Chapter 2

A Journey into Identification

2.2

1. $y[k] = \sqrt{u[k]}$ and $y[k] = \sin(u[k])$ are examples of models that are not identifiable globally, but identifiable locally.

2.5

Assume system is of the form

$$y[k] = a_1 u[k-1] + a_2 u[k-2] + a_3 u[k-3]$$

Given input contains two frequencies

$$u[k] = \sin(\omega_0 k) + \sin(\omega_1 k)$$

where $\omega_0 \neq \omega_1$. The output $y[k]$ of the system for the input $u[k]$ is obtained as

$$\begin{aligned} y[k] &= a_1 \sin(\omega_0 (k-1)) + a_1 \sin(\omega_1 (k-1)) + a_2 \sin(\omega_0 (k-2)) \\ &\quad + a_2 \sin(\omega_1 (k-2)) + a_3 \sin(\omega_0 (k-3)) + a_3 \sin(\omega_1 (k-3)) \end{aligned}$$

$$
\begin{aligned}
\implies y[k] = \sin(\omega_0 k) & \underbrace{(a_1 \cos(\omega_0) + a_2 \cos(2\omega_0) + a_3 \cos(3\omega_0))}_{b'_1} \\
- \cos(\omega_0 k) & \underbrace{(a_1 \sin(\omega_0) + a_2 \sin(2\omega_0) + a_3 \sin(3\omega_0))}_{b'_2} \\
+ \sin(\omega_1 k) & \underbrace{(a_1 \cos(\omega_1) + a_2 \cos(2\omega_1) + a_3 \cos(3\omega_1))}_{b'_3} \\
- \cos(\omega_1 k) & \underbrace{(a_1 \sin(\omega_1) + a_2 \sin(2\omega_1) + a_3 \sin(3\omega_1))}_{b'_4}
\end{aligned}
$$

There are four regressors $\sin(\omega_0 k), \sin(\omega_1 k), \cos(\omega_0 k), \cos(\omega_1 k)$ in this equation. From trigonometric properties, it is known that all regressors are uncorrelated with each other. Hence it is possible to estimate all the parameters b'_1,b'_2,b'_3,b'_4 uniquely which means it is possible to identify b_1, b_2, b_3 uniquely.