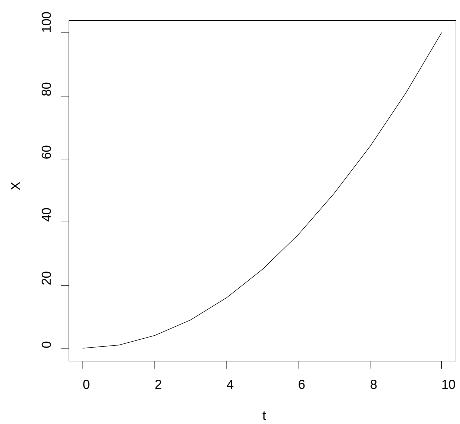
2 Review of basic mathematical concepts and introduction to R

Exercise 2-1

Use a variable X to denote human population on Earth. Explain why it varies and give an example of a value.
Solution:
Human population can be considered a variable that takes discrete values and it varies in continuous time due to deaths and births. An example is $X(t)$=6,23 × 10^9 people or 6,23 billion people at time t.

Exercise 2-2

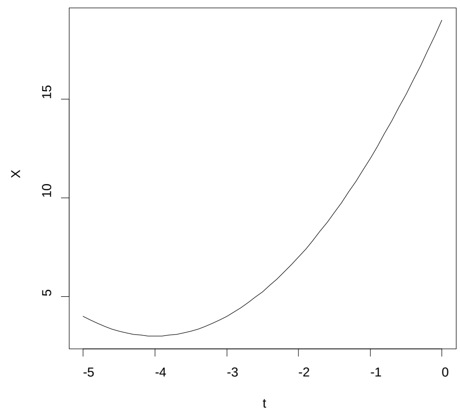
Assume that a= 1 and b= 2. Evaluate the derivative of X with respect to t for the $X(t)$ given in Equation 2.2 and plot it. Is the derivative a constant with respect to t? Is the derivative a linear function with respect to t?
Solution:
Substituting the values given in equation 2.2 we have $X(t)=t^2$. This is a simple parabola. A plot for values of t in between 0 and 10 would look like.



This plot can be produced by the code `t <- seq(0,10,1); X <- t^2; plot(t,X,type="l").` The derivative is $\frac{dX}{dt}=2t$, and therefore it is not constant with t. However, it is linear with t.

Exercise 2-3

Plot Equation 2.8 when b=−4 and a=3. Find the values of the function and its derivative at $t = b$.
Solution:
Write equation 2.8 substituting these values $X(t)=3+(t+4)^2$. The plot looks like

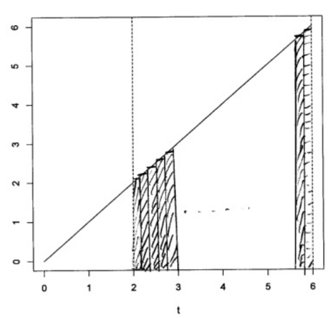


Which can be produced by the code `t <- seq(-5,0,1); X <- 3+(t+4)^2; plot(t,X,type="l").` At t=b, the function is $X(t)=3+(-4+4)^2=3$ and the derivative is $dX/dt=2(t-b)=0$

Exercise 2-4

Assume a=b=1 in equation 2.2. What is the area under the curve between t=2 and t=6? Illustrate the area using a graph like the one in Figure 2.6.
Solution:

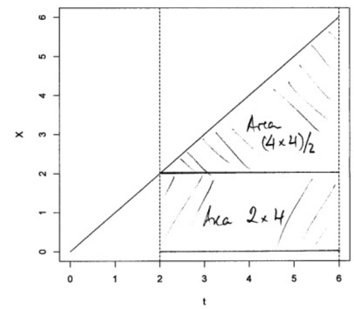
Equation 2.2 becomes $X(t)=t$. Because $b \neq -1$, the integral is $a\frac{t^2}{2}$ and because a=1, we have that the integral is just $t^2/2$. The area is obtained by evaluating the integral, that is $A=(6^2-2^2)/2=(36-4)/2=32/2=16$. A plot looks like



Produced by the code:

```
t<- seq(0,6,0.1); X <- t; plot(t,X,type="l");
abline(v=2,lty=2);abline(v=6,lty=2)
```

then segments are added by hand.
Note: It is simple to confirm the calculated area using the plot. As shown below, the area is composed of two parts a rectangle of height 2 and width 4, with area 8; and a triangle of sides 4 and 4, with area $(4\times 4)/2=8$. Thus, the total area is 8+8=16.

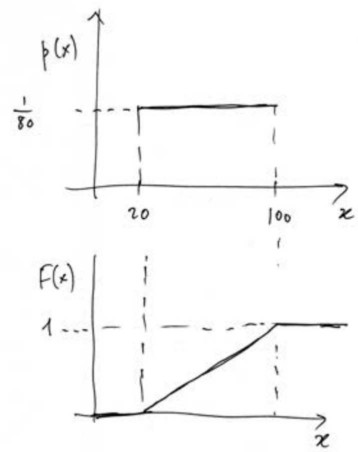


Exercise 2-5

Assume t_1=t and t_0=0. Write the solution for the ODE using the initial condition $X(0)$=1.
Solution:
Write $X(t) = a(t-0) + X(0) = a \times t + 1$

Exercise 2-6

Define a RV from the outcome of soil moisture measurements in the range of 20-100 % in volume. Give an example of an event. Assuming that it can take values in [20,100] uniformly, plot pdf and cdf. Calculate the mean and variance.
Solution:
Event: soil moisture is in range 40-45 %.



Use equation 2.31 $\mu_X = \dfrac{100+20}{2} = 60 \qquad \sigma_X^2 = \dfrac{(100-20)^2}{12} = \dfrac{80^2}{12} = 533.33$

Exercise 2-7

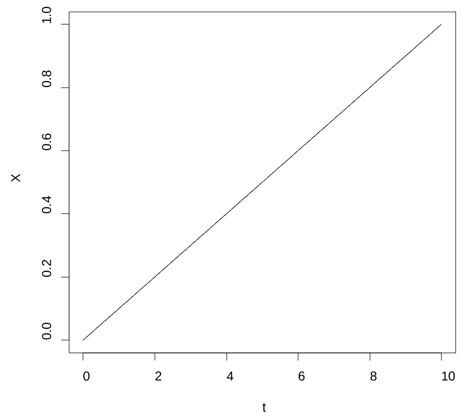
At a site monthly air temperature is normally distributed. It averages to 20 °C with standard deviation 4 °C. What is the probability that a value of air temperature in a given month exceeds 24 °C? What is the probability that it is below 16 °C or above 24 °C?
Solution:
Note that 24 °C is the mean plus one sd, and 16 °C is mean minus one sd. Thus, values above 24 or below 16 °C have probability 1-0.68=0.32. Now, only values above have half of that probability, then the probability is $\dfrac{1-0.68}{2} = 0.16$.

Exercise 2-8

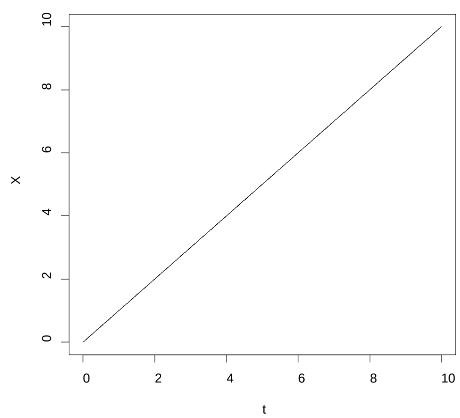
Draw line graphs of the function given in equation 2.1 for several values of coefficient *a*. Use *a*=0.1, 1, 10. Produce three graphs.

Solution:

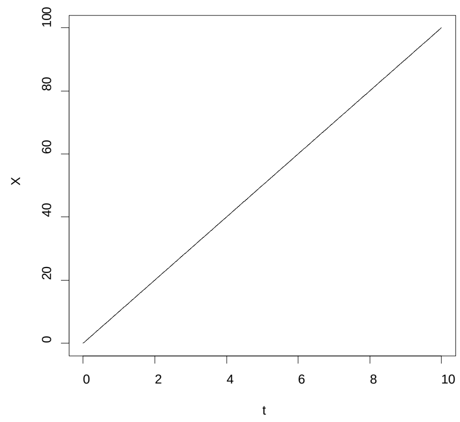
```
t <- seq(0,10,0.01)
a <- 0.1
X <-a*t
plot(t,X, type="l")
```




```
a <- 1
X <-a*t
plot(t,X, type="l")
```




```
a <- 10
X <-a*t
plot(t,X, type="l")
```

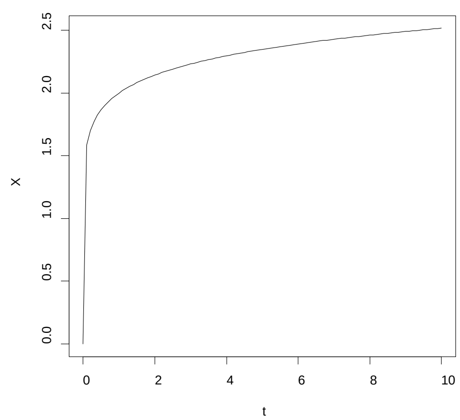


Exercise 2-9

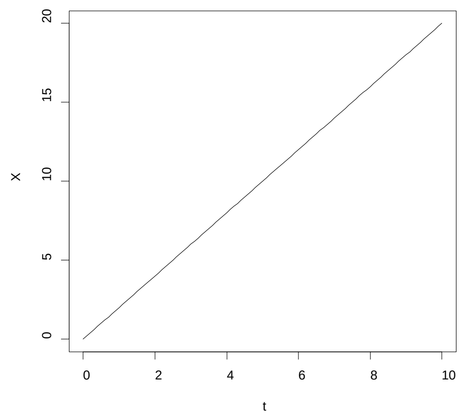
Plot equation 2.2 using *a*=2 and three different values of *b:* 0.1, 1, and 3.

Solution:

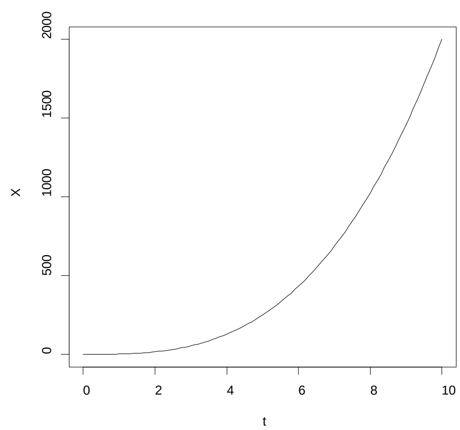
```
t <- seq(0,10,1)
a <- 2; b <-0.1
X <- a*t^b
plot(t,X, type="l")
```




```
t <- seq(0,10,0.1)
a <- 2; b <-1
X <- a*t^b
plot(t,X, type="l")
```




```
t <- seq(0,10,0.1)
a <- 2; b <-3
X <- a*t^b
plot(t,X, type="l"
```

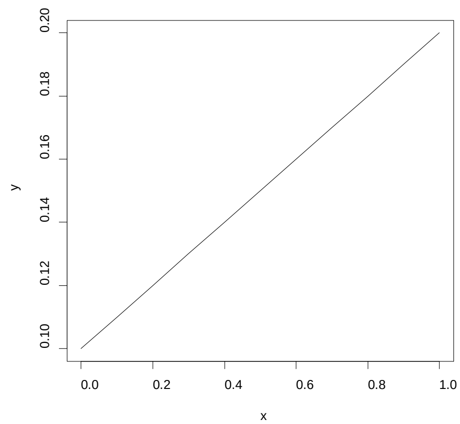


Exercise 2-10

Generate values for a function $y = ax + b$, a=0.1, b=0.1, Plot y for values of x in 0 to 1

Solution:

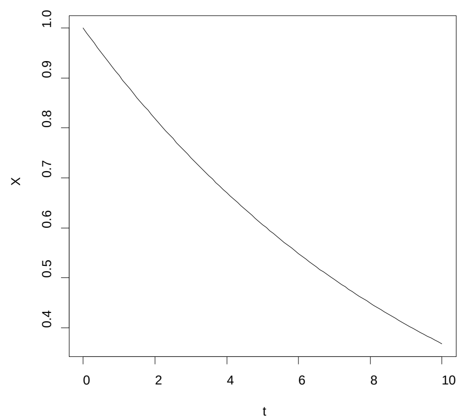
```
a=0.1;b=0.1
x=seq(0,1,0.1)
y <- a*x+b
plot(x,y,type="l")
```



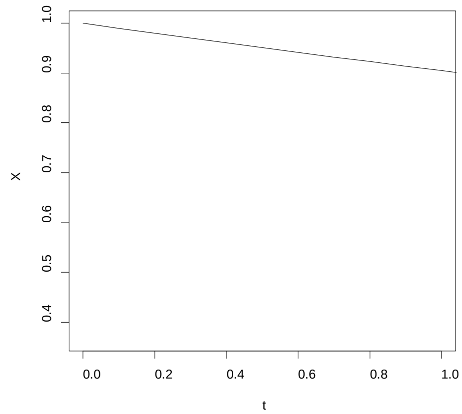
Exercise 2-11

Generate values for an exponential function with r=-0.1. Plot X for values of t in 0 to 10. Then limit t axis to interval [0,1].
Solution:

```
t <- seq(0,10,0.1)
r<--0.1
X <- exp(r*t)
plot(t,X, type="l")
```




```
plot(t,X, type="l", xlim=c(0,1))
```

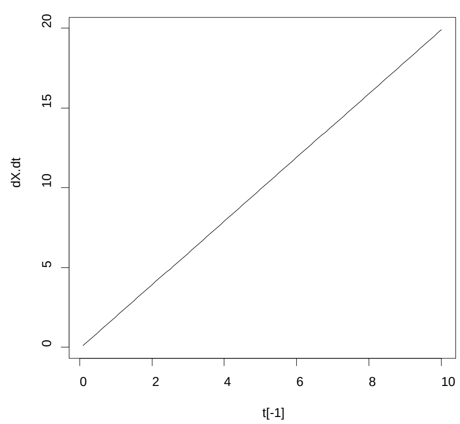


Exercise 2-12

Assume *a*=1 and *b*=2. Calculate the derivative of *X* with respect to *t* for *X* given in equation 2.2 using diff and plot it. Compare to exercise 2-2. Hint: the first entry of t should be removed so that dX.dt and t have the same length and are compatible for function plot.

Solution:

```
dt <- 0.1 # set time step
t <- seq(0,10,dt)
a <- 1; b <- 2; X <- a*t^b
dX <- diff(X)
dX.dt <- dX/dt
plot(t[-1],dX.dt,type="l")
```



The derivative is linear as in exercise 2-2.