

## Solutions Chapter 2

**2.1** In an isothermal quasistatic compression process on an ideal gas, there is no change in internal energy because the temperature is constant. For  $n = 1$  mol, the work done is given by

$$W = -\int_i^f P dV = -RT \int_{V_i}^{V_f} dV/V = -RT \ln(V_f/V_i), \quad (2.1.1)$$

where use has been made of the ideal gas equation of state. In the isothermal compression process, the pressure is doubled and it follows from the ideal gas law that  $V_f/V_i = 0.5$ . For  $T = 273$  K, Equation (2.1.1) gives  $W = 1573$  J.

Note that the work done is positive as expected for a compression process.

In order to maintain a constant temperature with  $E$  constant, it follows from the first law that  $Q = -W = -1573$  J.

**2.2** (a) In the isothermal expansion of the monatomic ideal gas at 280 K, the final pressure is  $P_f = P_i V_i/V_f = (30 \times 1.5)/3 = 15$  atm.

The number of moles is  $n = P_i V_i / RT = \left( \frac{30 \times 1.01 \times 10^5 \times 1.5 \times 10^{-3}}{8.314 \times 280} \right) = 1.95$  mol.

The work done *on* the gas is  $W = -nRT \ln(V_f/V_i) = -3147$  J.

(b) For the adiabatic expansion,  $PV^\gamma$  is constant, with  $\gamma = 5/3$  for a monatomic gas, and this leads to

$$P_f = P_i V_i^\gamma / V_f^\gamma = (30 \times 1.5^{5/3}) / 3^{5/3} = 9.45 \text{ atm.}$$

The final temperature is obtained with use of the ideal gas equation as  $T_f = 176$  K.

In the adiabatic process,  $\Delta Q = 0$  and it follows from the first law that the work done on the gas is  $W = \Delta E = \frac{3}{2} nR(T_i - T_f)$ , where use has been made of the equipartition theorem in order to determine  $\Delta E$ . This expression gives  $W = -2529$  J.

Note that the work done by the system is larger in the isothermal case than in the adiabatic case due to the larger decrease in pressure in the adiabatic process than in the isothermal process.

**2.3** The slopes of isothermals and adiabatics on a  $PV$  diagram are obtained by differentiation of the expressions for the pressure in terms of the volume in each case. For an isothermal, the ideal gas equation gives  $P = C/V$ , where  $C = nRT$  and the slope is given by

$$\left( \partial P / \partial V \right)_T = -C/V^2 = -P/V. \quad (2.3.1)$$

For an adiabatic process, the pressure expression is  $P = C'/V^\gamma$ , with  $C'$  a constant. The slope is

$$\left( \partial P / \partial V \right)_{\text{adiabatic}} = -\gamma C' / V^{\gamma+1} = -\gamma P / V. \quad (2.3.2)$$

Since  $\gamma > 1$ , it follows from Equations (2.3.1) and (2.3.2) that adiabatics are steeper than isothermals, with the ratio of the slopes given by  $\gamma$ .

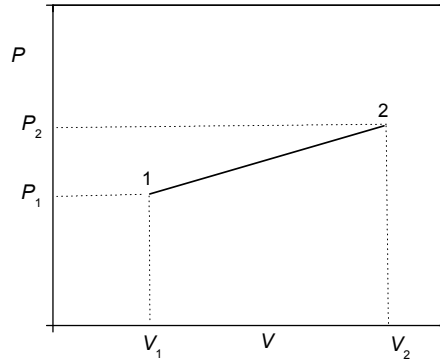
**2.4** For an adiabatic quasistatic process,  $P_1 V_1^\gamma = P_2 V_2^\gamma = C'$ , (2.4.1)

with  $C'$  a constant. The work done on the gas in an adiabatic process is

$$W = -\int_i^f P dV = -C' \int_{V_1}^{V_2} dV/V^\gamma = C'/(\gamma-1) \left[ 1/V_2^{\gamma-1} - 1/V_1^{\gamma-1} \right].$$

Use of Equation (2.4.1) for  $C'$  gives the required result  $W = 1/(\gamma-1) [P_2 V_2 - P_1 V_1]$ .

**2.5** The  $PV$  diagram shown in Figure 2.5.1 represents the process that is considered.



**Figure 2.5.1 Expansion process for an ideal gas from initial volume  $V_1$  to final volume  $V_2$ .**

The initial and final pressures and volumes are  $P_1 = 1.0$  atm,  $P_2 = 1.1$  atm,  $V_1 = 5$  L and  $V_2 = 10$  L. The number of moles  $n$  of argon is obtained from the ideal gas equation,

$$n = \frac{P_1 V_1}{RT_1} = \left( \frac{1.1 \times 10^5 \times 5 \times 10^{-3}}{8.314 \times 300} \right) = 0.22 \text{ mol.}$$

The work done on the system in the expansion process  $1 \rightarrow 2$  is given by the area under the line representing the process. From simple geometry this is

$$W = P_1 (V_2 - V_1) + \frac{1}{2} (P_2 - P_1) (V_2 - V_1) = -578 \text{ J.}$$

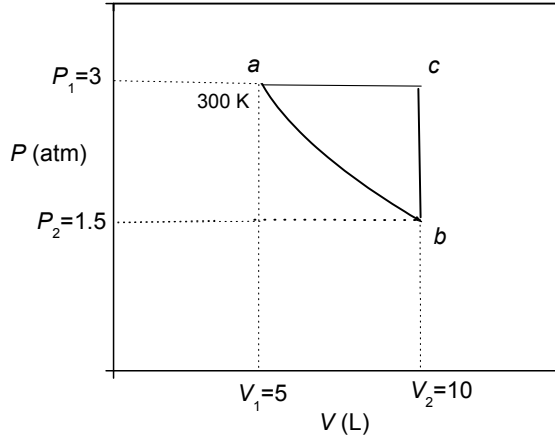
The final temperature  $T_2$  follows from the gas law and is given by  $T_2 = \left( \frac{P_2 V_2}{P_1 V_1} \right) T_1 = 660 \text{ K.}$

The change in internal energy is given by the equipartition theorem as

$$\Delta E = \frac{3}{2} nR (T_2 - T_1) = 988 \text{ J}$$

Finally, the heat input is obtained with use of the first law  $Q = \Delta E - W = 1566 \text{ J.}$

2.6 The  $PV$  diagram for the processes described has the form shown in Figure 2.6.1.



**Figure 2.6.1**  $PV$  diagram for an ideal gas cyclic process.

Let the temperature be  $T_1$  along the isothermal  $ab$ . The temperature at  $c$  is  $T_2 = (P_1/P_2)T_1 = 600$  K. This follows from the ideal gas equation with  $V_2$  held constant. The number of moles is also obtained from the ideal gas equation,

$$n = PV/RT = (3 \times 1.01 \times 10^5 \times 5 \times 10^{-3}) / (8.314 \times 300) = 0.607 \text{ mol}.$$

The work done *on* the system along the isothermal  $ab$  is

$$W_{ab} = -\int_a^b P dV = -nRT_1 \ln(V_2/V_1) = -1049.4 \text{ J}.$$

The minus sign shows that work is done by the system.

Along the isochoric ( $bc$ )  $W_{bc} = 0$  and for the isobaric process ( $ca$ )

$$W_{ca} = -P_2(V_1 - V_2) = 1515 \text{ J}.$$

The heat transferred in the constant energy isothermal process  $ab$  is obtained with the first law as  $\Delta E_{ab} = 0 = Q_{ab} + W_{ab}$  and this gives

$$Q_{ab} = nRT_1 \ln(V_2/V_1) = 1049.4 \text{ J}.$$

For the isobaric and isochoric processes, respectively, the molar specific heats are

$c_p = \frac{5}{2}R$  and  $c_v = \frac{3}{2}R$ . These expressions give for the isobaric process

$$Q_{ca} = \frac{5}{2}nR(T_1 - T_2) = -\frac{5}{2} \times 0.607 \times 8.314 \times 300 = -3784.9 \text{ J}$$

and for the isochoric process

$$Q_{bc} = \frac{3}{2}nR(T_2 - T_1) = 2271 \text{ J}.$$

In returning to the starting point at  $a$  after completion of the three processes, the net change in internal energy  $\Delta E_{\text{cycle}}$  is zero. It follows that  $\sum_{\text{cycle}} (W + Q) = 0$ , or heat out is equal to

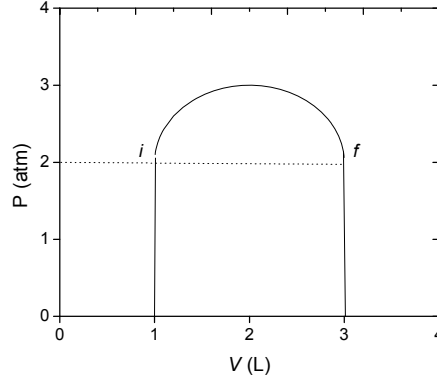
work in, and this requirement is consistent with the numerical values obtained.

**2.7** The  $PV$  diagram for the process described has the form shown in Figure 2.7.1. From the ideal gas law the change in temperature  $\Delta T$  for 0.1 mol of monatomic gas is

$$\Delta T = T_f - T_i = (P_f V_f - P_i V_i) / nR = (2 \times 3 - 2 \times 1) (1.01 \times 10^5 \times 10^{-3}) / (0.1 \times 8.314),$$

which gives  $\Delta T = 486$  K.

The change in internal energy is  $\Delta E = \frac{3}{2} nR \Delta T = \frac{3}{2} \times 8.314 \times 486 = 6061$  J.



**Figure 2.7.1**  $PV$  diagram for an ideal gas expansion process along a semicircular path.

The work done is

$$W_{if} = - \int_i^f P dV = - \left( 2 \times 2 + \frac{1}{2} \pi \times 1^2 \right) (1.01 \times 10^5 \times 10^{-3}) = -563 \text{ J}.$$

The integration has been carried out by determining the area under the curve geometrically.

The heat absorbed by the system follows from the first law,

$$Q_{if} = \Delta E_{if} - W_{if} = 1169 \text{ J}.$$

**2.8** The magnetic work done on the sample in an isothermal magnetization process  $1 \rightarrow 2$  shown in Figure 2.8.1, in which the magnetic field is increased from 0 to 10 T at 2 K, is

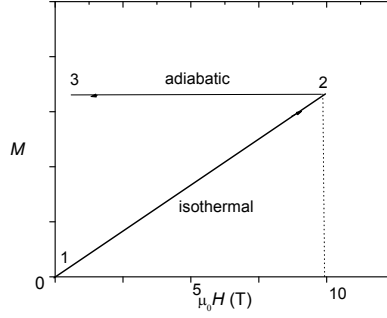
$$W = -(\mu_0 C V / T) \int_0^{H_2} H dH = \frac{1}{2} (\mu_0 C V / T) H_2^2. \quad (2.8.1)$$

The Curie constant  $C$  for the paramagnet is  $0.5 \text{ K/m}^3$ . The volume of the material of mass  $m$  and density  $\rho$  is  $m/\rho = 0.4/4 = 0.1 \text{ cm}^3$  and the field  $H_2 = B_2/\mu_0$ . Substituting in (2.8.1) gives for the work done

$$W = - (0.5 \times 10^{-7} \times 100) / (2 \times 4 \pi \times 10^{-7} \times 2) = -0.995 \text{ J}.$$

In the quasistatic adiabatic demagnetization process  $2 \rightarrow 3$  from 10 T to 0.1 T, the temperature of the thermally isolated sample decreases to a final value given by

$$T_3 = (H_3/H_2) T_2 = 0.02 \text{ K}.$$



**Figure 2.8.1** *MH* diagram for a paramagnet undergoing isothermal and adiabatic processes.

The change in internal energy is  $\Delta E = -\mu_0 CV \left[ H_3^2 / T_3 - H_2^2 / T_2 \right] = 1.97 \text{ J}$ .

Since no heat is exchanged in the adiabatic process, i.e.  $Q = 0$ , it follows from the first law that  $W = \Delta E$ .

**2.9** When a solid of mass  $m_o$  at temperature  $t_o$  is dropped into a well-lagged vessel containing a mass  $m_w$  of water at temperature  $t_w$ , heat is exchanged between the solid and the water until equilibrium is reached at a final temperature  $t_f$ . No work  $W$  is done and no heat  $Q$  is exchanged with the surroundings. The internal energy of the composite system does not change and from the first law it follows that

$$Q = m_s c_s (t_f - t_o) - m_w c_w (t_f - t_w) = 0.$$

The specific heat of the solid is given by  $c_s = \frac{m_w c_w (t_f - t_w)}{m_s (t_o - t_f)}$ .

This method provides a straightforward approach for the measurement of the specific heats of solids at temperatures above the freezing point of water.

**2.10** The heat required to heat a diamond from an initial temperature  $T_i$  to a final

temperature  $T_f$  is  $Q = n \int_{T_i}^{T_f} C dT$ , (2.10.1)

where  $n$  is the number of moles of diamond and the specific heat at the temperatures of interest is  $C = 3R(4\pi/5)(T/\theta_D)^3$ .

For a one carat (200 mg) diamond, the number of moles is  $n = 0.2/12 = 0.167 \text{ mol}$ .

Substitution for  $C$  in Equation (2.10.1), followed by integration, gives  $Q = \frac{1}{4} n C (T_f^4 - T_i^4)$  and insertion of the value of the Debye temperature  $\theta_D = 2200 \text{ K}$  plus the gas constant results in

$$Q = 3 \times 0.167 \times 8.314 \times (4\pi/5) (1/2200)^3 \left[ (300^4 - 4^4) / 4 \right] = 0.2 \text{ J}.$$

**2.11** The work done in compressing the copper is given by the familiar expression

$W = -\int_i^f P dV$ . The isothermal compressibility of a material is defined as

$\kappa = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T$  and use of the compressibility gives

$$W = \kappa \int_i^f V P dP. \quad (2.11.1)$$

In order to carry out the integral in Equation (2.11.1), it is convenient to make the approximation that  $V$  can be replaced by its average value and taken outside the integral. The justification for this step is the relatively small change in volume that occurs in the compression process. The fractional change in volume is readily estimated from the

relationship  $\int_{V_i}^{V_f} \frac{dV}{V} = -\kappa \int_{P_i}^{P_f} dP$ , which on integration gives  $\ln(V_f/V_i) = -\kappa(P_f - P_i)$ .

Replacing  $V_f$  by  $V_i + \Delta V$ , taking  $P_f - P_i = \Delta P$  and expanding the logarithm leads to the result  $\Delta V/V_i = -\kappa \Delta P = -(7.3 \times 10^{-5} \times 999) = -0.073$ . This shows that the volume decreases by just 7 % in the compression.

The volume  $V_i = m/\rho = 200/8.9 = 22.5 \text{ cm}^3$  and the relatively small volume change is  $\Delta V = -1.64 \text{ cm}^3$ .

The work done is obtained from Equation (2.11.1) by replacing  $V$  with  $\bar{V} = \frac{1}{2}(V_f + V_i)$  outside the integral as a good approximation so that the integral is simplified. This leads to

$$W \approx \kappa \bar{V} \int_i^f P dP = \frac{1}{2} \kappa \bar{V} (P_f^2 - P_i^2). \quad (2.11.2)$$

Inserting values in Equation (2.11.2) gives  $W = 80 \text{ J}$ .