

Chapter 2 Answers to Questions

1. The average can be calculated by looking up the % abundance and atomic mass of all three naturally occurring isotopes of Mg (Appendix C).

$$(0.7899)(23.9850 \text{ u}) + (0.1000)(24.9858 \text{ u}) + (0.1101)(25.9826 \text{ u}) = 24.30 \text{ u}$$

2. Let x = the fractional abundance of ^{85}Rb , then $1 - x$ = the fractional abundance of ^{87}Rb . Then:

$$(84.9118 \text{ u})x + (86.9092 \text{ u})(1 - x) = 85.4678 \text{ u}$$

$$x = 0.72164$$

^{85}Rb is 72.164% abundant, and ^{87}Rb is 27.836% abundant.

3. The percent abundance of ^{81}Br is 49.31%.

$$1.56 \text{ g Br}_2 \times \frac{1 \text{ mol Br}_2}{159.8 \text{ g Br}_2} \times \frac{2 \text{ mol Br}}{1 \text{ mol Br}_2} \times \frac{6.022 \times 10^{23} \text{ Br atoms}}{\text{mol Br}} \times \frac{49.31 \text{ } ^{81}\text{Br atoms}}{100 \text{ Br atoms}} \\ = 5.80 \times 10^{21} \text{ } ^{81}\text{Br atoms}$$

4. Count rate is the number of decays recorded by the detector per unit time. Typically, only a small fraction of all ionizing radiation being emitted by a radioactive source is picked up by the detector.

Decay rate is the total number of decays that take place for a given source per unit time. Decay rate is proportional to count rate; specifically, they are related by the counter's efficiency.

$$\% \text{ efficiency} = \frac{\text{count rate}}{\text{decay rate}} \times 100\%$$

5. Since each half-life reduces the activity by $1/2$, three half-lives would cut activity by $1/8$:

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

and five half-lives would cut activity to $1/32$ of the original value:

$$\left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

6. First convert days to years, then plug into equation 2.2.

$$2016 \text{ d} \times \frac{\text{a}}{365.24 \text{ d}} = 5.520 \text{ a}$$

$$A_2 = A_1 e^{-kt} = 1.5 \text{ } \mu\text{Ci} \times e^{-\frac{\ln 2}{5.271 \text{ a}} \times 5.520 \text{ a}} = 0.73 \text{ } \mu\text{Ci}$$

7. Since both activities are known, it's best to use the formula that allows easier solution of half-life:

$$\ln\left(\frac{A_1}{A_2}\right) = \left(\frac{\ln 2}{t_{1/2}}\right) \times t \quad \ln\left(\frac{5718 \text{ dpm}}{515 \text{ dpm}}\right) = \left(\frac{\ln 2}{t_{1/2}}\right) \times 24.0 \text{ h} \quad t_{1/2} = 6.91 \text{ h}$$

8. Using a half-life of 2.7476 days for ^{99}Mo :

$$A_2 = A_1 e^{-\left(\frac{\ln 2}{t_{1/2}}\right) \times t} = (15,000 \text{ dpm}) e^{-\left(\frac{\ln 2}{2.7476 \text{ d}}\right) \times 30.0 \text{ d}} = 7.7 \text{ dpm or 460 decays per hour}$$

9. We'll use equation 2.1, solved for time.

$$t = \frac{\ln \frac{A_1}{A_2}}{k} = \frac{\ln \frac{0.501 \mu\text{Ci} \times \frac{2.22 \times 10^6 \text{ dpm}}{\mu\text{Ci}}}{1.48 \times 10^5 \text{ dpm}}}{\frac{\ln 2}{3.78 \text{ a}}} = 11.0 \text{ a}$$

It was calibrated 11.0 years ago.

10. According to Appendix C, the half-life for ^{244}Pu is 8.11×10^7 years.

$$\begin{aligned} N_2 &= N_1 e^{-kt} \\ 20 \times 10^6 &= N_1 \times e^{-\frac{\ln 2}{8.11 \times 10^7 \text{ a}} \times 4.5 \times 10^9 \text{ a}} \\ N_1 &= 1.0 \times 10^{24} \text{ atoms} \end{aligned}$$

Convert to mass.

$$1.0 \times 10^{24} \text{ atoms} \times \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ atoms}} \times \frac{244 \text{ g}}{\text{mol}} = 410 \text{ g}$$

11. Using the percent efficiency equation:

$$\% \text{ efficiency} = \frac{\text{cpm}}{\text{dpm}} \times 100\% = \frac{2554 \text{ cpm}}{10756 \text{ dpm}} \times 100\% = 23.7\%$$

Notice the answer is rounded to three significant figures because count data are not usually good beyond this number.

12. 9.37 years had passed since this source was calibrated. Using a half-life of 3.78 years for ^{204}Tl , the activity on July 13, 2012 was:

$$A_2 = A_1 e^{-kt} = \left(1.05 \mu\text{Ci} \times \frac{2.22 \times 10^6 \text{ dpm}}{\mu\text{Ci}}\right) \times e^{-\frac{\ln 2}{3.78 \text{ a}} \times 9.37 \text{ a}} = 4.18 \times 10^5 \text{ dpm}$$

Shelf efficiencies were therefore:

$$\text{shelf 1: } \% \text{ efficiency} = \frac{13,652 \text{ cpm}}{4.18 \times 10^5 \text{ dpm}} \times 100\% = 3.27\%$$

$$\text{shelf 2: } \% \text{ efficiency} = \frac{7907 \text{ cpm}}{4.18 \times 10^5 \text{ dpm}} \times 100\% = 1.89\%$$

$$\text{shelf 3: } \% \text{ efficiency} = \frac{4010 \text{ cpm}}{4.18 \times 10^5 \text{ dpm}} \times 100\% = 0.960\%$$

13. The first step is to determine the activity of the source today:

$$3.45\% = \frac{1500 \text{ cpm}}{x \text{ dpm}} \times 100\%$$

$$\text{activity} = 4.35 \times 10^4 \text{ dpm}$$

Next solve for the activity 5.00 days from now:

$$A_2 = A_1 e^{-kt} = (4.35 \times 10^4 \text{ dpm}) e^{-\frac{\ln 2}{2.669 \text{ d}} \times 5.00 \text{ d}} = 1.19 \times 10^4 \text{ dpm}$$

14. First figure out how many potassium ions are in a 100 kg body:

$$100 \text{ kg} \times \frac{140 \text{ g K}}{70 \text{ kg}} \times \frac{\text{mol}}{39.10 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ ions}}{\text{mol}} = 3.08 \times 10^{24} \text{ atoms}$$

How many are ^{40}K ?

$$3.08 \times 10^{24} \text{ K ions} \times \frac{0.0117 \text{ }^{40}\text{K ions}}{100 \text{ K ions}} = 3.60 \times 10^{20} \text{ }^{40}\text{K ions}$$

Now calculate activity.

$$A = \frac{\ln 2}{t_{1/2}} N = \frac{\ln 2}{1.25 \times 10^9 \text{ a} \times \frac{5.259 \times 10^5 \text{ min}}{\text{a}}} \times (3.60 \times 10^{20} \text{ }^{40}\text{K ions}) = 3.8 \times 10^5 \text{ dpm}$$

Your results will vary depending on your mass, or the mass you decide to use. The math below is done for a 100 kg person. To check your answer, divide your mass (in kg!) by 100, and then multiply the result by $3.8 \times 10^5 \text{ dpm}$.

15. Like any specific activity problem, it doesn't matter what mass we start with. The clever approach would be to assume 1 g or 1 kg and just calculate activities. I'll take the slightly more cumbersome approach and assume a mass of 100 kg. This way we'll also see approximately how hot humans are. Let's start with tritium.

$$N = 9.5 \text{ kg H} \times \frac{1000 \text{ g H}}{\text{kg H}} \times \frac{\text{mol H}}{1.008 \text{ g H}} \times \frac{6.022 \times 10^{23} \text{ atoms H}}{\text{mol H}} \times \frac{5.0 \times 10^{-15} \text{ atoms } ^3\text{H}}{100 \text{ atoms H}}$$

$$= 2.8 \times 10^{11} \text{ atoms } ^3\text{H}$$

$$A = kN = \frac{\ln 2}{t_{1/2}} N = \frac{\ln 2}{12.32 \text{ a} \times \left(\frac{5.259 \times 10^5 \text{ min}}{\text{a}} \right)} \times (2.8 \times 10^{11} \text{ atoms } ^3\text{H})$$

$$= 3.0 \times 10^4 \text{ dpm}$$

Rinse and repeat with ^{12}C and ^{40}K .

$$N = 18.6 \text{ kg C} \times \frac{1000 \text{ g C}}{\text{kg C}} \times \frac{\text{mol C}}{12.01 \text{ g C}} \times \frac{6.022 \times 10^{23} \text{ atoms C}}{\text{mol C}} \times \frac{1.2 \times 10^{-10} \text{ atoms } ^{14}\text{C}}{100 \text{ atoms C}}$$

$$= 1.1 \times 10^{15} \text{ atoms } ^{14}\text{C}$$

$$A = kN = \frac{\ln 2}{t_{1/2}} N = \frac{\ln 2}{5715 \text{ a} \times \left(5.259 \times 10^5 \frac{\text{min}}{\text{a}} \right)} \times (1.1 \times 10^{15} \text{ atoms } ^{14}\text{C})$$

$$= 2.6 \times 10^5 \text{ dpm}$$

$$N = 0.35 \text{ kg K} \times \frac{1000 \text{ g K}}{\text{kg K}} \times \frac{\text{mol K}}{39.10 \text{ g K}} \times \frac{6.022 \times 10^{23} \text{ atoms C}}{\text{mol C}} \times \frac{0.0117 \text{ atoms } ^{40}\text{K}}{100 \text{ atoms K}}$$

$$= 6.3 \times 10^{20} \text{ atoms } ^{40}\text{K}$$

We can now calculate specific activity.

$$\frac{3.0 \times 10^4 \text{ dpm} + 2.6 \times 10^5 \text{ dpm} + 6.6 \times 10^5 \text{ dpm}}{100 \text{ kg}} = 9.5 \times 10^3 \frac{\text{dpm}}{\text{kg}}$$

Regardless of your mass, your specific activity is $\sim 9500 \text{ dpm/kg}$. That's a lot of radioactive decay going on inside each of us every day (~ 1 billion decays per day)!

16. First calculate the number of ^{99}Tc atoms that are present:

$$0.353 \text{ g NaTcO}_4 \times \frac{1 \text{ mol NaTcO}_4}{185.9 \text{ g NaTcO}_4} \times \frac{1 \text{ mol Tc}}{1 \text{ mol NaTcO}_4} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol Tc}}$$

$$= 1.14 \times 10^{21} \text{ atoms } ^{99}\text{Tc}$$

Now determine the activity for 1.14×10^{21} atoms of ^{99}Tc ($t_{1/2} = 2.13 \times 10^5$ years):

$$A = \left(\frac{\ln 2}{2.13 \times 10^5 \text{ a}} \right) \times \left(\frac{\text{a}}{5.259 \times 10^5 \text{ min}} \right) \times (1.14 \times 10^{21} \text{ atoms } ^{99}\text{Tc}) = 7.08 \times 10^9 \text{ dpm}$$

Finally, calculate specific activity:

$$\text{S.A.} = \frac{7.14 \times 10^9 \text{ dpm} \times \frac{1 \text{ Ci}}{2.22 \times 10^{12} \text{ dpm}}}{0.353 \text{ g}} = 9.11 \times 10^{-3} \frac{\text{Ci}}{\text{g}} = 9.11 \frac{\text{mCi}}{\text{g}}$$

17. In order to do the problem, you need to know that several isotopes of platinum occur naturally, but only one is radioactive (the others are all stable). The one radioactive isotope is ^{190}Pt , and it makes up only 0.014% of all platinum. In other words, if you went and dug up some Pt, only 0.014% of it would be ^{190}Pt . You also need to know the half-life of ^{190}Pt , which is $6.5 \times 10^{11} \text{ a}$. With all of that in mind, assume exactly 1 gram of Pt. Since only one isotope of Pt is radioactive, we should first determine the number of ^{190}Pt atoms present in one gram of Pt (*all* isotopes).

$$1 \text{ g Pt} \times \frac{\text{mol Pt}}{195.1 \text{ g Pt}} \times \frac{6.022 \times 10^{23} \text{ Pt atoms}}{\text{mol Pt}} \times \frac{0.014 \text{ }^{190}\text{Pt atoms}}{100 \text{ Pt atoms}} \\ = 4.3 \times 10^{17} \text{ }^{190}\text{Pt atoms}$$

The activity is then:

$$A = kN = \frac{\ln 2}{6.5 \times 10^{11} \text{ a} \times \frac{5.259 \times 10^5 \text{ min}}{\text{a}}} \times 4.3 \times 10^{17} \text{ atoms} \\ = 0.88 \text{ dpm}$$

Specific activity is:

$$\frac{0.88 \text{ dpm} \times \frac{\mu\text{Ci}}{2.22 \times 10^6 \text{ dpm}}}{1 \text{ g}} = 3.9 \times 10^{-7} \frac{\mu\text{Ci}}{\text{g}}$$

This problem could also be done starting with the assumption that there is some other mass of Pt. Since specific activity is characteristic of a particular element or compound, it does matter how much of it you have, the specific activity remains the same.

18. 20% by weight means that if we have 100 g of fertilizer, 20 g will be K_2O . Since potassium is naturally radioactive (and oxygen is not), we should first calculate the number of ^{40}K atoms present in 20 g of K_2O .

$$20 \text{ g K}_2\text{O} \times \frac{\text{mol K}_2\text{O}}{94.2 \text{ g K}_2\text{O}} \times \frac{2 \text{ mol K}}{1 \text{ mol K}_2\text{O}} \times \frac{0.0117 \text{ mol }^{40}\text{K}}{100 \text{ mol K}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} \\ = 3.0 \times 10^{19} \text{ atoms }^{40}\text{K}$$

Activity is then:

$$A = kN = \frac{\ln 2}{1.25 \times 10^9 \text{ a} \times \frac{5.259 \times 10^5 \text{ min}}{\text{a}}} \times 3.0 \times 10^{19} \text{ atoms} \\ = 3.2 \times 10^4 \text{ dpm}$$

Specific activity is:

$$\frac{3.2 \times 10^4 \text{ decays}}{\text{min}} \times \frac{\text{min}}{60 \text{ s}} \times \frac{\text{Bq}}{\text{dps}} = 5.3 \times 10^3 \frac{\text{Bq}}{\text{kg}} \\ \frac{100 \text{ g} \times \frac{\text{kg}}{1000 \text{ g}}}{\text{kg}}$$

Whoa! That's 26× higher than allegedly allowed by the Japanese government. It looks like the journalist was incorrect, and was perhaps more interested in raising fear than awareness.

5,300 Bq/kg is a conservative value, because we're ignoring other radioactive components of fertilizers. Two other common components are ammonium (NH_4^+)

and urea ($\text{CH}_4\text{N}_2\text{O}$). Since both carbon and hydrogen are naturally radioactive, the specific activity of our fertilizer is likely higher.

19. The ratio of the current activity to the original activity is given in this problem (0.559). Using a half-life for ^{14}C of 5715 years:

$$\ln\left(\frac{A_1}{A_2}\right) = \left(\frac{\ln 2}{t_{1/2}}\right) \times t \quad \ln\left(\frac{100}{55.9}\right) = \left(\frac{\ln 2}{5715 \text{ a}}\right) \times t \quad t \approx 4800 \text{ years ago}$$

20. This is like a dating problem (dinner or a movie?) only backwards. Equation 2.2 is probably the easiest to use on this one.

$$A_2 = A_1 e^{-kt} = 14 \text{ dpm/g} \times e^{-\frac{\ln 2}{5715 \text{ a}} \times 19,000 \text{ a}} = 1.4 \text{ dpm/g}$$

21. Over time, some of the ^{40}K decays to ^{40}Ar . We need to know how much ^{40}K was present in the rock when it was formed. First, let's figure out how much ^{40}K would produce 0.530 g ^{40}Ar – don't forget that only 10.72% of all ^{40}K decays produce ^{40}Ar .

$$0.530 \text{ g Ar} \times \frac{\text{mol } ^{40}\text{Ar}}{40.0 \text{ g } ^{40}\text{Ar}} \times \frac{1 \text{ mol } ^{40}\text{K}}{1 \text{ mol } ^{40}\text{Ar}} \times \frac{100 \text{ decays (all)}}{10.72 \text{ decays (to } ^{40}\text{Ar)}} \times \frac{40.0 \text{ g } ^{40}\text{K}}{\text{mol } ^{40}\text{K}} \\ = 4.94 \text{ g } ^{40}\text{K}$$

Now calculate the amount of ^{40}K that was present when the rock was formed.

$$0.092 \text{ g } ^{40}\text{K} + 4.94 \text{ g } ^{40}\text{K} = 5.04 \text{ g } ^{40}\text{K}$$

We can use the same formula as was used in the Rb-Sr example in the chapter to determine when the rock was formed. We can use masses instead of activities because mass is proportional to activity for the same nuclide:

$$t = \frac{\ln \frac{N_1}{N_2}}{\frac{\ln 2}{t_{1/2}}} = \frac{\ln \frac{5.04 \text{ g}}{0.092 \text{ g}}}{\frac{\ln 2}{1.25 \times 10^9 \text{ a}}} = 7.2 \times 10^9 \text{ a}$$

We assume that all of the ^{40}Ar found in the rock came from decay of ^{40}K that had also been in the rock. This seems unlikely, as this rock appears to be quite a bit older than our solar system.

22. ^{36}Cl decays by β^- emission to ^{36}Ar , and by β^+ (or EC) to ^{36}S with branch ratios of 98.10% and 1.90% respectively. The two partial half-lives are therefore:

$$\frac{3.01 \times 10^5 \text{ a}}{0.9810} = 3.07 \times 10^5 \text{ a for } \beta^- \quad \frac{3.01 \times 10^5 \text{ a}}{0.0190} = 1.58 \times 10^7 \text{ a for } \beta^+ \text{ and EC}$$

23. This is very similar to the example in Section 2.5. The atomic mass of ^{64}Cu is 63.93 g/mole, therefore N is:

$$N = (9.76 \times 10^{-15} \text{ g}) \times \frac{\text{mol}}{63.93 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} = 9.19 \times 10^7 \text{ atoms}$$

Now, activity – remember to adjust for efficiency and branch ratio. Only 39.00% of all ^{64}Cu decays emit a beta particle (table 2.1).

$$A = 2345 \text{ cpm beta} \times \frac{\text{dpm beta}}{0.0720 \text{ cpm beta}} \times \frac{\text{dpm all}}{0.3900 \text{ dpm beta}} = 8.35 \times 10^4 \text{ dpm}$$

Finally, we can determine half-life:

$$8.35 \times 10^4 \text{ dpm} = \frac{\ln 2}{t_{1/2}} \times (9.19 \times 10^7 \text{ atoms})$$

$$t_{1/2} = 763 \text{ min} = 12.7 \text{ h}$$

24. a. ^{231}Pa is part of the ^{235}U decay series, i.e., it is a radioactive daughter. The half-lives are $7.04 \times 10^8 \text{ a}$ (^{235}U) and $3.28 \times 10^4 \text{ a}$ (^{231}Pa). Since it is a rock, it has likely been sitting around for a few billion years ($> 7 \times 3.28 \times 10^4 \text{ a}$), and is therefore at equilibrium. The equilibrium is secular because we will only be observing it for a short time compared to the half-life of the parent ($7.04 \times 10^8 \text{ a}$).
- b. ^{47}Ca decays to ^{47}Sc , which is also radioactive. Equilibrium is attained because 35 days is longer than seven times the half-life of ^{47}Sc (3.349 d). It is at transient equilibrium because the parent (^{47}Ca) has decayed significantly during this time period ($t_{1/2} = 4.536 \text{ d}$).
- c. Neither, the daughter (^{72}Ga) has a half-life of 14.10 h and the time of observation is less than 3.5 daughter half-lives.
- d. Neither, ^{19}F is stable.
25. Since the uranium was separated from its daughters and the activity ratio is not one, this is not at equilibrium. We'll have to use equation 2.8 to solve this one.

$$\frac{A_{\text{Th-230}}}{A_{\text{U-238}}} = 0.290 = 1 - e^{-\frac{\ln 2}{7.56 \times 10^4 \text{ a}} \times t}$$

$$t = 3.73 \times 10^4 \text{ a}$$

The correction is important because ^{234}U would've also been deposited in the calcite along with the ^{238}U . ^{234}U will also produce ^{230}Th . The correction is done by measuring the $^{234}\text{U}/^{238}\text{U}$ activity ratio, which is greater than 1!

26. Since this is a rock, we can safely assume it is at secular equilibrium, therefore the activities of all radioactive nuclides in the series will be equal. Unfortunately there is a branch in this decay series at ^{212}Bi —it only decays to ^{212}Po 64.06% of the time.

$$A_B = B.R. \times \frac{N_A \times \ln 2}{t_{1/2(A)}} = 0.6406 \times \frac{\left(10.1 \text{ g Th} \times \frac{\text{mol}}{232.0 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}}\right) \times \ln 2}{1.40 \times 10^{10} \text{ a} \times \frac{5.259 \times 10^5 \text{ min}}{\text{a}}}$$

$$= 1.58 \times 10^6 \text{ dpm}$$

27. Since this is a rock, we can safely assume it is at secular equilibrium, therefore the activities of all radioactive nuclides in the series will be equal. Ignoring the branch at ^{212}Bi , ten radioactive nuclides are present in this decay series. We can ignore the branch because the sum of the activities of the two nuclides formed by ^{212}Bi should equal the activity of each of the other nuclides. All we have to do now is calculate the activity of 1.77 g of ^{232}Th and multiply by ten.

$$A_{\text{Th-232}} = \frac{\ln 2}{1.40 \times 10^{10} \text{ a} \times \frac{5.259 \times 10^5 \text{ min}}{\text{a}}} \times 1.77 \text{ g} \times \frac{\text{mol}}{232 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}}$$

$$= 4.33 \times 10^5 \text{ dpm}$$

$$A_{\text{total}} = 4.33 \times 10^5 \text{ dpm} \times 10$$

$$= 4.33 \times 10^6 \text{ dpm}$$

28. These two nuclides are formed in the ^{238}U decay series, and this sample is at secular equilibrium.

$$\frac{N_{\text{Th-230}}}{N_{\text{Th-234}}} = \frac{t_{1/2(\text{Th-230})}}{t_{1/2(\text{Th-234})}} = \frac{7.56 \times 10^4 \text{ a} \times \frac{365.24 \text{ d}}{\text{a}}}{24.10 \text{ d}} = 1.15 \times 10^6$$

Unfortunately, N_B/N_A is a ratio of the numbers of nuclides, and this problem asks for a mass ratio.

$$\frac{1.14 \times 10^6 \text{ mol } ^{230}\text{Th}}{\text{mol } ^{234}\text{Th}} \times \frac{\frac{230 \text{ g } ^{230}\text{Th}}{\text{mol } ^{230}\text{Th}}}{\frac{234 \text{ g } ^{234}\text{Th}}{\text{mol } ^{234}\text{Th}}} = 1.13 \times 10^6$$

29. a and c are true.

b is false because $k = \ln 2 / t_{1/2}$ therefore $k_{(A)} < k_{(B)}$.

d is false because it takes approximately 7 daughter half-lives after isolation of the parent for secular equilibrium to be reached.

30. Given the time of observation (56 h) and the two half-lives involved, this is an example of transient equilibrium (common for radionuclide generators). The only formulas we know for transient equilibrium apply at equilibrium. Fortunately 56 h is greater than $7 \times t_{1/2(\text{Tc-99m})}$, so we can assume we are at equilibrium. To use the equation, we'll first need to find the parent activity (^{99}Mo) after 56 h.

$$A_2 = A_1 e^{-\frac{\ln 2}{t_{1/2}} t} = (7.67 \times 10^{13} \text{ dpm}) \times e^{-\frac{\ln 2}{2.7476 \text{ d}} \times 2.33 \text{ d}} = 4.26 \times 10^{13} \text{ dpm}$$

If only 86% of the decays follow this path, then only 3.66×10^{13} dpm form $^{99\text{m}}\text{Tc}$.

$$\frac{A_{\text{Mo-99}}}{A_{\text{Tc-99m}}} = 1 - \frac{t_{1/2(\text{Tc-99m})}}{t_{1/2(\text{Mo-99})}} \quad A_{\text{Tc-99m}} = \frac{A_{\text{Mo-99}}}{1 - \frac{t_{1/2(\text{Tc-99m})}}{t_{1/2(\text{Mo-99})}}} = \frac{3.66 \times 10^{13} \text{ dpm}}{1 - \frac{6.008 \text{ h}}{65.9 \text{ h}}} = 4.0 \times 10^{13} \text{ dpm}$$

31. The half-life for ^{32}Si (parent) is 160 a, while ^{32}P (daughter) is 14.28 d. Since the time of observation is short compared of the parental half-life, and the parent's half-life is enormously long compared to the daughter, this is clearly secular equilibrium, although the sample will not have reached equilibrium after only four weeks since this is only equivalent to (roughly) 2 daughter half-lives. We'll have to use the equation below.

$$\begin{aligned} A_p &= A_{\text{Si}} (1 - e^{-k_p \cdot t}) = \frac{\ln 2}{t_{1/2(\text{Si})}} \times N_{\text{Si}} \times \left(1 - e^{-\frac{\ln 2}{t_{1/2(\text{P-32})}} \times t} \right) \\ &= \frac{\ln 2}{160 \text{ a} \times \frac{5.259 \times 10^5 \text{ min}}{\text{a}}} \times (7.0 \times 10^{-6} \text{ g}) \times \frac{\text{mol}}{32.0 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} \times \left(1 - e^{-\frac{\ln 2}{14.28 \text{ d}} \times 28 \text{ d}} \right) \\ &= 8.1 \times 10^8 \text{ dpm} \end{aligned}$$

32. Two isotopes of Ra are formed in this decay series, ^{228}Ra and ^{224}Ra . To find the total amount of radium present, we'll have to calculate both and add them together.

$$\begin{aligned} N_B &= \frac{N_A t_{1/2(\text{B})}}{t_{1/2(\text{A})}} = \frac{\left(1.00 \text{ g Th} \times \frac{\text{mol}}{232.0 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} \right) \times 5.76 \text{ a}}{1.40 \times 10^{10} \text{ a}} \\ &= 1.07 \times 10^{12} \text{ atoms of } ^{228}\text{Ra} \end{aligned}$$

$$\begin{aligned} N_B &= \frac{\left(1.00 \text{ g Th} \times \frac{\text{mol}}{232.0 \text{ g}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} \right) \times 3.63 \text{ d}}{\left(1.40 \times 10^{10} \text{ a} \right) \times \frac{365.24 \text{ d}}{\text{a}}} \\ &= 1.84 \times 10^9 \text{ atoms of } ^{224}\text{Ra} \end{aligned}$$

$$1.07 \times 10^{12} \text{ atoms} \times \frac{\text{mol}}{6.022 \times 10^{23} \text{ atoms}} \times \frac{228 \text{ g}}{\text{mol}} = 4.04 \times 10^{-10} \text{ g } ^{228}\text{Ra}$$

$$1.84 \times 10^9 \text{ atoms} \times \frac{\text{mol}}{6.022 \times 10^{23} \text{ atoms}} \times \frac{224 \text{ g}}{\text{mol}} = 6.85 \times 10^{-13} \text{ g } ^{224}\text{Ra}$$

Total radium = $4.04 \times 10^{-10} \text{ g} + 6.85 \times 10^{-13} = 4.05 \times 10^{-10} \text{ grams of radium}$

There is no ^{219}Rn formed in this decay series, therefore no activity will be observed from it.

33. The half-life for ^{48}Cr is 21.6 h, while the half-life for ^{48}V is 15.98 d. Since the daughter half-life is longer than the parent, they will not reach equilibrium. Even so, maximum daughter activity can still be calculated. First we need to get the two half-lives into the same units.

$$t_{1/2(\text{Cr})} = 21.6 \text{ h} \times \frac{\text{d}}{24 \text{ h}} = 0.900 \text{ d}$$

$$t_{\text{max}} = \left[\frac{1.44 t_{1/2(\text{Cr})} t_{1/2(\text{V})}}{(t_{1/2(\text{Cr})} - t_{1/2(\text{V})})} \right] \times \ln \frac{t_{1/2(\text{Cr})}}{t_{1/2(\text{V})}} = \left[\frac{1.44 \times 0.900 \text{ d} \times 15.98 \text{ d}}{(0.900 \text{ d} - 15.98 \text{ d})} \right] \times \ln \frac{0.900 \text{ d}}{15.98 \text{ d}} = 3.95 \text{ d}$$

34. The mean (\bar{x}) is 2115 cpm and is simply the average of the sample data.

The standard deviation is:

$$\sigma = \sqrt{\bar{x}} = \sqrt{2115} = 46 \text{ cpm}$$

For a Gaussian distribution, the standard deviation defines a range over which we might expect another measurement to fall. If our data fit a Gaussian distribution, then ~68% of the data should lie within 1σ of the mean value ($2115 \pm 46 \text{ cpm}$) and ~4% should lie outside of 2σ of the mean ($2115 \pm 92 \text{ cpm}$). Only 50% of the data given in this problem lie within 1σ , and 20% lie outside of 2σ . These data do not appear to fit a Gaussian distribution.

The standard deviation of the mean is:

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}} = \frac{46}{\sqrt{20}} = 10 \text{ cpm}$$

The RSD is:

$$\text{RSD} = \frac{\sigma}{\bar{x}} = \frac{46 \text{ cpm}}{2115 \text{ cpm}} = 0.022$$

Chi-squared must be calculated to determine if the data fit a Poisson distribution.

$$\chi^2 = \frac{1}{\bar{n}} \sum (\bar{n} - n)^2 = 37.3$$

Chi-squared is a measure of how well a set of data fit a Poisson distribution. For 20 data, there is a 1% chance that this value will exceed 36.1 and the data still fit a Poisson distribution. Therefore, it is quite likely that the data presented here do not fit a Poisson distribution and there is a serious problem with the instrument that determined them.

35. Using Excel to do the math, the following can be obtained.

	Run 1 (cpm)	Run 2 (cpm)
\bar{x}	89	1904
2σ	19	87
% error	21%	4.6%
RSD	0.11	0.023

Both the percent error and the RSD are lower for the higher count rates. This is consistent with the general statement that with any measurement, the larger the value measured, the lower the relative error. Higher count rates have lower relative errors.