Chapter 2

2.1 $A_e = \dfrac{G\lambda^2}{4\pi}$; $\lambda = \dfrac{c}{f} = \dfrac{3\times10^8}{9\times10^9} = 0.0333\,m$

G_{dB}	10	20	30
G	10	100	1000
$A_e\,m^2$	8.84×10^{-4}	8.84×10^{-3}	8.84×10^{-2}

2.2 $\lambda = \dfrac{c}{f} = \dfrac{3\times10^8}{1500\times10^6} = 0.2\,m$

$$A_e = \frac{G\lambda^2}{4\pi} = \frac{10^3 * 0.2^2}{4\pi} = 3.18\,m^2$$

$$P_t = \frac{P_{av}}{d_t} = \frac{25\times10^3}{0.2} = 1.25\,MW$$

$$P_D = \frac{P_t G}{4\pi R^2} = \frac{1.25\times10^6 * 10^3}{4\pi * (50\times10^3)^4} = 39.79\,MW/m^2$$

2.3 $S_{min} = 5.0\,dBm \Rightarrow S_{min} = 5-30 = -25\,dB \equiv 10^{-2.5} = 3.162\,mW$

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right)^{1/4} \Rightarrow R_{max}^4 = \frac{125\times10^4 * 1000^2 * 0.2^2\,\sigma}{(4\pi)^3 * 3.162\times10^{-3}}$$

$\Rightarrow$

for $\sigma = 1m^2 \Rightarrow R_{max} = 298.78m$

$\sigma = 10m^2 \Rightarrow R_{max} = (298.78)(10^{.25}) = 531.3m$

$\sigma = 20m^2 \Rightarrow R_{max} = (298.78)(20^{.25}) = 631.8m$

2.4 $F = 5dB \equiv 10^{.5} = 3.162$

$$S_{min} = kT_0BF\,(SNR)_{min} = 1.38\times10^{-23} * 290 * 5\times10^6 * 3.162 * 34.6736$$
$$= 2.194286\,PW$$

$$P_t = \frac{R^4 (4\pi)^3 S_{min}}{G^2 \lambda^2 \sigma} = \frac{(150\times10^3)^4 (4\pi)^3 (2.194286\times10^{-12})}{(5000)^2 (0.2)^2 (10)} = 220.440\,KW$$

$$\tau = \frac{1}{B} = \frac{1}{5\times10^6} = 0.2\,\mu sec.$$

2.5 Repeat example

2.6 A pulse train with period $T = \frac{1}{f_r}$ can be expressed as an exponential Fourier Series by

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2n\pi f_r t}$$

where

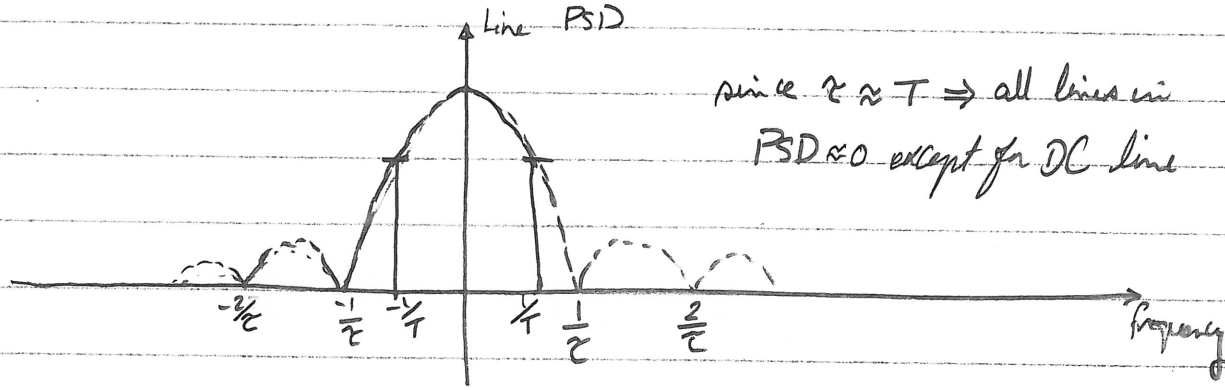
$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j2n\pi f_r t}\, df \Big|_{f=\frac{n}{T}}$$

$\Rightarrow$

$C_n = \frac{\tau}{T}\, \text{Sinc}\left(n\pi \frac{\tau}{T}\right)$ $\Leftrightarrow$ use Fourier transform table

$\Rightarrow$ DC component $(f=0=n) \Rightarrow C_0 = \frac{\tau}{T}$

and corresponding DC power is $|C_0|^2 = \left(\frac{\tau}{T}\right)^2 = (d_t)^2 \equiv$ duty cycle^2.
For high PRF the duty cycle, $d_t \cong 1$, and since the envelope of the Fourier series is maximum @ 0 & smaller everywhere else (Sinc function envelope) $\Rightarrow$ DC is dominant.



2.7 Repeat example

2.8 $\lambda = \frac{c}{f} = \frac{3\times10^8}{5000\times10^6} = 0.06\ m$

$R_u = \frac{c}{2f_r} = \frac{3\times10^8}{2*250} = 600\ Km$

$(SNR)_o = 0\ dB \equiv 1$

2-8 Cont. Aperture area $= \pi r^2 = \pi * 2^2 = 12.5664\, m^2$

$$\Rightarrow G = \frac{4\pi A}{\lambda^2} = \frac{(4\pi)(12.5664)}{(0.06)^2} = 43.865 \times 10^3$$

$\Rightarrow$

$$R_0 = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_0 B L (SNR)_0} = \frac{(10^6)(43.865)^2 (.06)^2 (10)}{(4\pi)^3 (1.38 \times 10^{-23})(600)\left(\frac{1}{2 \times 10^{-6}}\right)(31.6227)}$$

$$\Rightarrow R_0 = 718.5818\ Km$$

In general $(SNR)_{R_0} = \dfrac{Q\sigma}{R_0^4}$ and @ range R $(SNR)_R = \dfrac{Q\sigma}{R^4}$

$\Rightarrow$

$$\frac{(SNR)_R}{(SNR)_{R_0}} = \frac{R_0^4}{R^4} \Rightarrow (SNR)_R = (SNR)_{R_0} \left(R_0/R\right)^4$$

Therefore for $R = 0.75 R_0 \Rightarrow (SNR)_{.75R_0} = (SNR)_{R_0}\left(\frac{R_0}{.75 R_0}\right)^4 = \left(\frac{1}{.75}\right)^4 \approx 5dB$

2.9 The 2-way atmospheric loss at 20km is

$$L_a = 2 * 20\,km * \frac{.25\,dB}{Km} = 10dB \equiv 10$$

$SNR = \dfrac{Q}{R^4 L_a}$ since the SNR remains constant then so does the ratio $Q/R^4 L_{at} \Rightarrow$

$$R_0^4 L_a = R^4 * 1 \Rightarrow \left(\frac{R}{R_0}\right)^4 = L_a \Rightarrow 40 \log_{10} \frac{R}{R_0} = 10$$

$$\Rightarrow \frac{R}{R_0} = 1.77828 \Rightarrow R = 35.5656\ Km$$

2-10 In general $SNR = \dfrac{Q\sigma}{R^4} \Rightarrow \dfrac{(SNR)_{3/4\,R}}{(SNR)_{1/2\,R}} = \dfrac{Q\sigma/(3/4 R)^4}{Q\sigma/(\frac{1}{2}R)^4}$

but $\dfrac{(\frac{1}{2})^4}{(\frac{3}{4})^4} = 0.19753 \equiv -7dB$

$\Rightarrow$

$$SNR = \frac{Q(\sigma = 10)}{(\frac{3}{4}R)^4} = \frac{Q(\sigma_x = ?)}{(\frac{1}{2}R)^4} \Rightarrow \sigma_x = \frac{(10)(.5)^4 (R)^4}{(.75)^4 (R)^4} = 1.975$$

$$\Rightarrow \sigma_x = 1.569\, m^2$$

2.11 $\lambda = \frac{c}{f} = \frac{3\times10^{8}}{94\times10^{9}} = 3.19\,mm$

$\Delta R = \frac{c\tau}{2} = \frac{3\times10^{8} * .05\times10^{-3}}{2} = 15\,Km$

$B = \frac{1}{\tau} = \frac{1}{.05\times10^{-3}} = 20\,KHz$.

$$SNR = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 K T_0 B F L} = \frac{10 * 1000^2 * (3.19\times10^{-3})^2 (1)}{(R^4)(4\pi)^3(1.38\times10^{-23})(290)(20\times10^{3})(3.162)(6.309)}$$

$\Rightarrow$

$$SNR = \frac{1}{R^4}\, 32.11603\times10^{12}$$

for $SNR = 15\,dB = 31.62278 \Rightarrow R = \left(\frac{32.11603\times10^{12}}{31.62278}\right)^{1/4} \approx 1.004\,Km$

$\theta_{3dB} \approx \frac{\lambda}{D} \Rightarrow$

$$\theta_{3dB} = \frac{3.19\times10^{-3}}{0.254} \cdot \frac{180^\circ}{\pi} = 0.719^\circ$$

$$\dot{\theta} = \frac{\text{angular coverage}}{\text{frame time}} = \frac{200}{3} = 66.667\ deg/sec$$

$$T_i = \frac{\theta_{3dB}}{\dot{\theta}} = \frac{0.719}{66.667} = 10.794\ m\,sec$$

atmospheric attenuation is 3dB/Km

$\Rightarrow$ total atmospheric loss in SNR is $L_a = 2 * R * 3 = 6.02328\,dB$
$\equiv 4.00247$

$$(SNR)_{R_{new}} = (SNR)_{R_0} - 6.02328 \Rightarrow R_{new} = \frac{R_0}{(4.0024)^{1/4}} = 709.7439\,m$$

2.12 Repeat example

2.13

$$R_{co} = \sqrt{\frac{P_t G \sigma B_J L_J}{4\pi P_J G_J B L}} = \sqrt{\frac{10 * 100 * 1\times10 * 1.585 * B_J}{4\pi * 100 * 10 * 6.30957 * B}}$$

$\Rightarrow$

$$R_{co} = 1.41\sqrt{B_J/B}\quad m$$

2.14 $$R_{CO} = \left(\frac{P_t G^2 R_J^2 \sigma B_J L_J}{(4\pi)(P_J) G_J G' B L}\right)^{1/4} = \left(\frac{10 * 1000^2 * (25\times10^3)^2 * 1 * 1.585}{4\pi * 200 * 31.62 * 15.85 * 6.31}\right)^{1/4}\left(\frac{B_J}{B}\right)^{1/4}$$

$\Rightarrow$

$$R_{CO} = 187.88\,(B_J/B)^{1/4} \quad m$$

2.15 $$R_{CO} = \sqrt{\frac{P_t G \sigma B_J L_J}{4\pi P_J G_J B L}} = \sqrt{\frac{55\times10^3 * 1000 * 5 * 50\times10^6 * 1.259}{4\pi * 150 * 15.85 * 500\times10^3 * 10}}$$

$\Rightarrow$ $R_{CO} = 340.43$ m

2.16 $$SNR = \frac{P_{av} A_e \sigma}{4\pi R^4 k T_0 F L}\,\frac{T_s}{\Omega}$$

$$\Omega = 2\,\theta_{az} \cos\alpha \sin\left(\frac{\theta_{el}}{2}\right)$$

$\theta_{az} = 360° \equiv 2\pi$ rad. ; $\theta_{el} = (65° - 5°)\cdot\left(\frac{\pi}{180}\right) = 1.0472$ rad

$\alpha \equiv$ elevation center of the search sector is

$$\alpha = \left[5° + (65° - 5°)/2\right]\cdot\left(\frac{\pi}{180}\right) = 0.6109 \text{ rad}$$

$\Rightarrow$

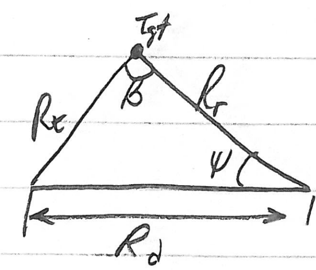
$$\Omega = 5.1469 \text{ steradians}$$

$$\Rightarrow P_{av} A_e = \frac{SNR}{\sigma}\, 4\pi R^4 k T_0 F . L\, \frac{\Omega}{T_s}$$

$$= \frac{(10^{6})(4\pi)(75\times10^3)^4(1.38\times10^{-23})(290)(10^{.8})(10^{.6})}{(10^{.5})}\;\frac{.6109}{2}$$

$\Rightarrow$ $P_{av} A_e = 3251.09 \equiv 35.1203$ dB

2.17



applying the law of cosines yields

$$R_t^2 = R_r^2 + R_d^2 - 2R_r R_d \cos\psi$$

let $S = R_r + R_t \Rightarrow R_r = \dfrac{S^2 - R_d^2}{2(S - R_d)\cos\psi}$.

2-18 $P_J = \frac{P_t G G_J \lambda^2}{(4\pi)^2 R^4}$ is the power density @ the jammer and the power received by the jammer is

$$\frac{P_t G G_J \lambda^2}{(4\pi)^2 R^2} \cdot \frac{G_J}{4\pi R^2}$$

so the power density received by the radar is

$$P_r = \frac{P_t G G_J \lambda^2}{(4\pi)^2 R^2} \frac{G_J}{4\pi R^2} \frac{G\lambda^2}{4\pi} \cdot \frac{3}{4}$$

$\Rightarrow$

$$\frac{P_J}{P_r} = \frac{(4\pi)^2 R^2}{G G_J \lambda^2} \cdot \frac{4}{3}$$

$\Rightarrow$

for $G = G_J = 200$ & $\frac{R}{\lambda} = 10^4 \Rightarrow \frac{P_J}{P_r} = \frac{(4\pi)^2 (10^4)^2 (4)}{(200 \times 200)(3)} = 57.213$ dB

2-19 The power density @ the target is

$$\frac{P_t G_t}{4\pi R_t^2} = \frac{4000 * 10^{2.5}}{4\pi * (35 \times 10^3)^2} = 8.21697 \times 10^{-5} \text{ W/m}^2$$

The effective radiated power from the target is

$$8.21697 \times 10^{-5} * 3 = 2.46509 \times 10^{-3} \text{ W}$$

$\Rightarrow$ @ the missile

$$SNR = \frac{P_t G_t \sigma_B A_e}{4\pi R_t^2 R_r^2} \frac{1}{k T_0 B F L}$$

$\Rightarrow$

$$SNR = 2.46509 \times 10^{-3} * \frac{.1}{(17 \times 10^3)^2} \frac{1}{(1.38 \times 10^{-23})(290)(750 \times 10^{-2})(10^{-7})} = 35 \text{ dB}$$

2.20 when 0.1 dB/Km atmospheric attenuation is present $\Rightarrow$

$$SNR = 35 - 0.1 * (17 + 35) = 29.8 \text{ dB}$$

$x = \frac{\pi r}{\lambda} \sin\theta$

2.21 expand the function $\frac{\sin x}{x}$ as a series yields

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots$$

since $x = \frac{\pi r}{\lambda} \sin\theta \Rightarrow$

$$x = \frac{\pi r}{\lambda}\left(\theta - \theta^3/3! + \theta^5/5! - \cdots\right)$$

use small angle approximation i.e., $\sin\theta \approx \theta$ or $\theta^n \approx 0$ for $n \geq 3$

$$\Rightarrow G = \left|\frac{\sin x}{x}\right|^2 = \left(1 - \frac{(\pi\theta r/\lambda)^2}{3!}\right)^2 \approx 1 - \frac{1}{3}\left(\frac{\pi r \theta}{\lambda}\right)^2$$

and the average antenna gain over any angular region around the boresight can be computed.

2.22 $f(y) = \left[\exp\left(-2\ln 2\left(y/\theta_{3d}\right)^2\right]^4\right.$

assuming circular symmetry and averaging the above pattern over the one-way half power beamwidth yield

$$\bar{\alpha}_{1\text{-way}} = \frac{2\pi}{\pi\left(\frac{\theta_{3d}}{2}\right)^2}\int_0^{\theta_{3d}/2} \exp\left[-8\ln 2\left(y/\theta_{3d}\right)^2\right] y\, dy$$

make change of variable $u = y^2 \Rightarrow du = 2y\,dy$

$\Rightarrow$

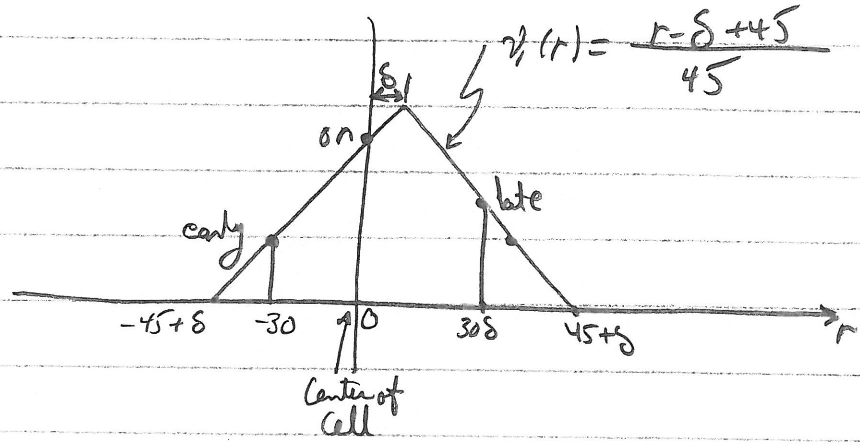
$$\bar{\alpha}_{1\text{-way}} = \frac{4}{\theta_{3d}^2}\int_0^{\left(\frac{\theta_{3d}}{2}\right)^2} \exp\left(-8\ln\frac{2u}{\theta_{3d}}\right) du$$

$$= \frac{4}{\theta_{3d}^2}\left[-\frac{\theta_{3d}^2}{8\ln 2}\right]\left[\exp\left\{-8\frac{\ln 2}{\theta_{3d}^2}\frac{\theta_{3d}^2}{4}\right\} - 1\right]$$

$$= \frac{1}{2\ln 2}\left(3/4\right) = 0.51$$

$$\Rightarrow \bar{\alpha}_{1\text{-way}} = 2.67\,dB$$

2.23



for early measurement $A_{early} = V_1(r=-30) = \frac{+30-\delta+45}{45} = \frac{15-\delta}{45}$

for late measurement $A_{late} = V_2(r=+30) = \frac{-30+\delta+45}{45} = \frac{15+\delta}{45}$

$\Rightarrow$

the sum of early + late measurements $\Rightarrow \frac{15-\delta}{45} + \frac{15-\delta}{45} = \frac{30}{45} = \frac{2}{3}$

and the difference is $\frac{15-\delta}{45} - \frac{15-\delta}{45} = \frac{2\delta}{45}$

$\Rightarrow$

$$\frac{A_{late} - A_{early}}{A_{late} + A_{early}} = \frac{2\delta/45}{2/3} = \delta/15$$

Therefore, $\delta_{meters} = 15 \frac{A_{late} - A_{early}}{A_{late} + A_{early}}$

by normalizing with respect to $A_{late} + A_{early}$

Now, if $A_{early} = 4/6$ & $A_{late} = 1/6$

we get

$$\delta = 15 \frac{1/6 - 4/6}{1/6 + 4/6} = 15 \frac{-3}{5} = -9 \text{ meters}$$

the minus sign means that we are left of the center of the on-cell.

2.24 since $H(f) = \frac{1}{1+a^2f^2}$ then the power attenuation is

$$|H(f)|^2 = \left(\frac{1}{1+a^2f^2}\right)^2$$

Solving for the ½ power point yields

$$\frac{1}{1+a^2f^2_{3dB}} = \frac{1}{\sqrt{2}} \Rightarrow a^2 = \frac{\sqrt{2}-1}{f^2_{3dB}}$$

The average power loss when the signal is between the peak value and the crossover frequency, f_c is

$$L = \frac{1}{f_c}\int_0^{f_c} \frac{df}{1+a^2f^2} = \frac{1}{2(1+a^2f_c^2)} + \frac{1}{2af_c}\tan^{-1}(af_c)$$

Therefore, for $f_{3dB} = 500\ Hz$ & $f_c = 350\ Hz \Rightarrow$

$$af_c = \frac{\sqrt{\sqrt{2}-1}}{500} * 350 = 0.451$$

and

$$L = \frac{1}{(2)(1+.451^2)} + \frac{1}{(2)(.451)}\tan^{-1}(.451) = 0.885 \equiv 0.53\,dB$$

2.25 The radar equation is $SNR = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 B F L}$

a. $P_t = 65\,KW \equiv 48.13\,dB$

$f_r = 4KHz \Rightarrow T = \frac{1}{4\times10^3} = 0.25\ msec$

$d_t = \frac{\tau}{T} \Rightarrow \tau = d_t \cdot T = 0.3 * .25\times10^{-3} = 75\,\mu sec$

$\Rightarrow B = \frac{1}{\tau} = \frac{1}{75\times10^{-6}} = 13.333\,KHz$

$G = \frac{4\pi A_e}{\lambda^2}$; $\lambda = \frac{c}{f} = \frac{3\times10^8}{8\times10^9} = 0.0375 = 37.5mm$

$A = \pi\frac{D^2}{4} = \pi\cdot\left(\frac{1}{2}\right)^2 = 0.7854\,m^2$

& $A_e = \rho A \Rightarrow A_e = 0.7 * 0.7854 = 0.54978\,m^2$

2.25 Cont. Thus, $G = \frac{4*\pi*0.54978}{(0.0375)^2} = 4912.87 \equiv 36.91\,dB$

b. $R_u = \frac{c}{2f_r} = \frac{3\times10^8}{2*4\times10^3} = 37.5\,Km$

c. Use the MATLAB function "radar_eq.m"

d.

P_t	G	σ	λ^2	$(4\pi)^3$	kT_0	B	F	L
48.13	36.91	6	-28.98	32.98	-203.98	41.25	8	5

$\Rightarrow$

$R^4 = 48.13 + 2*36.91 + 6 - 28.98 - 32.98 + 203.98 - 41.25 - 8 - 5 - 14 = 202.181\,dB$

$\Rightarrow R = 10^{(202.181/40)} = 113.377\,m \Rightarrow R = 11.34\,Km$

so if the SNR becomes 18dB $\Rightarrow$

$R^4 = 202.181 + 14 - 18 = 199.181\,dB$

$\Rightarrow R = 10^{\frac{199.181}{40}} = 9.54\,Km.$

2.26 a. $G = \frac{26000}{\theta_A\theta_E} = \frac{26000}{(1)(5)} = 5200 \equiv 37.16\,dB$

b. $G = \frac{4\pi A_e}{\lambda^2} \Rightarrow A_e = 37.16 + 10\log\lambda^2 - 10\log(4\pi)$

but $\lambda = 0.0534\,m$

$\Rightarrow A_e = 37.16 + 20\log(.0534) - 10\log(4\pi) = .71873\,dB \equiv A_e = 1.18\,m^2$

$\Rightarrow A = A_e/\rho = 1.18/.6 = 1.967\,m^2$

c. $B = \frac{1}{\tau} = \frac{1}{10\times10^{-6}} = 10\,MHz \equiv 70\,dB$

P_t	G^2	σ	λ^2	$(4\pi)^3$	kT_0	B	F	L
46.99	74.32	ϕ	-25.45	32.98	-203.98	70	10	5

$\Rightarrow$

$R^4 = 46.99 + 74.32 + \phi - 25.45 - 32.98 + 203.98 - 70 - 10 - 5 - 14 = 166.86\,dB$

$\Rightarrow R = 10^{166.86/40} = 14.842\,Km.$

2.27 $\Omega = 2\pi \frac{3}{4} = \frac{6\pi}{4} = \frac{3\pi}{2} = 4.7124$ Sterradians

$$P_{av} A_e = \frac{SNR\ 4\pi R^4\ kT_0\ FL\ \Omega}{\sigma\ T_s}$$

SNR	4π	R^4	kT_0	F	L	σ	T_s	Ω
15	10.99	196.12	-203.98	10	5	6	4.77	6.20

$\Rightarrow$

$$P_{av} A_e = 15 + 10.99 + 196.12 - 203.98 + 10 + 5 + 6.20 - 6 - 4.77$$
$$= 28.56 \text{ dB}$$

2.28 a. $G = \frac{26000}{\theta_A \theta_E} = \frac{26000}{(2)(4)} = 3250 \equiv 35.12 \text{ dB}$

b. $\lambda = \frac{c}{f} = \frac{3\times10^8}{7\times10^9} = 0.042857 \text{ m}$

$$G = \frac{4\pi A_e}{\lambda^2} \Rightarrow A_e = G\lambda^2/4\pi = \frac{(3250)(.042857)^2}{4\pi} = 0.475 \text{ m}^2$$

c.

P_t	G^2	σ	λ^2	$(4\pi)^3$	kT_0	B	F	L
50	70.24	6	-27.36	32.98	-203.98	70	12	2

$$R^4 = 50 - 70.24 + 6 - 27.36 - 32.98 + 203.98 - 70 - 12 - 2 - 13 = 166.88 \text{ dB}$$
$$\Rightarrow R = 10^{166.88/40} = 14.86 \text{ Km.}$$

2.29 a. $P_{r_{dB}} = \left(P_t + G^2 + \lambda^2 + \sigma - 4\pi^3 - R^4 - L\right)_{dB}$

$P_t = 50$ dB, $L = 1$ dB, $G = 38$ dB, $f_c = 5\times10^9$ Hz $\Rightarrow \lambda = -12.218$ dB

$R = 44.77$ dB

$\Rightarrow P_{r_{dB}} = -111.5$ dB

b. $P_r = -110.37$ dB

2-30

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} \quad ; \; T_o = 290^\circ K$$

$\Rightarrow$

$$F_{total} = 2.0345 + \frac{10 - 1}{10^{.2}} + \frac{10^{1.5} - 1}{(10^{.2})(10^{2.2})} = 2.6145 \equiv 4.174\,dB$$

note that $T_e = (F_1 - 1)T_o \Rightarrow F_1 = \frac{T_e + T_o}{T_o} = 2.0345$

$$T_{total} = (F_{total} - 1)T_o = (2.6145 - 1)(290) = 336.82^\circ K$$

and

$$F_o = (F - 1) + \frac{T_s}{T_o} \Rightarrow F_o = (2.6145 - 1) + \frac{500}{290} = 3.34 \Rightarrow 5.24\,dB$$

2.31 a. $F_t = F_1 + \frac{F_2 - 1}{G_1} = \frac{10^{.2} - 1}{10^{-.2}} = 5.0119 \equiv 7\,dB$

b. $F_t = F_2 + \frac{F_3 - 1}{G_2} + \frac{F_4 - 1}{G_2 G_3}$

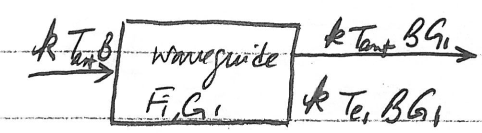
$$= 3.1623 + \frac{31.6228 - 1}{630.9573} + \frac{1000 - 1}{(630.9573)(.5012)} = 6.3699 \equiv 8.04\,dB$$

c. $T_e = (F - 1)T_o$

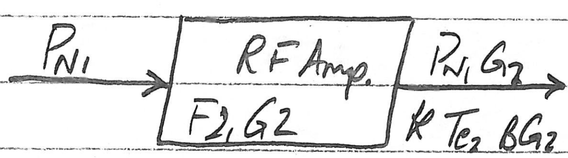
$$T_{e_t} = T_{e_1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3}$$

$$= 169.6 + \frac{8880.6}{(.631)(630.9573)} + \frac{289710}{(.631)(630.9573)(0.5012)} = 2637.7^\circ K$$

d.



$P_{N_1} = (k G_1 T_{ant} + k T_{e_1} G_1)B$



$P_{N_2} = k(G_1 G_2 T_{ant} B) +$
$k(T_{e_1} G_1 G_2 + T_{e_2} G_2)B$

and so forth $\Rightarrow$

2.31 Cont. $\Rightarrow$ total noise power $P_N = (K_1 T_{ant} + K_2)B$ where

Component	K_1	K_2
waveguide	kG_1	$k(T_{e_1} G_1)$
RF Amp	kG_1G_2	$k(T_{e_2}G_2 + T_{e_1}G_1G_2)$
1st mixer	$kG_1G_2G_3$	$k(T_{e_3}G_3 + T_{e_2}G_2G_3 + T_{e_1}G_1G_2G_3)$
IF Amp	$kG_1G_2G_3G_4$	$k(T_{e_4}G_4 + T_{e_3}G_3G_4 + T_{e_2}G_2G_3G_4 + T_{e_1}G_1G_2G_3G_4)$

2.32 In general an FM signal can be written as

$$x(t) = A\sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_0 + n\omega_m)t$$

it follows that

$$\langle x^2(t)\rangle = \frac{1}{2}A^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

However, the same signal can be written as

$$x(t) = A\cos(\omega_0 t + \phi(t)) \Rightarrow \langle x^2(t)\rangle = \frac{1}{2}A^2$$

$$\Rightarrow \frac{1}{2}A^2\sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{1}{2}A^2 \Rightarrow \sum_{n=-\infty}^{\infty} J_n^2(\beta) \equiv 1$$

2.33 From Hint

$$J_n(z) = \frac{1}{\pi}\int_0^{\pi} \cos(z\sin y - ny)\,dy$$

let $u = \pi - y \Rightarrow J_n(z) = \frac{1}{\pi}\int_{\pi}^{0} \cos(z\sin(\pi-u) - n(\pi-u))(-1)\,du$

$$\Rightarrow J_n(z) = \frac{1}{\pi}\int_0^{\pi} \cos[z\sin(\pi-u) - n\pi + nu]\,du$$

however since $\sin(\pi-u) = -\sin(u-\pi) = \sin u$ we get

$$J_n(z) = \frac{1}{\pi}\int_0^{\pi} \cos[z\sin u + nu - n\pi]\,du$$

2.33 Cont. and since $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

let $q = \beta \sin u + nu$ & $v = \pi n$

$\Rightarrow$

$$J_n(z) = \frac{1}{\pi}\int_0^\pi \cos(\beta \sin u + nu)\overset{(-1)^n}{\cancel{\cos n\pi}}\, du + \frac{1}{\pi}\int_0^\pi \sin(\beta \sin u + nu)\overset{0}{\sin n\pi}\, du$$

$$= (-1)^n \frac{1}{\pi}\int_0^\pi \cos(\beta \sin u + nu)\, du = (-1)^n J_{-n}(z)$$

$\Longrightarrow$

$$J_{-n}(z) = (-1)^n J_n(z)$$

2.34 $R_{un} = \frac{c}{2\Delta f} = \frac{3\times 10^8}{2(115-105)\times 10^3} = 30\text{ km}$

2.35 In general $f_b = \frac{2R\dot{f}}{c} \Rightarrow R = (f_b \cdot c)/2\dot{f}$

$\Rightarrow$

for $f_b = 1200\text{ Hz}$ & $\dot{f} = 20\times 10^6\text{ Hz} \Rightarrow R = \frac{(1200)(3\times 10^8)}{(2)(20\times 10^6)} = 9.0\text{ Km}$

and

for $f_b = 1200\text{ Hz}$ & $\dot{f} = 10\times 10^6\text{ Hz} \Rightarrow R = \frac{(1200)(3\times 10^8)}{(2)(10\times 10^6)} = 18.0\text{ Km}$

2.36 $\bar{f}_b = \frac{4R f_m \Delta f}{c}$

let $\bar{f}_{b_1}$ correspond to R_1 & $\bar{f}_{b_2}$ to R_2

$\Rightarrow$

$\Delta \bar{f}_b = \frac{4\,\Delta R\, f_m\, \Delta f}{c}$ where $\Delta \bar{f}_b = \bar{f}_{b_2} - \bar{f}_{b_1}$ & $\Delta R = R_2 - R_1$

it follows

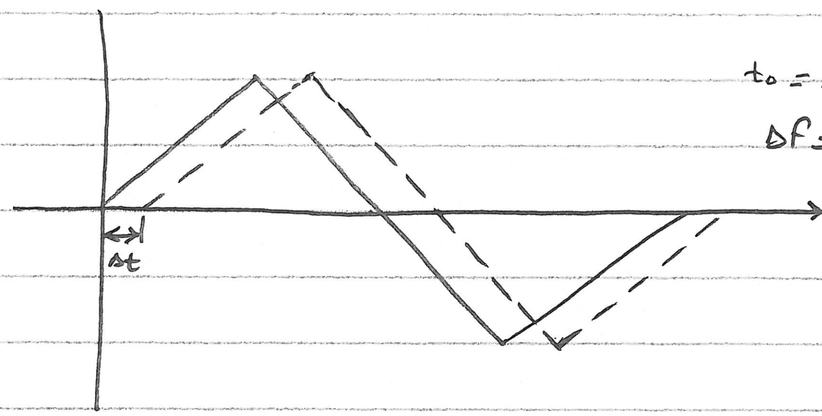
$$\Delta \bar{f}_b = \frac{(4)(3000)(50\times 10^6)}{3\times 10^8}\,\Delta R = 200\,\Delta R$$

so for $\Delta R = 10\text{m} \Longrightarrow \Delta \bar{f}_b = 200 * 10 = 2000\text{ Hz}$

and

for $\Delta R = 15\text{m} \Longrightarrow \Delta \bar{f}_b = 200 * 15 = 3000\text{ Hz}$.

2.37



$t_0 = 20$ msec

$\Delta f = 200$ KHz ; $\lambda = 3$ cm

a. $\dot{f} = \dfrac{200 \times 10^3}{20 \times 10^{-3}} \Rightarrow \dot{f} = 10$ MHz (from slope)

b. $\dot{R} = \dfrac{\lambda(f_{bd} - f_{bu})}{4}$ and $R = \dfrac{c\,(f_{bu} - f_{bd})}{4\dot{f}}$; ($\lambda = c/f$)

$\Rightarrow$

$f_{bu} = \dfrac{2R\dot{f}}{c} - \dfrac{2\dot{R}}{\lambda}$ & $f_{bd} = \dfrac{2R\dot{f}}{c} + \dfrac{2\dot{R}}{\lambda}$

but $R = 350$ km & $v = 250$ m/sec $\equiv \dot{R}$

$\Rightarrow$

$f_{bu} = \dfrac{2 * 350 \times 10^3 * 10 \times 10^6}{3 \times 10^8} - \dfrac{2 * 250}{3 \times 10^{-2}} = 6.667$ KHz

&

$f_{bd} = \dfrac{2 * 350 \times 10^3 * 10 \times 10^6}{3 \times 10^8} + \dfrac{2 * 250}{3 \times 10^{-2}} = 40$ KHz

$\Rightarrow$ and

$\Delta t = \dfrac{2R}{c} = \dfrac{2 * 350 \times 10^3}{3 \times 10^8} = 2.33$ msec.

2-38 The output signal from mixer A is

$E_A = k_a E_0 \cos(\pm \omega_d t + \varphi)$ where in general the transmitted signal is

$E_t = E_0 \cos \omega_0 t$

while the echo signal is

$E_r = k E_0 \cos((\omega_0 \pm \omega_d)t + \varphi)$

2.38 Cont. The output signal from mixer B is

$$E_B = k_B E_0 \cos(\pm\omega_d t + \varphi + \tfrac{\pi}{2})$$

So if the target is closing (+ve Doppler) then the outputs are

$$E_A^{+} = k_a E_0 \cos(\omega_d t + \varphi) \quad \text{and} \quad E_B^{+} = k_B E_0 \cos(\omega_d t + \varphi + \pi/2)$$

Alternatively if the target is receding (-ve Doppler) we get

$$E_A^{-} = k_a E_0 \cos(\omega_d t + \varphi) \quad \& \quad E_B^{-} = k_B E_0 \cos(\omega_d t - \varphi - \pi/2)$$

Hence, the target direction can be determined by checking whether the output E_B leads or lags the output E_A. One way of doing this is by applying both of E_A & E_B to a synchronous two-phase indicator or detector.