

Chapter 2 - Thermodynamics of Airflow

2.1 THE FIRST LAW OF THERMODYNAMICS

A reversible thermodynamic process is one in which the process produces exactly the same result when taken in either of two directions when traversing around a pressure-volume curve (referred to as the P - V curve) for the particular gas of interest. Such a curve is illustrated in Figure 2.1. The directional convention is that work done by the gas is positive when the curve is followed in a clockwise direction.

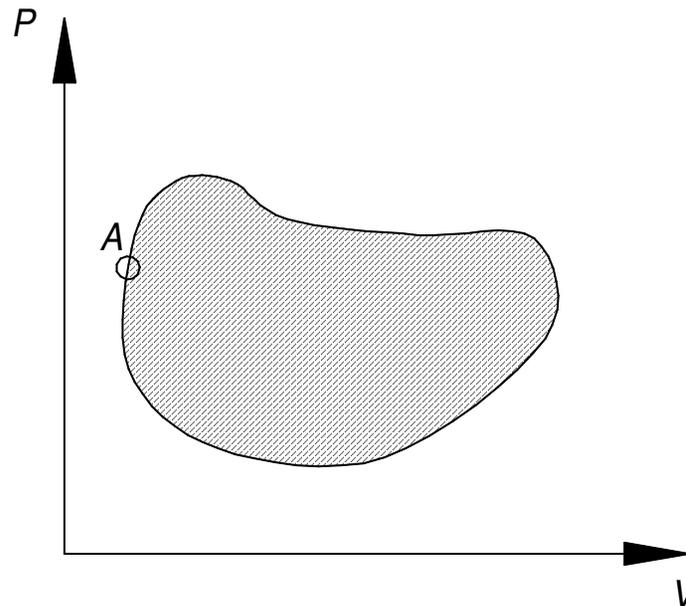


Figure 2.1. Thermodynamic P - V curve for an arbitrary gas.

Empirical studies have always shown that a quantity of work

$$W = \oint \bar{d}W = \oint P dV$$

appears or disappears for each cycle around the curve, beginning and ending at point A , for example.

The barred symbol \bar{d} means that the result depends on the path taken, the direct mathematical implication being that $\bar{d}W$ is not an exact differential and cannot be obtained by differentiating a function.

If we define an infinitesimal quantity of thermal energy, i.e. heat, as $\bar{d}q$, then the conservation of energy in a cyclic, closed (thermally isolated) system is

Equation (2.1):
$$\oint (\bar{d}q - \bar{d}W) = 0$$

which means that heat and work taken together are equivalent. Equation (2.1) cannot be proven analytically, but it has also never been shown to be false if all forms of work are included. Now suppose we are dealing with an open system, i.e. one that is not thermally isolated. In such a system the heat minus the work in a complete cycle is not conserved so that we say

$$\text{Heat into system} - \text{Work done by system} \neq 0$$

This is not a concern because *the system has changed state* and a balancing amount is taken from the system.

Therefore the mathematical equivalent to Eq. (2.1) is

Conservation of energy, (2.2): $dE = \bar{d}q - \bar{d}W$
 open system, First Law

which is our expression for *The First Law of Thermodynamics*. The quantity E is the *total system energy*. The change in energy also means a *change of state* for the system.

Consider a *quasi-static* system, by which we mean that any motion is very slow, so slow that each successive step is calculated using static processes.

Figure 2.2 shows a piston moving an infinitesimal distance L within a cylinder. Because the process is so slow, the internal pressure is very nearly equal to the external hydrostatic pressure. Within this static system, the internal force therefore equals the external force and, of course, P is constant.

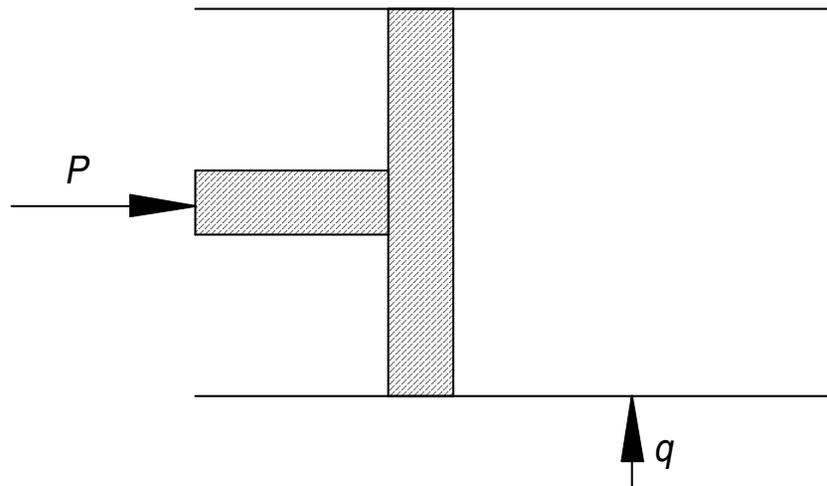


Figure 2.2. A quasi-static process with only hydrostatic processes.

The total energy E for the gas in our piston system is the sum of the internal (thermal) energy U , the kinetic energy KE , and the potential energy PE , etc. Then

$$E = U + KE + PE + \dots$$

But since we shall consider thermodynamic systems where the only changes are in the internal (thermal) energy, i.e. $dE = dU$, we write

Equation (2.3):
$$\bar{d}q = dE + \bar{d}W = dU + \bar{d}W$$

We can rewrite Equation (2.3) using $\bar{d}W = PdV$ to get *

$$dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV$$

$$\bar{d}q = dU + PdV = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV + PdV$$

Equation (2.4):
$$\bar{d}q = \left(\frac{\partial U}{\partial T} \right)_V dT + \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right] dV$$

* Remember - an equation of state has three variables: P , V , T . Only two can be independent.

2.2 HEAT CAPACITY AT CONSTANT VOLUME

The definition of heat capacity at constant volume, using Eq. (2.4) is

$$C_V \equiv \left(\frac{\bar{dq}}{dT} \right)_V = \left(\frac{\partial U}{\partial T} \right)_V$$

but it is more common practice to use $u \equiv$ internal energy per unit mass, so that

$c_V \equiv$ Heat capacity per unit mass

$$c_V \equiv \text{Specific heat capacity} = \left(\frac{\partial u}{\partial T} \right)_V$$

2.3 HEAT CAPACITY AT CONSTANT PRESSURE

The first steps here are to define the quantity of enthalpy H and to temporarily remove any conditions of constant P , V . Then

Enthalpy (2.5): $H = U + PV$

$$U = H - PV$$

and
$$dU = dH - PdV - VdP = \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP - PdV - VdP$$

Then
$$dU_P = \left(\frac{\partial H}{\partial T}\right)_P dT - PdV$$

and using
$$\bar{dq}_P = dU_P + \bar{d}W = dU_P + PdV$$

we obtain
$$\bar{dq}_P = \left(\frac{\partial H}{\partial T}\right)_P dT - PdV + PdV = \left(\frac{\partial H}{\partial T}\right)_P dT$$

We now define the heat capacity at constant pressure and use the preceding result to get

$$C_P \equiv \left(\frac{\bar{dq}}{dT}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P$$

Using the usual heat capacity per unit mass, i.e. specific heat, c_p , we have

Specific heat, constant P (2.6):
$$c_P = \left(\frac{\partial h}{\partial T} \right)_P$$

where we have used $u \equiv$ internal energy per unit mass and $h \equiv$ enthalpy per unit mass. This ends the specific heat capacity discussion with the result that we will use in the next section.

2.4 STEADY GAS FLOW AS AN OPEN, STEADY, SINGLE STREAM

We make the assumption of a reversible process. Consider the stream shown in Figure 2.3 with the definitions of

$\dot{m} \equiv$ mass flow rate

$u \equiv$ internal energy per unit mass

$\bar{V} \equiv$ volume per unit mass, $\bar{V} = \bar{V}_1 = \bar{V}_2$

$Q \equiv$ net heat rate input

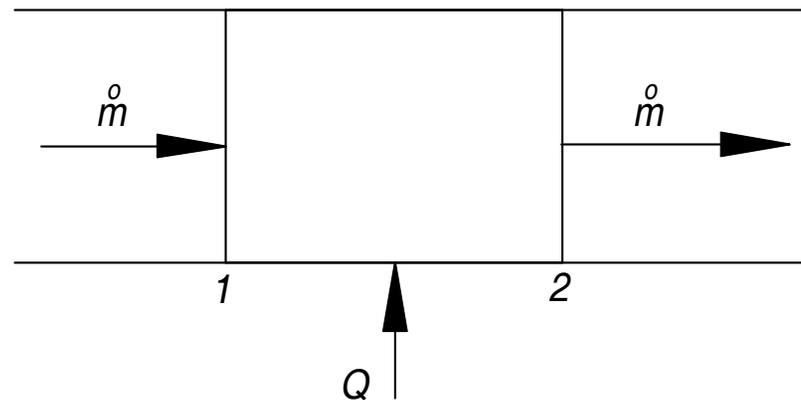


Figure 2.3. A steady, open, single stream flow.

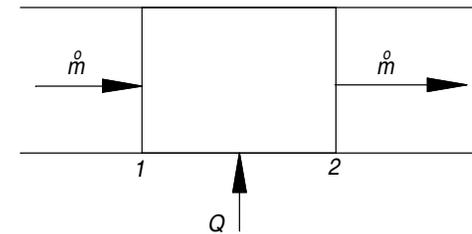
We apply the conservation of energy, which is written as

Transport-rate of internal energy into control volume + net heat rate added + work-rate to push \bar{V} into control volume = Transport-rate of internal energy out of control volume + work-rate to push volume \bar{V} out of control volume.

The preceding statement of conservation of energy as a mathematical statement is

$$\dot{m}u_1 + Q + \dot{m}P_1\bar{V} = \dot{m}u_2 + \dot{m}P_2\bar{V}$$

$$\dot{m}(u_1 + P_1\bar{V}) + Q = \dot{m}(u_2 + P_2\bar{V})$$



Substituting the enthalpy definition, Eq. (2.5), $h = u + P\bar{V}$ as a per/unit mass into the preceding,

$$\dot{m}h_1 + Q = \dot{m}h_2$$

$$Q = \dot{m}(h_2 - h_1) = \dot{m}\Delta h$$

Enthalpy change in stream flow (2.7): $\Delta h = Q / \dot{m}$

If a gas obeys the ideal gas law, as air certainly does at modest pressures, the internal energy and enthalpy are functions of temperature only (see J.P. Holman, 1974 or E. Fermi, 1937) and we can write $(\partial h/\partial T)_P = dh/dT$ and using our result, Eq. (2.6), $c_P = (\partial h/\partial T)_P = dh/dT$, we can then integrate

$$\Delta h = h_2 - h_1 = \int_{T_1}^{T_2} c_P dT$$

If c_p is temperature-independent, $\Delta h = c_P(T_2 - T_1)$. Setting this equal to Eq. (2.7), $\Delta h = Q/\dot{m}$, we obtain a very important equation for single stream, ideal gas flow.

Gas flow temperature rise, (2.8):

 general formula 1

$$Q = \dot{m} c_P (T_2 - T_1)$$

2.5 AIR TEMPERATURE RISE: TEMPERATURE DEPENDENCE

The formula for a gas flow temperature rise has been derived as Eq. (2.8) and is rewritten as

Gas flow temperature rise, (2.9):
 general formula 2

$$\Delta T = Q / \dot{m} c_P = Q / \rho G c_P$$

where

$\dot{m} \equiv$ mass flow rate

$c_P \equiv$ specific heat, constant pressure

$Q \equiv$ heat convected into air stream

$\rho \equiv$ air density

$G \equiv$ volumetric air flow

We use the ideal gas law:

$$PV = (m/M)RT'$$

where

$P =$ gas pressure, $V =$ gas volume,

$m =$ mass, $M =$ gas molecular weight

$R =$ gas constant

$T' = T + 273.16$, gas temperature T [°C]

Then $\rho = (m/V) = PM/RT'$ = the gas density at an absolute temperature T' and

$\rho_0 = PM/RT'_0$ = the gas density at some reference temperature T'_0 .

Assuming negligible pressure change from temperature T_0 to temperature T , we have an expression for the density at temperature T .

$$\rho = \rho_0 \left(\frac{T_0 + 273.16}{T + 273.16} \right)$$

Inserting the preceding into Eq. (2.9), $\Delta T = Q/\dot{m}c_p = Q/\rho Gc_p$,

Gas flow temperature (2.10):
rise, formula

$$\Delta T = \frac{Q(T + 273.16)}{\rho_0 c_p (T_0 + 273.16) G}$$

Now we are ready to insert appropriate values for ρ_0 , c_P , and T_0 into Eq. (2.10). Referring the Appendix *ii*, a table of air properties, and taking $T_0 = 0^\circ\text{C}$ we get

$$\Delta T = \frac{Q(T + 273.16)}{\rho_0 c_P (T_0 + 273.16) G} = \frac{Q[\text{J/s}](T + 273.16)[\text{K}]}{(0.021\text{gm/in.}^3)[1.01\text{J}/(\text{gm}\cdot\text{K})](273.16)[\text{K}]G[\text{in.}^3/\text{s}]}$$

$$\Delta T = \frac{0.17Q[\text{J/s}](T + 273.16)[\text{K}]}{G[\text{in.}^3/\text{s}]}$$

The author's preference is to use G in the units of $[\text{ft}^3/\text{min}]$. Then we must write

$$G\left[\frac{\text{in.}^3}{\text{s}}\right] = G\left[\frac{\text{ft}^3}{\text{min}}\right]\left(\frac{\text{min}}{60\text{s}}\right)\left(\frac{12\text{in.}}{\text{ft}}\right)^3$$

so that

$$\Delta T = \frac{Q[\text{J/s}](T + 273.16)[\text{K}]}{(0.021\text{gm/in.}^3)(1.01\text{J}/\text{gm}\cdot\text{K})(273.16)[\text{K}]G[\text{in.}^3/\text{s}]} = \frac{0.1726Q[\text{J/s}](T + 273.16)[\text{K}]}{G\left[\frac{\text{ft}^3}{\text{min}}\right]\left(\frac{\text{min}}{60\text{s}}\right)\left(\frac{12\text{in.}}{\text{ft}}\right)^3}$$

Air (2.11):
$$\Delta T[^\circ\text{C}] = \frac{5.99 \times 10^{-3} (T[^\circ\text{C}] + 273.16) Q[\text{W}]}{G[\text{ft}^3/\text{min}]}$$

2.6 AIR TEMPERATURE RISE: T IDENTIFIED USING DIFFERENTIAL FORMS OF $\Delta T, \Delta Q$

The temperature T in Eq. (2.11) is the “cup mixing temperature” air temperature. But where in the air stream is this temperature taken? The next two sections help us to resolve what number to insert for T in Eq. (2.11).

If we shrink the finite T, Q in Figure 2.3 down to the infinitesimals $\Delta T, \Delta Q$ in Figure 2.4,

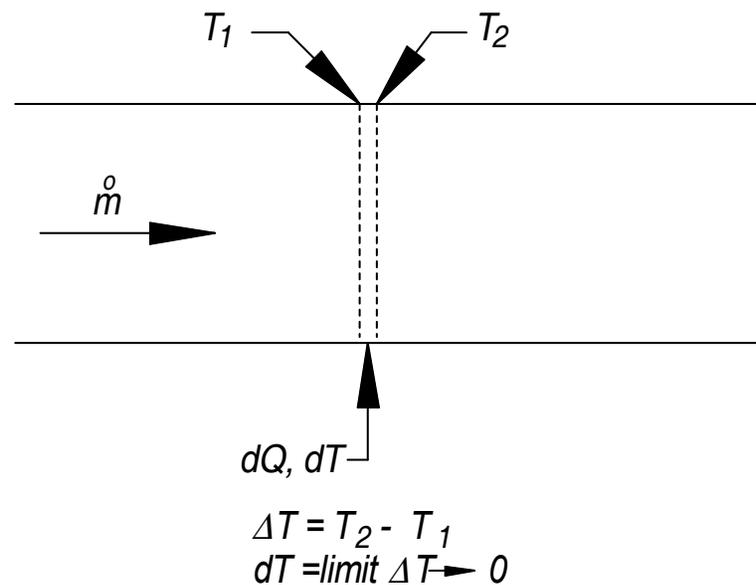


Figure 2.4. Infinitesimal heat transfer into fluid with infinitesimal temperature rise.

then

$$dT = \frac{C}{G}(T + 273.16)dQ, \quad C = 5.99 \times 10^{-3}$$

$$\frac{dT}{(T + 273.16)} = C \frac{dQ}{G}, \quad \int \frac{du}{u} = \frac{C}{G} \int dQ + B$$

$$\ln\left(\frac{T + 273.16}{A}\right) = \frac{C}{G}Q, \quad T = Ae^{CQ/G} - 273.16,$$

where $A, B = \text{constants}$. At $Q = 0, T = T_I$ and $A = T_I$, where T_I is the duct inlet air temperature. Then we finally get

$$(T + 273.16) = (T_I + 273.16)e^{CQ/G}$$

which, in a more useful form, is

Air temperature rise (2.12):
$$\Delta T = (T_I + 273.16)\left(e^{CQ/G} - 1\right)$$

$$C = 5.99 \times 10^{-3}$$

You may view Eq. (2.12) as an unexpected result, particularly if you have previous heat transfer experience. One way to check this result is to expand the exponential portion in a series as in

$$e^{CQ/G} = 1 + \left(\frac{CQ}{G}\right) + \frac{1}{2!}\left(\frac{CQ}{G}\right)^2 + \dots$$

Equation (2.12) then becomes

$$\Delta T = (T_I + 273.16)(e^{CQ/G} - 1) = (T_I + 273.16) \left[\left(\frac{CQ}{G} \right) + \frac{1}{2!} \left(\frac{CQ}{G} \right)^2 + \dots \right]$$

$$\Delta T \approx (T_I + 273.16) \left(\frac{CQ}{G} \right), \quad \frac{CQ}{G} \ll 2$$

and at $T_I = 20^\circ\text{C}$ we get our useful result of

Air temperature rise (2.13): $\Delta T = 1.76Q/G$

where the units are $\Delta T [^\circ\text{C}]$, $Q [\text{W}]$, and $G [\text{ft}^3/\text{min}]$. If you are rather particular, you might wish to replace the 1.76 with a different value when your inlet air temperature is different than 20°C .

2.7 AIR TEMPERATURE RISE: T IDENTIFIED AS AVERAGE BULK TEMPERATURE

Now let us look at Eq. (2.11) in a different way by identifying the bulk air temperature T , as the *average bulk temperature* \bar{T}_B so that

$$\Delta T [^{\circ}\text{C}] = \frac{5.99 \times 10^{-3} (\bar{T}_B + 273.16) Q [\text{W}]}{G [\text{ft}^3/\text{min}]}$$

and $\bar{T}_B = (T_I + T_E)/2$ where T_I and T_E are the duct inlet and exit air temperatures, respectively.

We then substitute \bar{T}_B into the preceding formula for ΔT to get

$$\Delta T [^{\circ}\text{C}] = \frac{5.99 \times 10^{-3} \left[\left(\frac{T_I + T_E}{2} \right) + 273.16 \right] Q [\text{W}]}{G [\text{ft}^3/\text{min}]} = \frac{5.99 \times 10^{-3} \left[\left(\frac{2T_I + \Delta T}{2} \right) + 273.16 \right] Q [\text{W}]}{G [\text{ft}^3/\text{min}]}$$

Solving for ΔT ,

Air temperature rise (2.14):
$$\Delta T = \frac{2(T_I + 273.16)}{\left(\frac{2G}{CQ} - 1 \right)}; C = 5.99 \times 10^{-3}$$

which you may also consider to be another unexpected result.

We again resort to one of our check methods for these strange looking results. In the case of Eq. (2.14), we make a small change to our last result and expand the resulting $()^{-1}$ term.

$$\begin{aligned} \Delta T &= \frac{2(T_I + 273.16)}{\left(\frac{2G}{CQ} - 1\right)} = \frac{2(T_I + 273.16)}{\left(1 - \frac{CQ}{2G}\right)} \left(\frac{CQ}{2G}\right) \\ &= (T_I + 273.16) \left(\frac{CQ}{G}\right) \left(1 - \frac{CQ}{2G}\right)^{-1} \\ &= (T_I + 273.16) \left(\frac{CQ}{G}\right) \left[1 + \left(\frac{CQ}{2G}\right) + \left(\frac{CQ}{2G}\right)^2 + \dots\right] \\ \Delta T &\approx (T_I + 273.16) \left(\frac{CQ}{G}\right), \quad \frac{CQ}{G} \ll 2 \end{aligned}$$

Substitute $T_I = 20^\circ\text{C}$ and the value for the constant C , we get the same result as Eq. (2.13),

$$\Delta T = 1.76Q/G$$

Both Eqs. (2.12) and (2.14) lead to the same approximate result, Eq. (2.13), when $CQ/G \ll 2$ or $Q/G \ll 300$ or $Q/G \leq 30$.

It is evident that when $2G/CQ$ is exactly equal to one in

$$\Delta T = \frac{2(T_I + 273.16) \left(\frac{CQ}{2G} \right)}{\left(1 - \frac{CQ}{2G} \right)}$$

the expression “blows up”.

When Q/G is such that the denominator is negative we also have nonsense.

The only instances when this has been a problem for myself were when I used Eq. (2.14) in a computer program where iterative calculations were such that there was one iterative step where $2G/CQ$ was exactly 1.0. This can be avoided by using Eq. (2.12) in your code.