Chapter 2 - Thermodynamics of Airflow

Exercise 2.1. Derive the equivalent of Eq. (2.11) for helium gas, bulk gas T. Use the properties of density $\rho_0 = 3.15 \cdot 10^{-3}$ gm/in.^{3}, $c_P = 5.18$ J/(gm K), temperature $T_0 = -17.78\,^0$C ($0\,^0$F).

Check on Conversion Using Air:

ρ_{Air} = Density of Air at 0 ^{0}C = 0.081lb$_m$/ft^3

ρ_0= Density of Air at 0 ^{0}C in units of gm/in.^{3}

$$\rho_{Air} := 0.081 \qquad \rho_0 := \frac{\rho_{Air}}{62.43} \cdot 2.54^3 \qquad \rho_0 = 0.021$$

Input:

ρ_{He}= Helium density in lb$_m$/ft^3 at T = -17.78 ^{0}C (0 ^{0} F). $\rho_{He} := 0.012$

ρ_0= Helium density in gm/in.^{3} at T = -17.78 ^{0}C (0 ^{0} F).

c_P = Helium specific heat at constant pressure in J/gm K. $c_P := 5.18$

Calculate Helium Density at -17.77^{0}C:

$$\rho_0 := \frac{\rho_{He}}{62.43} \cdot 2.54^3 \qquad \rho_0 = 3.15 \times 10^{-3}$$

The Desired Formula:

$\Delta T = Q(T+273.16)/[\rho_0 c_P(T_0+273.16)G]]$

$$C := \frac{1}{\rho_0 \cdot c_P \cdot (-17.78 + 273.16)} \cdot \frac{60}{12^3} \qquad C = 8.333 \times 10^{-3}$$

Dummy values of T, Q and G to get rid of "apparent" errors:

$$T := 1 \qquad Q := 1 \qquad G := 1$$

$$\boxed{\Delta T := \frac{8.33 \cdot 10^{-3} (T + 273.16)\, Q}{G}}$$

Q [W], G [ft^3/min.], T [^{0}C], ΔT [K or ^{0}C]

Exercise 2.2. Compare the representations of the three air temperature rise formulae, Eqs. (2.12)-(2.14), assuming an inlet air temperature of 20 ^{0}C. Label the calculated air temperature rises with subscripts *E*, *A*, and *B* for Eqs. (2.12), (2.13), and (2.14), respectively. Use the ratio $Q/G = 1, 10, 30, 50, 100, 200$, and 300 as your independent variable, and additional values if you wish. Present your results in both tabular and graphical form. Be sure to use logarithmic axes for both Q/G and the temperature rise, otherwise your results will not be very meaningful. Assuming that Eq. (2.12) is the most exact, calculate the percent error that you get from Eq. (2.13) vs. Q/G for 1 to 100. Comment on the latter result and how it compares to the series approximations for $Q/G \le 30$.

Equation 2.12: $\Delta T_E := (T_I + 273.16)\cdot\left(e^{\frac{CQ}{G}} - 1\right)$

Equation 2.13: $\Delta T_A := \dfrac{1.76 \cdot Q}{G}$

Equation 2.14: $\Delta T_B := \dfrac{2\cdot(T_I + 273.16)}{\left(\dfrac{2\cdot G}{C\cdot Q} - 1\right)}$

$C := 5.99\cdot 10^{-3}$

Input ΔT_I=20 ^{0}C: $T_I := 20$

Write Equations with r=Q/G:

$\Delta T_E(r) := (T_I + 273.16)\cdot\left(e^{C\cdot r} - 1\right)$ $\quad \Delta T_A(r) := 1.76\cdot r$ $\quad \Delta T_B(r) := \dfrac{2\cdot(T_I + 273.16)}{\dfrac{2}{C\cdot r} - 1}$

Calculate Table Values of ΔT for r=Q/G of 1, 10, 30, 50, 100, 200, 300:

$\Delta T_E(1) = 1.761$ $\quad \Delta T_E(10) = 18.097$ $\quad \Delta T_E(30) = 57.711$ $\quad \Delta T_E(50) = 102.367$

$\Delta T_E(100) = 240.478$ $\quad \Delta T_E(200) = 678.221$ $\quad \Delta T_E(300) = 1.475\times 10^3$

$\Delta T_A(1) = 1.76$ $\quad \Delta T_A(10) = 17.6$ $\quad \Delta T_A(30) = 52.8$ $\quad \Delta T_A(50) = 88$

$\Delta T_A(100) = 176$ $\quad \Delta T_A(200) = 352$ $\quad \Delta T_A(300) = 528$

$\Delta T_B(1) = 1.761$ $\quad \Delta T_B(10) = 18.102$ $\quad \Delta T_B(30) = 57.882$ $\quad \Delta T_B(50) = 103.265$

$\Delta T_B(100) = 250.682$ $\quad \Delta T_B(200) = 875.825$ $\quad \Delta T_B(300) = 5.19\times 10^3$

r=Q/G	ΔT_E		ΔT_A
1	1.76	1.76	1.761
10	18.09	17.60	18.10
30	57.71	52.8	57.88
50	102.37	88.00	103.27
100	240.48	176.00	250.68
200	678.22	352.00	875.83
300	1.48×10^3	528.00	5.19x10

Calculate Table of Error:

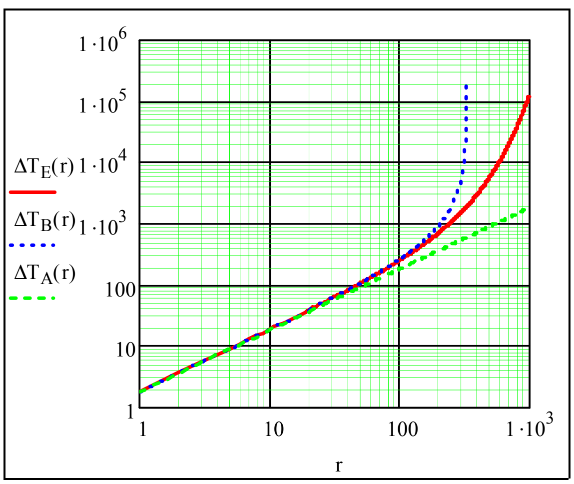
Define Equation and Calculate Values:

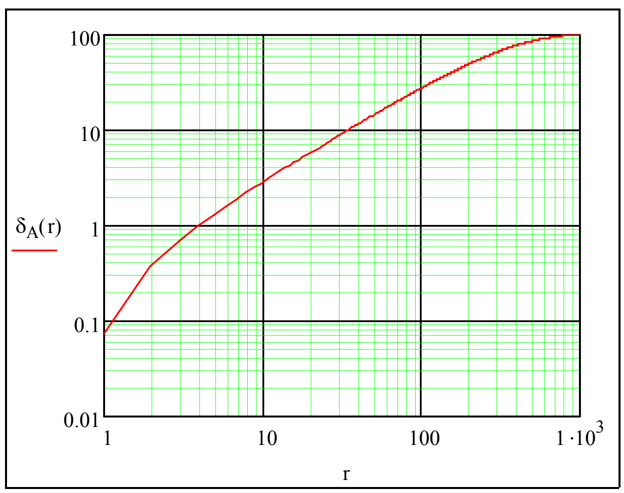
$$\delta_A(r) := \left| \frac{\Delta T_A(r) - \Delta T_E(r)}{\Delta T_E(r)} 100 \right|$$

$\delta_A(1) = 0.074$ $\delta_A(10) = 2.746$ $\delta_A(30) = 8.51$ $\delta_A(50) = 14.035$ $\delta_A(100) = 26.813$

r=Q/G	δ_A
1	-0.07
10	-2.75
30	-8.51
50	-14.04
100	-26.81

$r := 1, 2 .. 1000$





From the Table or Graph of r=Q/G, we see that $\Delta T = 1.76 Q/G$ is within 10% accuracy for Q/G approximately less than or equal to 30.

Exercise 2.3. In those situations where you wish to use an equation like Eq. (2.13) to calculate air temperature rise, with what values would you replace the 1.76 for inlet air temperature of 30 ^{0}C and 50 ^{0}C?

Basic Equation to Use from Text:

$$\Delta T := (T_I + 273.16)\left(\frac{C \cdot Q}{G}\right)$$

The Required Constant:

$$K(T_I) := (T_I + 273.16) \cdot 5.99 \cdot 10^{-3}$$

Values of the Constant for T_I of 20, 30, and 50 ^0C:

$\boxed{K(20) = 1.76}$ $\boxed{K(30) = 1.82}$ $\boxed{K(50) = 1.94}$