CHAPTER 2 MATERIALS

SOLUTION (**2.1**)

$$A_0 = \tfrac{\pi}{4}(0.5)^2 = 196.35(10^{-3})\ in.^2\ ,\quad A_f = \tfrac{\pi}{4}(0.5-0.00024)^2 = 196.16(10^{-3})\ in.^2$$

We have $\varepsilon_a = \frac{12(10^{-3})}{8} = 1500\ \mu,\quad \varepsilon_t = \frac{0.24(10^{-3})}{0.5} = 480\ \mu$

Thus

$$S_p = \frac{P}{A_0} = \frac{4(10^3)}{196.35(10^{-3})} = 20.37\ ksi$$

$$E = \frac{S_p}{\varepsilon_a} = \frac{20.37(10^3)}{1500(10^{-6})} = 13.58(10^6)\ psi,\quad \nu = \frac{\varepsilon_t}{\varepsilon_a} = 0.32 \qquad \blacktriangleleft$$

Also

$$\%elongation = \frac{12(10^{-3})}{8}(100) = 0.15$$

$$\%\ reduction\ in\ area = \frac{196.35-196.16}{196.35}(100) = 0.097 \qquad \blacktriangleleft$$

SOLUTION (**2.2**)

Normal stress is

$$\sigma = \frac{P}{A} = \frac{500}{\frac{\pi}{4}(1/8)^2} = 4744\ ksi$$

This is below the yield strength of 50 ksi (Table B.1).
We have

$$\varepsilon = \frac{\delta}{L} = \frac{0.3}{18.5\times 12} = 0.001351 = 1351\ \mu$$

Hence

$$E = \frac{\sigma}{\varepsilon} = \frac{40{,}744}{135(10^{-6})} = 30\times 10^6\ \ psi \qquad \blacktriangleleft$$

SOLUTION (**2.3**)

The cross-sectional area: $A = w_o t_o = 0.5(0.25) = 0.125\ in.^2$

(**a**) Axial strain and axial stress are

$$\varepsilon_a = \frac{0.00331}{2.5} = 0.01324 = 1324\ \mu$$

$$\sigma_a = \frac{P}{A} = \frac{4.8}{0.125} = 38.4\ ksi$$

Because $\sigma_a < S_y$ (See Table B.1), Hooke's Law is valid.

(**b**) Modulus of elasticity,

$$E = \frac{\sigma_a}{\varepsilon_a} = \frac{38{,}400}{1324(10^{-6})} = 29\times 10^6\ \ psi$$

(**c**) Decrease in the width and thickness

$$\Delta w = \nu w_o = 0.3(0.5) = 0.15\ in. \qquad \blacktriangleleft$$

$$\Delta t = \nu t_o = 0.3(0.24) = 0.072\ in.$$

SOLUTION **(2.4)**

Assume Hooke's Law applies. We have

$$\varepsilon_t = -\tfrac{1.5}{5} = -300\ \mu$$

$$\varepsilon_a = -\tfrac{\varepsilon_t}{\nu} = -\tfrac{-300}{0.34} = 822\ \mu$$

Thus,

$$\sigma = E\varepsilon_a = (105\times10^9)(822\times10^{-9}) = 92.61\ MPa$$

Since $\sigma < S_y$, our assumption is valid.

So

$$P = \sigma A = (92.61)(\pi/4)(5)^2 = 1.818\ kN \quad ◄$$

SOLUTION (**2.5**)

We obtain

$$L_{AC} = L_{BD} = \sqrt{15^2 + 15^2} = 21.21\ mm$$

$$\varepsilon_x = \tfrac{\Delta L_{AC}}{L_{AC}} = \tfrac{21.17-21.21}{21.21} = -1886\ \mu$$

$$\varepsilon_y = \tfrac{\Delta L_{BD}}{L_{BD}} = \tfrac{21.22-21.21}{21.21} = 471\ \mu$$

(**a**) $E = \tfrac{\sigma_x}{\varepsilon_x} = \tfrac{100(10^6)}{-1886(10^{-6})} = 53\ GPa$ ◄

(**b**) $\nu = \tfrac{\varepsilon_y}{\varepsilon_x} = \left|\tfrac{471}{1886}\right| = 0.25$ ◄

(**c**) $G = \tfrac{53}{2(1+0.25)} = 21.2\ GPa$ ◄

SOLUTION (**2.6**)

Use generalized Hooke’s law:

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \tfrac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) \qquad (1)$$

For a constant triaxial state of stress:

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon, \quad \sigma_x = \sigma_y = \sigma_z = \sigma$$

Then, Eq. (1) becomes $\varepsilon = \tfrac{1-2\nu}{E}\sigma$. Since σ and ε must have identical signs:

$$1 - 2\nu \geq 0 \quad \text{or} \quad \nu = \tfrac{1}{2} \quad ◄$$

SOLUTION (**2.7**)

We have $\sigma_x = \tfrac{100(10^3)}{3(2)} = 16.67\ ksi$

(CONT.)

2.7 (CONT.)

(**a**) $\varepsilon_x = \frac{0.02}{10} = 2000\ \mu, \quad \varepsilon_y = -\frac{0.001}{2} = -500\ \mu$

$\nu = \left|\frac{500}{2000}\right| = 0.25$ ◄

(**b**) $E = \frac{\sigma_x}{\varepsilon_x} = \frac{16.67(10^3)}{2000(10^{-6})} = 8.335(10^6)\ psi$ ◄

(**c**) $\varepsilon_z = -\frac{\nu\sigma_x}{E} = -0.25\frac{16.67(10^3)}{8.335(10^6)} = -500\mu$

$\Delta a = -500(10^{-6})3 = -1.5(10^{-3})in.; \qquad a' = 3.0 - 0.0015 = 2.9885\ in.$ ◄

(**d**) $G = \frac{8.335(10^6)}{2(1+0.25)} = 3.334(10^6)\ psi$ ◄

SOLUTION (**2.8**)

We have

$$\varepsilon_y = \varepsilon_z = 0 \qquad\qquad \sigma_x = \frac{25(10^3)}{20\times10(10^{-6})} = 125\ MPa$$

Thus

$$\varepsilon_y = 0 = \tfrac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \tag{1}$$

$$\varepsilon_z = 0 = \tfrac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \tag{2}$$

$$\varepsilon_x = \tfrac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \tag{3}$$

Equations (1) and (2) become

$$\sigma_y - \nu\sigma_z = \nu\sigma_x \tag{1'}$$

$$\sigma_z - \nu\sigma_y = \nu\sigma_x \tag{2'}$$

Adding : $\nu(\sigma_y + \sigma_z) = 2\nu^2\sigma_x/(1-\nu)$. Then, Eq. (3):

$$\varepsilon_x = \frac{1-\nu-2\nu^2}{1-\nu}\frac{\sigma_x}{E}$$ ◄

Substituting the data:

$$\varepsilon_x = \frac{1-0.3-0.18}{0.7}\frac{125(10^6)}{70(10^9)} = 1327\ \mu$$ ◄

SOLUTION (**2.9**)

<u>Hooke's Law.</u> We have $\sigma_y = 0$ and

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{10^6}{72\times10^9}[(80) - 0 - 0.3(140)] = 0.000528 = 528\ \mu$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{10^6}{72\times10^9}[-0.3(80) + 0 - 0.3(140)] = -917\ \mu$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$= \frac{10^6}{72\times10^9}[-0.3(80) - 0 + 140] = 1611\ \mu$$

(CONT.)

2.9 (CONT.)

(**a**) Change in length $\Delta L_{AB} = \varepsilon_x a$,

$$\Delta L_{AB} = (528\times 10^{-6})(320) = 0.169 \ mm$$

(**b**) Change in thickness

$$\Delta t = \varepsilon_y t = (-917\times 10^{-6})(15) = -0.014 \ mm$$

(**c**) Change in volume,

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 528 - 917 + 1611 = 1.222$$

$$\Delta V = eV_o = 1.222(320\times 320\times 15) = 1.877 \ mm^3$$

SOLUTION (**2.10**)

By assumptions, rubber in triaxial stress:

$$\sigma_x = \sigma_z = -p, \quad \sigma_y = -\tfrac{F}{\pi d^2/4} = -\tfrac{4F}{\pi d^2}$$

Stains are $\varepsilon_x = \varepsilon_z = 0$. Hooke's law gives

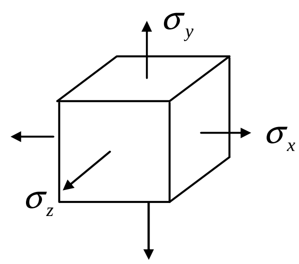
$$\varepsilon_x = 0 = \tfrac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

or

$$0 = p - \nu \tfrac{4F}{\pi d^2 (1-\nu)}$$

Solving,

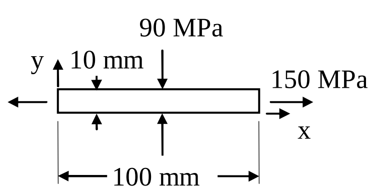
$$p = \tfrac{4\nu F}{\pi d^2 (1-\nu)}$$ Q.E.D.



Substitute the data:

$$P = \tfrac{4(0.5)(2\times 10^3)}{\pi (2.5)^2 (1-0.5)} = 407.4 \ psi \quad \text{(C)}$$

SOLUTION (**2.11**)



Hooke's law gives

$$\varepsilon_x = \tfrac{1}{E}(\sigma_x - \nu\sigma_y) = \tfrac{10^6}{100(10^9)}(150 + \tfrac{90}{3}) = 1800 \ \mu$$

$$\varepsilon_y = \tfrac{1}{E}(\sigma_y - \nu\sigma_x) = \tfrac{10^6}{100(10^9)}(-90 - \tfrac{150}{3}) = -1400 \ \mu$$

$$\varepsilon_z = -\tfrac{\nu}{E}(\sigma_x + \sigma_y) = -\tfrac{(1/3)10^6}{100(10^9)}(150 - 90) = -200 \ \mu$$

Thus

$$\Delta L = 1800\,\mu(100) = 180 \ \mu m$$

$$\Delta a = -1400\,\mu(50) = -70 \ \mu m$$

$$\Delta b = -200\,\mu(10) = -2 \ \mu m$$

and

$$L' = 100.018 \ mm, \quad a' = 49.993 \ mm, \quad b' = 9.9998 \ mm$$

SOLUTION (**2.12**)

We have

$$\sigma_x = \sigma_y = \sigma_z = -p$$

Gen. Hooke's law:

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = -\tfrac{p}{E}(1-2\nu) = -\tfrac{120(10^6)}{100(10^9)}\tfrac{1}{3} = -400\ \mu$$

Thus

$$\Delta L = -400\,\mu(100) = -40\ \mu m$$
$$\Delta a = -400\,\mu(50) = -20\ \mu m$$
$$\Delta b = -400\,\mu(10) = -4\ \mu m$$

and

$$L' = 99.96\ mm, \quad a' = 49.98\ mm, \quad b' = 9.996\ mm$$ ◄

SOLUTION (**2.13**)

We have

$\sigma_x = \sigma_y = \sigma_z = -p$. The volume is

$$V_o = \tfrac{4}{3}\pi r^3 = \tfrac{4\pi}{3}(5)^3 = 523.6\ in.^3$$

(**a**) $\varepsilon_x = -\tfrac{1}{E}[\sigma - \nu(\sigma+\sigma)] = -\tfrac{\sigma}{E}(1-2\nu)$

$$= \tfrac{24(10^3)}{10(10^6)}(1-0.5) = -1200\ \mu$$

Change in diameter,

$$\Delta d = \varepsilon_x d = -1200(10^{-6})10 = -0.012\ in.$$

Decrease in circumference:

$$\pi(\Delta d) = -0.012\pi = -0.038\ in.$$ ◄

(**b**) $\Delta V = eV_o = (1-2\nu)\varepsilon_x V_o$

$$= (0.5)(-1200\times 10^{-6})(523.6) = -0.314\ in.^3$$ ◄

SOLUTION (**2.14**)

From Fig.2.3b and Eq.2.20:

$$U_t = \tfrac{S_y+S_u}{2}\varepsilon_f \approx \tfrac{250+440}{2}(0.27) \approx 93\ MPa$$ ◄

We have $L_f = 50 + 50(0.27) = 63.5\ mm$

Using Eq.(2.1): $\%\ elongation = \tfrac{63.5-50}{50}(100) = 27\ \%$ ◄

SOLUTION (**2.15**)

Table B.1: $S_y = 260\ MPa, \qquad E = 70\ GPa$

We have

$$V = AL = \tfrac{\pi}{4}(0.005)^2(3) = 58.9 \times 10^{-6}\ m^3$$

$$U_r = \tfrac{S_y^2}{2E} = \tfrac{(260\times10^6)^2}{2(70\times10^9)} = 482.9\ kJ/m^3$$

$$U_{app} = U_r V = 482.9\times10^3(58.9\times10^{-6}) = 28.44\ J$$

For $U_{app} = 9\ J$:

$$n = \tfrac{28.44}{9} = 3.16 \qquad ◄$$

SOLUTION (**2.16**)

(**a**) <u>ASTM-A242.</u> $E = 200\ GPa$ and $\sigma_y = 345\ MPa$

$$U_o = \tfrac{S_y{}^2}{2E} = \tfrac{(345\times10^6)^2}{2(200\times10^9)} = 298\ \tfrac{kJ}{m^3}$$

$$= \tfrac{298}{6.895} = 43.2\ in.-lb/in.^3 \qquad ◄$$

(**b**) <u>Stainless (302).</u> $E = 190\ GPa$ and $S_y = 520\ MPa$

$$U_o = \tfrac{S_y{}^2}{2E} = \tfrac{(520\times10^6)^2}{2(200\times10^9)} = 712\ \tfrac{kJ}{m^3}$$

$$= \tfrac{712}{6.895} = 103\ in.-lb/in.^3 \qquad ◄$$

SOLUTION (**2.17**)

(**a**) <u>Aluminum 2014-T6.</u> $E = 72\ GPa$ and $\sigma_y = 410\ MPa$

$$U_o = \tfrac{S_y{}^2}{2E} = \tfrac{(410\times10^6)^2}{2(72\times10^9)}$$

$$= 1167\ \tfrac{kJ}{m^3} = \tfrac{1167}{6.895} = 169\ in.-lb/in.^3 \qquad ◄$$

(**b**) <u>Annealed yellow brass.</u> $E = 105\ GPa$ and $S_y = 105\ MPa$

$$U_o = \tfrac{S_y{}^2}{2E} = \tfrac{(105\times10^6)^2}{2(105\times10^9)} = 52.5\ \tfrac{kJ}{m^3}$$

$$= \tfrac{52.5}{6.895} = 7.61\ in.-lb/in.^3 \qquad ◄$$

SOLUTION (**2.18**)

Referring to Fig. P.2.18: $E = \tfrac{15\times10^3}{0.0025} = 6\times10^6\ \ psi, \quad S_y = 27.5\ ksi.$

(**a**) $U_o = \tfrac{S_y{}^2}{2E} = \tfrac{(27.5\times10^3)^2}{2(6\times10^6)} = 63.02\ in.-lb \qquad ◄$

(**b**) Total area under $\sigma - \varepsilon$ diagram:

$$U_t \approx 37.5(10^3)(0.176) = 6.6\ in.-kip/in.^3 \qquad ◄$$

SOLUTION (**2.19**)

(**a**) $V = 50 \times 50 \times 1{,}500 = 3.75(10^6)\ mm^3$

Thus $nU = \frac{S_y^2}{2E} V$

or $S_y = [\frac{2EnU}{V}]^{\frac{1}{2}}$

$= [\frac{2 \times 200 \times 10^9 \times 1.5 \times 400}{3.75(10^{-3})}]^{\frac{1}{2}} = 253\ MPa$ ◄

(**b**) $U_r = \frac{S_y^2}{2E} = \frac{(253 \times 10^6)^2}{2(200 \times 10^9)} = 160\ kPa$ ◄

SOLUTION (**2.20**)

Table B.1: $S_y = 36\ ksi, \qquad E = 29 \times 10^6\ psi$

We have

$$U = nU_{app} = 5(150) = 750\ in.\cdot lb$$

$$U_r = \frac{S_y^2}{2E} = \frac{(36 \times 10^3)^2}{2(29 \times 10^6)} = 22.34\ in.\cdot lb/in.^3$$

Therefore

$$V = \frac{U}{U_r} = \frac{750}{22.34} = 33.57\ in.^3$$

Also $V = AL$: $\quad 33.57 = \frac{\pi}{4} d^2 (8 \times 12)$

or

$d = 0.667\ in.$ ◄

SOLUTION (**2.21**)

Refer to Fig. P2.21. We have

$$E = \frac{190(10^6)}{0.001} = 190\ GPa, \quad S_y \approx 245\ MPa$$

(**a**) $U_o = \frac{S_y^{\ 2}}{2E} = \frac{(245 \times 10^3)^2}{2(190 \times 10^9)} = 158\ \frac{kJ}{m^3}$

$= \frac{158}{6.895} = 22.9\ in.-lb/in.^3$ ◄

(**b**) Total area under $\sigma - \varepsilon$ diagram:

$$U_t \approx 350 \times 10^3 (0.28) = 98\ \frac{MJ}{m^3}$$

$\approx \frac{98}{6.895} = 14.2\ in.-kip/in.^3$ ◄

SOLUTION (**2.22**)

$V = (0.05)(0.05)(1.2) = 0.003\ m^3$ and $S_y \approx S_p$

(**a**) $n \cdot U = \frac{S_p^{\ 2}}{2E} V, \quad S_p^{\ 2} = \frac{nU(2E)}{V}$

Substituting the data given,

(CONT.)

1.22 (CONT.)

$$S_p^{\ 2} = \tfrac{1.8(150)(2\times 210\times 10^9)}{0.003} = 37.8\times 10^{15}$$ ◄

or

$$S_p = 194.4\ MPa$$

(**b**) $U_o = \frac{S_p^{\ 2}}{2E} = \frac{37.8\times 10^{15}}{2(210\times 10^9)} = 90\ \frac{kJ}{m^3}$ ◄

SOLUTION (**2.23**)

Applying Eq. (2.22), we find

$$S_u = 500 H_B\ \ psi = 500(149) = 74.5\ ksi$$ ◄

Equation (2.24):

$$S_y = 525(149) - 30000 = 48.225\ ksi$$ ◄

SOLUTION (**2.24**)

Using Eq. (2.22),

$$S_u = 500 H_B\ \ psi = 500(179) = 89.5\ ksi$$ ◄

Formula (2.24):

$$S_y = 525(179) - 30,000 = 63.975\ ksi$$ ◄

SOLUTION (**2.25**)

Formula (2.22),

$$S_u = 500 H_B\ \ psi = 500(156) = 78\ ksi$$ ◄

Equation (2.24):

$$S_y = 525(156) - 30,000 = 51.9\ ksi$$ ◄

SOLUTION (**2.26**)

Equation (2.22) gives

$$S_u = 500 H_B\ \ psi = 500(293) = 146.5\ ksi$$ ◄

Formula (2.24):

$$S_y = 525(293) - 30,000 = 123.825\ ksi$$ ◄

End of Chapter 2