2.1 $P_{old} = 12$ kW P.F. = 0.6 lag $S_{old} = 12/0.6 = 20$ kVA $Q_{old} = 16$ kVAR

(a) Add P_x.

$P_{new} = 12 + P_x$ $Q_{new} = 16$ $S_{new} = 25$

$(25)^2 = (P_x + 12)^2 + (16)^2$

$(P_x + 12) =$ $P_x = 7.21$ kW

(b) Add $P_x = S_x$ $Q_x = - 0.5 S_x$

$P_{new} = 12 - Q_x$ $Q_{new} = 16 + Q_x$ $S_{new} = 25$

$(25)^2 = (- Q_x + 12)^2 + (Q_x + 16)^2$

$4 Q_x^2 + 8Q_x(- 3 + 4) = 625 - 256 - 144 = 225$

$Q_x^2 - 2(1.196) Q_x + (1.196)^2 = 56.25 + (1.196)^2 = 57.68$

$Q_x = - 7.595 + 1.196 = -6.399$ kVAR

$P_x =$ 11.083 kW

$S_x =$ 12.798 kVA @ 0.866 P.F.lead

Check: $S_{new} =$

2.2 $S_{old} = 500$ kVA P.F.= 0.6 lag $P_{old} = 300$ kW $Q_{old} = 400$ kVAR

P.F.$_{new}$ = 0.9 lag $P_{new} = 300$ kW $Q_{new} = 300 \tan o = 145.3 = 400 + Q_x$

(a) $Q_x = 145.3 - 400 = - 254.7$ kVAR

(b) $S_{new} = 300/0.9 = 333.33$ kVA = 66.67% of 500 kVA.

[PEAS2-1]

2.3 Actual system: Given ohms. Then ohms.

Base system:

MVA_{base} MVA_{base}

KV_{1base} $KV_{2base} = (N_2/N_1)KV_{1base}$

$Z_{1base} = (KV_{1base})^2/MVA_{base}$ $Z_{2base} = (KV_{2base})^2/MVA_{base}$

$= (N_2/N_1)^2 Z_{1base}$

Per Unit system:

2.4 Single-phase transformer: 1 kVA, 208/120 V.

Base system:

1 kVA	1 kVA
208 V	120 V
4.81 A	8.33 A
43.26 ohms	14.40 ohms

Short Circuit Test

	V	I	P	S	Q
	-----	----	-----	------	-------
	10.4	4.81	20	50.0	45.8
p.u.	0.05	1.00	0.02	0.050	0.0458

[PEAS2-1]

Open Circuit Test

	V	I	P	S	Q
LV side	120	0.25	10	30	28
p.u.	1.00	0.03	0.01	0.03	0.028
HV side	208	0.1443	10	30	28

(a) First step should always be to define base system and then express test parameters in per unit as shown above.

(b) Shown above.

(c)

(d) & (e) To convert to ohms, multiply per unit values by base impedances.

There will be one value as seen from the high side, another value as seen from the low side.

2.5 Single-phase transformer: 10 kVA, 2400/240 V

<u>Base</u> system:

10 kVA	10 kVA
2400 V	240 V
4.17 A	41.7 A
576 ohms	5.76 ohms

<u>Short Circuit Test</u>

	V	I	P	S	Q
	67	4.17	146	279	238
p.u.	0.0279	1.00	0.0146	0.0279	0.0238

<u>Open Circuit Test</u>

	V	I	P	S	Q
	240	0.75	72	180	165
p.u.	1.00	0.018	0.0072	0.018	0.0165

(a) First step should always be to define base system and then express test parameters in per unit as shown above.

(b)

2.6 (a) Let $V_2 = 1.0$. Then $I_2 = 0.8 - j\,0.6$.

Then:

$V_m = 1.0 + (0.8 - j\,0.6)(0.0025 + j0.03) = 1.02 + j\,0.0225$

I_{exc} at $V_m = 1.0$ is:

$0.031(0.12 - j\,0.9928)$

I_{exc} at $V_m = 1.02 + j\,0.0225$ is:

$0.031(1.02 + j\,0.0225)(0.12 - j\,0.9928) = 0.0045 - j0.0313$

Then:

$I_1 = 0.8045 - j\,0.6313 = 1.0226$

$I_2 = 0.8 - j\,0.6 \quad = 1.0$

(b) Let $V_2 = 1.0$. Then $I_2 = 0.8 + j\,0.6$.

Then:

$V_m = 1.0 + (0.8 + j\,0.6)(0.0025 + j0.03) = 0.984 + j\,0.0255$

I_{exc} at $V_m = 1.0$ is:

$0.031(0.12 - j\,0.9928)$

I_{exc} at $V_m = 0.984 + j\,0.0255$ is:

$0.031(0.984 + j\,0.0255)(0.12 - j\,0.9928) = 0.0044 - j0.0302$

Then:

$I_1 = 0.8044 + j\,0.5698 = 0.9858$

$I_2 = 0.8 + j\,0.6 \quad = 1.0$

[PEAS2-1]

2.7

f	V	i_{exc}	S_c	P_c	P_e	P_h	Q_c	i_m
30	110	1.2	132	53	14	39	121	1.10

B_{max} remains constant: $P_e = 56$; $P_h = 78$; $i_m = 1.10$

f	V	i_{exc}	S_c	P_c	P_e	P_h	Q_c	i_m
60	220	1.26	277	134	56	78	242	1.10

$S_c =$ $i_{exc} = 277/220 = 1.26$

2.8 For the autotransformer:

$$Z_{base} = (230)^2/200 = 264.5 \quad \text{[ohms]}$$

on the output base of 200 MVA. The short-circuit reactance of the autotransformer is therefore:

$$= 0.08(264.5) = 21.16 \quad \text{[ohms]}$$

The winding base MVA is 100 MVA. For the two-winding transformer:

$$Z_{base} = (115)^2/100 = 132.25 \quad \text{[ohms]}$$

so that:

$$= 21.16/132.25 = 0.16$$

2.9 The series winding resistance is 0.06 ohms, and the common winding resistance is 0.04 ohms. For an output current of 10 amperes, the input current is 5 amperes. On the basis of ampere-turn balance, the current is 5 amperes in each winding. Therefore:

$P = (5)^2(0.06 + 0.04) = 2.5$ [watts]

2.10 For a three-winding transformer there are three pairs of windings for which six short-circuit impedances (in ohms) may be obtained. When expressed in terms of per unit, only three short-circuit impedances remain. If a wye equivalent network is to be determined, we have:

2.11 (a)

	P	F	C
MVA_{base}	4.42	4.42	4.42
kV_{base}	3.006	1.169	0.167
Z_{base}	2.0444	0.3092	0.0063

[PEAS2-1]

(b) = 0.250/2.0444 = 0.1223

= 0.029/0.3092 = 0.0938

= 0.066/2.0444 = 0.0323

(c)

[PEAS2-1]

```
P
o
 \
  \
   \
    \___________ o F
    /
   /
  /
 /
o
C



-------------------------------
//////////////////////////////
```

2.12 For the new three-winding transformer:

Obtain per equation (2.9.17).

Obtain per equation (2.9.10).

 = 0.28 as given.

[PEAS2-1]

(a) Thus:

= 0.1466667 = 0.1866667

= 0.0533333 = -0.04

= 0.28 = 0.0933333

(b) Load: 360 I_b = 120 I_c; 3 I_b = I_c

Winding: $I_b + I_c = 4 I_b = 40$ (max).

I_b = 10 [A] I_c = 30 [A]

I_a = (360/480) 10 + (120/480) 30 = 7.5 + 7.5 = 15.0 [A]

S_a = 0.480(15) = 7.2 kVA

2.13 For the new four-winding transformer:

Obtain per equation (2.9.17). Obtain per equation (2.9.10). Obtain per equation (2.9.10). = 0.12 as given.

= 0.28 as given. = 0.12 as given.

= 0.1466667 = 0.12

= 0.0533333 = 0.0133333

= 0.28 = 0.12

K_1 = 0.12 K_2 = 0.02666665

= 0.0983824 = -0.0682843 = 0.0517157

= -0.0482843 = 0.1765685 = 0.0832352

Check:

When terminal #4 is eliminated, there is left a network including a delta network. This may be converted to an equivalent wye consisting of , , and . When these are added to , , and , respectively, the result should be the same as for the three-winding transformer in

Exercise 2.12: = 0.0882843 = 0.0282843 = 0.0416176

2.14

```
                                I
                                 1           -----
           ................................  |     | ........
           . |------------------------------| Z   |----------|
           . |                              |  1  |          |
           . |                               -----           |
           . |                                              |
           . |                                              |
 .......... |                                               |
 .  -----   |                                               |    V  = 1.0
... |     |  |                                              |      L
.  -| X   |--| V                                                |----
. | |  d  |  |  g                                             |    |
. |  -----   |                                            |    |
. | ...... . |                                            |    |
. | .      . |                                            |    |
. | .      . |                                            |    |
. | .      . |      1/n                    -----          |    |
. | .      . |                          |     |          |    |
. | .      . |-----     --------------------| Z   |----------|    |
          ....... |   (      ............   |  2  | .......     -----
E               .  -- (  .                  -----            |     |
 i       I     ..... (  .     I                            | Z   |
           3        . (  .      2                            |  L  |
 |                 . (  .                                   -----
 |                 . |  .                                      |
 |                 . |  . n V                                   |
 |                 . |  .                                      |
 |                   |  .                                      |
```

[PEAS2-1]

-----   -----   -----

/////   /////   /////

$$0 = Z_1 I_1 - (Z_2/n) I_2 + (n - 1)/n$$

[PEAS2-1]

[PEAS2-1]

(a) $n = 110/110 = 1$

(b) $n = 111.5/110 = 1.0136$

[PEAS2-1]

[PEAS2-1]

```
                    I1


        ................................         ........→
        . |--------------------------------------------
        . |                                         |
        . |                                         |
        . |                                         |
        . |                                         |
 .......... |                                         |
 .          |                                         |    VL = 1.0
...          |                                         |
.  -------                                          ----
. |          |                                         |    |
. |          |                                         |    |
. | ...... . |                                         |    |
. | .      . |                                         |    |
. | .      . |                                         |    |
. | .      . |      1/n                                 |    |
. | .      . |                                         |    |
. | .      . |-----      ---------------------------------
        ....... |   (   ...................    ...........→     |
```

[PEAS2-1]

```
    E_i       .  --→   .
         I_3    ..... (  .     I_2
                      .    .
 |                    . (  . ↑
 |                    .    .                               |
 |                      (  . n △V                            |
 |                    ↓     .                               |
 |                      |  . |                             |
-----                  -----                              -----
/////                 /////                              /////
```

[PEAS2-1]

3.1

```
                    \  38.1  = 66 kV          P + j Q =  0 - j 9   MVA
                     \
                      ------
                     /
      /\            /
     /  \
    /    \          \  65.8  = 114 kV         P + j Q = 12 + j 9   MVA
   /______\          \                          -----------------------
                      ------                    P + j Q = 12 + j 0   MVA
   13.86 kV          /
                    /
```

I_{in} = 12/13.86 = 500 A

3.2

```
      /\                 (   (   (
     /  \                (   (   (  220 V
    /    \               (   (   (
   /______\                                secondaries
                         (   (   (
  2200 V primary         (   (   (  220 V
                         (   (   (
```

(a)

```
                                             _______________________o
       ___________________o                 |
      /\        /\                          |    /\               220 V
     /__\      /__\    220 V                |   /  \
    |   |_____|___|_______o                ---/    \----------------o
    |         |                              /      \
    |_________|___________o                 /________\
                                                 |_________________o
```

(b)

```
     _____________________o                      ___________________o
     \            \                             |
      \____        \____   381 V                |____          381 V
      /    |       /    |                      /     \
     /_____|______/_____|_o                   /       \_____________o
           |            |                     \       /
           |____________|_o                    \ ____/
                                                |
                                                |___________________o
```

```
                ____o____________o
                    \
          ___o       \____
         |    \  /    /        381 V
         |     \/    /
         |          o_____________o
         |
         |________________________o
```

[PEAS3-2]

(c)

```
           ___________________o          _______________________o
          /\                            o     ___o______________o
         /  \                            \   /\
        o    o         440 V              \/__\
       /      \                               /          440 V
      /________\______________o              /
     |                                      o____________________o
     |________________________o
```

(d)   (e)

```
                      o_______o
                     /
                    /
                   o                                o______________o
                  /\      582 V                    /
                 /  \                             /
          o ____o____o                            \          660 V
           |          \                            \_____
           |           \                           /     \
           |            o_____o               ____/       \
           |                                 |             o______o
           |__________________o              |____________________o
```

(f)

```
          o__________________________________o
           \
            \
             o
              \
               \_____o_____o_________________o
               /
              /
             o                      762 V
            /
           /
          o__________________________________o
```

3.3

```
                    /\                              /\
       c'          /  \                   c'   /  \
        \         /    \                   \ /    \
         \       /______\                   \      \
          ------          a                 b ======
         /                                   /
        /                                   /
       b'                                  b'
```

$V_{max} = V_{a-b'} = V_{a-c'} = 2125$ V $V_{min} = V_{b-b'} = V_{b-c'} = 569.4$ V

```
3.4         o-------    o------
                   )          )    120 V
                   )          )
         2400 V    )          )    2.5 kVA
                   )          )
            5 kVA  )    o------  20.833 A
                   )
       2.0833 A    )    o------
                   )          )    120 V
                   )          )
                   )          )    2.5 kVA
                   )          )
            o-------    o------  20.833 A
```

(a)

```
o_____________o
             | \                       | \
             |  \                      |  \
             |   \                     |aI \
             |    \  a^2 I/20         |   \
       I_w |     \                   |     \
             |      \                  |      o______________o
             |       \                 |       \
             |        \                |        \   (1 - a)I
             |         \               |         \
       I/20  |          \o__     ______o        I \
             |          /   |   |      |          /
             |         /    |   |      |         /
             |        /     |   |      |        /
             |       /      |   |      |       /    (a^2 - 1)I
             |      /       |   |      |     o______________o
             |     /        |   |      |a^2 I /
             |    /  aI/20  |   |      |    /
             |   /          |   |      |   /
             |  /           |   |      |  /
             | /            |   |      | /
o_____________o             |   |
                            |   |                  (a - a^2)I
o___________________________|   |____________________________o
```

Let $I = 20.833$ A. $I_w = I/20 = 1.04165$ A = (1/2)(2.0833)

$S = (1/2)15 = 7.5$ kVA

(b) With only one 120 V winding connected in delta, and carrying rated kVA, the current in the 120 V winding is 20.833 A and the current in the high voltage winding is 1.04165 A. Thus, again, S = 7.5 kVA.

(c) The high voltage winding current is 1.04165 A in both cases.

(d) Given a load of = P + j Q = 4000 + j 0 at 120 V connected as in part (a). Then: = 4000 + j 0 = 3(2400) = 8000/720 [A]

[PEAS3-4]

```
             o_____________o
                           |  \                        |  \
                           |   \                       |   \
                           |    \                      |aI  \
                           |     \   a2 I/20         |     \
                  Iw |      \                      |      \
                           |       \                   |       o______________o
                           |        \                  |        \
                           |         \                 |         \    (1 - a)I
                           |          \                |          \
                I/20   |           \o__     ______o        I  \
                           |           /   |   |       |           /
                           |          /    |   |       |          /
                           |         /     |   |       |         /
                           |        /      |   |       |        /    (a2 - 1)I
                           |       /       |   |       |       o______________o
                           |      /        |   |       |a2 I  /
                           |     /  aI/20  |   |       |     /
                           |    /          |   |       |    /
                           |   /           |   |       |   /
                           |  /            |   |       |  /
             o_____________o               |   |
                                           |   |                (a - a2)I
             o_____________________________|   |____________________________o
```

Given a load of = P'+ j Q' = S' = -jaS' connected across the 240 V windings as shown on the next page. Then:

-jaS' = 3(2400)(/10) = ja^2 S'/720 [A]

For the top 120 V winding we have:

$$I_{rated} = \frac{2500}{120} = \frac{15000}{720} \quad [A]$$

$$(a\,\tilde{I} - \tilde{I}') = a\,\frac{(8000 - j\,a\,S')}{720} \qquad (a\,\tilde{I} - \tilde{I}')^{*} = a^{2}\,\frac{(8000 + j\,a^{2}S')}{720}$$

$$|a\,\tilde{I} - \tilde{I}'|^{2} = \frac{(S')^{2} + 8000\,\sqrt{3}\,S' + (8000)^{2}}{(720)^{2}} = \left(\frac{15000}{720}\right)^{2}$$

$$S' = -4000\,\sqrt{3} \pm \sqrt{3(4000)^{2} - 4(4000)^{2} + (15000)^{2}} = 7.528629 \quad [kVA]$$

```
             o_____________o                         o____________________o
                          | \                       | \
                          |  \                      |  \   (1 - a^2)I'
                          |   \                     |   \
                          |    \                    |    \
                          |     \                I' |     \
                          |      \                  |      o
                          |       \                 |       \
                          |        \                |  a^2 I'\
                          |         \               |         \  (a^2- a)I'
                          |          \o___          o          \__________o
                    I'/10 |          /    |         |          /
                          |         /     |         |         /
                          |        /      |         |        /
                          |       /       |      I' |   aI' /
                          |      /        |         |      o
                          |     /         |         |     /
                          |    /          |         |    /
                          |   /           |         |   /
                          |  /            |         |  /
                          | /             |         | /  (a - 1)I'
             o_____________o              |          o____________________o
                                          |
                                          |
             o____________________________|
```

For the bottom 120 V winding we have:

$$(a^2 \tilde{I} - \tilde{I}') = a^2 \frac{(8000 - j S')}{720} \qquad (a^2 \tilde{I} - \tilde{I}')^* = a \frac{(8000 + j S')}{720}$$

$$|a^2 \tilde{I} - \tilde{I}'|^2 = \frac{(S')^2 + (8000)^2}{(720)^2} = \left(\frac{15000}{720} \right)^2$$

$$S' = \sqrt{(15000)^2 - (8000)^2} = 12.68858 \quad [kVA]$$

[PEAS3-6]

Thus, the top 120 V winding is restrictive, and we may add only S' = 7.53 kVA for
which the total phasor power is S + S' = 11.17 kVA.

3.5 The fundamental frequency winding voltages on the secondary are 200, . The third harmonic voltages on the secondary are 300/3 = 100.
(a) For one winding:
(b) For two windings:
(c) For three windings:

3.6 The rms line current is one ampere. This can only be fundamental frequency. Therefore the fundamental frequency winding current is 1/ amperes. But the total rms winding current is one ampere. Therefore the rms third harmonic winding current is:

$$I_{3\ rms} = \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}} = 0.8165 \quad [A]$$

3.7 System Base:

System Base:	30 MVA	30 MVA
	66 kV	22 kV
	145.2 ohms	16.13 ohms
Z_{sc}:	.07(145.2) = 10.164 ohms	.07(16.13) = 1.13 ohms

3.8

System Base:	15 MVA	15 MVA
	66 kV	13.2 kV
	290.4 ohms	11.616 ohms

$$\bar{Z} = \bar{R} + j\,\bar{X} = 0.0065 + j\ 0.08976$$

$$\bar{S} = \bar{P} + j\,\bar{Q} = 0.6666 + j\ 0.5000 = \quad \bar{I}^{*}$$

$$\bar{V}_{drop} = 0.04921 + j\ 0.05659$$

$$V_{in} = (1.04921 + j\ 0.05659)(66) = 69.35\ kV$$

```
3.9    __________o__________            _______o___________________
                 .          |          | \     .
                 .          |          |  \    .
                 .          |          |   \   .
                 .          |          |    \__._______o___________
                 .          |          |    /  .       .
                 .         / \         |   /   .       .
                 .        /   \        |  /    .       .
                 .       /     \       | /     .       .
        _o_______._____/         \       -------.-------.------o---
         .       .               |             .       .      .
        _._______._______________o             .       .      .
         .       .               |      ........       .      .
         .       ........        |      \              .      .
         .              /\       |       \             .      .
         .             /  \      |        \            .      .
         .            /    \     |         \____________      .
         .           /      \    |         /                  .
         .          /        \   |        /                   .
         ........../___________\_|       /                    .
                                        /......................

        _________________                    o..................
3.10                   / \                  /                  .
        2I            /   \                /                   .
                a^2 /     \ a         a^2 / I                .
                    / I   I \            /                     .
        0          /    I     \         /    I                 .
        __________/____________\     o /____________ o         .
                               |                     \         .
                       1       |             1        \        .
                               |                       \  I    .
        2I                     |                        \      .
        _______________________|                    -a   \     .
                                                          \ ....
                                                           o
```

[PEAS3-8]

5.1 (a) The $[\tilde{\hat{Z}}_c]$ matrices, in [ohms/km], are:

$$\begin{array}{cc} \text{b-a-c} & \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \quad \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 1.43330\angle{+80.369} & 0.01742\angle{+90.000} & 0.01742\angle{+90.000} \\ 0.01742\angle{+90.000} & 0.55549\angle{+83.577} & 0.03484\angle{-90.000} \\ 0.01742\angle{+90.000} & 0.03484\angle{-90.000} & 0.55549\angle{+83.577} \end{bmatrix} \end{array} \qquad \begin{array}{cc} \text{c-a-b} & \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \quad \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 1.43330\angle{+80.369} & 0.01742\angle{+90.000} & 0.01742\angle{+90.000} \\ 0.01742\angle{+90.000} & 0.55549\angle{+83.577} & 0.03484\angle{-90.000} \\ 0.01742\angle{+90.000} & 0.03484\angle{-90.000} & 0.55549\angle{+83.577} \end{bmatrix} \end{array}$$

$$\begin{array}{cc} \text{c-b-a} & \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \quad \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 1.43330\angle{+80.369} & 0.01742\angle{-30.000} & 0.01742\angle{+210.000} \\ 0.01742\angle{+210.000} & 0.55549\angle{+83.577} & 0.03484\angle{+150.000} \\ 0.01742\angle{-30.000} & 0.03484\angle{+30.000} & 0.55549\angle{+83.577} \end{bmatrix} \end{array} \qquad \begin{array}{cc} \text{a-b-c} & \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \quad \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 1.43330\angle{+80.369} & 0.01742\angle{-30.000} & 0.01742\angle{+210.000} \\ 0.01742\angle{+210.000} & 0.55549\angle{+83.577} & 0.03484\angle{+150.000} \\ 0.01742\angle{-30.000} & 0.03484\angle{+30.000} & 0.55549\angle{+83.577} \end{bmatrix} \end{array}$$

$$\begin{array}{cc} \text{a-c-b} & \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \quad \end{array} \\ 0 & \begin{vmatrix} 1.43330\angle{+80.369} & 0.01742\angle{+210.000} & 0.01742\angle{-30.000} \end{vmatrix} \end{array} \qquad \begin{array}{cc} \text{b-c-a} & \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \quad \end{array} \\ 0 & \begin{vmatrix} 1.43330\angle{+80.369} & 0.01742\angle{+210.000} & 0.01742\angle{-30.000} \end{vmatrix} \end{array}$$

$$
\begin{array}{c}
\\
1 \\
\\
2
\end{array}
\begin{vmatrix}
\dots & \dots & \dots \\
0.01742 \,\underline{/ -30.000} & 0.55549 \,\underline{/ +83.577} & 0.03484 \,\underline{/ +30.000} \\
0.01742 \,\underline{/ +210.000} & 0.03484 \,\underline{/ +150.000} & 0.55549 \,\underline{/ +83.577}
\end{vmatrix}
\quad
\begin{vmatrix}
\dots & \dots & \dots \\
0.01742 \,\underline{/ -30.000} & 0.55549 \,\underline{/ +83.577} & 0.03484 \,\underline{/ +30.000} \\
0.01742 \,\underline{/ +210.000} & 0.03484 \,\underline{/ +150.000} & 0.55549 \,\underline{/ +83.577}
\end{vmatrix}
$$

(b) For cases b-a-c and c-a-b we have:

$$\tilde{Z}^{\wedge\wedge}_{01} = \tilde{Z}^{\wedge\wedge}_{02} = \tilde{Z}^{\wedge\wedge}_{10} = \tilde{Z}^{\wedge\wedge}_{20}$$

because we have selected the a-phase such that there is geometric symmetry about this phase.

(c)

Case	$\tilde{m}^{\wedge}_{0}$	$\tilde{m}^{\wedge}_{2}$
b-a-c	$0.01215 \,\underline{/ +9.631}$	$0.06272 \,\underline{/ -173.577}$
c-b-a	$0.01215 \,\underline{/ -110.369}$	$0.06272 \,\underline{/ -53.577}$
a-c-b	$0.01215 \,\underline{/ +129.631}$	$0.06272 \,\underline{/ +66.423}$
c-a-b	$0.01215 \,\underline{/ +9.631}$	$0.06272 \,\underline{/ -173.577}$
a-b-c	$0.01215 \,\underline{/ -110.369}$	$0.06272 \,\underline{/ -53.577}$
b-c-a	$0.01215 \,\underline{/ +129.631}$	$0.06272 \,\underline{/ +66.423}$

5.2 (a) From the $[\tilde{\hat{Z}}]^{\wedge\wedge\,-1}_{c}$ matrices we obtain:

	$\tilde{m}^t_0$	$\tilde{m}^t_2$	$\tilde{m}^c_0$	$\tilde{m}^c_2$
bac-bac	0.00098/+207.97	0.03369/ +5.34	0.05870/ +94.01	0.00366/+280.28
bac-cba	0.01575/+308.91	0.04790/ +65.69	0.00375/ +30.58	0.05162/+336.00
bac-acb	0.01633/+249.87	0.05325/+306.09	0.05588/+154.29	0.04677/ +35.45
bac-cab	0.01913/+276.61	0.07534/ +12.96	0.00000/ +2.30	0.00000/+288.76
bac-abc	0.00925/+216.41	0.01864/ +14.29	0.05712/+150.99	0.04859/+331.87
bac-bca	0.00925/+336.41	0.01864/+254.29	0.05712/ +90.99	0.04859/ +31.87
cab-cab	0.00098/+207.97	0.03369/ +5.34	0.05870/+274.01	0.00366/+100.28
cab-abc	0.01633/+129.86	0.05325/ +66.09	0.05588/+214.29	0.04677/+335.45
cab-bca	0.01575/ +68.91	0.04790/+305.69	0.00375/+330.58	0.05162/ +36.00
cab-bac	0.01848/+102.47	0.07594/ -1.44	0.00000/+288.67	0.00000/ +41.92
cab-cba	0.00894/ +42.27	0.01796/+121.05	0.05753/+277.28	0.04807/+339.41
cab-acb	0.00894/+162.27	0.01796/ +1.05	0.05753/+217.28	0.04807/ +39.41

(b) For the cab-cab arrangement, the $[\tilde{Z}^{\wedge}]_c$ matrix in polar form is:

+0.92776 / +82.755	+0.01407 /+129.781	+0.01293 / +44.770	+0.23588 / +77.685	+0.03429 /+195.162	+0.03356 / -16.081
+0.01293 / +44.770	+0.55280 / +83.504	+0.03277 /+268.424	+0.03429 /+195.162	+0.01154 / -59.605	+0.01195 / +89.970
+0.01407 /+129.781	+0.03266 / -89.520	+0.55280 / +83.504	+0.03356 / -16.081	+0.01195 / +89.970	+0.01185 /+239.746
+0.23588 / +77.685	+0.03356 / -16.081	+0.03429 /+195.162	+0.92776 / +82.755	+0.01293 / +44.770	+0.01407 /+129.781
+0.03356 / -16.081	+0.01185 /+239.746	+0.01195 / +89.970	+0.01407 /+129.781	+0.55280 / +83.504	+0.03266 / -89.520
+0.03429 /+195.162	+0.01195 / +89.970	+0.01154 / -59.605	+0.01293 / +44.770	+0.03277 /+268.424	+0.55280 / +83.504

5.2 (b) (cont'd)

and in rectangular form:

$$
\begin{bmatrix} 0 \\ \tilde{\hat{V}}_1 \\ 0 \\ 0 \\ \tilde{\hat{V}}_1 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
+0.11700+j0.92035 & -0.00900+j0.01081 & +0.00918+j0.00911 & +0.05031+j0.23045 & -0.03310-j0.00897 & +0.03225-j0.00930 \\
+0.00918+j0.00911 & +0.06254+j0.54925 & -0.00090-j0.03276 & -0.03310-j0.00897 & +0.00584-j0.00995 & +0.00001+j0.01195 \\
-0.00900+j0.01081 & +0.00027-j0.03266 & +0.06254+j0.54925 & +0.03225-j0.00930 & +0.00001+j0.01195 & -0.00597-j0.01024 \\
+0.05031+j0.23045 & +0.03225-j0.00930 & -0.03310-j0.00897 & +0.11700+j0.92035 & +0.00918+j0.00911 & -0.00900+j0.01081 \\
+0.03225-j0.00930 & -0.00597-j0.01024 & +0.00001+j0.01195 & -0.00900+j0.01081 & +0.06254+j0.54925 & +0.00027-j0.03266 \\
-0.03310-j0.00897 & +0.00001+j0.01195 & +0.00584-j0.00995 & +0.00918+j0.00911 & -0.00090-j0.03276 & +0.06254+j0.54925
\end{bmatrix}
\begin{bmatrix} \tilde{\hat{I}}_0 \\ \tilde{\hat{I}}_1 \\ \tilde{\hat{I}}_2 \\ \tilde{\hat{I}}_{0'} \\ \tilde{\hat{I}}_{1'} \\ \tilde{\hat{I}}_{2'} \end{bmatrix}
$$

Eliminating small impedance terms leading to products of small terms we have:

```
|    |   |                                                                   || ~  |
| 0  |   | +0.11700  -0.00900    00000   +0.05031  -0.03310    00000  || I^ |
|    |   |                                                                   ||  0 |
|    |   |+j0.92035 +j0.01081    00000  +j0.23045 -j0.00897    00000  ||    |
|    |   |                                                                   ||    |
| ~  |   |                                                                   || ~  |
| V^ |   |   00000   +0.06254    00000     00000   +0.00584    00000   || I^ |
|  1 |   |                                                                   ||  1 |
|    |   |   00000  +j0.54925    00000     00000  -j0.00995    00000   ||    |
|    |   |                                                                   ||    |
|    |   |                                                                   || ~  |
| 0  |   |   00000   +0.00027  +0.06254    00000   +0.00001    00000  || I^ |
|    |   |                                                                   ||  2 |
|    |   |   00000  -j0.03266 +j0.54925    00000  +j0.01195    00000  ||    |
|    | = |                                                                   ||    |
|    |   |                                                                   || ~  |
| 0  |   | +0.05031  +0.03225    00000   +0.11700  +0.00918    00000  || I^ |
|    |   |                                                                   ||  0'|
|    |   |+j0.23045 -j0.00930    00000  +j0.92035 +j0.00911    00000  ||    |
|    |   |                                                                   ||    |
| ~  |   |                                                                   || ~  |
| V^ |   |   00000   -0.00597    00000     00000   +0.06254    00000   || I^ |
|  1 |   |                                                                   ||  1'|
|    |   |   00000  -j0.01024    00000     00000  +j0.54925    00000   ||    |
|    |   |                                                                   ||    |
|    |   |                                                                   || ~  |
| 0  |   |   00000   +0.00001    00000     00000   -0.00090  +0.06254  || I^ |
|    |   |                                                                   ||  2'|
|    |   |   00000  +j0.01195    00000     00000  -j0.03276 +j0.54925  ||    |
```

Separating groups of matrix equations we have:

```
|                      || ~  |   |                      || ~  |
| -0.11700  -0.05031  || I^ |   | -0.00900   -0.03310  || I^ |
|                      ||  0 |   |                      ||  1 |
|-j0.92035 -j0.23045  ||    |   |+j0.01081   -j0.00897 ||    |
|                      ||    | = |                      ||    |
|                      || ~  |   |                      || ~  |
| -0.05031  -0.11700  || I^ |   | +0.03225   +0.00918  || I^ |
|                      ||  0'|   |                      ||  1'|
|-j0.23045 -j0.92035  ||    |   |-j0.00930   +j0.00911 ||    |
```

```
| ~    |    |                          || ~  |
| V^ |   | +0.06254  +0.00584 || I^ |
|  1   |    |                          ||  1 |
|        |    |+j0.54925 -j0.00995 ||     |
|        | = |                          ||     |
|   ~  |    |                          || ~  |
| V^ |   | -0.00597  +0.06254 || I^ |
|     1 |    |                          ||  1'|
|        |    |-j0.01024 +j0.54925||     |
```

5.2 (b) (cont'd)

```
|                                || ~  |    |                             || ~  |
| -0.06254   0.00000 || I^ |   | +0.00027  +0.00001  || I^ |
|                                ||  2 |    |                             ||  1 |
|-j0.54925  j0.00000 ||     |   |-j0.03266 +j0.01195    ||     |
|                                ||     | = |                             ||     |
|                                || ~  |    |                             || ~  |
|  0.00000  -0.06254 || I^ |   | +0.00001  -0.00090  || I^ |
|                                ||  2'|                                  ||  1'|
| j0.00000 -j0.54925 ||     |   |+j0.01195 -j0.03276  ||     |
```

****

```
| ~  |   |                          || ~    |   |             | ~
| I^ |   | +0.20493  +0.02596 || V^ |   | +0.23089 |(V^)
|  1 |   |                          ||  1 |    |             |  1
|     |   |  -j1.79812 -j0.02744 ||       |   |-j1.82556 |
|     | = |                          ||       | = |             |
| ~  |   |                          ||  ~  |   |             |
| I^ |   | -0.01150  +0.20493 || V^ |   | +0.19343 |
|  1'|   |                          ||  1    |   |             |
|     |   |-j0.03706 -j1.79812 ||       |   |-j1.83518 |
```

$$
\begin{bmatrix} \tilde{\hat{I}}_0 \\ \tilde{\hat{I}}_{0'} \end{bmatrix}
=
\begin{bmatrix} -0.13127 + j1.14352 & +0.00755 - j0.29255 \\ +0.00755 - j0.29255 & -0.13127 + j1.14352 \end{bmatrix}
\begin{bmatrix} -0.00900 + j0.01081 & -0.03310 - j0.00897 \\ +0.03225 - j0.00930 & +0.00918 + j0.00911 \end{bmatrix}
\begin{bmatrix} \tilde{\hat{I}}_1 \\ \tilde{\hat{I}}_{1'} \end{bmatrix}
$$

$$
\begin{bmatrix} \tilde{\hat{I}}_2 \\ \tilde{\hat{I}}_{2'} \end{bmatrix}
=
\begin{bmatrix} -0.20466 + j1.79736 & +0.00000 + j0.00000 \\ +0.00000 + j0.00000 & -0.20466 + j1.79736 \end{bmatrix}
\begin{bmatrix} +0.00027 - j0.03266 & +0.00001 + j0.01195 \\ +0.00001 + j0.01195 & -0.00090 - j0.03276 \end{bmatrix}
\begin{bmatrix} \tilde{\hat{I}}_1 \\ \tilde{\hat{I}}_{1'} \end{bmatrix}
$$

****

$$
\begin{bmatrix} \tilde{\hat{I}}_0 \\ \tilde{\hat{I}}_{0'} \end{bmatrix}
=
\begin{bmatrix} -0.01366 - j0.02122 & +0.01734 - j0.03929 \\ +0.00950 + j0.04081 & -0.01450 + j0.01892 \end{bmatrix}
\begin{bmatrix} \tilde{\hat{I}}_1 \\ \tilde{\hat{I}}_{1'} \end{bmatrix}
=
\begin{bmatrix} -0.11064 - j0.01938 \\ +0.10861 + j0.02235 \end{bmatrix}
(\tilde{\hat{V}})_1
$$

$$\begin{bmatrix} \tilde{\hat{I}}_2 \\ \tilde{\hat{I}}_{2'} \end{bmatrix} = \begin{bmatrix} +0.05865 & -0.02148 \\ +j0.00717 & -j0.00243 \\ -0.02148 & +0.05907 \\ -j0.00243 & +j0.00509 \end{bmatrix} \begin{bmatrix} \tilde{\hat{I}}_1 \\ \tilde{\hat{I}}_{1'} \end{bmatrix} + \begin{bmatrix} +0.01802 \\ -j0.06646 \\ +0.01137 \\ -j0.06877 \end{bmatrix} (\tilde{\hat{V}}_1)$$

****

5.2 (b) (cont'd)

Through Current	Circulating Current
$(\tilde{\hat{I}}_1 + \tilde{\hat{\hat{I}}}_{1'})/\tilde{\hat{V}}_1 = 0.42432 - j3.66074$	$(\tilde{\hat{I}}_1 - \tilde{\hat{\hat{I}}}_{1'})/\tilde{\hat{V}}_1 = 0.03746 + j0.00962$
$(\tilde{\hat{I}}_0 + \tilde{\hat{\hat{I}}}_{0'})/\tilde{\hat{V}}_1 = -0.00203 + j0.00297$	$(\tilde{\hat{I}}_0 - \tilde{\hat{\hat{I}}}_{0'})/\tilde{\hat{V}}_1 = -0.21925 - j0.04173$
$(\tilde{\hat{I}}_2 + \tilde{\hat{\hat{I}}}_{2'})/\tilde{\hat{V}}_1 = 0.02939 - j0.13523$	$(\tilde{\hat{I}}_2 - \tilde{\hat{\hat{I}}}_{2'})/\tilde{\hat{V}}_1 = 0.00665 + j0.00231$

$$\tilde{\hat{\hat{m}}}_{0t} = 0.00098\angle{+207.74} \qquad \tilde{\hat{\hat{m}}}_{2t} = 0.03755\angle{+5.65}$$

$$\tilde{\hat{\hat{m}}}_{0c} = 0.06056\angle{+274.16} \qquad \tilde{\hat{\hat{m}}}_{2c} = 0.00191\angle{+102.54}$$

-PEA6 Solutions-

6.1

```
        x
       / \       ...\__
      /_ _\  _|_ /
             ///

                    j:1
o------xxxxx--------------o  o---------------o
                    | |
  j 0.10             )(
                     )(
                     )(
                    | |
o------------------------o  o---------------o
                    1:j
o------xxxxx--------------o  o---------------o
                    | |
  j 0.10             )(
                     )(
                     )(
                    | |
o------------------------o  o---------------o
                    1:1
o-------------o     o----o  o---xxxxx-------o
              |    | |
              |     )(   j 0.10
              |     )(
              |     )(
              |    | |
o-------------------o----o  o---------------o
```

Base: 60 MVA

13.8 kV   69 kV

3.174 ohms   79.35 ohms

(a) 7.935 ohms on 69 kV side.

(b) 0.3174 ohms on 13.8 kV side.

```
    -
(c) X^ = 0.10 from 69 kV side.
     0
    -
    X^ = oo from 13.8 kV side.
     0
```

6.2

```
                  x
                 / \     ...\__
                /_ _\  _|  /
                       ///

                      j:1
o---xxxxx--o--xxxxx-------------o  o--------------o-------------o
|              i1               | |        I1                   |
| j 0.05   j 0.10               )(                              |
                                )(                              |
|       e1                      )(           E1                 |
|                               | |                             |
o---------o----------===---------o  o--------------o            |
                      1:j                          |            |
o---xxxxx--o--xxxxx-------------o  o--------------o             |
|              i2               | |        I2                   |
| j 0.05   j 0.10               )(                              |
|                               )(                              |
|       e2                      )(           E2                 |
|                               | |                             |
o---------o---------------------o  o---------------------o      |
                      1:1                                |      |
o---xxxxx--o----------o GN  o----o  o---xxxxx------------o GN   |
|             i0              |      | |        I0              |
o  j 0.08                     |      )(   j 0.10                |
|                             |      )(                         |
o       e0                    |      )(           E0            |
|                G, N         |     | |  G, N                   |
o---------o-------------------o---o  o--------------o-----------o
```

(a) i1 = 1.0/j 0.40 = - j 2.5   I1 = - 2.5
    i2 =   + j 2.5   I2 = - 2.5
    i0 =   0   I0 = - 2.5

(b) e1 = 0.875   E1 = - j 0.625
    e2 = 0.125   E2 = + j 0.375
    e0 = 0   E0 = + j 0.250

(c) ia =   0   Ia = - 7.5
    ib = -2.50 -/3   Ib =   0
    ic = +2.50 -/3   Ic =   0

(d) ean =   1.0   Ean =   0
    ebn = - 0.5 - j 0.6495   Ebn = - 0.866 + j 0.375
    ecn = - 0.5 + j 0.6495   Ecn = + 0.866 + j 0.375

(e) eab =   1.5 + j 0.6495   Eab =   0.866 - j 0.375
    ebc =   - j 1.2990   Ebc = - 1.732
    eca = - 1.5 + j 0.6495   Eca =   0.866 + j 0.375

6.3

```
-                    -
Z^^^ = j 0.10        Z^ =    0
 H-L                  H
-                    -
Z^^^ = j 0.18        Z^ = j 0.10
 H-T                  L
-                    -
Z^^^ = j 0.28        Z^ = j 0.18
 L-T                  T
```

Base: 30 MVA

```
66 kV        11 kV        2.2 kV
145.2 ohms   4.033 ohms   0.16133 ohms
```

	(a)	(b)	(c)
Z^ = H	0	0	0
Z^ = L	14.52 ohms	0.4033 ohms	0.016133 ohms
Z^ = T	26.136 ohms	0.726 ohms	0.029043 ohms

6.4

```
o---xxxxx--------o---xxxxx---o----xxxxx---o------------------o
| j 0.10       L  j 0.10          x     j 0    H
|                                 x
|                 j 0.18          x
|                                 o---o    o---o T
|                                 |
o---------------------------------o---------------------------------o
```

(a) j (0.18)(0.20)/0.38 = j 0.09474

(b) j 0.18

6.5

```
o----------------o---xxxxx---o----xxxxx---o------------------o
               L  j 0.30          x   j 0.20   H
                                  x
                  - j 0.10        x
                                  o---o    o---o T
                                  |
o--------------------------------o---------------------------------o
```

Considering only the autotransformer and ignoring parallel return paths for zero sequence currents through the system:

(a) j 0.10

(b) j 0.20

```
                              -PEA7 Solutions-


                                           x---------o
7.1                                \__    / \        |
                                   /     /_ _\     __|___
                                                   /////

                                      1:j
     o-------------xxxxx--------------o  o------------------------o-----o
     |                                |  |                              |
     |  |                              )(                         |     |
        |e1                            )(                     v1  |     |
     |  |                              )(                         |     |
     |                                |  |                              |
     o--------------------------------o  o------------------------o     |
                                      j:1                         |     |
     o-------------xxxxx--------------o  o------------------------o     |
     |                                |  |                              |
     |                                 )(                         |     |
     |                                 )(                     v2  |     |
     |                                 )(                         |     |
     |                                |  |                              |
     o--------------------------------o  o------------------------o     |
                                      1:1                         |     |
     o-------------xxxxx--------------o  o------o    o------------o     |
     |                                |  |      |                       |
    ---                                )(       |                 |     |
   |   |                               )(       |             v0  |     |
    ---                                )(       |                 |     |
     |                                |  |      |                       |
     o--------------------------------o  o------o-----------------o-----o

                                         -
                        v^ = e^ = 13.2/-/3 = 7.621 kV
                         0    1
```

$\hat{v}_0 = \hat{e}_1 = 13.2/\sqrt{3} = 7.621$ kV

7.2 (a) SLGF

```
                                        1:1
    o------------xxxxx--------------o  o-----------------------o-----o
    |            j 0.10             |  |       i1 --                 |
    |  |                             )(                        |     |
       |e1 = 1.0                     )(                  v1    |     |
    |  |                             )(                        |     |
    |                               |  |                             |
    o-------------------------------o  o-----------------------o     |
                                        1:1                    |     |
    o------------xxxxx--------------o  o-----------------------o     |
    |               0               |  |       i2 --                 |
    |                                )(                        |     |
    |                                )(                  v2    |     |
    |                                )(                        |     |
    |                               |  |                             |
    o-------------------------------o  o-----------------------o     |
                                        1:1                    |     |
    o------------xxxxx--------------o  o-----------------------o     |
    |         0.10 + j 0.12         |  |       i0 --                 |
   ---                               )(                        |     |
  |   |                              )(                  v0    |     |
   ---                               )(                        |     |
    |                               |  |                             |
    o-------------------------------o  o-----------------------o-----o
```

$\hat{i}_0 = \hat{i}_1 = \hat{i}_2 = 1.0/(0.10 + j\,0.22) = 1.712 - j\,3.767 = 4.138$

$\hat{i}_a = 5.136 - j\,11.301 = 12.414$

$\hat{i}_b = \hat{i}_c = 0$

7.2 (b) LLGF

```
                                      1:1
     o-------------xxxxx---------------o  o------------------------o-------o
     |            j 0.10               |  |       i1 --                    |
     |  |                               )(                         |       |
        |e1 = 1.0                       )(                     v1  |       |
     |  |                               )(                         |       |
     |                                 |  |                                |
     o---------------------------------o  o------------------------o---o   |
                                      1:1                              |   |
     o-------------xxxxx---------------o  o------------------------o---)---o
     |                0                |  |       i2 --                |   |
     |                                  )(                         |   |   |
     |                                  )(                     v2  |   |   |
     |                                  )(                         |   |   |
     |                                 |  |                            |   |
     o---------------------------------o  o------------------------o---o   |
                                      1:1                              |   |
     o-------------xxxxx---------------o  o------------------------o---)---o
     |         0.10 + j 0.12           |  |       i0 --                |
    ---                                 )(                         |   |
   |   |                                )(                     v0  |   |
    ---                                 )(                         |   |
     |                                 |  |                            |
     o---------------------------------o  o------------------------o---o
```

$\hat{i}_1 = -\hat{i}_2 = 1.0/j\,0.10 = -j\,10.0$ $\qquad \hat{i}_0 = 0$

$\hat{i}_a = 0$ $\qquad \hat{i}_b = -j\,10.0(\hat{a}^2 - a) = -\,17.32$ $\qquad \hat{i}_c = +\,17.32$

7.2 (c) LLF

```
                                     1:1
  o-------------xxxxx--------------o  o-------------------------o-------o
  |            j 0.10              |  |       i1 --                     |
  |  |                              )(                         |        |
     |e1 = 1.0                      )(                   v1    |        |
  |  |                              )(                         |        |
  |                                |  |                                 |
  o--------------------------------o  o-------------------------o---o   |
                                     1:1                            |   |
  o-------------xxxxx--------------o  o-------------------------o---)---o
  |                0               |  |       i2 --                 |
  |                                 )(                         |    |
  |                                 )(                   v2    |    |
  |                                 )(                         |    |
  |                                |  |                             |
  o--------------------------------o  o-------------------------o---o
                                     1:1
  o-------------xxxxx--------------o  o-------------------------o
  |         0.10 + j 0.12          |  |       i0 --
 ---                                )(                         |
|   |                               )(                   v0    |
 ---                                )(                         |
  |                                |  |
  o--------------------------------o  o-------------------------o
```

Same results as for 7.2 (b)

7.2 (d) Three-phase fault

```
                                     1:1
  o-------------xxxxx--------------o  o-------------------------o-------o
  |            j 0.10              |  |       i1 --                     |
  |  |                              )(                         |        |
     |e1 = 1.0                      )(                   v1    |        |
  |  |                              )(                         |        |
  |                                |  |                                 |
  o--------------------------------o  o-------------------------o-------o
```

$\hat{i}_1 = \hat{i}_a = 1.0/j\,0.10 = -j\,10.0$

7.3 (a) $\hat{i}_a = 0.5$ for three-phase fault $= 1.0/j\,\hat{X}_1$. $\hat{X}_1 = 2.0$

(b) $\hat{i}_b = 0.7$ for LLF $= \sqrt{3}/(j\,\hat{X}_1 + j\,\hat{X}_2)$ $\hat{X}_2 = 0.4743$

(c) $\hat{i}_a = 1.1$ for SLGF $= 3/(j\,\hat{X}_1 + j\,\hat{X}_2 + j\,\hat{X}_0)$ $\hat{X}_0 = 0.2530$

7.4 (a) SLGF

```
                                            1:1
    o-------------xxxxx---------------o   o-------------------------o---------o
    |             j 1.45              |   |       i1 --             |         |
    |   |                              ) (                        |  -        |
        |e1 = 2.0                      ) (                   v1 | | |j 1.0    |
    |   |                              ) (                        |  -        |
    |                                 |   | GN, N, G                |         |
    o---------------------------------o   o-------------------------o         |
                                            1:1                     |         |
    o-------------xxxxx---------------o   o-------------------------o         |
    |             j 0.286             |   |       i2 --             |         |
    |                                  ) (                        |  -        |
    |                                  ) (                   v2 | | |j 1.0    |
    |                                  ) (                        |  -        |
    |                                 |   | GN, N, G                |         |
    o---------------------------------o   o-------------------------o         |
                                            1:1                     |         |
    o-------------xxxxx---------------o   o-------------------------o         |
    |             j 0.350             |   | GN    i0 --           |  -        |
   ---                                 ) (                  v0N | | |j 1.0    |
  |   |                                ) (                        |  -        |
   ---                                 ) (                        N  o--------o
    |                                 |   | G
    o---------------------------------o   o-------------------------o
```

$\hat{Z}_2 = j\ 0.286/1.286 = j\ 1.0/4.5$ $\quad \hat{Z}_0 = j\ 1.0$ $\quad \hat{Z}_2 + \hat{Z}_0 = j\ 5.5/4.5$

$j\ 1.0 \ || \ (\hat{Z}_2 + \hat{Z}_0) = j\ 5.5/10.0 = j\ 0.55$ $\quad \hat{\hat{Z}}_{eq} = j\ 2.0$

$\hat{i}_1 = 2.0/j\ 2.0 = -j\ 1.0$ $\quad \hat{v}_1 = 2.0 - 1.45 = 0.55$ $\quad \hat{\hat{i}}_{1L} = -j\ 0.55$

$\hat{i}_0 = 0$ $\quad \hat{\hat{v}}_0 = -\ 0.45$ $\quad \hat{\hat{i}}_{0L} = +j\ 0.45$

$\hat{i}_2 = -j\ 0.35$ $\quad \hat{v}_2 = -\ 0.10$ $\quad \hat{\hat{i}}_{2L} = +j\ 0.10$

==

$\hat{i}_a = -j\ (\ 1.35 \ + j\ \ 0 \ )$ $\quad \hat{\hat{v}}_{aN} = 0$ $\quad \hat{\hat{i}}_{aL} = 0$

$\hat{i}_b = -j\ (-0.675 - j\ 0.5629)$ $\quad \hat{\hat{v}}_{bN} = -0.675 - j\ 0.5629$ $\quad \hat{\hat{i}}_{bL} = \hat{i}_b$

$\hat{i}_c = -j\ (-0.675 + j\ 0.5629)$ $\quad \hat{\hat{v}}_{cN} = -0.675 + j\ 0.5629$ $\quad \hat{\hat{i}}_{cL} = \hat{i}_c$

==

$\hat{\hat{v}}_{ab} = \ 0.675 + j\ 0.5629$

$\hat{\hat{v}}_{bc} = \quad -\ j\ 1.1258$

$\hat{\hat{v}}_{ca} = -0.675 + j\ 0.5629$

```
Observe:
  i^ + i^ + i^ = 0      (no zero sequence in the line currents)
   a    b    c
  i^^ + i^^ + i^^ = j 1.35 = 3 i^^
   aL    bL    cL               0L
  v^^^^ = - v^^^^ = - v^^ = 0.45
   N-GN      GN-N      0N
  v^^^^ = 0   (ground and geometric neutral are the same)
   G-GN
```

7.5 (a)

```
o-------------------o------------------o     o---------------o
|                   |                  |     |               |
|  |                |                  |     |               )
   |e1            ===== ZC = -j 745    |     |    ZL = j 500 )
|  |                |                  |     |               )
|                   |                  |     |               |
o-------------------o------------------)-----)---------------o
                                       |     |
o-------------------o------------------o     o---------------o
|                   |                  |     |               |
|                   |                  |     |               )
|                 ===== ZC = -j 745    |     |    ZL = j 500 )
|                   |                  |     |               )
|                   |                  |     |               |
o-------------------o------------------)-----)---------------o
                                       |     |
o-------------------o------------------o     o---------------o
|                   |                                        |
o                   |                                        )
                  ===== ZC = -j 745               ZL = j 500 )
o                   o                                        o
|                   |                                        |
o-------------------o----------------------------------------o
```

```
    -            -            -   -
(b) Z^ = -j 245  Z^ = j 500  Z^||Z^ = - j 500(49/51)
     0            2           0   2
     -
    ^v^^ = - 3(49/51)/(2/51) = - 73.50
      rs
```

(c)

	Source Neutral	Capacitor Neutral	Load Neutral
1.	Ungrounded	Ungrounded	Solidly Grounded
2.	Ungrounded	Solidly Grounded	Ungrounded
3.	Resonant Grounded	Solidly Grounded	Solidly Grounded
4.	Resis. Grounded 0 < R < 100	Solidly Grounded	Solidly Grounded

7.6

```
                                                                   1/a
  o----------------o------------------o     o---------------o---     ------
  |                |                  |     |               |   |  |        |
  |  |             |                  |     |               )    )(         |
     |e1         ===== ZC = -j 745    |     |    ZL = j 500 )    )(         |
  |  |             |                  |     |               )    )(         |
  |                |                  |     |               |   |  |        |
  o----------------o------------------)-----)---------------o--     ----    |
                                      |     |                           |   |
                                      |     |                      2    |   |
                                      |     |                   1/a^    |   |
  o----------------o------------------o     o---------------o---     ---    |
  |                |                  |     |               |   |  |        |
  |                |                  |     |               )    )(         |
  |              ===== ZC = -j 745    |     |    ZL = j 500 )    )(         |
  |                |                  |     |               )    )(         |
  |                |                  |     |               |   |  |        |
  o----------------o------------------)-----)---------------o---     ---    |
                                      |     |                           |   |
                                      |     |                           |   |
                                      |     |                   1/1     |   |
  o----------------o------------------o     o---------------o---     ---    |
  |                |                                        |   |  |        |
  o                |                                        )    )(         |
                 ===== ZC = -j 745               ZL = j 500 )    )(         |
  o                o                                        o    )(         |
  |                |                                        |   |  |        |
  o----------------o----------------------------------------o---     ------
```

7.7 (a)

$$\hat{\hat{i}}_{ca} = \hat{\hat{e}}_{ca}/Z = (a - 1)\hat{e}_a/Z = -j\sqrt{3}\,\hat{a}^2\,\hat{e}_a/Z$$

$$\hat{\hat{i}}_{ba} = \hat{\hat{e}}_{ba}/Z = (a - 1)\hat{e}_a/Z = j\sqrt{3}\,\hat{a}\,\hat{e}_a/Z$$

$$\hat{i}_a = 3\,\hat{e}_a/Z$$

$$\hat{i}_b = j\sqrt{3}\,\hat{a}\,\hat{e}_a/Z$$

$-\quad 2$

```
                             i^ = -j -/3 a^ e^/Z
                              c              a
7.7(b)(1)       -------------------------o----------
               |   i^ --->               |          |
               |    1                    |          |
               |                        ---         |    i^ = 2 e^/Z
           e^                          | Z |        |     1      a
            a                           ---         |
               |                         |          |
               |                         |          |
                -------------------------o          |    i^ = e^/Z
                                         |          |     2    a
                                         |          |
                                         |          |
                                         |          |
                -------------------------o          |    i^ = 0
               |   i^ --->               |          |     0
               |    2                    |          |
               |                        ---         |
               |                       | Z |        |
               |                        ---         |
               |                         |          |   i^ =         3 e^/Z

               |                         |          |     a               a
                -------------------------o          |
                                         |          |
                                         |          |          -
                                         |          |    i^ =  j -/3 a^ e^/Z
                                         |          |     b               a
                -------------------------o          |
               |   i^ --->               |          |
               o    0                   ---         |          -  2
                                       | Z |        |    i^ = -j -/3 a^ e^/Z
                                        ---         |     c               a
               o                         o----------
               |
               |
                -------------------------o
```

```
7.7(b)(2)                                    ----
                -------------------------o--| -Z |--
               |   i^ --->               |   ----   |
               |    1                    |          |
               |                       -----        |    i^ = 2 e^/Z
           e^                         | Z/3 |       |     1      a
            a                          -----        |
               |                         |          |
               |                         |          |
                -------------------------o----      |    i^ = e^/Z
                                              |     |     2    a
                                              |     |
                                              |     |
                                              |     |
                -------------------------o----)-----     i^ = 0
               |   i^ --->               |    |           0
               |    2                    |    |
               |                       -----  |
               |                      | Z/3 | |
               |                       -----  |
               |                         |    |          i^ =         3 e^/Z
               |                         |    |           a              a
                -------------------------o----

                                                                   -
                                                         i^ =  j -/3 a^ e^/Z
                                                          b              a
                -------------------------o
               |   i^ --->               |
               o    0                  -----                       -  2
                                      | Z/3 |            i^ = -j -/3 a^ e^/Z
                                       -----              c              a
               o                         o
               |
               |
                -------------------------o
```

```
                                             2
7.7(b)(3)                    1/a^         1/a^

                -------------o------------o
               |   i^ --->   |   ----     |   ----
               |    1        |  |    |    |  |    |
               |              )(    ---    )(    ---      i^ = 2 e^/Z
           e^                 )(   | Z |   )(   | Z |      1      a
            a                 )(    ---    )(    ---
               |             |  |    |    |  |    |
               |             |    -- |    |    -- |
                -------------o-----)-)----o     | |       i^ = e^/Z
                                   | |          | |        2    a
                                2  | |          | |
                             1/a^  | |    1/a^  | |
                                   | |          | |
                -------------o-----)-)----o     | |       i^ = 0
               |   i^ --->   |   --)-     |   --)-         0
               |    2        |  |  |      |  |  |
               |              )(   |       )(   |
               |              )(   |       )(   |
               |        2     )(   |       )(   |
               |       a^i^/2|  |  | ai^/2|  |  |         i^ =         3 e^/Z
               |          1  |    --   1  |    --          a              a
                -------------o------------o


                                                                    -
                                                          i^ =  j -/3 a^ e^/Z
                                                           b              a

                -------------o------------o
               |   i^ --->
               o    0                                               -  2
                                                          i^ = -j -/3 a^ e^/Z
               o                          o                c              a
               |
                -------------o------------o
```

Solution 9.1

The per-unit short-circuit impedance of the transformers is j 0.10 and the per-unit load impedance is j 0.90. Assume a balanced source voltage of 1.0 per-unit.

```
                   a          B
o-----------------            -------------------------- o--------------o
                  |          /|                          |              |
                  |         / |                          |              |
                  |     A _/  |                          |              |
                 / \     | \  |                          |              |
                /   \    |  \ |                          |              |
               /     \   |   \|                          |              |
o-------------- c   b |  |    ----------------------o----)--------------o
                      |  |    C                     |    |
                      |  |                          |    |
o---------------------     --------------------o    |    |
                                               |    |    |
                                               |    |    |
                                               -    -    -
                                              | |  | |  | | Z^
                                               -    -    -   L
                                               |    |    |
                                                ---------
                                                /////////
```

The symmetrical component sequence network representation for the LLF is as shown below.

```
                    1/j
  ------------()()()--  ----------------------------o-----------------o-----
 |           j 0.10   )(                            |                      |
 |                    )(                            |                      |
 |                    )(                            |                      |
 |                                                  -                      |
   e^ = 1.0           )(                           | | j 0.90              |
    1                                               -                      |
 |                    )(                            |                      |
 |                    )(                            |                      |
 |                    )(                            |                      |
  --------------------  ----------------------------o-----------------o--  |
                                                                       | |
                                                                       | |
                   -1/j                                                | |
  ------------()()()--  ----------------------------o-----------------o--)--
 |           j 0.10   )(                            |                   |
 |                    )(                            |                   |
 |                    )(                            |                   |
 |                                                  -                   |
 |                    )(                           | | j 0.90           |
 |                                                  -                   |
 |                    )(                            |                   |
 |                    )(                            |                   |
 |                    )(                            |                   |
  --------------------  ----------------------------o-----------------o--
```

Ignoring phase-shifting transformers:

$$\hat{\hat{Z}}_{eq} = j\,[0.10 + 0.45(0.10)/0.55] = j\,2/11$$

At source:

$$\hat{I}_1 = -j\,5.50$$

At transformer secondary:

$$\hat{I}_1 = 5.50 \qquad \hat{V}_1 = j\,0.45$$

At load:

$$\hat{\hat{V}}_{L1} = \hat{\hat{V}}_{L2} = j\,0.45 \qquad \hat{\hat{I}}_{L1} = \hat{\hat{I}}_{L2} = 0.50$$

In fault:

$$\hat{\hat{I}}_{F1} = 5.00 \qquad \hat{\hat{I}}_{F2} = -\,5.0$$

At transformer secondary:

$$\hat{I}_2 = -4.50$$

At source:

$$\hat{I}_2 = -j\,4.5$$

```
                      1/j
------------()()()--  ----------------------------o----------------o-----
| -j 5.5    j 0.10   )(          5.5              |     5.0             |
|                    )(                           |                     |
|                    )(                           |                     |
|                                                 -                     |
  e^ = 1.0      0.45 )(  j 0.45                  | | j 0.90             |
   1                                             -                     |
|                    )(                           |                     |
|                    )(                           | 0.5                 |
|                    )(                           |                     |
--------------------  ----------------------------o----------------o--   |
                                                                   |   |
                                                                   |   |
                     -1/j                                          |   |
------------()()()--  ----------------------------o----------------o--)--
| -j 4.5    j 0.10   )(         - 4.5             |    - 5.0         |
|                    )(                           |                  |
|                    )(                           |                  |
|                                                 -                  |
|             -0.45  )(  j 0.45                  | | j 0.90          |
|                                                 -                  |
|                    )(                           |                  |
|                    )(                           | 0.5              |
|                    )(                           |                  |
--------------------  ----------------------------o----------------o--
```

At source:

$\hat{I}_a = -j\,10.0$ $\qquad \hat{I}_b = j\,5.0\,[1.0 + j\,0.1\sqrt{3}]$ $\qquad \hat{I}_c = j\,5.0\,[1.0 - j\,0.1\sqrt{3}]$

Before the load:

$\hat{I}_a = 1.0$ $\qquad \hat{I}_b = 5.0\,[-j\sqrt{3} - 0.1]$ $\qquad \hat{I}_c = 5.0\,[+j\sqrt{3} - 0.1]$

In the fault:

$\hat{I} = 0.0$ $\qquad \hat{I} = -j\,5.0\sqrt{3}$ $\qquad \hat{I} = +j\,5.0\sqrt{3}$

a b c

In the load:

$\hat{I}_a = 1.0$ $\hat{I}_b = -0.5$ $\hat{I}_c = -0.5$

```
                                        -                          -
                   a          B   -j 5-/3 - 0.5              -j 5-/3
o-----------------            -------------------------- o--------------o
   -j 10.0        |          /|                          |              |
                  |         / |                          |              |
                  |     A _/  |                          |              |
                 / \     | \  |                          |              |
                /   \    |  \ |                          |              |
             -                         -                           -
j 5.0 + 0.5-/3 /     \   |   \|   j 5-/3 - 0.5           |   +j 5-/3    |
o-------------- c   b |  |    ----------------------o----)--------------o
                      |  |    C                     |    |
                      |  |                     -0.5 |    | -0.5
                      |  |                          |    |
o---------------------     --------------------o    |    |
             -
j 5.0 - 0.5-/3                     1.0         |    |    |
                                               |    |    |
                                               -    -    -
                                              | |  | |  | | Z^
                                               -    -    -   L
                                               |    |    |
                                                ---------
                                                /////////
```

The currents in the delta windings are $1/\sqrt{3}$ times the currents in the associated wye windings on the basis of ampere-turn balance.

The per-unit short-circuit impedance of the transformers is j 0.10 and the per-unit load impedance is j 0.90. Assume a balanced source voltage of 1.0 per-unit.

a B

```
o-----------------            -------------------------- o--------------o
                  |          /|                          |              |
                  |         / |                          |              |
                  |     A _/  |                          |              |
                 / \     | \  |                          |              |
                /   \    |  \ |                          |              |
               /     \   |   \|                          |              |
o-------------- c   b |  |    ----------------------o----)--------------o
                      |  |    C                     |    |
                      |  |                          |    |
o---------------------     --------------------o    |    |
                                               |    |    |
                                               |    |    |
                                               -    -    -
                                              | |  | |  | | Z^
                                               -    -    -   L
                                               |    |    |
                                                ---------
                                                /////////
```

The alpha-beta component sequence network representation for the LLF is as shown below.

```
                     1/1
  ------------()()()--  ---o                   o-----o----------------o
 |           j 0.10   )(     \                 |     |
 |                    )(      \                |     |
 |                    )(       \              /      |
 |                                                   -
   e^ = 1.0           )(        \            /      | | j 0.90
    ^                                                -
 |                    )(         \          /        |
 |                    )(          \        /         |
 |                    )(           \      /          |
  --------------------  ---o     o--)----)-----------o----------------o
                             \  /    \  /
                               /       /
                              /       /
                     1/1     /  \    /  \
  ------------()()()--  ---o     \  /    o-----------o----------------o
 |           j 0.10   )(          \                  |                |
 |                    )(           \                 |                |
 |                    )(         /  \                |                |
 |                                                   -                |
   e^ = -j 1.0        )(        /    \              | | j 0.90        |
    ^                                                -                |
 |                    )(       /      \              |                |
 |                    )(      /        \             |                |
 |                    )(     /          \            |                |
  --------------------  ---o             o-----------o----------------o
```

At source:

```
                         I^ = -j 10.0     I^ = - 1.0
                          ^                ^
```

At transformer secondary:

```
                         I^ = 1.0       I^ = -j 10.0
                          ^               ^
```

At load:

```
              V^^ = j 0.90    V^^ = 0     I^^ = 1.0     I^^ = 0
               L               L^          L^            L^
```

In fault:

```
                          I^^ = 0.0    I^^ = -j 10.0
                           F^           F^
```

```
                     1/1                       1.0          0
  ------------()()()--  ---o                  o-----o----------------o
 | -j 10.0   j 0.10   )(    \                 |     |
 |                    )(     \                |     |
 |                    )(      \              /      |
 |                                                  -
   e^ = 1.0           )(       \            /      | | j 0.90
    ^                                        j 0.9  -
 |                    )(        \          /        |
 |                    )(         \        /         | 1.0
 |                    )(          \      /          |
  --------------------  ---o    o--)----)-----------o----------------o
                            \  /    \  /
                              /      /
                             /      /
                     1/1    /  \    /  \
  ------------()()()--  ---o    \  /    o-----------o----------------o
 |  - 1.0    j 0.10   )(         \        -j 10.0   |   -j 10.0      |
 |                    )(          \                 |                |
 |                    )(        /  \                |                |
 |                                                  -                |
   e^ = -j 1.0        )(       /    \          0   | | j 0.90        |
    ^                                               -                |
 |                    )(      /      \              |                |
 |                    )(     /        \             | 0              |
 |                    )(    /          \            |                |
  --------------------  ---o            o-----------o----------------o
```

Returning to phase quantities we have:

```
                                     -                           -
        I^ = I^   I^ = 0.5(-I^ + j -/3 I^)   I^ = 0.5(-I^ - j -/3 I^)
         a         b                          c
```

At source:

$\hat{I}_a = -j\,10.0 \qquad \hat{I}_b = j\,5.0\,[1.0 + j\,0.1\sqrt{3}] \qquad \hat{I}_c = j\,5.0\,[1.0 - j\,0.1\sqrt{3}]$

Before the load:

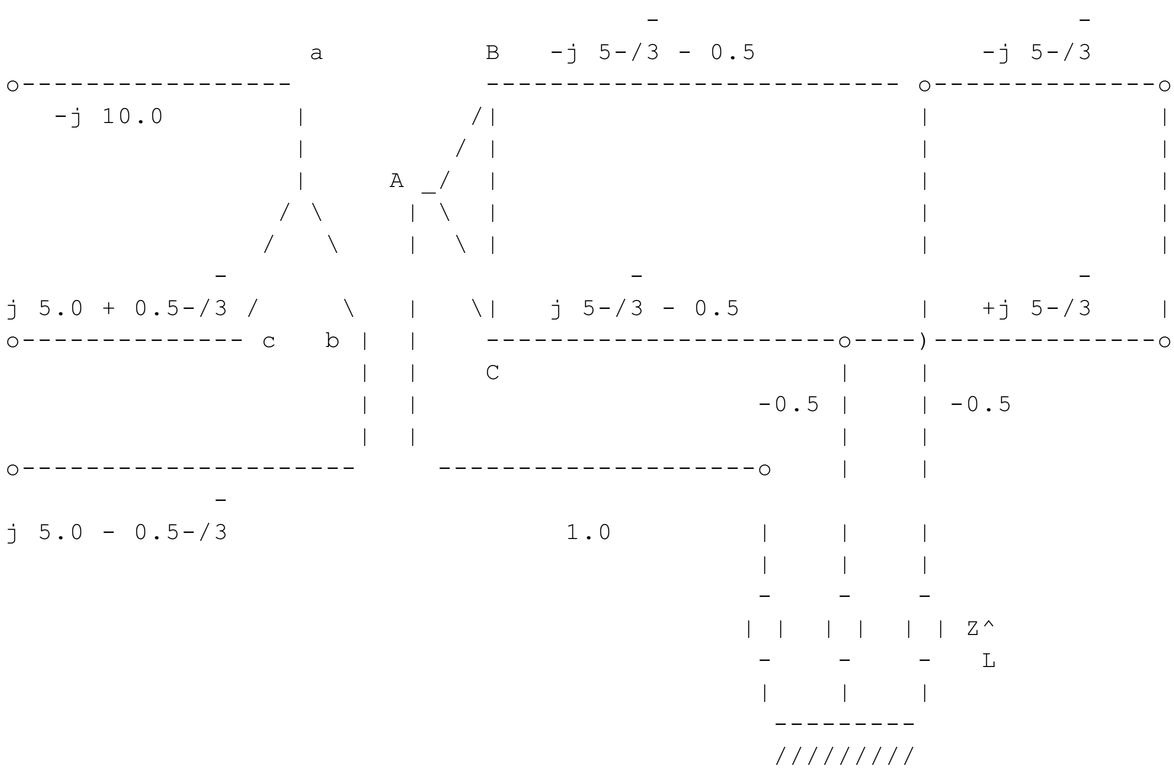
$\hat{I}_a = 1.0 \qquad \hat{I}_b = 5.0\,[-j\sqrt{3} - 0.1] \qquad \hat{I}_c = 5.0\,[+j\sqrt{3} - 0.1]$

In the fault:

$\hat{I}_a = 0.0 \qquad \hat{I}_b = -j\,5.0\sqrt{3} \qquad \hat{I}_c = +j\,5.0\sqrt{3}$

In the load:

$\hat{I}_a = 1.0 \qquad \hat{I}_b = -\,0.5 \qquad \hat{I}_c = -\,0.5$



The currents in the delta windings are $1/\sqrt{3}$ times the currents in the associated wye windings on the basis of ampere-turn balance.