Chapter 2, Section 2.6

1. The relationship between disintegration constant (λ), meanlife (τ) and the half-life ($t_{1/2}$) are given by

$$T_{1/2} = 0.693 \times \tau = \frac{0.693}{\lambda}$$

Time conversion is 1 year = 3.156×10^7 s
1 day = 86400 s
1 hour = 3600 s
1 minute = 60 s

a)

Isotope	$T_{1/2}$	τ	λ (s^{-1})
14-C	5730 yrs	8268 yrs	3.83×10^{-12}
22-Na	2.6 yrs	3.75 yrs	8.45×10^{-9}
99-Mo	65.9 hours	95.13 hours	2.92×10^{-6}
56-Co	77.2 days	111.38 days	1.5×10^{-7}
238-U	4.47×10^9 yrs	6.45×10^9 yrs	4.91×10^{-18}

Note: We expressed the disintegration constant (λ) in second^{-1}, since it comes handy when we need to calculate the activities in Becquerels or Curies.

b) The table below lists the partial or multiples of meanlife (τ) lapses for each isotope per one week, one year, 10 years or 1000 years.

Isotope	τ	1week	1 year	10 years	1000 years
14-C	8268 yrs	2.32×10^{-6}	1.2×10^{-4}	1.2×10^{-3}	0.124
22-Na	3.75 yrs	5.13×10^{-3}	0.27	2.7	270
99-Mo	95.13 hours	1.766	89.8	898	89814
56-Co	111.38 days	0.091	4.61	46.1	4611
238-U	6.45×10^9 yrs	2.98×10^{-12}	1.5×10^{-10}	1.5×10^{-9}	1.5×10^{-7}

The activity of an isotope in a time lapse of $t = n\tau$, where 'n' is real number (integer or fraction), is given by $A(t = n\tau) = A_0 \times^{(-t/\tau)} = A_0 \times^{(-n\tau/\tau)} = A_0 \times e^{-n} = \frac{A_0}{e^n}$

Given the activity of each isotope at t=0 is $A_0 = 1\ \mu\text{Ci} = 3.7 \times 10^4$ Bq
For $A_0 = 1\ \mu\text{ci}$, and $t > 8\tau$, the activity is less than 1 Bq or less than 0.01% of the initial activity. We will show it as $A(t>8\tau) < 1$ Bq and not show insignificant figures

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Isotope	A(t=1 week)	A(t= 1 year)	A(t=10 years)	A (t=1000 years)
14-C	1 μCi	1 μCi	1 μCi	0.88 μCi
22-Na	1 μCi	0.76 μCi	0.067 μCi	<1 Bq
99-Mo	0.17 μCi	<1 Bq	<1 Bq	<1 Bq
56-Co	0.94 μCi	0.04 μCi	<1 Bq	<1 Bq
238-U	1 μCi	1 μCi	1 μCi	1 μCi

2. The 137-Cs has a half life of 30 years. During the period of September 1, 2000 till February 1, 2013, the time lapse is 12.5 years
Or, in terms of half lifes, 12.5 year = m $t_{1/2}$= m x 30 years or m = 0.42 halflifes.

Eq 2.5 : $\frac{A(mt_{1/2})}{A(0)} = 0.5^m$ gives, Activity on Feb.1, 2013 = $0.5^{0.42}$ μCi= 0.75 μCi.

The activity is 0.75 μCi = 2775 Bq. (1 μCi= 3.7 x 10^4 Bq)

3. The data for isotopes of interest:

Isotope	Half-life $T_{1/2}$	Mean life $\tau = T_{1/2}/0.693$
11-C	20 m	28.9 m
13-N	10 m	14.4 m
18-F	110 m	158.6 m
99-Mo	66 h	95.2 h
123-I	13 h	18.8 h

Equation 2.13 gives activity at time 't', A(t) is $A(t) = R\left(1-e^{-\lambda t}\right) = R\left(1-e^{-t/\tau}\right)$
where 'R' is the saturation activity.

Thus the time 't' to produce an activity A(t) is given by
$t = -\tau \ln\left[1-\frac{A(t)}{R}\right]$

For 99% activity, A(t)/R=0.99, t= 4.6 τ
For 60% activity, A(t)/R=0.6, t= 0.9 τ
For 20% activity, A(t)/R=0.2, t= 0.22τ

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Isotope	Meanlife τ= $T_{1/2}/0.693$	For 99%, t=4.6τ	For 60%, t = 0.9τ	For 20%, t= 0.22τ
11-C	28.9 m	133 m (2.2 h)	26 m	6.4 m
13-N	14.4 m	66 m (1.1 h)	13 m	3.2 m
18-F	158.6 m	730 m (12.1 h)	145 m (2.4 h)	35 m
99-Mo	95.2 h	438 h (18days)	87 h (3.6 d)	21 h
123-I	18.8 h	86.5 h (3.6days)	17 h	4.2 h

From the above equations and the numbers in the table above, we find

$$\frac{A(t)/R=0.99}{t}:\frac{A(t)/R=0.6}{t}:\frac{A(t)/R=0.2}{t}::4.6:1.5:1.1$$

or the efficiencies are 1: 2.8: 4.2 for 99%, 60% and 20%, respectively.

It is most effective to produce the 20%, and least effective to irradiate to get 99% activity, if we consider only the growth and decay of radiation. There may be other operational conditions such as times to change the samples, switching on and off of the activation processes and other operational procedures to be considered. We must take into consideration all these aspects as we determine the irradiation protocols.

4.

i) The parent ^{137}Cs is of half-life $T_{½}$ = 30 years >> daughter ^{137}Ba's excited state half life of 2.55 minutes. From section 2.3, we know the daughter ^{137}Ba level is in secular equilibrium with its parent.

ii) When an isotope of short half-life is in secular equilibrium with its parent of long half life, it will follow its parent's half-life. Thus, if we measure the half life of the 661keV photon emitting level in secular equilibrium with its parent (^{137}Cs ground state), we will measure its half life to be 30 years, the same as that of the parent.

iii) If we separate the ^{137}Ba from its parent, then the daughter has no feeding and it has no memory of its parent. It follows its own decay time. The activity will exhibit the 2.55 minutes half-life.
If the separation is not efficient, then the sample will exhibit a short lived component of the daughter's half life (2.55 minutes) and a long lived component of 30 years. The activity levels are proportional to the percentage of each species of atoms.

Below, we allow for arbitrary amount of contamination and give an example with specific numbers.

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Let us say, there are N_0 atoms in the sample at time t=0, an admixture of two species of atoms:
$N_{10} = N_0 x$ atoms of mean life τ_1 with $\lambda_1 = 1/\tau_1$.
$N_{20} = N_0(1-x)$ atoms of mean life τ_2 with $\lambda_2 = 1/\tau_2$.
Above, $0 \le x \le 1$.

Then we have

$$N(t) = N_{10}e^{-\lambda_1 t} + N_{20}e^{-\lambda_2 t} = N_0 x e^{-\lambda_1 t} + N_0(1-x)e^{-\lambda_2 t}$$

and the overall activity of the sample,

$$A(t) = -\lambda_1 N_0 x e^{-\lambda_1 t} - \lambda_2 N_0(1-x)e^{-\lambda_2 t}$$

We drop the − sign in the above equation to write the magnitude of activity

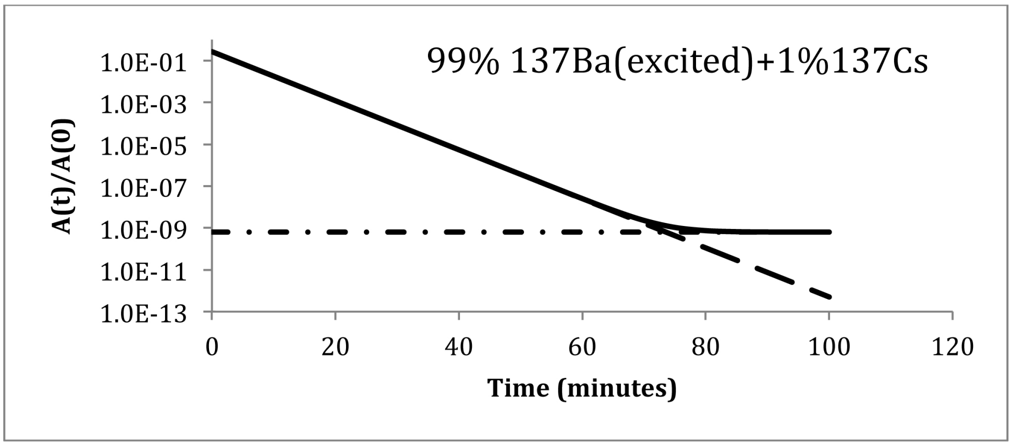
$$A(t) = \lambda_1 N_0 x e^{-\lambda_1 t} + \lambda_2 N_0(1-x)e^{-\lambda_2 t}$$

or $A(t) = A_1(t) + A_2(t)$, the sum of the activities of the two components.
In secular equilibrium, the daughter is very short lived compared to the parent's life. In the beginning, the daughter's activity is important. The parent's activity will become important for later times.

In our example, the daughter's half life is 2.55 minutes, corresponding to mean life $\tau_1 = 3.68$ minutes of disintegration constant $\lambda_1 = 0.27\ m^{-1}$.

The parent's half life is 30 years, mean life $\tau_2 = 43.3$ year and disintegration constant $\lambda_2 = 6.34 \times 10^{-8}\ m^{-1}$.
Note that we are using units of (1/minute) for the disintegration constant in this problem, as the short life time is in minutes.

Here $\lambda_1 >> \lambda_2$ and very likely x>> (1-x).



Above is a plot of the activity of the sample for 1% contamination of ^{137}Cs at t=0. At a given time 't', the solid curve is the total activity due to contributions of ^{137}Cs and ^{137}Ba. In the figure, the horizontal dash-dotted line represents the small, constant activity due to ^{137}Cs. Initially, the ^{137}Cs contribution is negligibly small due to the large difference in the lifetimes. The steep dashed line is the contribution of the separated ^{137}Ba. As we see in the figure, the contribution of separated ^{137}Ba comes down to about less than 10^{-8} of initial activity in an about an hour and becomes less than ^{137}Cs activity. For later times, ^{137}Cs is the main activity and ^{137}Ba becomes negligibly small.

5.

(i) The parent ^{99}Mo has a half-life of 66 hours, while the daughter ^{99m}Tc is of 6 hours. If we prepare a pure ^{99m}Tc at the point of production and transport it, the product will decay with the half-life of 6 hours. However, if we transport the daughter in equilibrium with parent, the half-life is that of the long living parent. So, the effective half-life is 11 times longer than the pure ^{99m}Tc transport. Say, it takes 66 hours (^{99}Mo half-life) to transport the isotope to a place. ^{99}Mo-^{99m}Tc would decay to 50% of initial activity, while only 0.05% activity will remain if we transport pure ^{99m}Tc. Also, at the clinic we can extract much more ^{99m}Tc over an extended period of time from the parent-daughter mixture.

(ii) From eq. (2.10), we have activity

$\frac{A(mt_{1/2})}{A(0)} = 0.5^m$, where 'm' is a positive number

The time of handling and transport is one week. With the parent's half life Of 66 hours, m = No. of hours in a week/66 = 2.546

Thus, A (2.546 x $t_{1/2}$) = $0.5^{2.546}$ x A(0)= 0.17.
Or the activity of the parent and daughter is 0.17 Ci for the production point activity of 1Ci assuming secular equilibrium.

In fact, it turns out that the daughter is in transient equilibrium with its parent, as the daughter's half-life is not too short lived compared to the parent's half-life.

The daughter activity in transient equilibrium is

$$A(t)^{daughter} = \frac{T_{1/2}^{parent}}{T_{1/2}^{parent} - T_{1/2}^{daughter}} A(t)^{parent}$$

After one week, ^{99m}Tc activity is 0.17 x 1.1 = 0.187 Ci for a parent activity of 1Ci at t=0

(iii) Again from eq. (2.10), we have activity

$\frac{A(mt_{1/2})}{A(0)} = 0.5^m$, where 'm' is a positive number.

We want $A(mt_{1/2})/A(0) > 0.1$, or $(0.1) > 0.5^m$
Taking logarithm both sides, $-1 > -0.3010\ m$
or $m < 1/0.3010 \Rightarrow m < 3.32$ half-lives

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For ^{99m}Tc half-life of 6 hours, time of handling and transport should be less than 6 * 3.32 < 20 hours.

If we transport ^{99}Mo-^{99m}Tc in equilibrium the time constraint is 220 hours since the parent half life is 66 hours.

6. In the 4n series of ^{232}Th, the progenitor ^{232}Th has the longest half-life. If we remove ^{232}Th, its daughter ^{228}Ra of 5.7 year half life is the longest lived isotope. So, it will determine the time period of this series. If we remove the ^{228}Ra from the sample, ^{228}Th with half life of 1.9 years is the next candidate, to be followed by the ^{224}Ra of 3.6 days half life. If these isotopes are removed, we will find the series follows the half life of ^{212}Pb ($T_{1/2}$=10.8 hours). The last significant half life is that of ^{212}Bi of about 1hour.

In 4n+2 series with ^{238}U of $T_{1/2}$= 4.5 billion years as the progenitor, the next longest lived isotope is ^{234}U of $T_{1/2}$= 0.25 million years. If this is also removed, then ^{230}Th ($T_{1/2}$= 75 thousand years) and ^{226}Ra ($T_{1/2}$=1600 year), in that order. Subsequently, ^{210}Pb of 22 years half life will determine the activity levels of decay products. The ultimate activity is due to ^{210}Po of $T_{1/2}$= 138 days.

In 4n+3 series, the progenitor is ^{235}U of 0.7 billion years of half life. If it is removed, the subsequent activities are due to the products in secular equilibrium with the isotopes ^{231}Pa (32760 years), ^{227}Ac (22 years), ^{227}Th (18.7 days), ^{223}Ra(11 days), ^{211}Pb(36 minutes) and ^{207}Tl (5 minutes).