Chapter 2

2.1. We can define a pure strategy as a choice to play Rock, Scissor or Paper at any position of the game. If the position of the game is determined by the current score (number of rounds won, irrespective of the order of results), then a pure strategy is a six-tuple (x_{p_1,p_2}) where $p_1 \in \{0,1,2\}$ is the score of Player 1 and $p_2 \in \{0,\dots,2-p_1\}$ is a score of Player 2. If a position of the game is given by the current score and the round number, then a pure strategy is a countable collection $(x_{p_1,p_2,n})$ $p_1 \in \{0,1,2,2\}$ is the score of Player 1 and $p_2 \in \{0,\dots,2-p_1\}$ is a score of Player 2 and $n = p_1+p_2, p_1+p_2+1,\dots$ is the round number.

2.2. Let C_P be the cost of colicin production, C_I be the cost of being immune ad $H > C_P + C_I$ be the harm. We then have

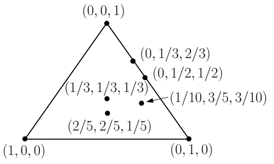
$$\begin{array}{l} \\ \text{Producing} \\ \text{Immune} \\ \text{Neither} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Producing} & \text{Immune} & \text{Neither} \end{array} \\ \begin{pmatrix} -(C_P+C_I) & -(C_P+C_I) & H-(C_P+C_I) \\ -C_I & -C_I & -C_I \\ -H & 0 & 0 \end{pmatrix} \end{array}.$$

Adding $C_P + C_I$ to the first collum and C_I to the second column yields

$$\begin{array}{l} \\ \text{Producing} \\ \text{Immune} \\ \text{Neither} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Producing} & \text{Immune} & \text{Neither} \end{array} \\ \begin{pmatrix} 0 & -C_P & H-(C_P+C_I) \\ C_P & 0 & -C_I \\ C_P+C_I-H & C_I & 0 \end{pmatrix} \end{array}.$$

2.3. Let p_{XY} be payoff to Paul if he went to bar X while John went to bar Y. The payoff matrix to Paul is then given by $\begin{pmatrix} p_{AA} & p_{AB} \\ p_{BA} & p_{BB} \end{pmatrix}$ where, $p_{AA} > p_{BB} > p_{AB} \approx p_{BA}$. Similarly for John.

2.4. Let E be the option for Paul to go somewhere else. Let j_{XY} be the payoff to John if he went to bar X while Paul went to bar Y. The payoff for John is then $\begin{pmatrix} j_{AA} & j_{AB} & j_{AE} \\ j_{BA} & j_{BB} & j_{BE} \end{pmatrix}$ where $j_{AA} > j_{BB} > j_{AB} \approx j_{BA} \approx j_{AE} \approx j_{BE}$. The payoff to Paul is given by $\begin{pmatrix} p_{AA} & p_{AB} \\ p_{BA} & p_{BB} \\ p_{EA} & p_{EB} \end{pmatrix}$ where $p_{BA} = p_{AB} > p_{EA} = p_{EB} > p_{AA} = p_{BB}$.

2.5. 

2.6. If a σ-strategist plays in the population $\sum \alpha_j \delta_{\mathbf{p}_j}$ where $\mathbf{p}_j$ is a mixed strategy $\mathbf{p}_j = \sum_i p_{j,i} S_i$, then the σ-strategist plays against S_i with probability $\sum_j \alpha_j p_{j,i}$. It is thus the same as playing in the population $\delta_{\bar{\mathbf{p}}}$.

2.7. $E[\mathbf{p}, \mathbf{q}] = \mathbf{p} A \mathbf{q}^{\mathbf{T}} = (-p_2 + p_3)q_1 + (p_1 - p_3)q_2 + (-p_1 + p_2)q_3 = p_1(q_2 - q_3) + p_2(q_3 - q_1) + p_3(q_1 - q_2)$. Let $m = \max\{q_2 - q_3, q_3 - q_1, q_1 - q_2\}$. The best reply to $\mathbf{q}$ is $(1, 0, 0)$ if $q_2 - q_3 = m$; it is $(0, 1, 0)$ if $q_3 - q_1 = m$; and it is $(0, 0, 1)$ if $q_1 - q_2 = m$.

2.8. (i) generic, (ii) technically non-generic but does not affect the analysis, (iii) non-generic, (iv) generic, (v) technically non-generic but does not affect the analysis, (vi) technically non-generic but does not affect the analysis, (vii) non-generic, (viii) non-generic.

2.9. By Exercise 2.6, the payoffs are the same as if the game is played against an individual playing $p = 0.4(0.5, 0.5) + 0.3(1, 0) + 0.3(0.2, 0.8) = (0.56, 0.44)$. We thus get $\mathcal{E}[(0.5, 0.5); \Pi] = (0.5, 0.5)A(0.56, 0.44)^{\mathbf{T}} = 0.4$ and similarly for other strategies, giving payoffs 0.56 and 0.464.

2.10. For the matrix games, the mean strategy is as in Exercise 2.9. $p = 0.4(0.5, 0.5) + 0.3(1, 0) + 0.3(0.2, 0.8) = (0.56, 0.44)$. If the opponent is selected based on its probability of playing S_1 the mean strategy is $\widetilde{p} = \frac{0.5}{0.5+1+0.2}(0.5, 0.5) + \frac{1}{0.5+1+0.2}(1, 0) + \frac{0.2}{0.5+1+0.2}(0.2, 0.8)$.

Chapter 3

3.1.