

2.1. (a) Use three 1-D bar elements,

For element 1 (connecting nodes 1 and 2),

$$\mathbf{K}_1 = \frac{EA}{L} \begin{bmatrix} & u_1 & & u_2 \\ 1 & & -1 & \\ -1 & & & 1 \end{bmatrix}$$

For element 2 (connecting nodes 2 and 3),

$$\mathbf{K}_2 = \frac{2EA}{L} \begin{bmatrix} & u_2 & & u_3 \\ 1 & & -1 & \\ -1 & & & 1 \end{bmatrix}$$

For element 3 (connecting nodes 3 and 4),

$$\mathbf{K}_3 = \frac{EA}{L} \begin{bmatrix} & u_3 & & u_4 \\ 1 & & -1 & \\ -1 & & & 1 \end{bmatrix}$$

We can assemble the global FE equation as follows:

$$\frac{EA}{L} \begin{bmatrix} & u_1 & & u_2 & & u_3 & & u_4 \\ 1 & & -1 & & 0 & & 0 & \\ -1 & & & 3 & & -2 & & 0 \\ 0 & & & -2 & & & 3 & & -1 \\ 0 & & & & 0 & & -1 & & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

Load and boundary conditions are

$$u_1 = 0, \quad F_2 = 0, \quad F_3 = 0, \quad F_4 = P$$

FE equations become:

$$\frac{EA}{L} \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P \end{bmatrix}$$

$$\therefore \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \frac{PL}{EA} \begin{bmatrix} 1 \\ 1.5 \\ 2.5 \end{bmatrix}$$

(b) Stress in element 1:

$$\sigma_1 = E \varepsilon_1 = E \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$= \frac{E}{L} (u_2 - u_1) = \frac{E}{L} \left(\frac{PL}{EA} - 0 \right) = \frac{P}{A}$$

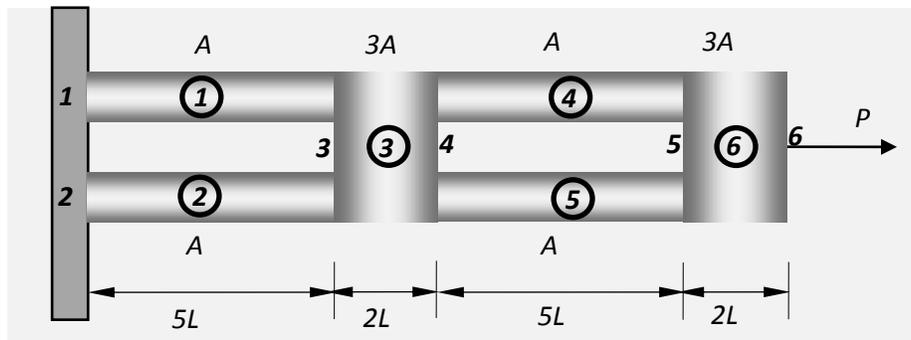
Stress in element 2:

$$\begin{aligned}\sigma_2 &= E\varepsilon_2 = E \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} \\ &= \frac{E}{L}(u_3 - u_2) = \frac{E}{L} \left(\frac{1.5PL}{EA} - \frac{PL}{EA} \right) = \frac{P}{2A}\end{aligned}$$

Stress in element 3:

$$\begin{aligned}\sigma_3 &= E\varepsilon_3 = E \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} \\ &= \frac{E}{L}(u_4 - u_3) = \frac{E}{L} \left(\frac{2.5PL}{EA} - \frac{1.5PL}{EA} \right) = \frac{P}{A}\end{aligned}$$

2.2. Use six 1-D bar elements to discretize the simple structure as shown below:



For element 1:

$$K_1 = \frac{EA}{5L} \begin{bmatrix} u_1 & u_3 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$$

For element 2:

$$K_2 = \frac{EA}{5L} \begin{bmatrix} u_2 & u_3 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$$

For element 3:

$$K_3 = \frac{3EA}{2L} \begin{bmatrix} u_3 & u_4 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$$

For element 4:

$$\mathbf{K}_4 = \frac{EA}{5L} \begin{bmatrix} & u_4 & u_5 \\ 1 & & -1 \\ -1 & & 1 \end{bmatrix}$$

For element 5:

$$\mathbf{K}_5 = \frac{EA}{5L} \begin{bmatrix} & u_4 & u_5 \\ 1 & & -1 \\ -1 & & 1 \end{bmatrix}$$

For element 6:

$$\mathbf{K}_6 = \frac{3EA}{2L} \begin{bmatrix} & u_5 & u_6 \\ 1 & & -1 \\ -1 & & 1 \end{bmatrix}$$

The assembled global FE equations are:

$$\begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ 0.2 & 0 & -0.2 & 0 & 0 & 0 \\ 0 & 0.2 & -0.2 & 0 & 0 & 0 \\ -0.2 & -0.2 & 1.9 & -1.5 & 0 & 0 \\ 0 & 0 & -1.5 & 1.9 & -0.4 & 0 \\ 0 & 0 & 0 & -0.4 & 1.9 & -1.5 \\ 0 & 0 & 0 & 0 & -1.5 & 1.5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}$$

Load and boundary conditions are:

$$u_1 = 0, \quad u_2 = 0, \quad F_3 = 0, \quad F_4 = 0, \quad F_5 = 0, \quad F_6 = P$$

FE equations become:

$$\frac{EA}{L} \begin{bmatrix} 1.9 & -1.5 & 0 & 0 \\ -1.5 & 1.9 & -0.4 & 0 \\ 0 & -0.4 & 1.9 & -1.5 \\ 0 & 0 & -1.5 & 1.5 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ P \end{bmatrix}$$

$$\therefore \begin{bmatrix} u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \frac{PL}{EA} \begin{bmatrix} 2.500 \\ 3.167 \\ 5.667 \\ 6.333 \end{bmatrix}$$

The stress in element 1:

$$\sigma_1 = E\varepsilon_1 = E[-1/5L \quad 1/5L] \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = \frac{E}{5L} (u_3 - u_1) = \frac{E}{5L} \times \frac{PL}{EA} (2.5 - 0) = \frac{P}{2A}$$

The stress in element 2:

$$\sigma_2 = E\varepsilon_2 = E[-1/5L \quad 1/5L] \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \frac{E}{5L}(u_3 - u_2) = \frac{E}{5L} \times \frac{PL}{EA}(2.5 - 0) = \frac{P}{2A}$$

The stress in element 3:

$$\sigma_3 = E\varepsilon_3 = E[-1/2L \quad 1/2L] \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \frac{E}{2L}(u_4 - u_3) = \frac{E}{2L} \times \frac{PL}{EA}(3.167 - 2.5) = 0.33 \frac{P}{A}$$

The stress in element 4:

$$\sigma_4 = E\varepsilon_4 = E[-1/5L \quad 1/5L] \begin{bmatrix} u_4 \\ u_5 \end{bmatrix} = \frac{E}{5L}(u_5 - u_4) = \frac{E}{5L} \times \frac{PL}{EA}(5.667 - 3.167) = \frac{P}{2A}$$

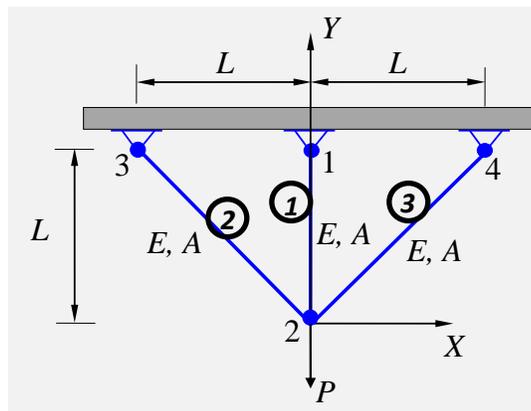
The stress in element 5:

$$\sigma_5 = E\varepsilon_5 = E[-1/5L \quad 1/5L] \begin{bmatrix} u_4 \\ u_5 \end{bmatrix} = \frac{E}{5L}(u_5 - u_4) = \frac{E}{5L} \times \frac{PL}{EA}(5.667 - 3.167) = \frac{P}{2A}$$

The stress in element 6:

$$\sigma_6 = E\varepsilon_6 = E[-1/2L \quad 1/2L] \begin{bmatrix} u_5 \\ u_6 \end{bmatrix} = \frac{E}{2L}(u_6 - u_5) = \frac{E}{2L} \times \frac{PL}{EA}(6.333 - 5.667) = 0.33 \frac{P}{A}$$

2.3.



Element 1: 1 – 2, $\theta = 90^\circ$, $l = 0$, $m = L$

$$\mathbf{K}_1 = \frac{EA}{L} \begin{bmatrix} u_2 & v_2 & u_1 & v_1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Element 2: 2 – 3, $\theta = 135^\circ$, $l = -\sqrt{2}/2$, $m = \sqrt{2}/2$

$$\mathbf{K}_2 = \frac{EA}{\sqrt{2}L} \begin{bmatrix} u_2 & v_2 & u_3 & v_3 \\ 1/2 & -1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{bmatrix}$$

Element 3: 2 – 4, $\theta = 45^\circ$, $l = \sqrt{2}/2$, $m = \sqrt{2}/2$

$$\mathbf{K}_3 = \frac{EA}{\sqrt{2}L} \begin{bmatrix} u_2 & v_2 & u_4 & v_4 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix}$$

Assemble the global FE system:

$$\frac{EA}{2\sqrt{2}L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 & u_4 & v_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\sqrt{2} & 0 & -2\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 1 & -1 & -1 \\ 0 & -2\sqrt{2} & 0 & 2\sqrt{2} + 2 & 1 & -1 & -1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{bmatrix} \quad (1)$$

B.C.: $u_1 = v_1 = u_3 = v_3 = u_4 = v_4 = 0$, $F_{2X} = 0$, $F_{2Y} = -P$,

Reduced global FE equation:

$$\frac{EA}{2\sqrt{2}L} \begin{bmatrix} 2 & 0 \\ 0 & 2\sqrt{2} + 2 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -P \end{bmatrix}$$

$$\therefore \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{PL}{EA} \begin{bmatrix} 0 \\ -0.5858 \end{bmatrix} \quad (2)$$

Substituting (2) into (1), we have the following reduced equation:

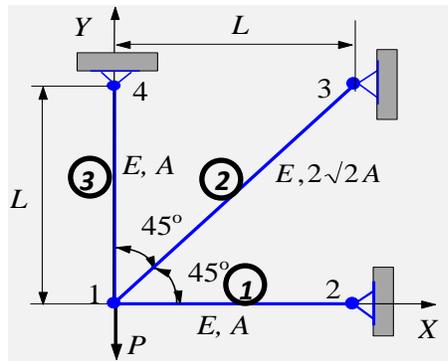
$$\begin{bmatrix} F_{1X} \\ F_{1Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{bmatrix} = \frac{EA}{2\sqrt{2}L} \begin{bmatrix} 0 & 0 \\ 0 & -2\sqrt{2} \\ -1 & 1 \\ 1 & -1 \\ -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = P \begin{bmatrix} 0 \\ 0.5858 \\ -0.2071 \\ 0.2071 \\ 0.2071 \\ 0.2071 \end{bmatrix}$$

$$\sigma_1 = \frac{E}{L} [0 \quad -1 \quad 0 \quad 1] \begin{bmatrix} u_2 \\ v_2 \\ u_1 \\ v_1 \end{bmatrix} = 0.5858 \frac{P}{A}$$

$$\sigma_2 = \frac{E}{\sqrt{2}L} \left[\frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2} \right] \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = 0.2929 \frac{P}{A}$$

$$\sigma_3 = \frac{E}{\sqrt{2}L} \left[-\frac{\sqrt{2}}{2} \quad -\frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2} \right] \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = 0.2929 \frac{P}{A}$$

2.4.



Element 1: 1 – 2, $\theta = 0^\circ$, $l = 1$, $m = 0$;

$$\mathbf{K}_1 = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2: 1 – 3, $\theta = 45^\circ$, $l = m = \sqrt{2}/2$, $L = \sqrt{2}L$;

$$\mathbf{K}_2 = \frac{E(2\sqrt{2}A)}{\sqrt{2}L} \begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_3 & v_3 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

Element 3: 1 – 4, $\theta = 90^\circ$, $l = 0$, $m = l$;

$$\mathbf{K}_3 = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_4 & v_4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Assemble the global FE system:

$$\frac{EA}{L} \begin{bmatrix} 2 & 1 & -1 & 0 & -1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 & -1 & -1 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{bmatrix} \quad (1)$$

B.C.s: $u_2 = v_2 = u_3 = v_3 = u_4 = v_4 = 0$, $F_{1X} = 0$, $F_{2Y} = -P$

$$\Rightarrow \frac{EA}{L} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -P \end{bmatrix}$$

Solving,

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{PL}{3EA} \begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad (2)$$

Reaction forces:

From (1)

$$\begin{bmatrix} F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ -1 & -1 \\ -1 & -1 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{P}{3} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 2 \end{bmatrix},$$

$$i.e. \begin{bmatrix} F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{bmatrix} = \begin{bmatrix} -P/3 \\ 0 \\ P/3 \\ P/3 \\ 0 \\ 2P/3 \end{bmatrix}.$$

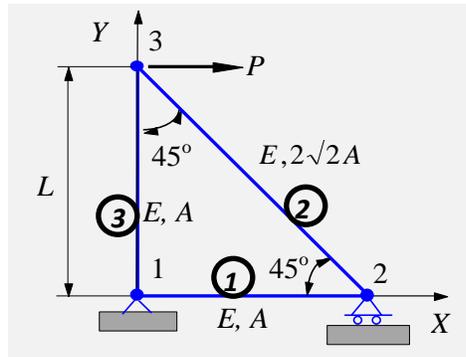
Stresses:

$$\sigma_1 = \frac{E}{L} \begin{bmatrix} -l & -m & l & m \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \frac{E}{L} \begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} \frac{PL}{3EA} = -\frac{P}{3A}$$

$$\sigma_2 = \frac{E}{\sqrt{2}L} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} \frac{PL}{3EA} = \frac{P}{6A}$$

$$\sigma_1 = \frac{E}{L} [0 \quad -1 \quad 0 \quad 1] \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} \frac{PL}{3EA} = \frac{2P}{3A}$$

2.5.



Element 1: 1-2, $\theta = 0, l = 1, m = 0$

$$\mathbf{K}_1 = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2: 2-3, $\theta = 135^\circ, l = -\frac{\sqrt{2}}{2}, m = \frac{\sqrt{2}}{2}$

$$\mathbf{K}_2 = \frac{E(2\sqrt{2}A)}{\sqrt{2}L \cdot (2)} \begin{bmatrix} u_2 & v_2 & u_3 & v_3 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Element 3: 1-3, $\theta = 90^\circ, l = 0, m = 1$

$$\mathbf{K}_3 = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_3 & v_3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Global stiffness matrix:

$$\mathbf{K} = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 2 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & -1 & 1 & -1 & -1 & 2 \end{bmatrix}$$

Applying B.C., where:

$$u_1 = v_1 = v_2 = 0, F_{2x} = 0, F_{3x} = P, F_{3y} = 0$$

$$\frac{EA}{L} \begin{bmatrix} u_2 & u_3 & v_3 \\ 2 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ P \\ 0 \end{bmatrix}$$

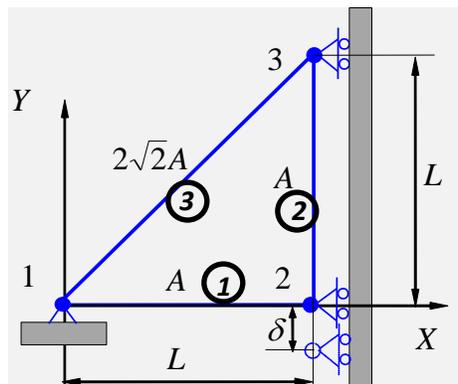
$$\begin{bmatrix} u_2 \\ u_3 \\ v_3 \end{bmatrix} = \frac{PL}{EA} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$\sigma_1 = \frac{E}{L} \begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \frac{E}{L} u_2 = \frac{P}{A}$$

$$\sigma_2 = \frac{E}{\sqrt{2}L} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \frac{E}{2L} (1 - 3 + 1) \frac{PL}{EA} = -\frac{P}{2A}$$

$$\sigma_3 = \frac{E}{L} \begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{bmatrix} = \frac{E}{L} v_3 = \frac{P}{A}$$

2.6.



Element stiffness matrices are

Element 1: $\theta = 0, l = 1, m = 0$

$$k_1 = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2: $\theta = 90^\circ, l = 0, m = 1$

$$k_2 = \frac{EA}{L} \begin{bmatrix} u_2 & v_2 & u_3 & v_3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Element 3: $\theta = 45^\circ, l = 1\sqrt{2}, m = 1\sqrt{2}$

$$k_1 = \frac{E(2\sqrt{2}A)}{2\sqrt{2}L} \begin{bmatrix} u_1 & v_1 & u_3 & v_3 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

Global FE equation

$$\frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 \\ 2 & 1 & -1 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & -1 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ -1 & -1 & 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \end{bmatrix}$$

Boundary Conditions:

$$u_1 = v_1 = u_2 = u_3 = 0, F_{3Y} = 0, v_2 = -\sigma$$

$$\frac{EA}{L} \begin{bmatrix} v_2 & v_3 \\ 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} F_{2Y} \\ 0 \end{bmatrix}$$

$$\text{Or } -v_2 + 2v_3 = 0$$

$$v_3 = \frac{1}{2} v_2 = -\frac{1}{2} \sigma$$

$$\therefore \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\sigma \\ -\sigma/2 \end{bmatrix}$$

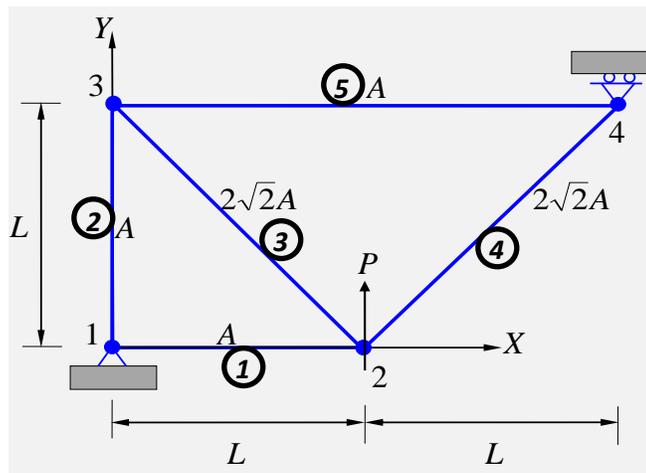
Stresses:

$$\sigma_1 = \frac{E}{L} \begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = 0$$

$$\sigma_2 = \frac{E}{L} \begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \frac{E}{L} \left(\sigma - \frac{\sigma}{2} \right) = \frac{E\sigma}{2L}$$

$$\sigma_3 = \frac{E}{\sqrt{2}L} \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{bmatrix} = \frac{E}{2L} \left(-\frac{\sigma}{2} \right) = -\frac{E\sigma}{4L}$$

2.7.



Element stiffness matrices:

Element 1: 1 – 2, $\theta = 0^\circ$, $l = 1$, $m = 0$

$$k_1 = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2: 1 – 3, $\theta = 90^\circ$, $l = 0$, $m = 1$

$$k_2 = \frac{EA}{L} \begin{bmatrix} u_1 & v_1 & u_3 & v_3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Element 3: 2 – 3, $\theta = 135^\circ$, $l = -2\sqrt{2}$, $m = \sqrt{2}/2$

$$k_3 = \frac{E(2\sqrt{2}A)}{\sqrt{2}L} \begin{bmatrix} u_2 & v_2 & u_3 & v_3 \\ 1/2 & -1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{bmatrix}$$

Element 4: 2 – 4, $\theta = 45^\circ$, $l = \sqrt{2}/2$, $m = \sqrt{2}/2$

$$k_4 = \frac{E(2\sqrt{2}A)}{\sqrt{2}L} \begin{bmatrix} u_2 & v_2 & u_4 & v_4 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix}$$

Element 5: 3 – 4, $\theta = 0^\circ$, $l = 1$, $m = 0$

$$k_5 = \frac{EA}{2L} \begin{bmatrix} u_3 & v_3 & u_4 & v_4 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Assemble the global FE system:

$$\begin{bmatrix} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 & u_4 & v_4 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0+1 & 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1+1+1 & 0-1+1 & -1 & 1 & -1 & -1 \\ 0 & 0 & 0-1+1 & 0+1+1 & 1 & -1 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0+1+0.5 & 0-1 & -0.5 & 0 \\ 0 & -1 & 1 & -1 & 0-1 & 1+1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -0.5 & 0 & 1+0.5 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{bmatrix} \quad (1)$$

B.C.: $u_1 = v_1 = v_4 = 0$, $F_{2X} = F_{3X} = F_{3Y} = 0$, $F_{2Y} = P$, $F_{4X} = 0$,

Applying B.C. to equation (1), we have:

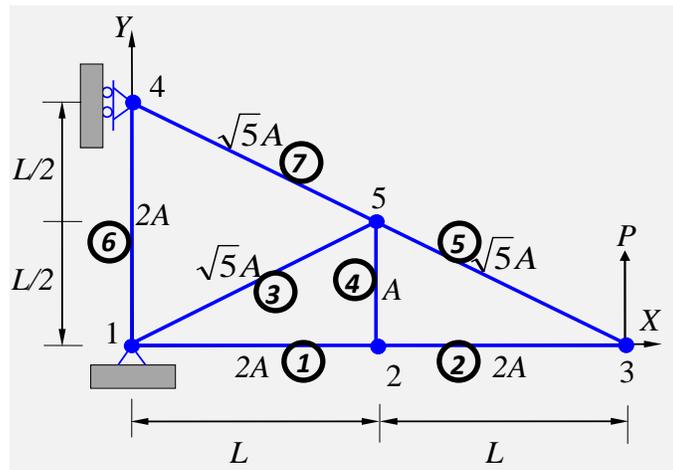
$$\frac{EA}{L} \begin{bmatrix} u_2 & v_2 & u_3 & v_3 & u_4 \\ 3 & 0 & -1 & 1 & -1 \\ 0 & 2 & 1 & -1 & -1 \\ -1 & 1 & 1.5 & -1 & -0.5 \\ 1 & -1 & -1 & 2 & 0 \\ -1 & -1 & -0.5 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ P \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \end{bmatrix} = \frac{PL}{EA} \begin{bmatrix} 0 \\ 1.25 \\ -0.25 \\ 0.5 \\ 0.75 \end{bmatrix} \quad (2)$$

Substituting (2) into (1), we have:

$$\begin{bmatrix} F_{1X} \\ F_{1Y} \\ F_{4Y} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ -1 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \end{bmatrix} = P \begin{bmatrix} 0 \\ -0.5 \\ -0.5 \end{bmatrix}$$

2.8.



Element stiffness matrices:

Element 1: 1-2, $\theta = 0, l = 1, m = 0$

$$k_1 = \frac{E(2A)}{L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 2: 2-3, $\theta = 0, l = 1, m = 0$

$$k_2 = \frac{E(2A)}{L} \begin{bmatrix} u_2 & v_2 & u_3 & v_3 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Element 3: $1-5, l = \frac{2}{\sqrt{5}}, m = \frac{1}{\sqrt{5}}$

$$k_3 = \frac{E(\sqrt{5}A)}{\left(\frac{\sqrt{5}L}{2}\right)} \begin{bmatrix} u_1 & v_1 & u_5 & v_5 \\ 4 & 2 & 4 & 2 \\ 5 & 5 & -5 & -5 \\ 2 & 1 & 2 & 1 \\ 5 & 5 & -5 & -5 \\ -4 & -2 & 4 & 2 \\ -5 & -5 & 5 & 5 \\ 2 & 1 & 2 & 1 \\ -5 & -5 & 5 & 5 \end{bmatrix}$$

Element 4: $2-5, \theta = 90^\circ, l = 0, m = 1$

$$k_4 = \frac{EA}{\left(\frac{L}{2}\right)} \begin{bmatrix} u_2 & v_2 & u_5 & v_5 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Element 5: $3-5, l = -\frac{2}{\sqrt{5}}, m = \frac{1}{\sqrt{5}}$

$$k_5 = \frac{E(\sqrt{5}A)}{\left(\frac{\sqrt{5}L}{2}\right)} \begin{bmatrix} u_3 & v_3 & u_5 & v_5 \\ 4 & 2 & 4 & 2 \\ 5 & 5 & -5 & -5 \\ -2 & 1 & 2 & 1 \\ 5 & 5 & -5 & -5 \\ -4 & -2 & 4 & 2 \\ -5 & -5 & 5 & 5 \\ 2 & 1 & 2 & 1 \\ 5 & 5 & -5 & -5 \end{bmatrix}$$

Element 6: $1-4, \theta = 90^\circ, l = 0, m = 1$

$$k_6 = \frac{E(2A)}{L} \begin{bmatrix} u_1 & v_1 & u_4 & v_4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Element 7: $5-4, l = -\frac{2}{\sqrt{5}}, m = \frac{1}{\sqrt{5}}$

$$k_7 = \frac{E(\sqrt{5}A)}{\left(\frac{\sqrt{5}L}{2}\right)} \begin{bmatrix} u_5 & v_5 & u_4 & v_4 \\ \frac{4}{5} & -\frac{2}{5} & -\frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} & -\frac{1}{5} \\ -\frac{4}{5} & \frac{2}{5} & \frac{4}{5} & -\frac{2}{5} \\ -\frac{2}{5} & -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Assemble the global FE system:

$$\frac{2EA}{L} \begin{bmatrix} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 & u_4 & v_4 & u_5 & v_5 \\ 1.8 & 0.4 & -1 & 0 & 0 & 0 & 0 & 0 & -0.8 & -0.4 \\ 0.4 & 1.2 & 0 & 0 & 0 & 0 & 0 & -1 & -0.4 & -0.2 \\ -1 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1.8 & -0.4 & 0 & 0 & -0.8 & 0.4 \\ 0 & 0 & 0 & 0 & -0.4 & 0.2 & 0 & 0 & 0.4 & -0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & -0.4 & -0.8 & 0.4 \\ 0 & -1 & 0 & 0 & 0 & 0 & -0.4 & 1.2 & 0.4 & -0.2 \\ -0.8 & -0.4 & 0 & 0 & -0.8 & 0.4 & -0.8 & 0.4 & 2.4 & -0.4 \\ -0.4 & -0.2 & 0 & -1 & 0.4 & -0.2 & 0.4 & -0.2 & -0.4 & 1.6 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{bmatrix} = \begin{bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \\ F_{5X} \\ F_{5Y} \end{bmatrix}$$

Boundary Conditions:

$$u_1 = v_1 = u_4 = 0, F_{2X} = F_{2Y} = F_{3X} = F_{4Y} = F_{5X} = F_{5Y} = 0, F_{3Y} = P$$

Applying BCs to the global FE system, we have:

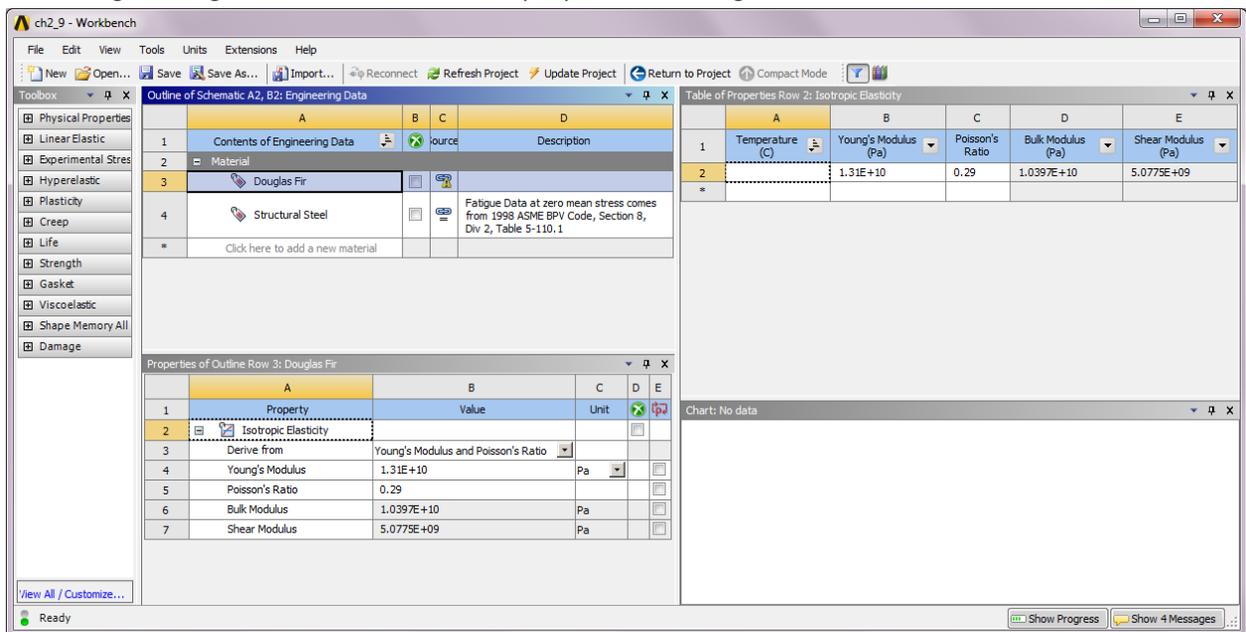
$$\frac{2EA}{L} \begin{bmatrix} u_2 & v_2 & u_3 & v_3 & v_4 & u_5 & v_5 \\ 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1.8 & -0.4 & 0 & -0.8 & 0.4 \\ 0 & 0 & -0.4 & 0.2 & 0 & 0.4 & -0.2 \\ 0 & 0 & 0 & 0 & 1.2 & 0.4 & -0.2 \\ 0 & 0 & -0.8 & 0.4 & 0.4 & 2.4 & -0.4 \\ 0 & -1 & 0.4 & -0.2 & -0.2 & -0.4 & 1.6 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \\ v_4 \\ u_5 \\ v_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ P \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \\ v_4 \\ u_5 \\ v_5 \end{bmatrix} = \frac{PL}{2EA} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 19 \\ 1 \\ -1 \\ 3 \end{bmatrix}$$

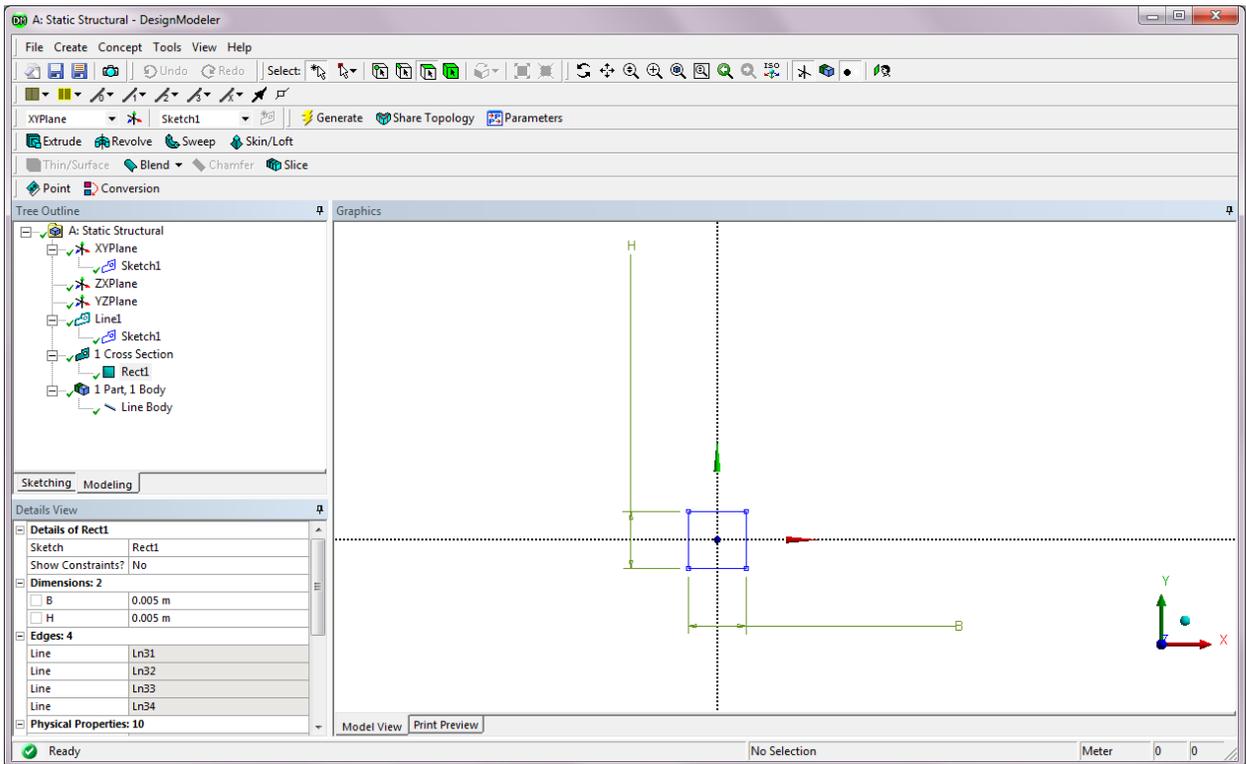
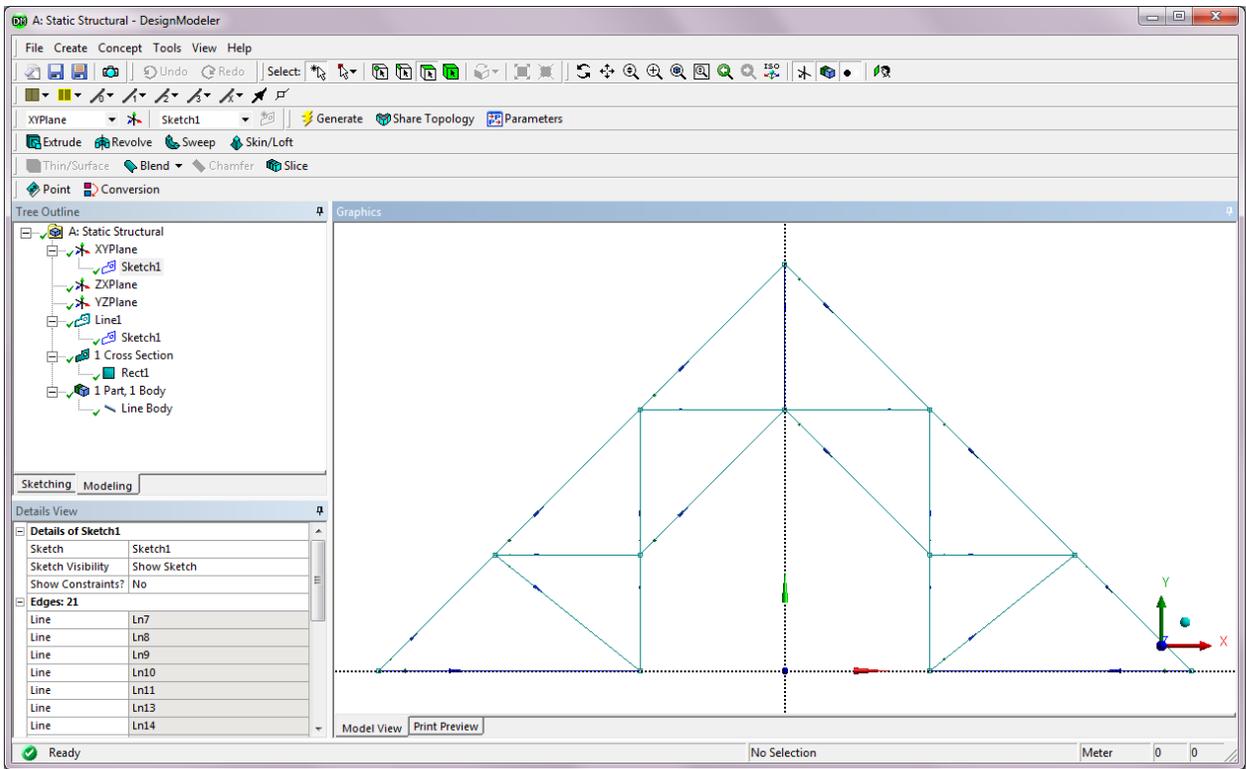
The reaction forces are:

$$\begin{bmatrix} F_{1X} \\ F_{1Y} \\ F_{4X} \end{bmatrix} = \frac{2EA}{L} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & -0.8 & -0.4 \\ 0 & 0 & 0 & 0 & -1 & -0.4 & -0.2 \\ 0 & 0 & 0 & 0 & -0.4 & -0.8 & 0.4 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \\ v_4 \\ u_5 \\ v_5 \end{bmatrix} = \begin{bmatrix} -2P \\ -P \\ 2P \end{bmatrix}$$

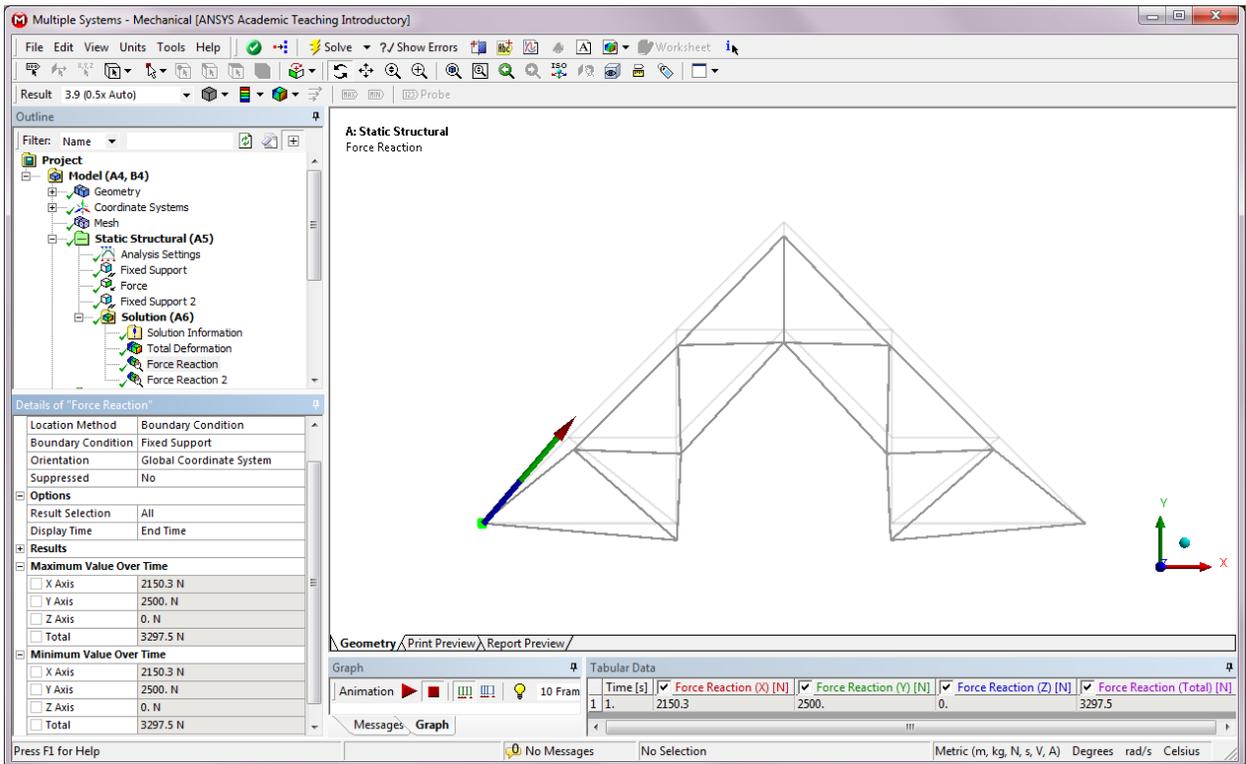
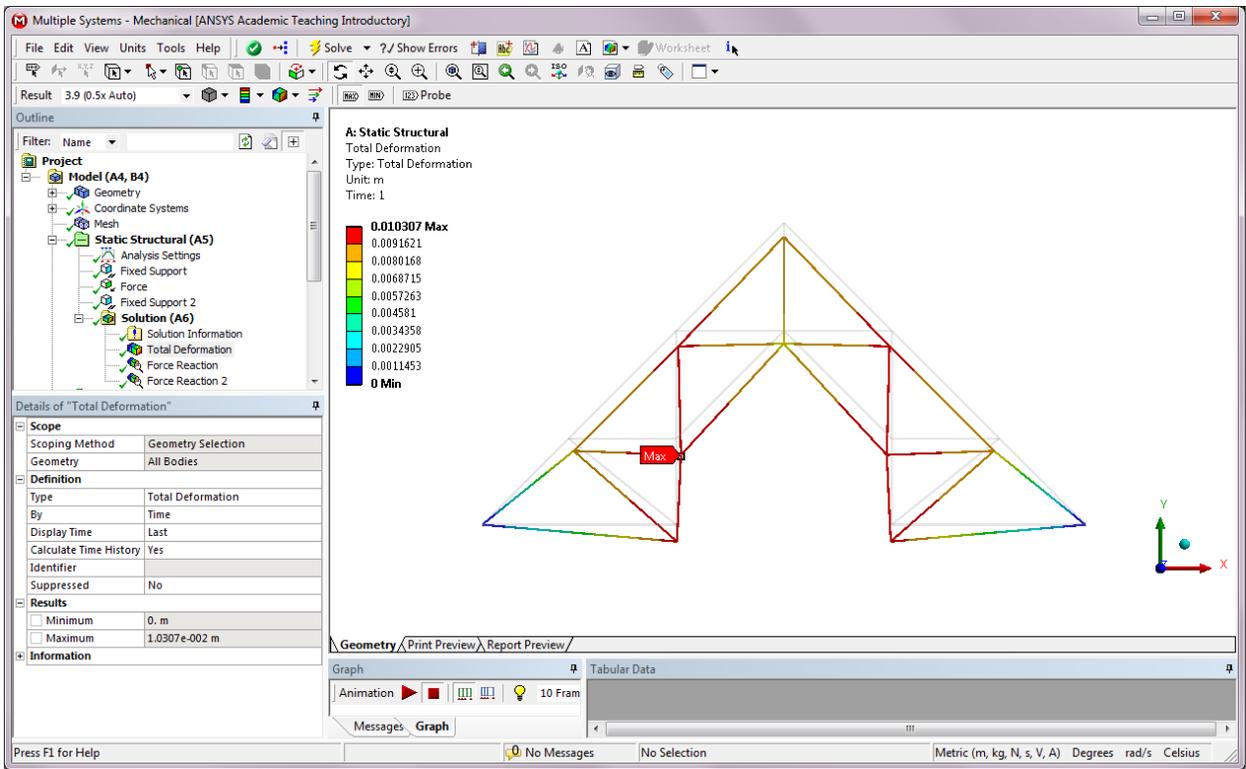
2.9. In the Engineering Data table, add material properties of Douglas Fir:

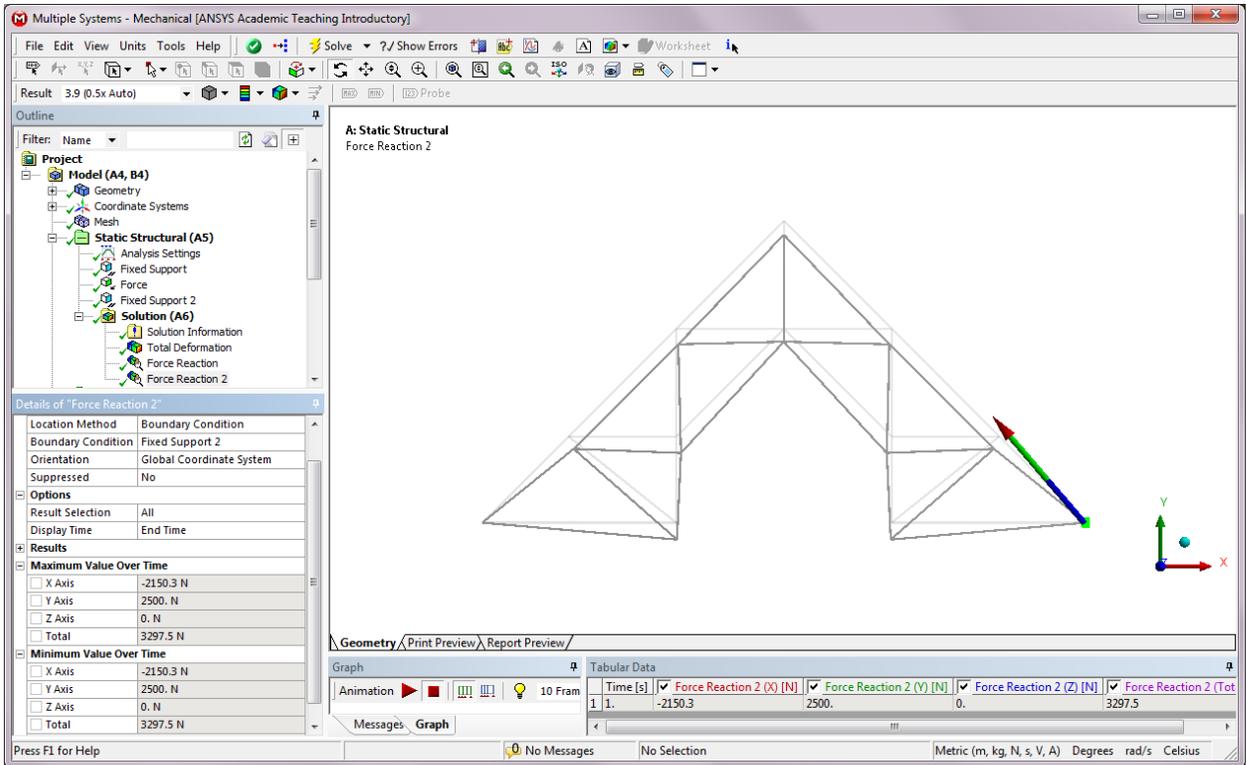


In the DesignModeler program, create a line sketch of the roof truss and a cross section definition of the truss member. Then create a line body concept model from the sketch:



In the Multiple Systems – Mechanical program, retrieve results after applying loads and boundary conditions:



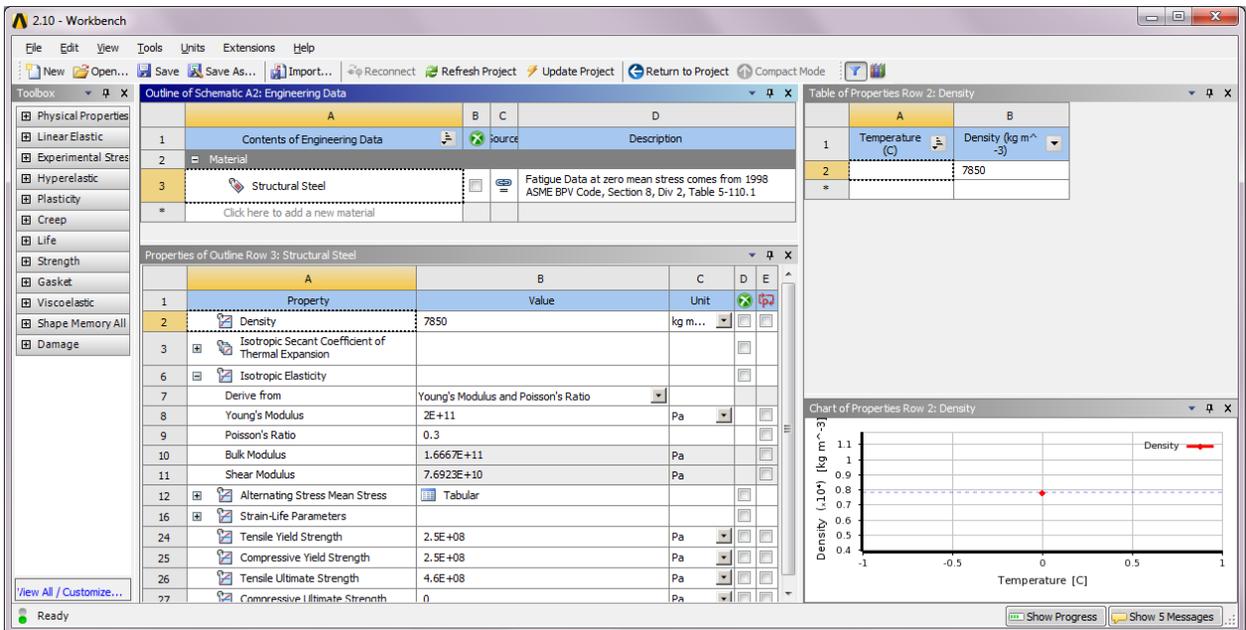


The maximum total deformation of the roof truss is: **10.307 mm**

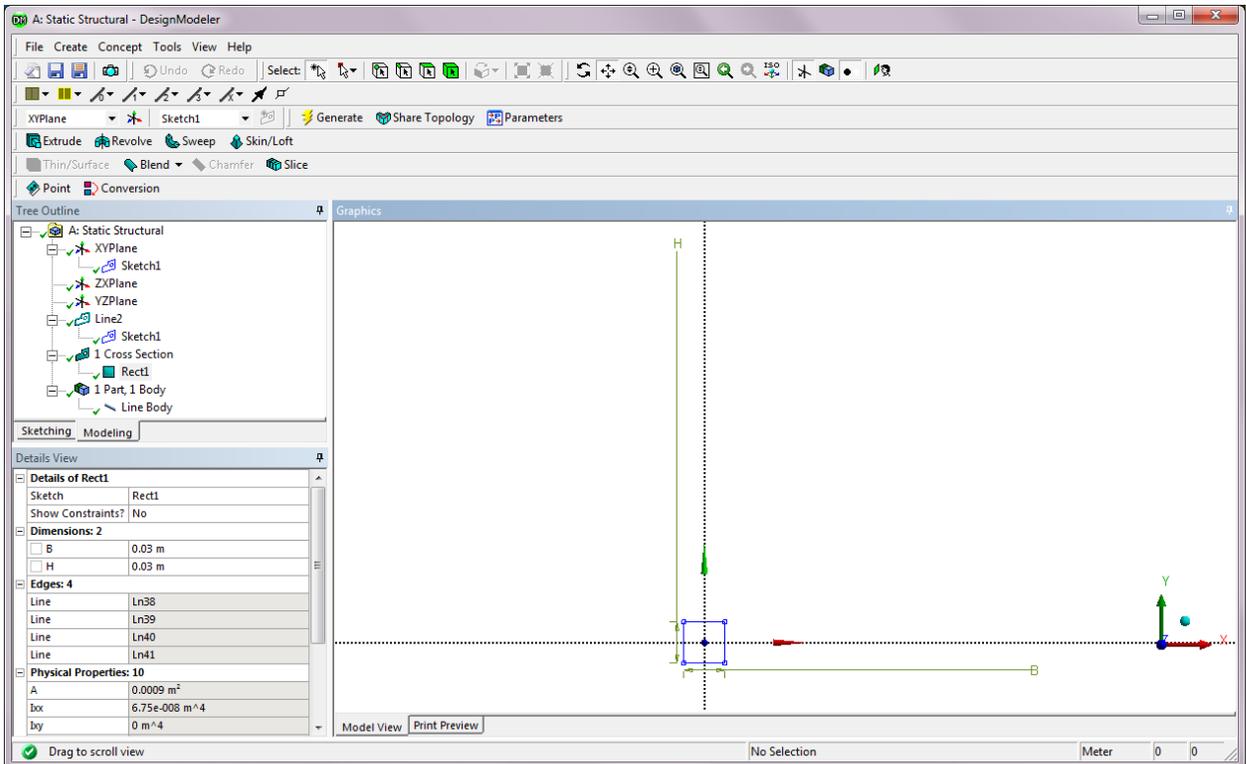
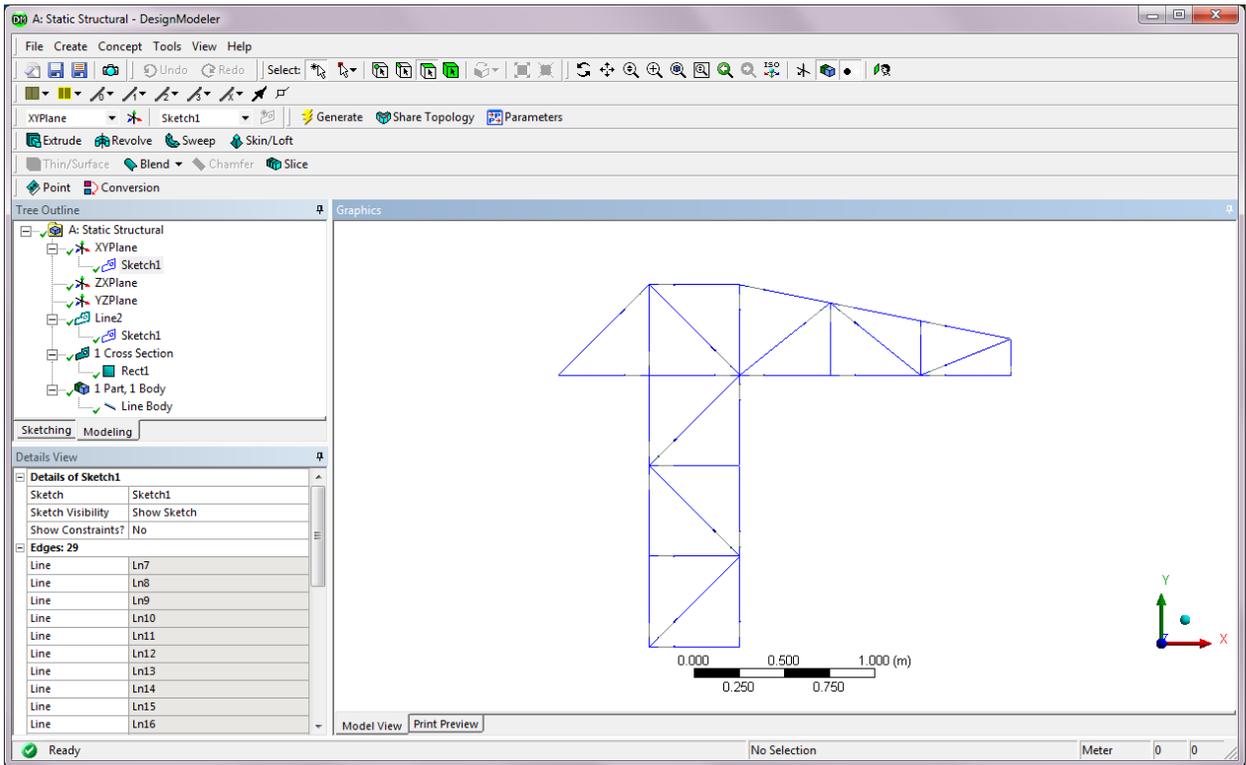
X, Y and Z components of the reaction force at the left support are: **(2150.3 N, 2500 N, 0)**

X, Y and Z components of the reaction force at the right support are: **(-2150.3 N, 2500 N, 0)**

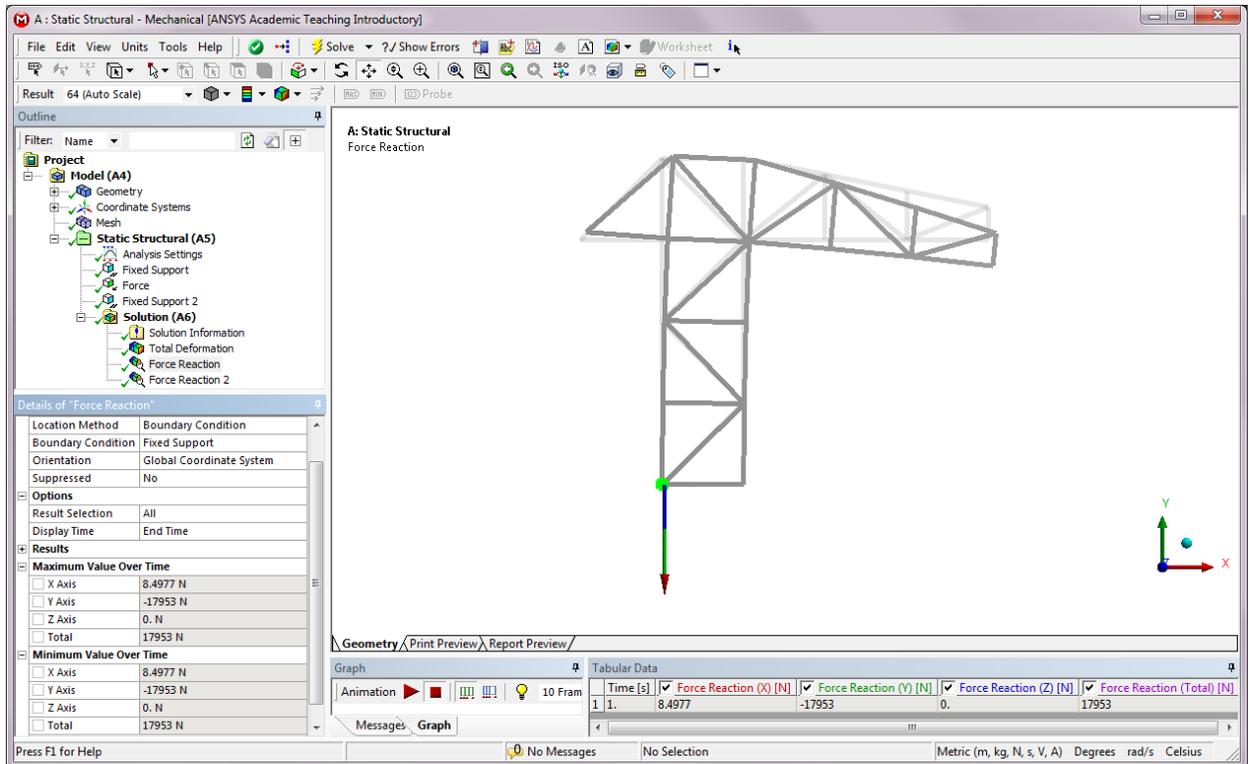
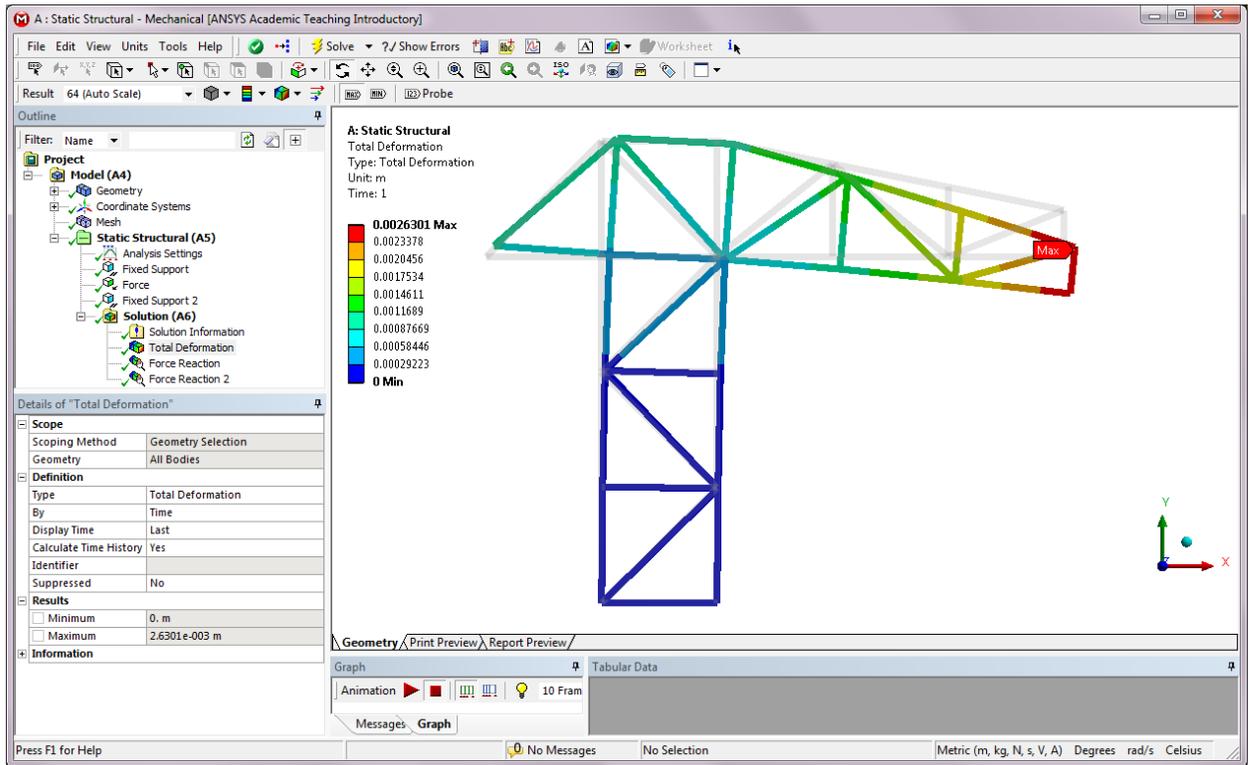
2.10. In the Engineering Data table, use material properties of Structural Steel:

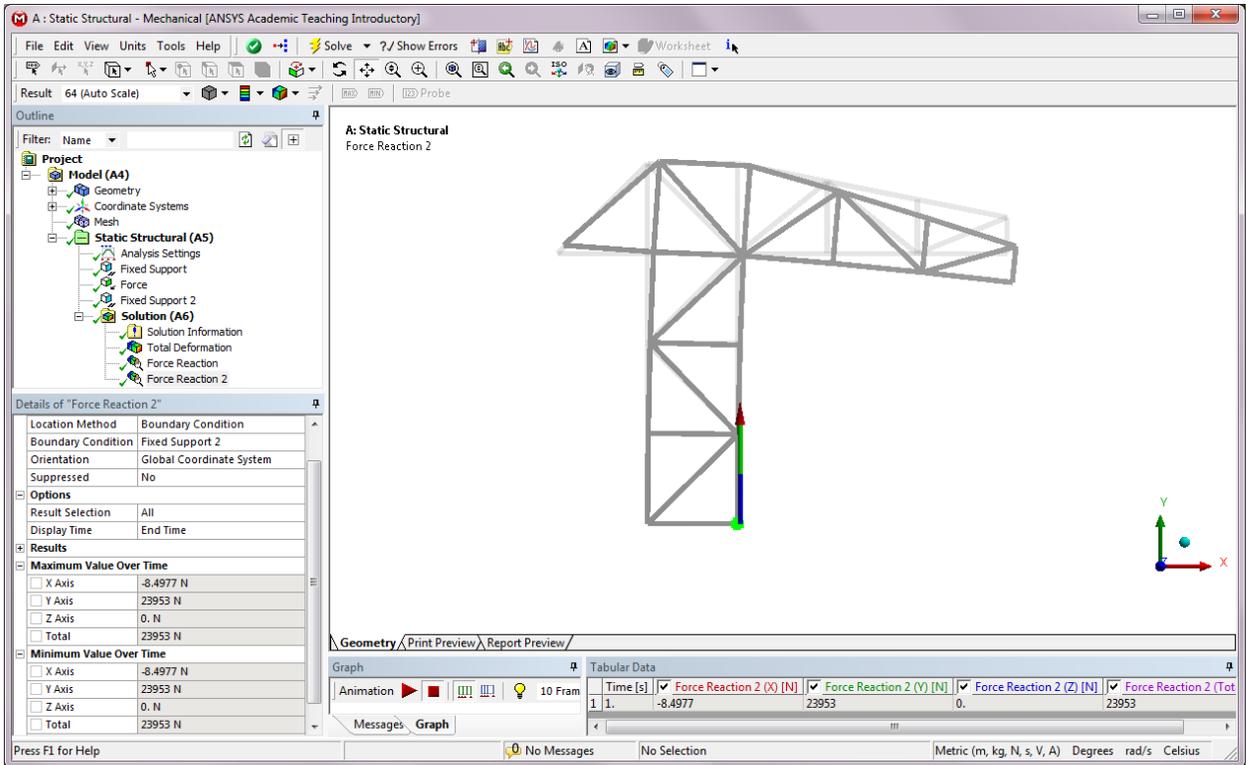


In the DesignModeler program, create a line sketch of the truss tower crane and a cross sectional definition of the truss member. Then create a line body concept model from the sketch:



In the Multiple Systems – Mechanical program, retrieve results after applying loads and boundary conditions:



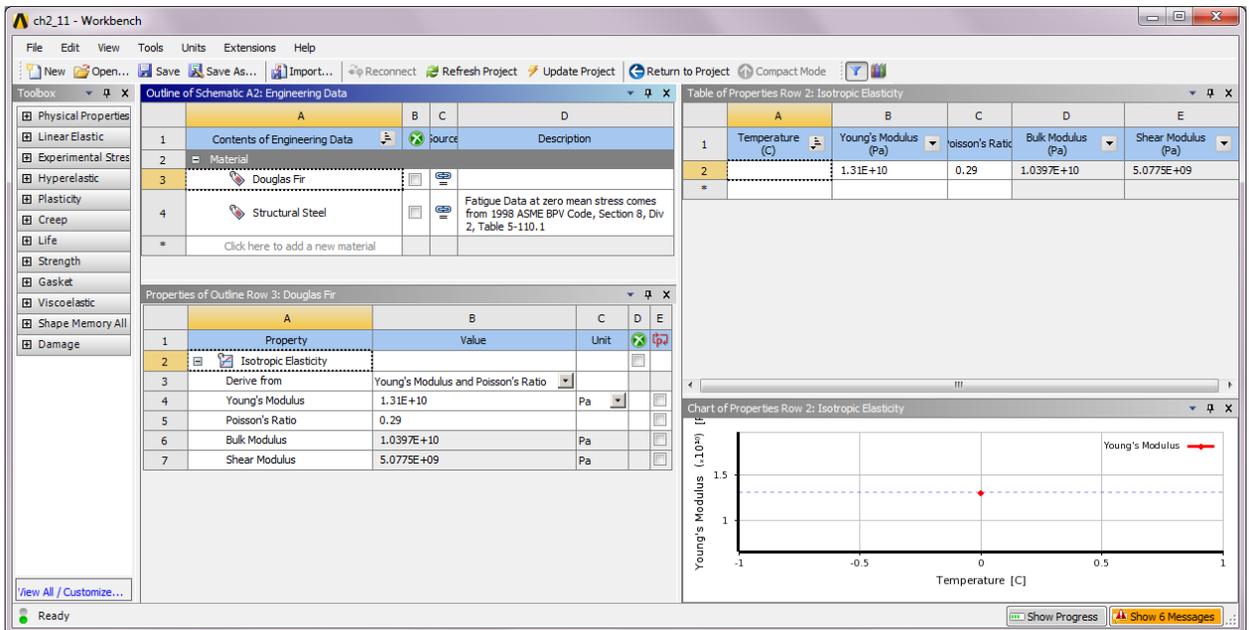


The maximum total deformation of the truss tower crane is: **2.6301 mm**

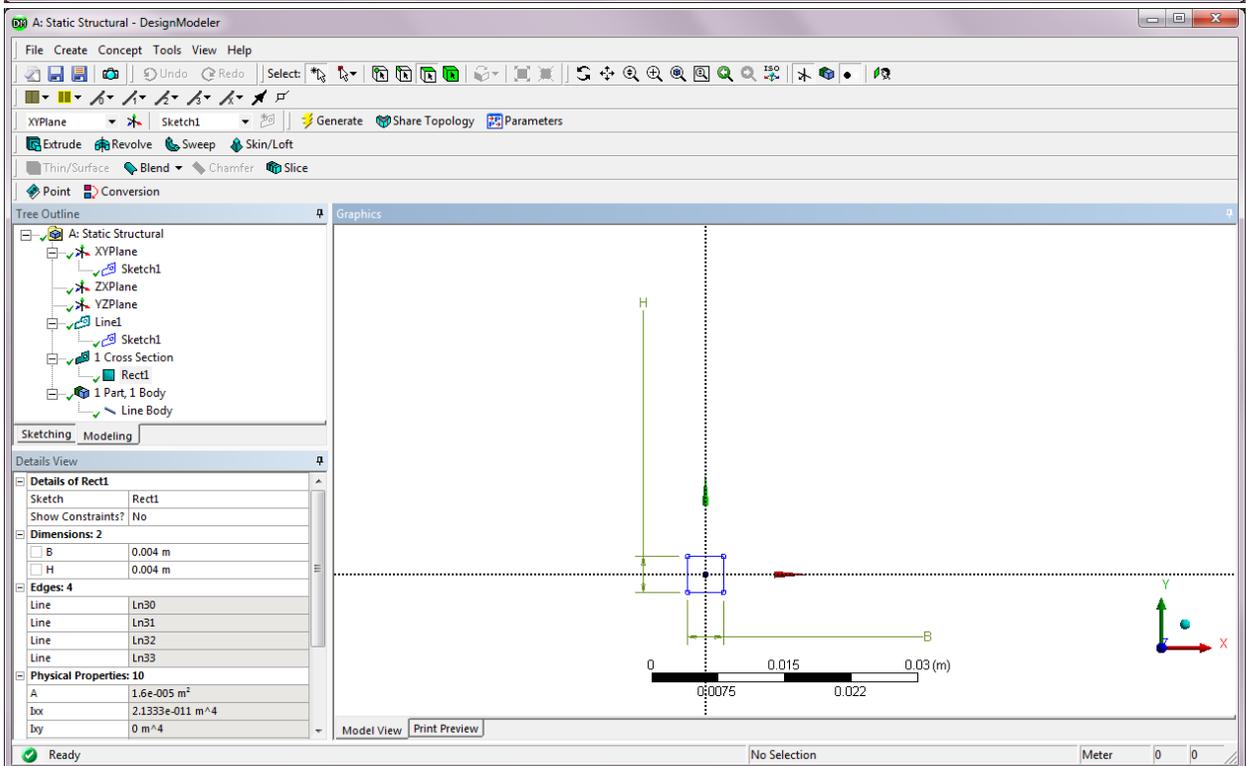
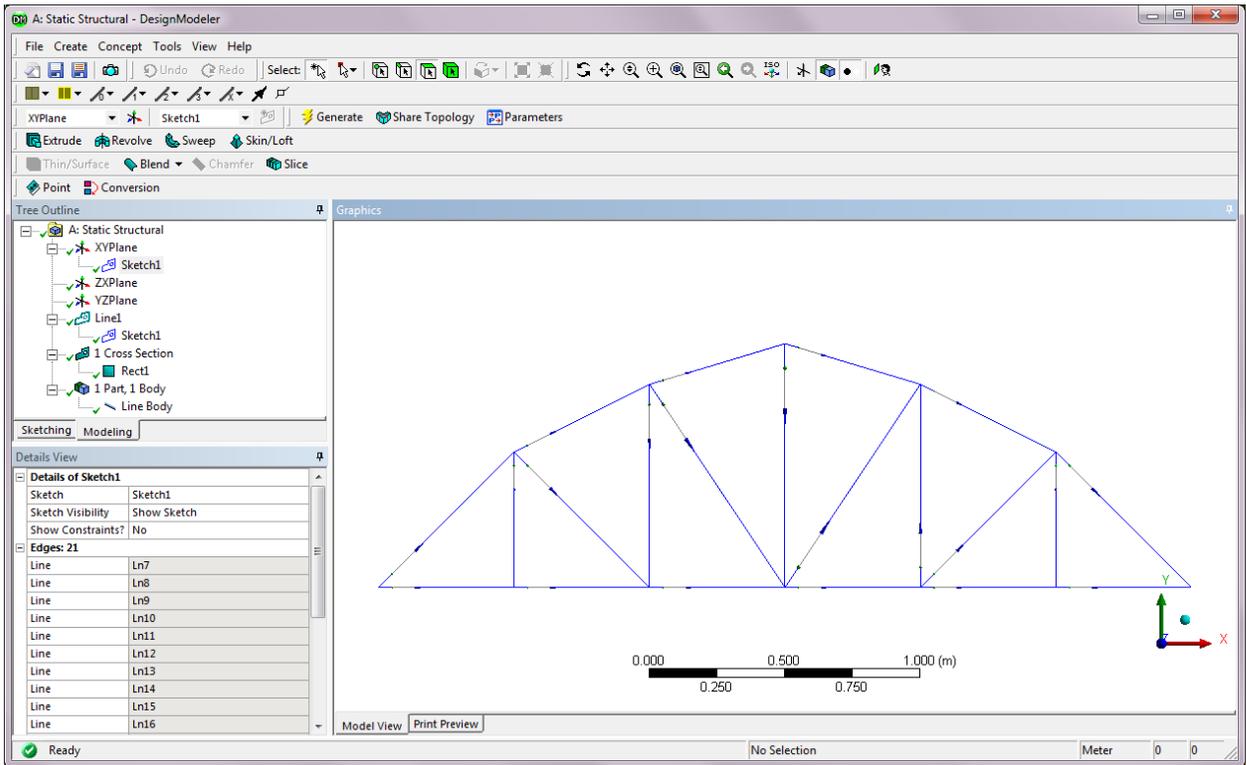
X, Y and Z components of the reaction force at the left support are: **(8.4977 N, -17953 N, 0)**

X, Y and Z components of the reaction forces at the right support are: **(-8.4977 N, 23953 N, 0)**

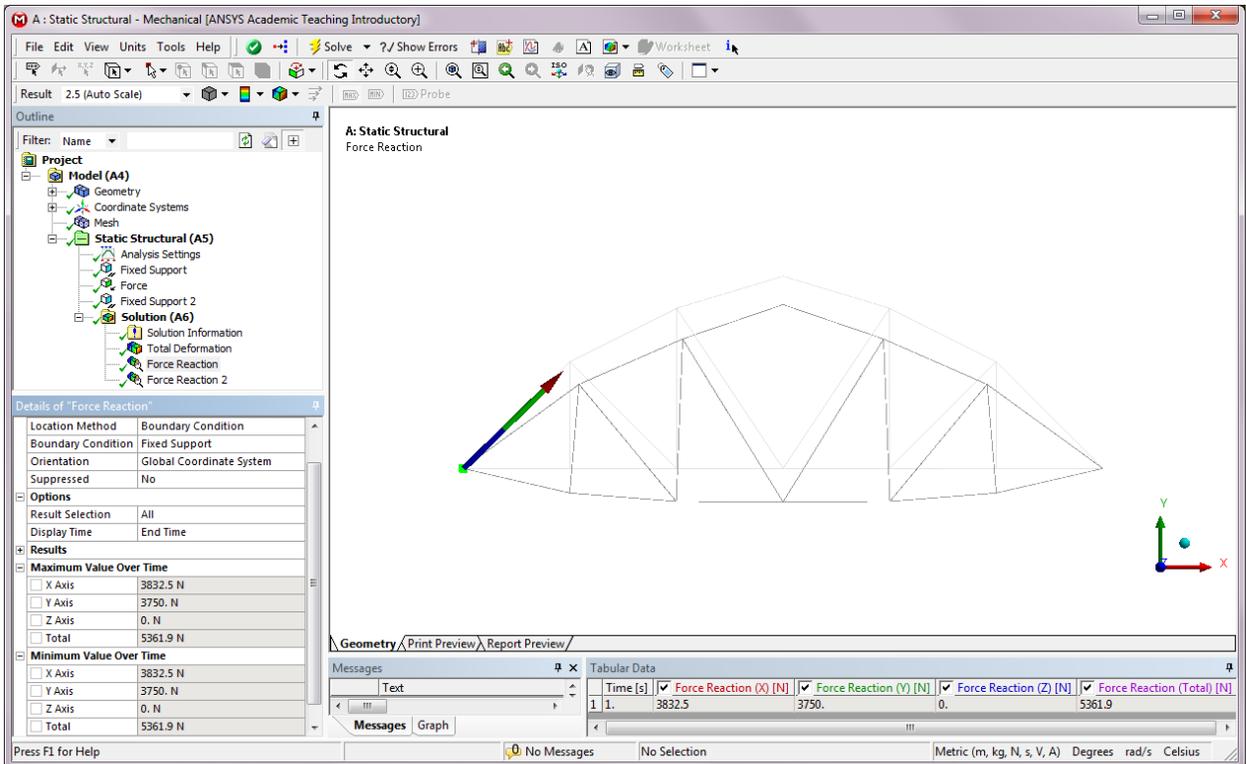
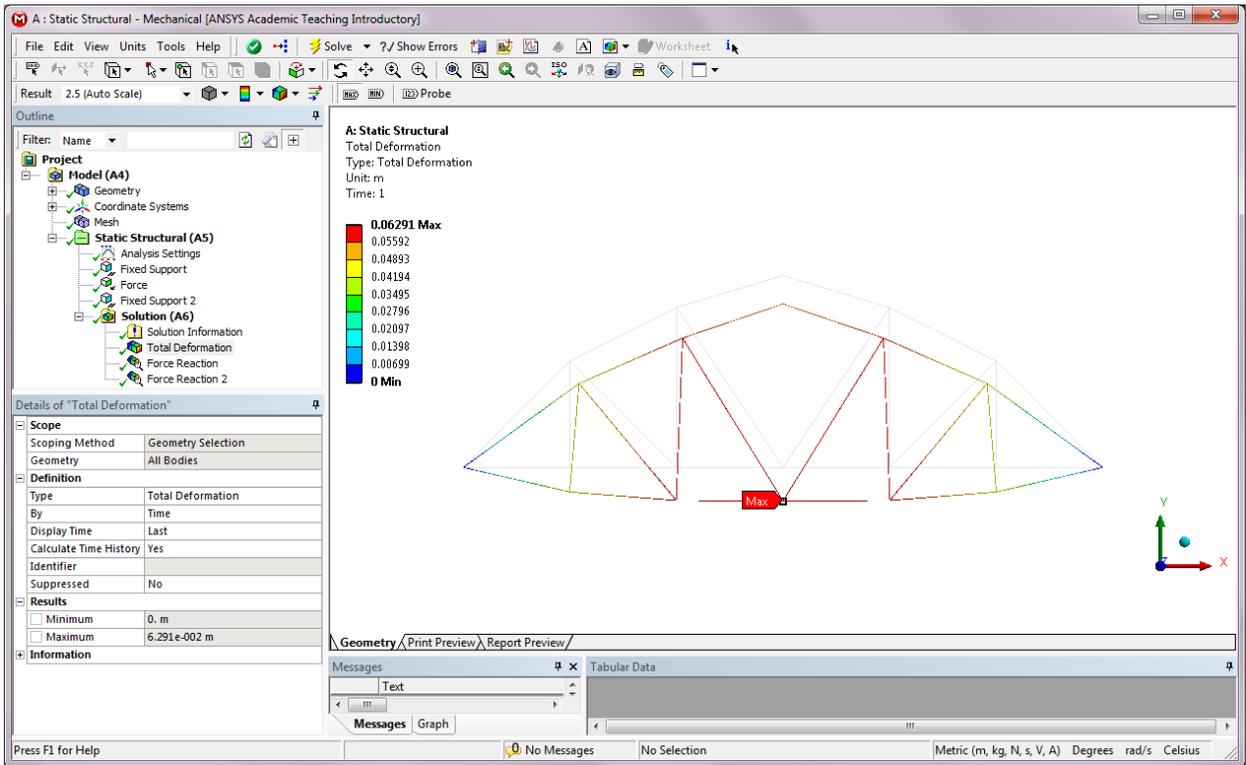
2.11. In the Engineering Data table, add material properties of Douglas Fir:

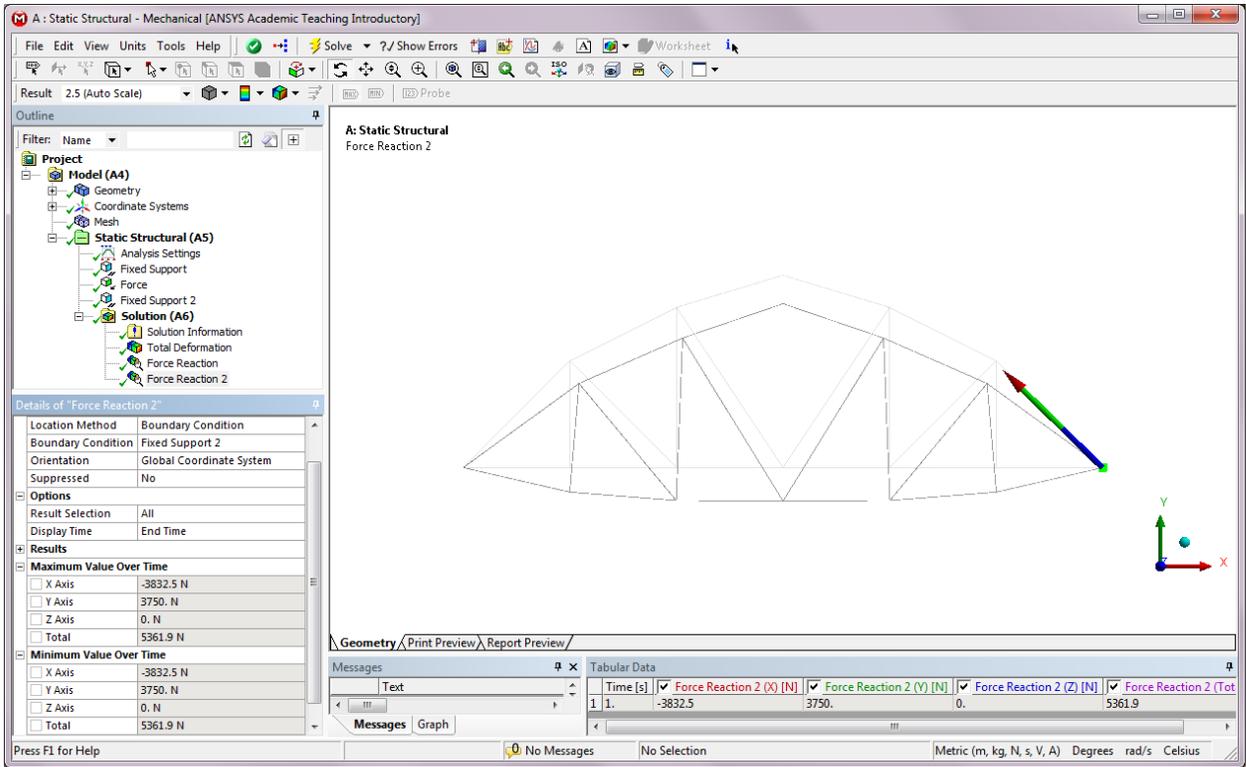


In the DesignModeler program, create a line sketch of the truss bridge and a cross section definition of the truss member. Then create a line body concept model from the sketch:



In the Multiple Systems – Mechanical program, retrieve results after applying loads and boundary conditions:



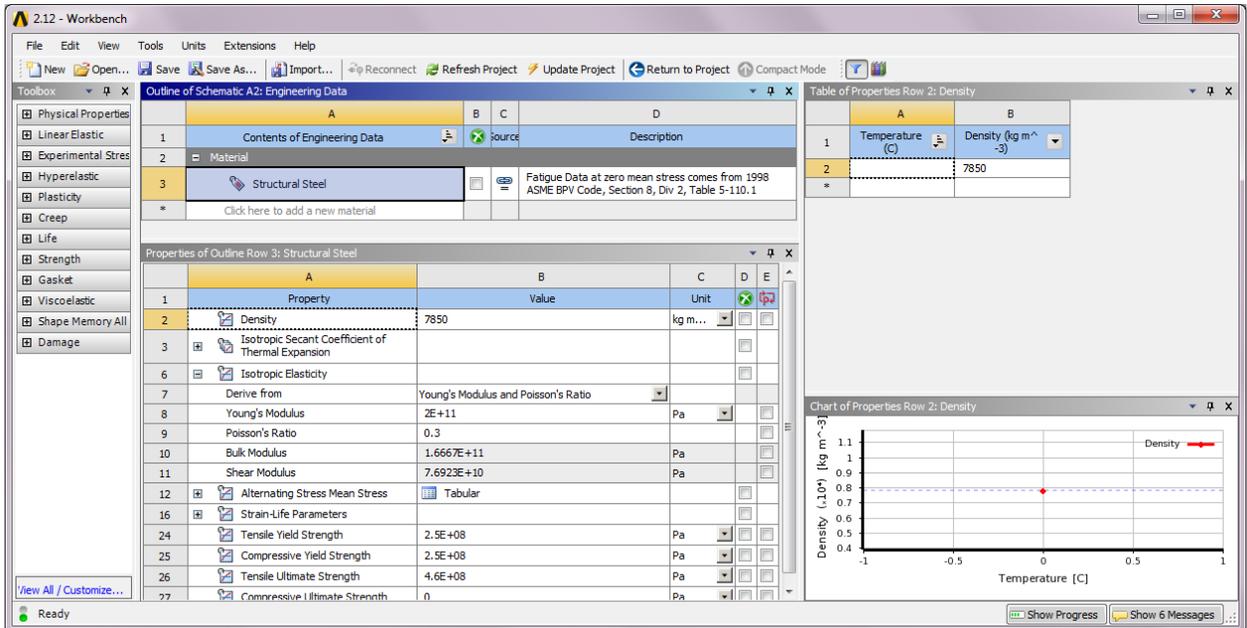


The maximum total deformation of the truss tower crane is: **62.91 mm**

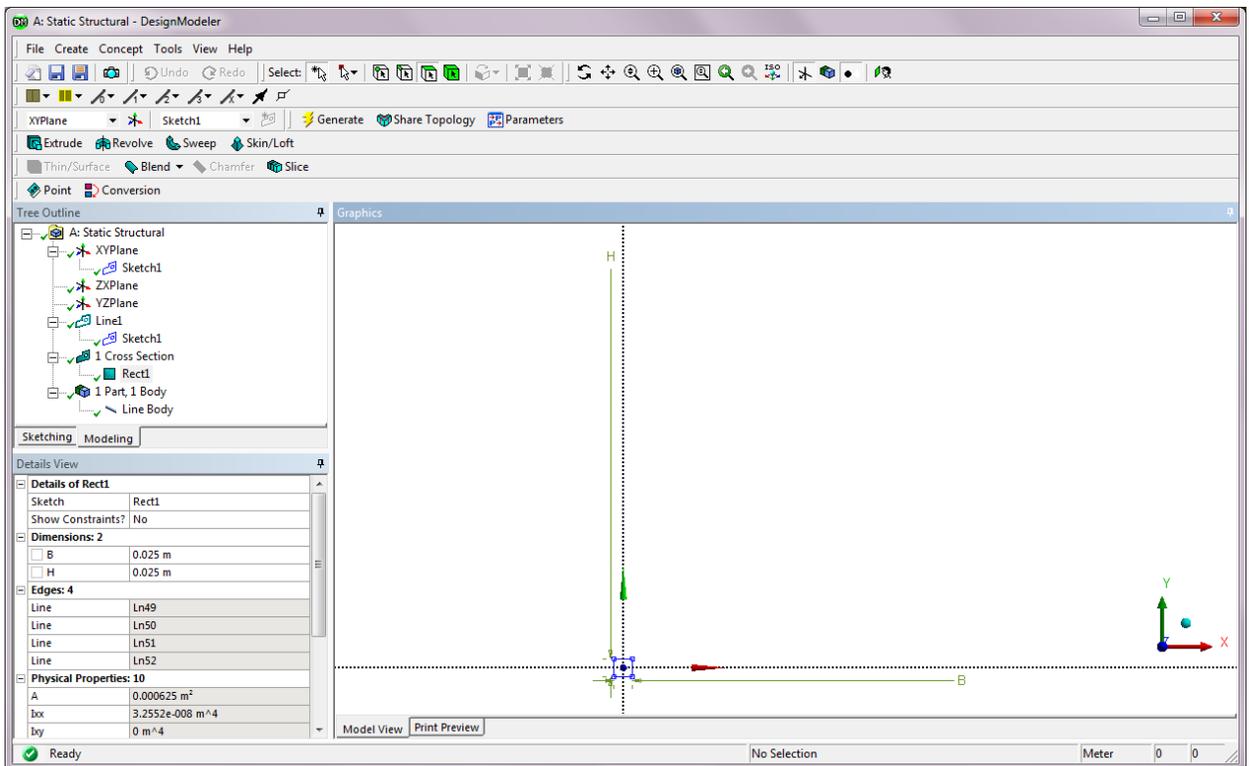
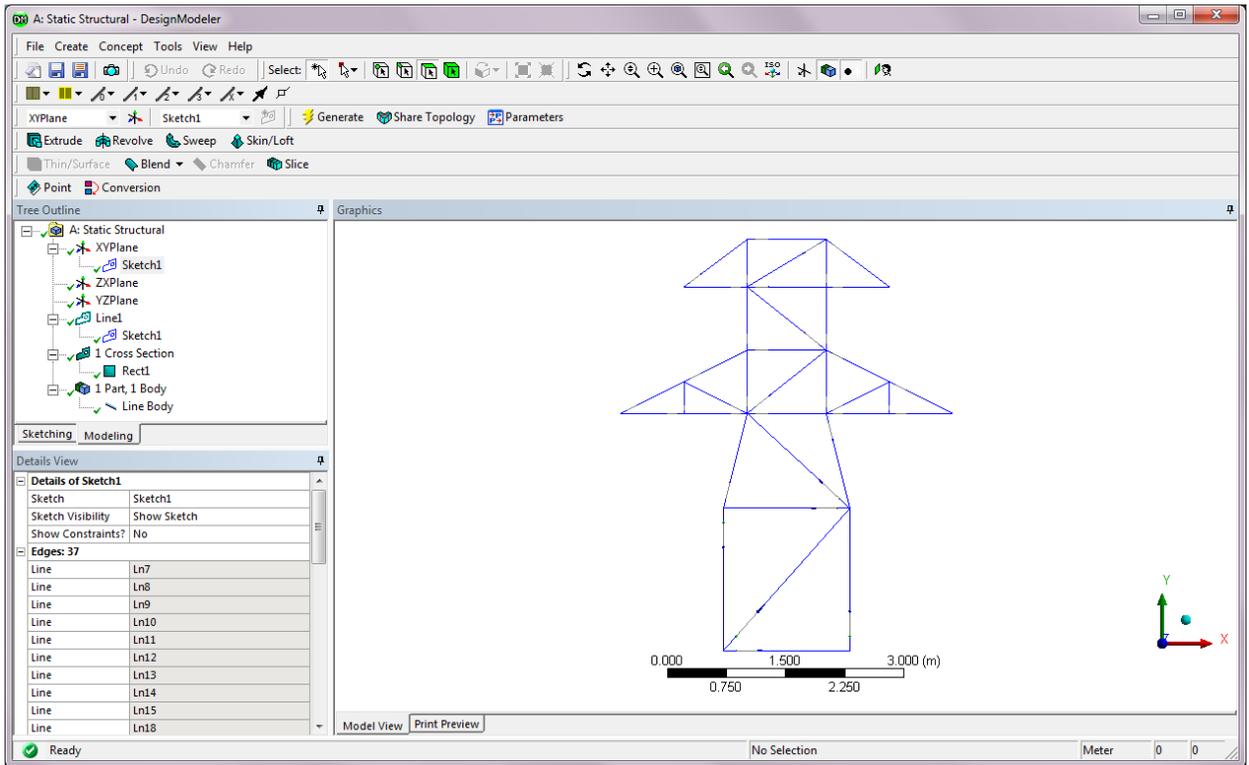
X, Y and Z components of the reaction force at the left support are: **(3832.5 N, 3750 N, 0)**

X, Y and Z components of the reaction forces at the right support are: **(-3832.5 N, 3750 N, 0)**

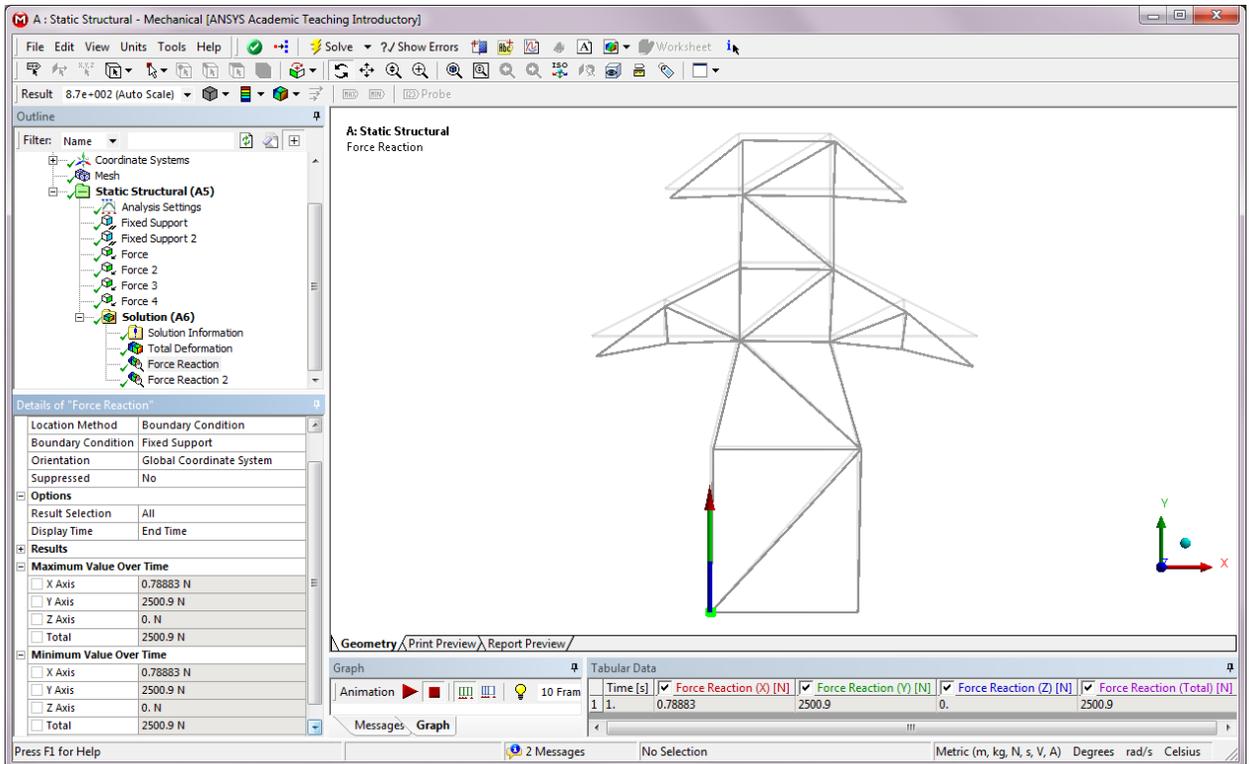
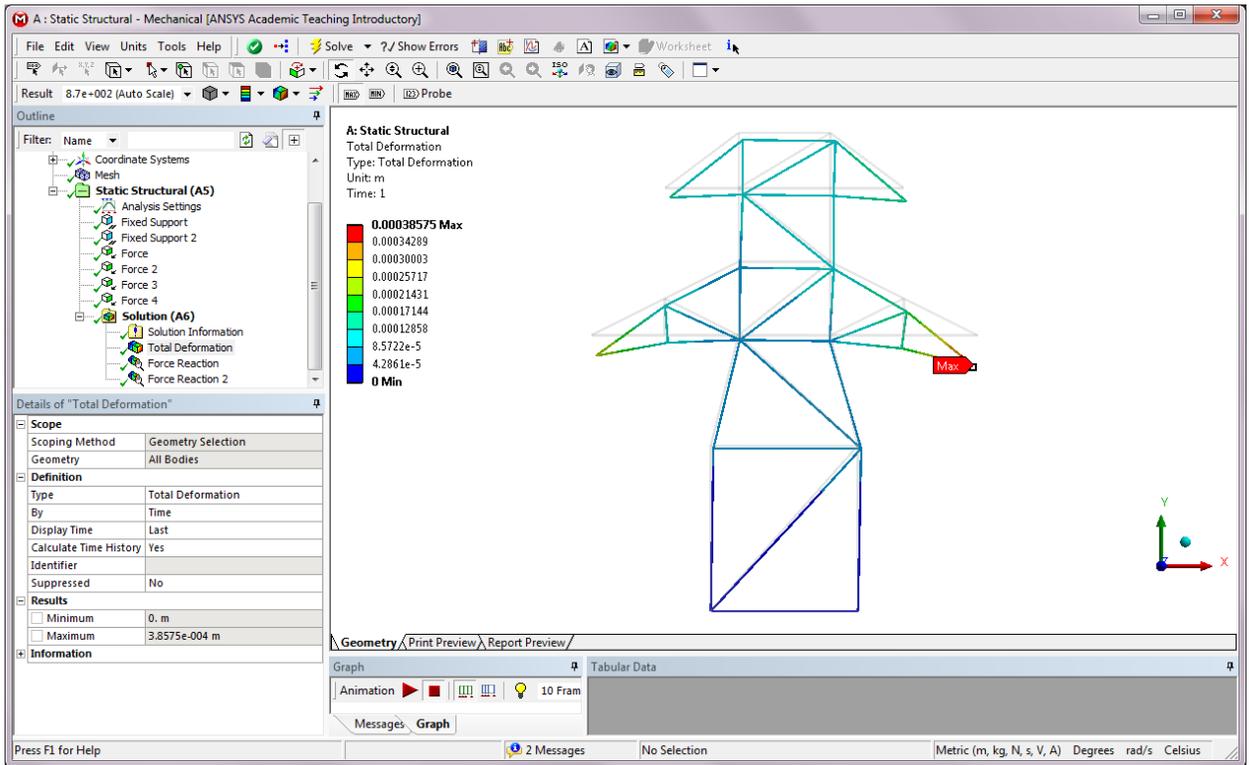
2.12. In the Engineering Data table, use material properties of Structural Steel:

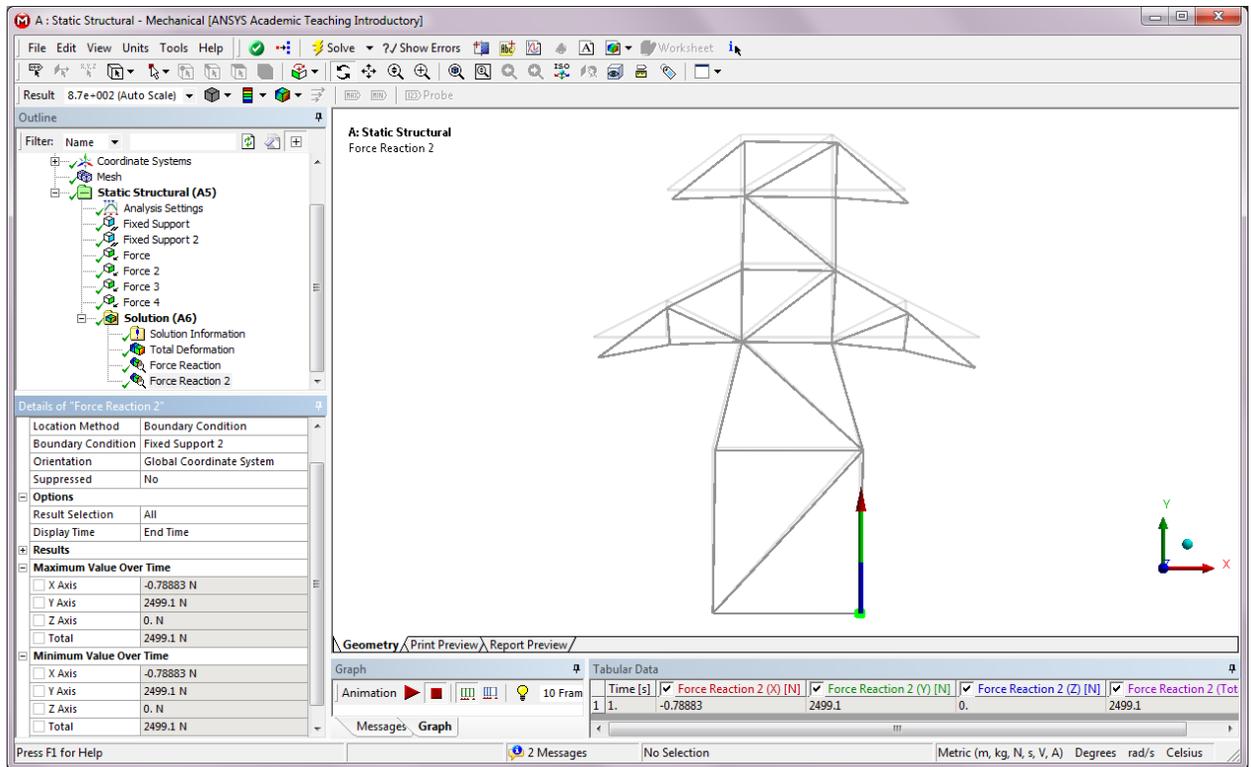


In the DesignModeler program, create a line sketch of the truss transmission tower and a cross sectional definition of the truss member. Then create a line body concept model from the sketch:



In the Multiple Systems – Mechanical program, retrieve results after applying loads and boundary conditions:





The maximum total deformation of the truss transmission tower is: **0.38575 mm**

X, Y and Z components of the reaction force at the left support are: **(0.78883 N, 2500.9 N, 0)**

X, Y and Z components of the reaction forces at the right support are: **(-0.78883 N, 2499.1 N, 0)**