Chapter 2

Conduction Heat Transfer

2.1 Given, $\Delta T = 420°\text{C}$, $\Delta x = 25$ mm, $k = 0.04$ W/(m °C).

By Fourier's law, Equation (2.1), we have the rate of heat transfer as

$$\begin{aligned}
\dot{q} &= -k\frac{dT}{dx} \\
&= -k\frac{\Delta T}{\Delta x} \\
&= -(0.04) \times \frac{420}{25 \times 10^{-3}} \\
&= \boxed{-672\,\text{W/m}^2}
\end{aligned}$$

The negative sign indicates that heat is lost.

2.2 Given, $\Delta x = 10$ cm, $\Delta T = 160°\text{C}$, $\dot{q} = -80$ W/m^2.

By Equation (2.1), we have

$$\begin{aligned}
\dot{q} &= -k\frac{dT}{dx} \\
k &= -\dot{q}\,\frac{\Delta x}{\Delta T} \\
&= -(-80) \times \frac{0.1}{160} \\
&= \boxed{0.05\,\text{W/(m °C)}}
\end{aligned}$$

2.3 Given, $k = 0.04$ W/(m °C), $\Delta T = 45$°C, $\dot{q} = -40$ W/m^2.

By Equation (2.1), we have

$$\begin{aligned} \dot{q} &= -k\frac{\Delta T}{\Delta x} \\ \Delta x &= -\frac{k\,\Delta T}{\dot{q}} \\ &= -\frac{0.04 \times 45}{-40} \\ &= 0.045\,\mathrm{m} \\ &= \boxed{45\,\mathrm{mm}} \end{aligned}$$

2.4 Given, $\Delta x = 50$ mm, $k = 1.2$ W/(m °C), $T_1 = 230$°C, $T_2 = 20$°C, $A_s = 6$ m^2.

By Equation (2.1), the heat transfer rate is

$$\begin{aligned} \dot{q} &= -k\frac{T_1 - T_2}{\Delta x} \\ &= -1.2 \times \frac{20 - 230}{0.05} \\ &= 5040\,\mathrm{W/m^2} \end{aligned}$$

Therefore, the rate at which heat is transferred across 6 m^2 becomes

$$\begin{aligned} \dot{Q} &= \dot{q}A_s \\ &= 5040 \times 6 \\ &= \boxed{30.24\,\mathrm{kW}} \end{aligned}$$

2.5 Given, $\Delta x = 5$ cm, $k = 0.13$ W/(m °C), $T_h = 200$°C, $\dot{q} = -200$ W/m^2.

By Equation (2.1), the heat transfer rate is

$$\dot{q} = -k\frac{\Delta T}{\Delta x}$$

$$
\begin{aligned}
-200 &= -k\frac{T_h - T_c}{\Delta x} \\
200 &= 0.13\,\frac{T_h - T_c}{0.05} \\
T_h - T_c &= \frac{200 \times 0.05}{0.13} \\
200 - T_{\text{cold}} &= \frac{200 \times 0.05}{0.13} \\
&= 76.92 \\
T_{\text{cold}} &= 200 - 76.92 \\
&= \boxed{123.08^\circ\text{C}}
\end{aligned}
$$

2.6 Given, $\Delta x = 15$ mm, $h = 90$ cm, $k = 0.8$ W/(m °C), $T_i = 22$°C, $T_o = -10$°C, $\dot{Q} = -2000$ kJ/h.

Therefore, the heat loss per second is

$$
\begin{aligned}
\dot{Q} &= -\frac{2000 \times 10^3}{3600} \\
&= -555.56\,\text{J/s}
\end{aligned}
$$

But, $\dot{Q} = \dot{q}\,A_s$, thus,

$$
\begin{aligned}
\dot{Q} &= \dot{q}\,A_s \\
&= -k\frac{\Delta T}{\Delta x}\,A_s \\
-555.56 &= -0.8 \times \frac{32}{0.015}\,A_s \\
A_s &= \frac{555.56 \times 0.015}{0.8 \times 32} \\
&= 0.3255\,\text{m}^2
\end{aligned}
$$

Let w be the width of the glass window. Then $A_s = h\,w$. Therefore,

$$\begin{aligned} h \times w &= 0.3255 \\ w &= \frac{0.3255}{0.9} \\ &= 0.36\,\text{m} \\ &= \boxed{36\,\text{cm}} \end{aligned}$$

2.7 Given, $L = 20$ cm, $k = 1.75$ W/(m K), $T_i = 1200$ K, $T_o = 900$ K, $w = 2.5$ m, $h = 0.75$ m.

The heat transfer rate is given by

$$\begin{aligned} \dot{Q}_x &= A\,\dot{q}_x = -\frac{kA}{L}\,(T_o - T_i) \\ &= \frac{kA}{L}\,(T_i - T_o) \\ &= \frac{k\,(h \times w)}{L}\,(T_i - T_o) \\ &= \frac{1.75\,(2.5 \times 0.75)}{0.20} \times (1200 - 900) \\ &= \boxed{4921.88\,\text{W}} \end{aligned}$$

2.8 Given, $T_w = T_\infty = 20°$C, $D = 100$ mm, $T_s = 250°$C, $\epsilon = 0.8$, $h = 15$ W/(m^2 K).

Rate of heat loss $\dot{Q}$ from the pipe is by convection to the room air, and by radiation exchange with the walls. Thus,

$$\begin{aligned} \dot{Q} &= h\,A\,(T_s - T_\infty) + \epsilon\,A\,\sigma\,\left(T_s^4 - T_{\text{surr}}^4\right) \\ &= h\,(\pi DL)(T_s - T_\infty) + \epsilon\,(\pi DL)\,\sigma(T_s^4 - T_{\text{surr}}^4) \end{aligned}$$

where L is the length of the pipe.

The rate heat loss per unit length of pipe is given by

$$
\begin{aligned}
\dot{q} &= \frac{\dot{Q}}{L} \\
&= 15\,(\pi \times 0.1)(250 - 20) + 0.8\,(\pi \times 0.1)\,(5.67 \times 10^{-8})(523.15^4 - 293.15^4) \\
&= 1083.86 + 962.16 \\
&= \boxed{2046\,\mathrm{W/m}}
\end{aligned}
$$