - \Phi(2.31) = 1.00 - 0.9896 = 0.01(*100)$$

$\Sigma(e_i - a_i)^2 = 52.85$

$\chi^2_{2,0.05} = 5.99$

The $\chi^2_{obs} > \chi^2_{crit.} \Rightarrow$ Reject H_0

The data doesn't seem to come from a normal population.

2.54 $n = 72$, $X_{\min} = 47$, $X_{\max} = 74$, $R = 74 - 47 = 27$
To find the # of cells and cell width: 3.3 log (72) = 6.18.
Use 7 cells with width = 4
$\bar{x} = 53.8$ s = 5.4

Cell limits	Actual Frequency(a_i)	Exp. Freq.(e_i)	Adjusted exp. Freq. (e_i)	$(a_i\text{-}e_i)^2$
44 – 48	3	9.7	9.7	0.49
49 – 53	42	22.5	22.5	380.25
54 – 58	18	23.9	23.9	34.81
59 – 63	3 (adj.=9)	11.3	13.9	24.01
64 – 68	4	2.3		
69 – 73	1	0.3		
74 – 78	1	0		
	72			439.56

H_0: The data came from a population which is $N(\mu = 53.8, \sigma^2 = 5.4^2)$
To calculate the expected frequencies in the intervals:

$$P(43.5 \le x \le 48.5) = \Phi\left(\frac{48.5 - 53.8}{5.4}\right) - \Phi\left(\frac{43.5 - 53.8}{5.4}\right) = 0.1635 - 0.0281 = 0.1354(*72)$$
$$P(48.5 \le x \le 53.5) = \Phi(-0.06) - \Phi(-0.98) = 0.4761 - 0.1635 = 0.3126(*72)$$
$$P(53.5 \le x \le 58.5) = \Phi(0.87) - \Phi(-0.06) = 0.8078 - 0.4761 = 0.3317(*72)$$
$$P(58.5 \le x \le 63.5) = \Phi(1.80) - \Phi(0.87) = 0.9647 - 0.8078 = 0.1563(*72)$$
$$P(63.5 \le x \le 68.5) = \Phi(2.72) - \Phi(1.80) = 0.9967 - 0.9641 = 0.0326(*72)$$
$$P(68.5 \le x \le 73.5) = \Phi(3.65) - \Phi(2.72) = 1.00 - 0.9967 = 0.0033(*72)$$
$$P(73.5 \le x \le 78.5) = \Phi(4.57) - \Phi(3.65) = 0$$

$$\chi^2_{obs} = 439.56$$
$$\chi^2_{0.05,2} = 5.99 \Rightarrow$$

Reject H_0. The data do not seem to come from a normal population.