2 Truss Analysis: Force Method, Part I

1.1 Introduction

The formulation of the governing equations with the **forces** as **unknown variables.**

Provide insight on how the **externally applied loads** are **transmitted and taken up by the members of the truss.**

Statically determinate $\longrightarrow$ solved by the equilibrium equations alone

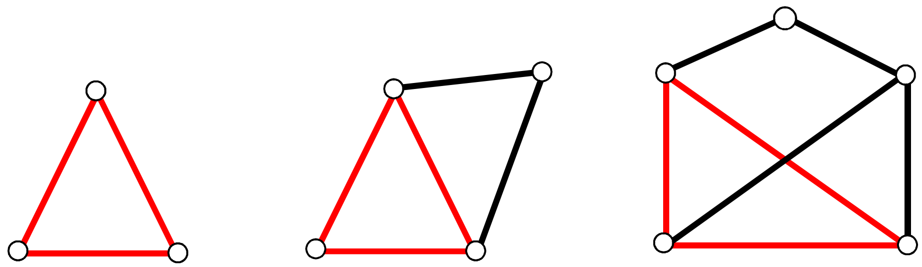
Statically indeterminate $\longrightarrow$ additional equations based on the geometric compatibility or consistent deformations

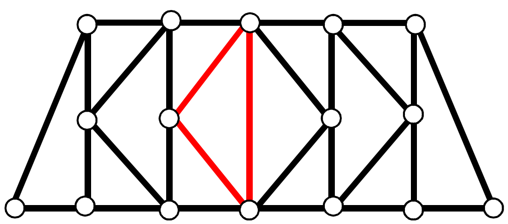
1.2 Statically Determinate Plane Truss Types

Simple Truss

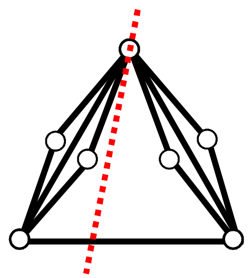
basic triangle of three bars and three nodes

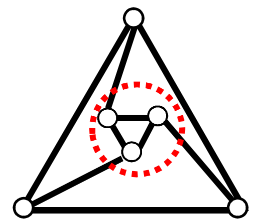
adding two-bar-and-a-node one at a time

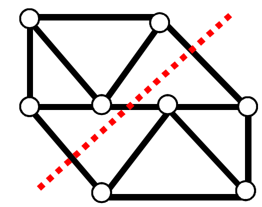




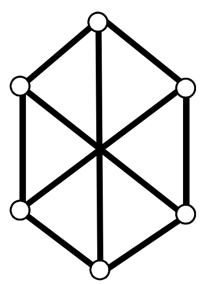
Compound Truss two or more simple trusses linked together

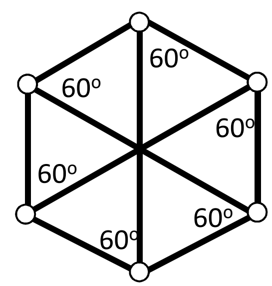






complex truss neither simple truss nor compound truss





Stable and unstable complex trusses.

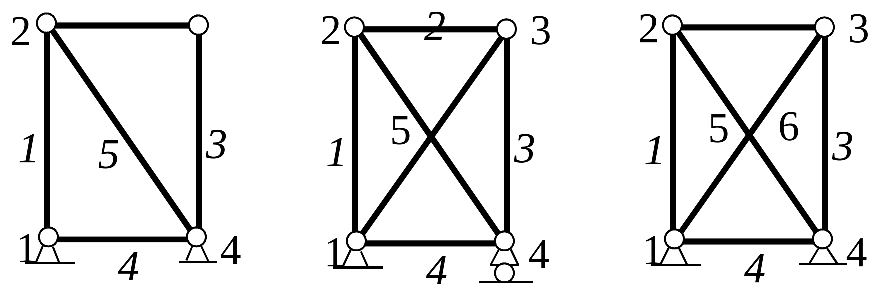
Conclusions:

(1) Stability depends on the adequacy of external supports and internal member connections. If ***M+R<2N, it is always unstable****, a truss turning into a* ***mechanism*** *under certain loads.*

(2)For a stable plane truss, if $M+R=2N$, *then it is statically determinate.*

(3)A simple truss is stable and determinate.

(4) For a stable plane truss, if $M+R>2N$, *then it is* ***statically indeterminate****. The discrepancy between the two numbers,* $M+R-2N$, *is* ***called the degrees of indeterminacy****, or* ***the number of redundant forces.***



Statically indeterminate trusses.

The truss at the **left** is **statically indeterminate to the first degree** because there is one redundant reaction force: M=5, R=4, and M+R-2N=1.

The truss in the **middle** is also **statically indeterminate to the first degree** because of one redundant member: M=6, R=3, *and* M+R-2N=1.

The truss at the right is **statically indeterminate to the second degree** because M=6, R=4 and M+R-2N=2.

1.3 Method of Joints and Method of Sections

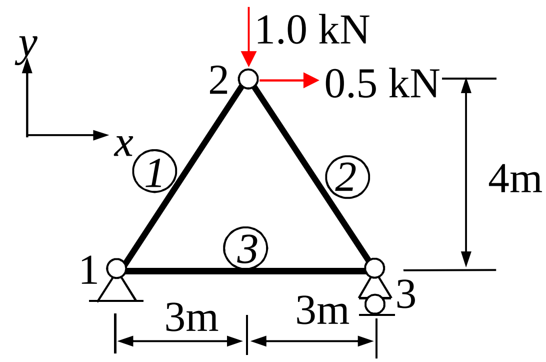
The method of joints

- ❖a FBD is selected: at the joints of a truss one at a time.
- ❖From each FBD, two equilibrium equations are derived.
- ❖provides insight on how the external forces are balanced by the member forces at each joint.

The method of sections

- ❖FBD is a portion of the structure created by cutting through one or more sections.
- ❖From each FBD, three equilibrium equations are derived.
- ❖provides insight on how the member forces resist external forces at each "section"

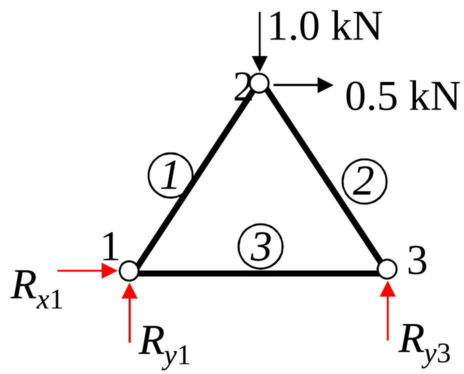
Example 2.1 Find all support reactions and member forces of the loaded truss shown.



Solution

(1) Identify all force unknowns.

The member forces are F_1, F_2, *and* F_3.

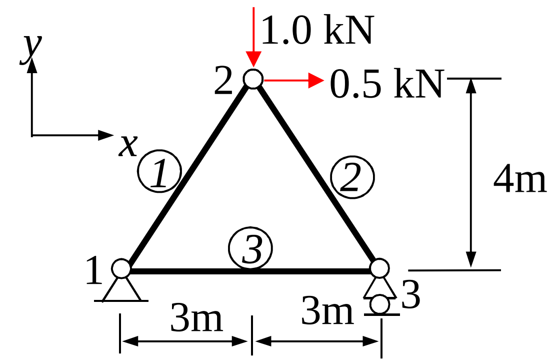


The reaction forces are R_{x1}, R_{y1} *and* R_{y3}.

Free-Body diagram of the three-bar-truss to expose the reaction forces.

(2) Examine the static determinacy of the structure.

number of all member force unknowns as *M*



number of reaction forces as *R*

total number of force unknowns is *M+R*

M=3, R=3 and M+R =6.

number of equilibrium equations available is 2*N*

N is the number of nodes in a truss.

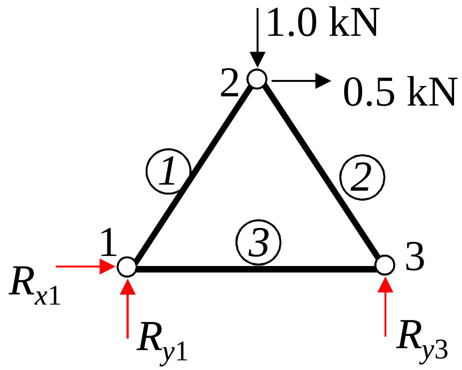
N=3 and 2N=6

M+R=2N **statically determinate**

Reach the same conclusion if we note that the truss is a simple truss.

(3) Solve for force unknowns.

(a) Find all reactions.

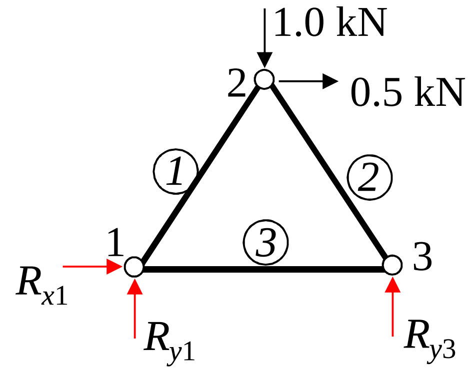


$\Sigma F_x = 0 \rightarrow R_{x1} + 0.5 = 0 \rightarrow \boxed{R_{x1} = -0.5 \text{ kN}}$

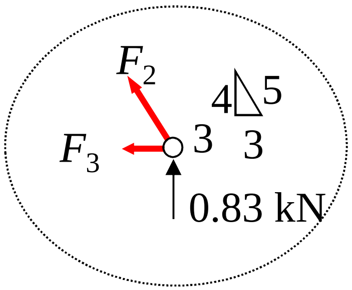
$\Sigma M_1 = 0 \rightarrow R_{y3}(6) - (1.0)(3) - (0.5)(4) = 0 \rightarrow \boxed{R_{y3} = 0.83 \text{ kN}}$

$\Sigma F_y = 0 \rightarrow R_{y1} + 0.83 - 1.0 = 0 \rightarrow \boxed{R_{y1} = 0.17 \text{ kN}}$

(b) Find member forces.



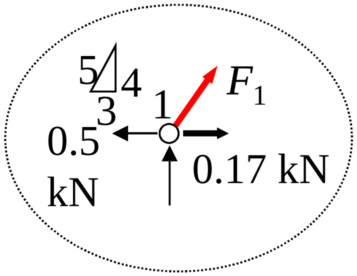
Joint 3.



$\Sigma F_y = 0,$ $\quad F_2(4/5)+0.83=0,$ $\boxed{F_2 = -1.04 \text{ kN}}$

$\Sigma F_x = 0,$ $\quad -F_2(3/5) - F_3=0,$ $\boxed{F_3 = 0.62 \text{ kN}}$

Joint 1.

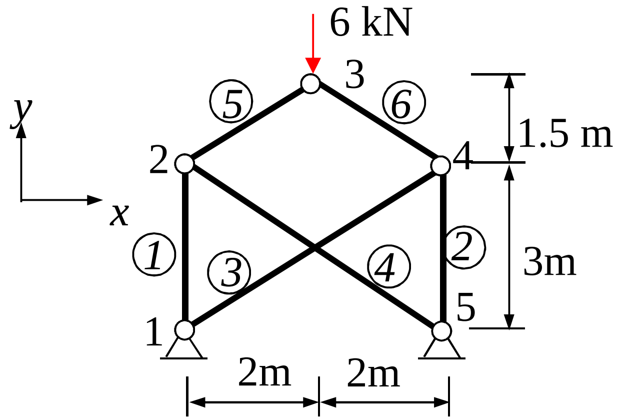


$\Sigma F_y = 0,$ $\quad F_1(4/5)+0.17=0,$ $\boxed{F_1 = -0.21 \text{ kN}}$

There are no more than six independent equilibrium equations.

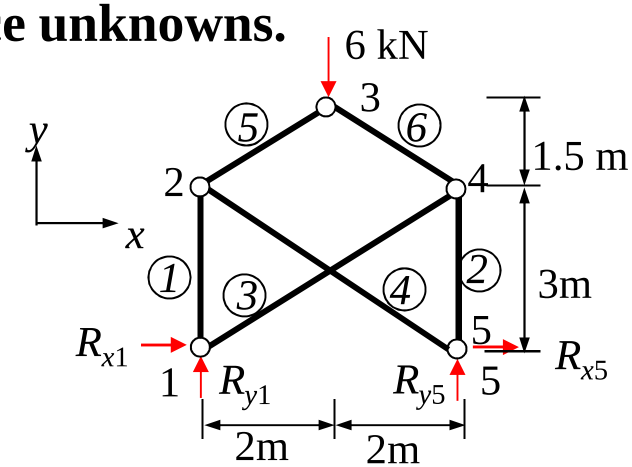
Select those equations that give us the easiest way of getting the answer to the unknown forces

Example 2.2 Find all reaction and member forces for the loaded truss shown.



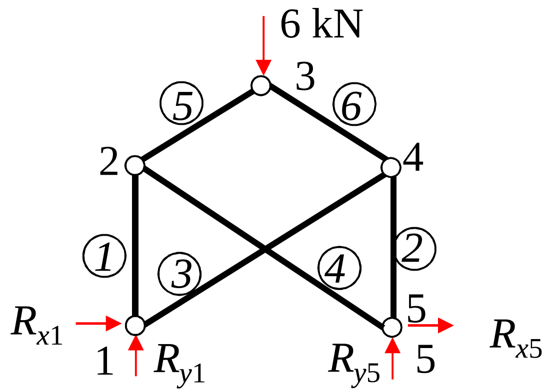
Solution A slightly different solution strategy is followed in this example.

(1) Identify all force unknowns.



M*=6, *R*=4** *and M*+*R*=10, a total of ***ten force unknowns.

(2) Examine the static determinacy of the structure.

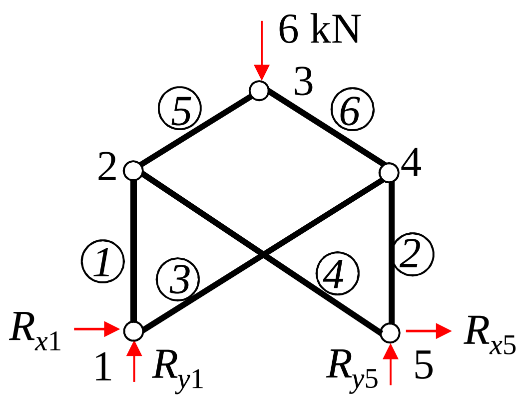


There are five nodes, N=5. *Thus*
M+R=2N=10. *This is a* ***statically determinate problem***.

(3) Solve for force unknowns.

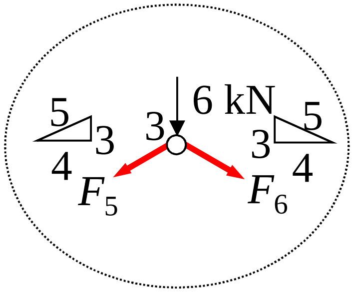
There is no advantage in solving for the reactions first, cannot solve from the FBD of the whole truss anyway.

Go from joint to joint in the following order, 3, 2, 4, 1, and 5, we will be able to solve for member forces one node at a time and eventually getting to the reactions.



This way each node has no more then 2 unknowns.

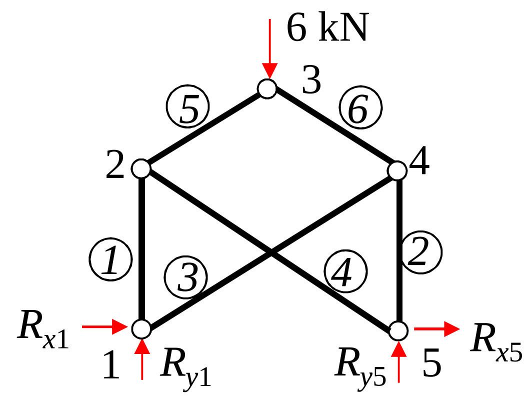
Joint 3.



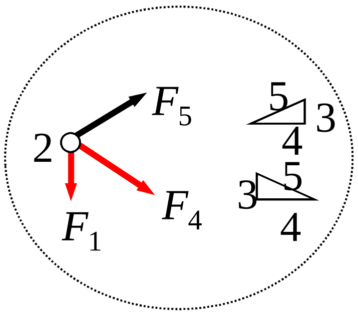
$$\Sigma F_y = 0,\ F_5(3/5) + F_6(3/5) = -6,$$

$$\Sigma F_x = 0, -F_5(4/5) + F_6(4/5) = 0$$

$$\Rightarrow \boxed{F_5 = -5\text{ kN},\quad F_6 = -5\text{ kN}}$$



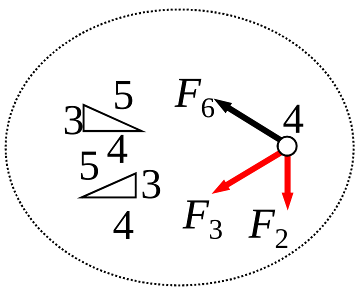
Joint 2.

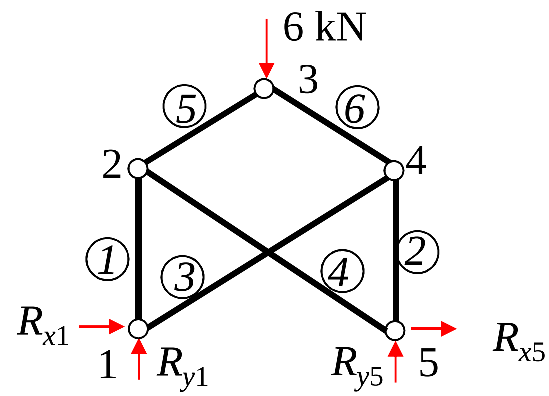


$$\Sigma F_x = 0, F_5(4/5) + F_4(4/5) = 0, \Rightarrow \boxed{F_4 = 5\text{ kN.}}$$

$$\Sigma F_y = 0, F_5(3/5) - F_4(3/5) - F_1 = 0, \Rightarrow \boxed{F_1 = -6\text{ kN}}$$

Joint 4.

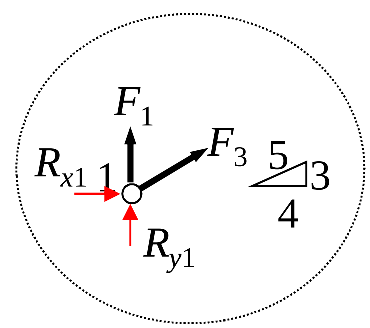




$$\Sigma F_x = 0,\ F_6(4/5) + F_3(4/5) = 0, \quad \Rightarrow \quad \boxed{F_3 = 5 \text{ kN.}}$$

$$\Sigma F_y = 0,\ F_6(3/5) - F_3(3/5) - F_2 = 0, \quad \Rightarrow \quad \boxed{F_2 = -6 \text{ kN.}}$$

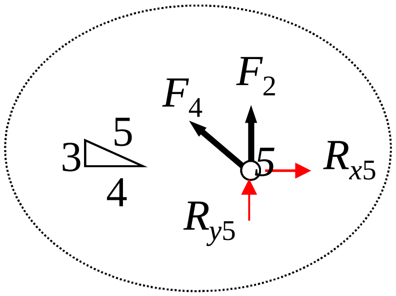
Joint 1.

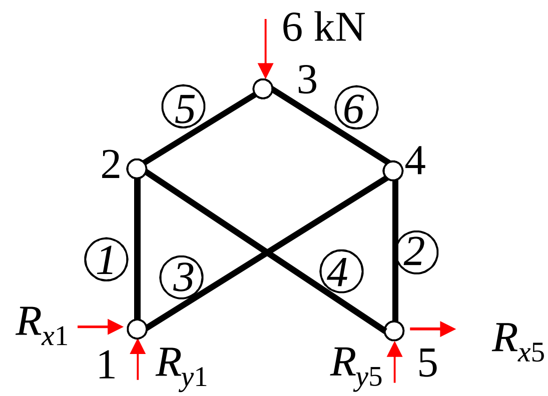


$$\Sigma F_x = 0,\ R_{x1} + F_3(4/5) = 0, \quad \Rightarrow \quad \boxed{R_{x1} = -4 \text{ kN.}}$$

$$\Sigma F_y = 0,\ R_{y1} + F_3(3/5) + F_1 = 0, \quad \Rightarrow \quad \boxed{R_{y1} = 3 \text{ kN.}}$$

Joint 5.



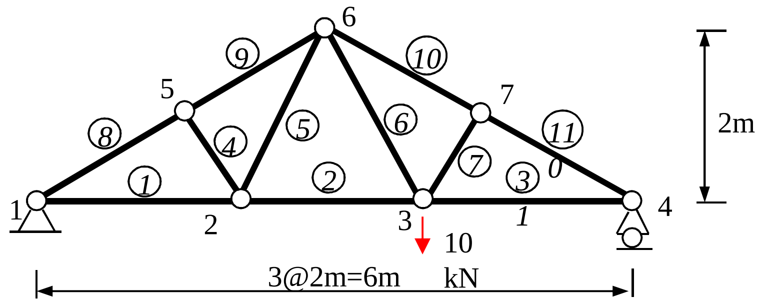


$\Sigma F_x = 0, R_{x5} – F_4(4/5) =0,$ ➡ $\boxed{R_{x5} = 4 \text{ kN.}}$

$\Sigma F_y = 0, R_{y5}+F_4(3/5) +F_2= 0,$ ➡ $\boxed{R_{y5}= 3 \text{ kN.}}$

Note in both example problems, we **always assume the member forces to be in tension.** This results in **FBDs that have member forces pointing away from the joints.** This is simply an **easy way to assign force directions.** It is **highly recommended** because it **avoids unnecessary confusion that often leads to mistakes** .

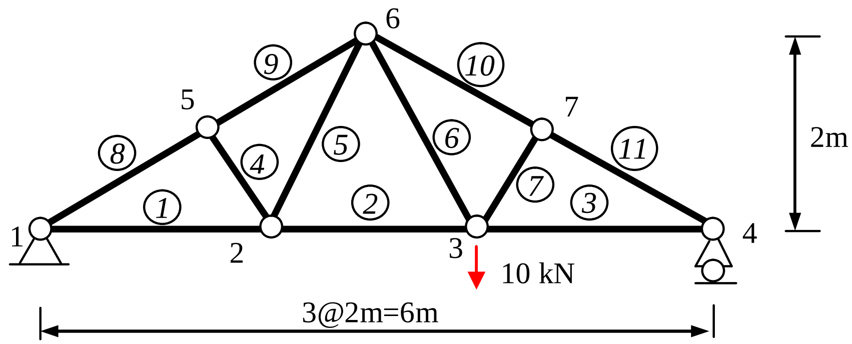
Example 2.3 Find the member forces in bars 4, 5, 6, and 7 of the loaded Fink truss shown.



Solution We shall illustrate a special feature of the method of joints.

(1) Identify all force unknowns. The FBD of the whole structure would have shown that there are **three reactions.** Adding the eleven member forces, we have ***M*=11, *R*=3 *and M*+*R*=14,** a total of **14 *force unknowns.***

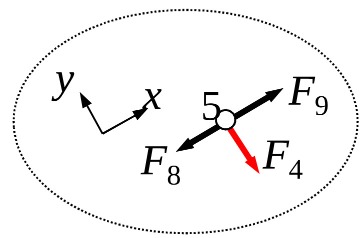
(2) Examine the static determinacy of the structure. There are seven nodes, ***N*=7.** Thus ***M*+*R*=2*N*=14.** This is a ***statically determinate problem.***



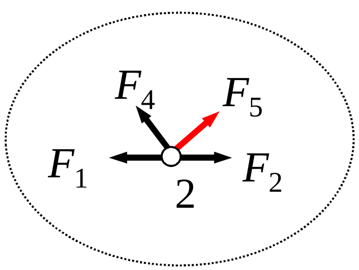
(3) Solve for force unknowns. Normally Fink trusses are used to take roof loading on the upper chord nodes. We deliberately apply a single load at a lower chord node in order to make a point about **a special feature of the method of joints: zero force members.** We start by concentrating on Joint 5.

Joint 5. $\Sigma F_y = 0,$ ➡ $\boxed{F_4 = 0}$ **zero force member**

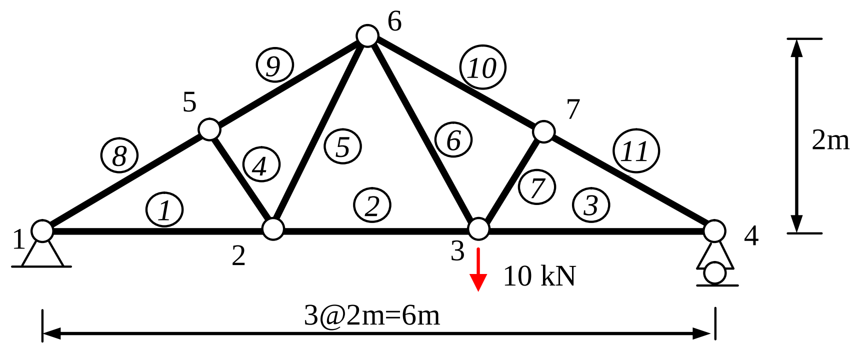
$\Sigma F_x = 0, -F_8 + F_9 = 0$ ➡ $\boxed{F_8 = F_9}$



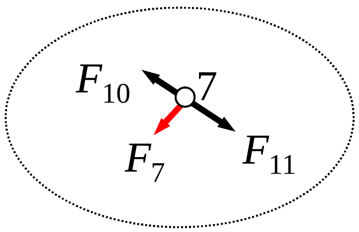
Joint 2. $\Sigma F_y = 0$, and $F_4 = 0$, ➡ $\boxed{F_5 = 0.}$



$\Sigma F_x = 0,$ ➡ $F_1 = F_2$



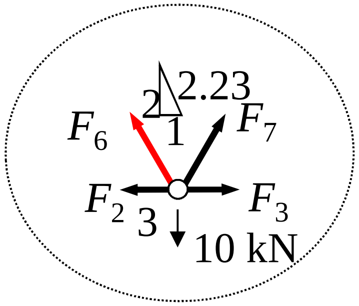
Joint 7.



$\boxed{F_7 = 0,}$ ➡ $F_{10} = F_{11}$

zero force member

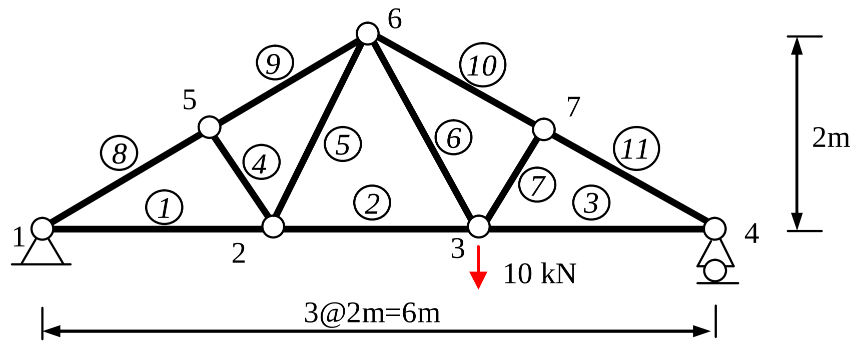
Joint 3.



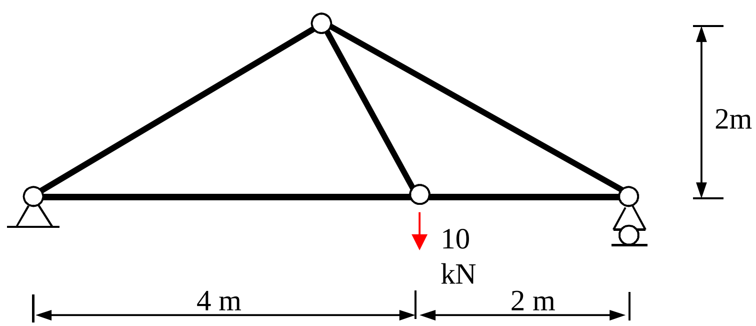
$F_7 = 0$, from equilibrium of Joint 7.

$\Sigma F_y = 0$, $F_6\,(2/2.23) = 10$ ➡ $\boxed{F_6 = 11.15 \text{ kN.}}$

That completes the solution for F_4,F_5,F_6 and F_7.

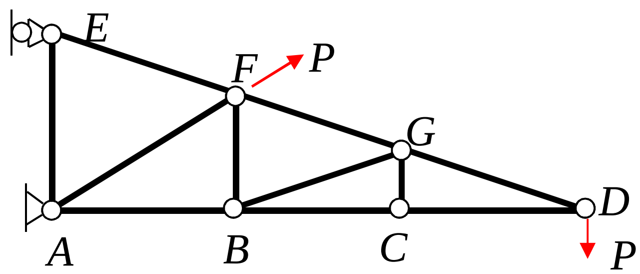


with the exception of member 6, all the web members are **zero-force members** for this particular loading case. For purpose of analysis under the given load the Fink truss is equivalent to the truss shown below.

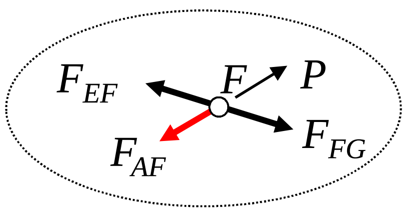


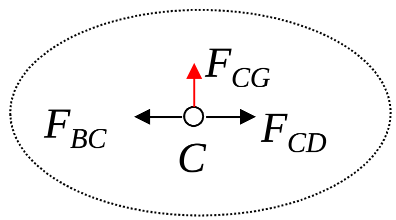
The zero force members are needed for other loading conditions. The **interesting feature** of the method of joints: we can **identify zero-force members easily** if they exist under a given loading condition. This feature is further illustrated in the **next example**.

Example 2.4 Identify zero-force members and equal-force members in the loaded trusses shown.

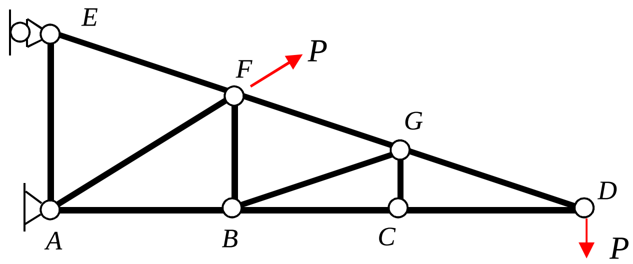


Solution

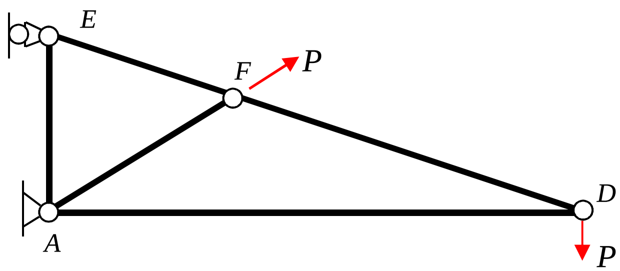




The equilibrium of forces at joint C leads to $F_{CG}=0$ and $F_{BC}=F_{CD}$. Once we know $F_{CG} = 0$, *it follows* $F_{BG} = 0$ *and then* $F_{BF} = 0$, based on the equilibrium of forces at node G and node B, respectively. The equilibrium of forces at joint F leads to $F_{AF}=P$ and $F_{EF}=F_{FG}$.

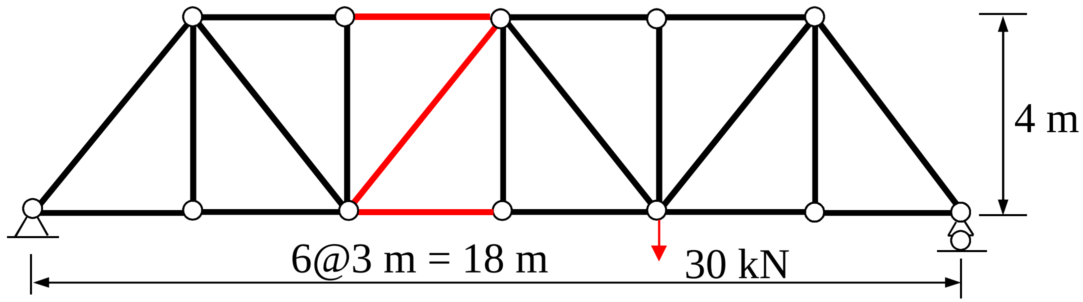


For practical purposes, the original truss problem is equivalent to the truss problem shown below for the given loading case.



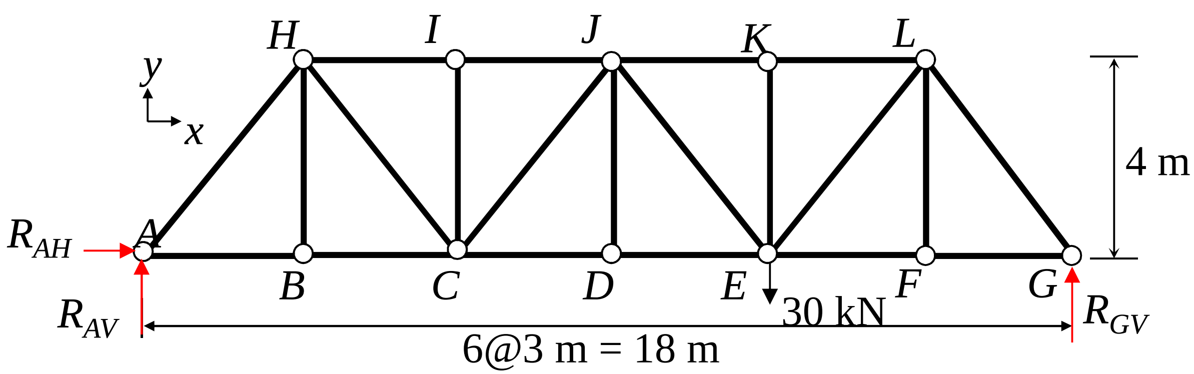
We can identify:
(1) zero force members. At each joint, all the forces are concurrent forces. If **all the forces are co-linear except one** then the lone exception must be zero.
(2) equal force members. If two forces at a joint are **co-linear** and all other forces at the joint are also co-linear in another direction, then the two forces must be equal.

Example 2.5 Find member forces in bars in the 3rd panel from the left of the truss shown.

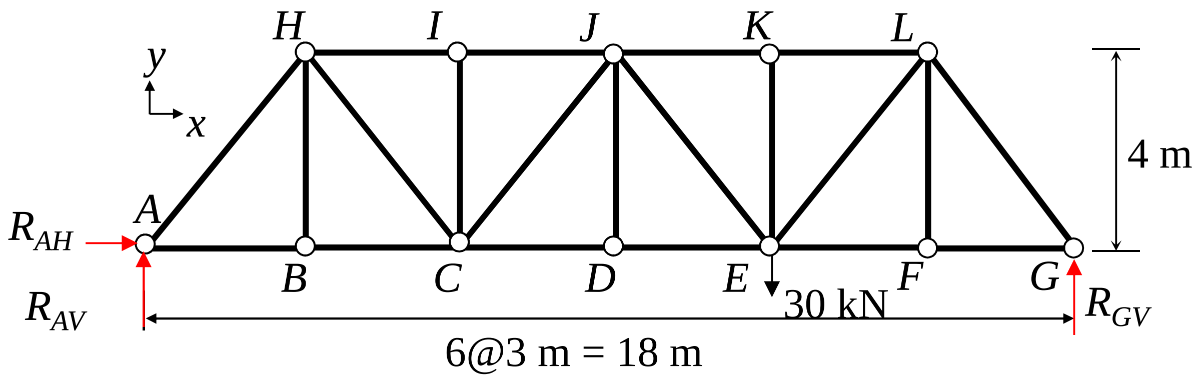


Solution We shall solve this problem by the method of sections with the following procedures.

(1) Name all joints. We can refer to each joint by a symbol and each member by the two end joints as shown in the figure below. We also define an *x*-*y* coordinate system as shown. We need to find F_{IJ}, F_{CJ}, and F_{CD}. The truss is stable and determinate.



(2) Find reactions. We have to look at the FBD of the whole truss.

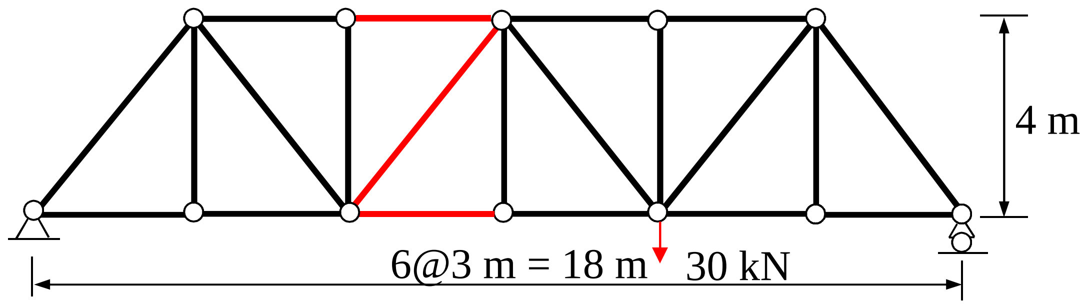


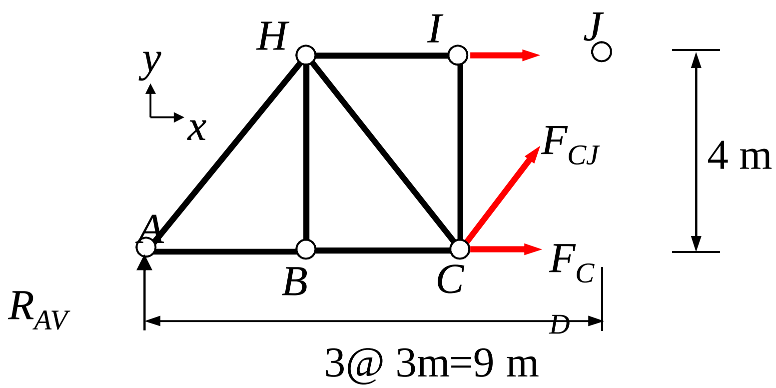
$\Sigma M_A=0$, $(12)(30) - (18)R_{GV} = 0$, $\quad R_{GV} = 20$ kN.

$\Sigma F_x=0$, $\quad R_{AH} = 0$.

$\Sigma M_G=0$, $(18)R_{AV} - (6)(30) = 0$, $\quad R_{AV} = 10$ kN.

(3) Establish FBD. We make a vertical cut through the 3rd panel from the left, thus exposing the member force of members *IJ, CJ and CD.*





R_{AV} = 10 kN.

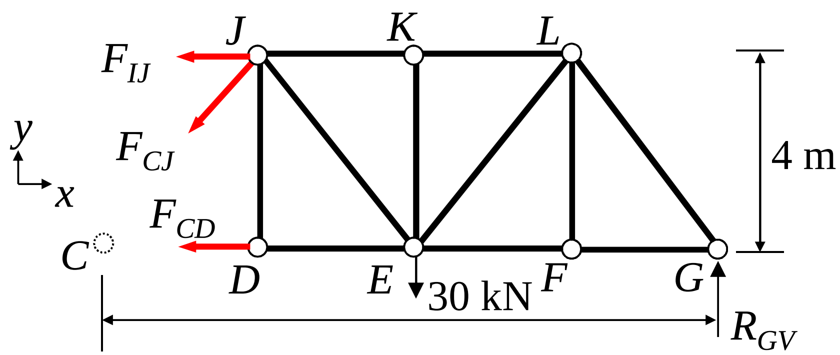
$\Sigma M_C = 0$, $(4)\ F_{IJ} + (6)\ R_{AV} = 0$, $F_{IJ} = -1.5\ R_{AV} = -15$ kN.

$\Sigma M_J = 0$, $-(4)\ F_{CD} + (9)\ R_{AV} = 0$, $F_{CD} = 2.25\ R_{AV} = 22.5$ kN.

$\Sigma F_y = 0$, $(0.8)\ F_{CJ} + R_{AV} = 0$, $F_{CJ} = -1.25\ R_{AV} = -12.5$ kN.

To illustrate the effect of taking a different FBD, let us choose the right part of the cut as the FBD. Note that we already know $R_{GV} = 20$ kN.

$R_{GV} = 20$ kN.

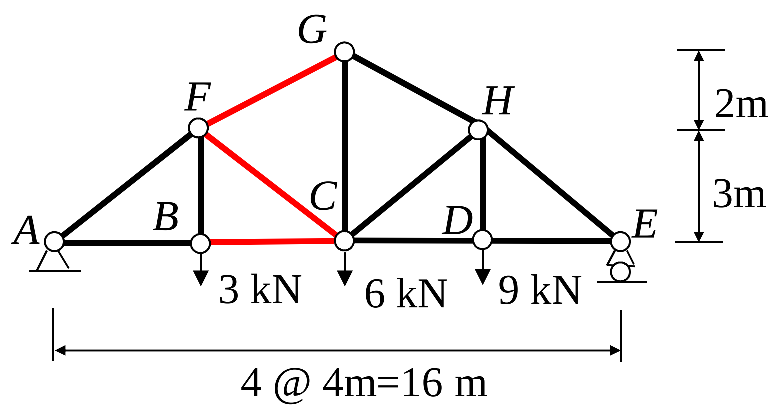


$\Sigma M_C = 0$, $-(4)\,F_{IJ} + (6)\,(30) - (12)\,R_{GV} = 0$, $\quad F_{IJ} = -\,3\,R_{GV} + 45 = -15$ kN.

$\Sigma M_J = 0$, $(4)\,\mathrm{F}_{CD} + (3)\,(30) - (9)\,R_{GV} = 0$, $\quad F_{CD} = 2.25\,R_{GV} - 22.5 = 22.5$ kN.

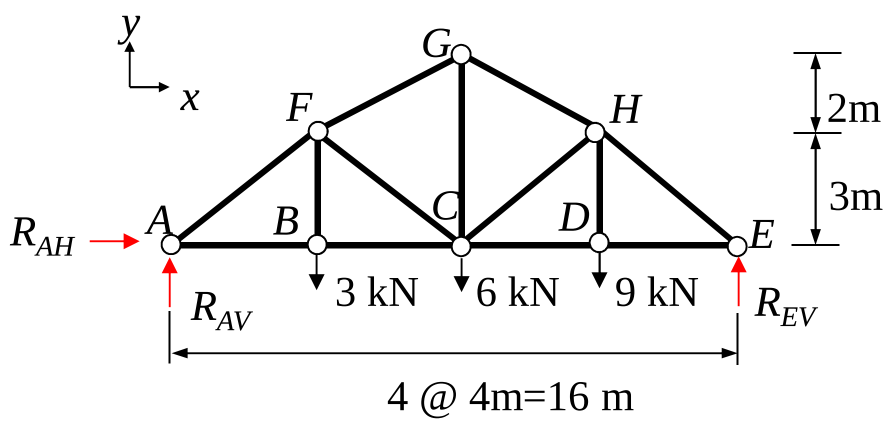
$\Sigma F_y = 0$, $\quad -(0.8)\,F_{CJ} - 30 + R_{GV} = 0$, $\quad F_{CJ} = -37.5 + 1.25\,R_{GV} = -12.5$ kN.

Example 2.6 Find member forces in bars in the 2nd panel from the left of the truss shown.



Solution The inclined chord geometry will cause complications in computation, but the process is the same as that of the last example.

(1) Find reactions. This is a simple truss, stable and determinate.

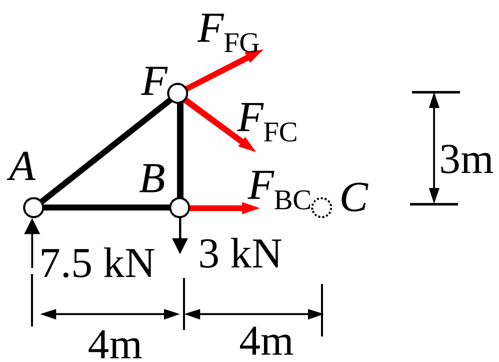


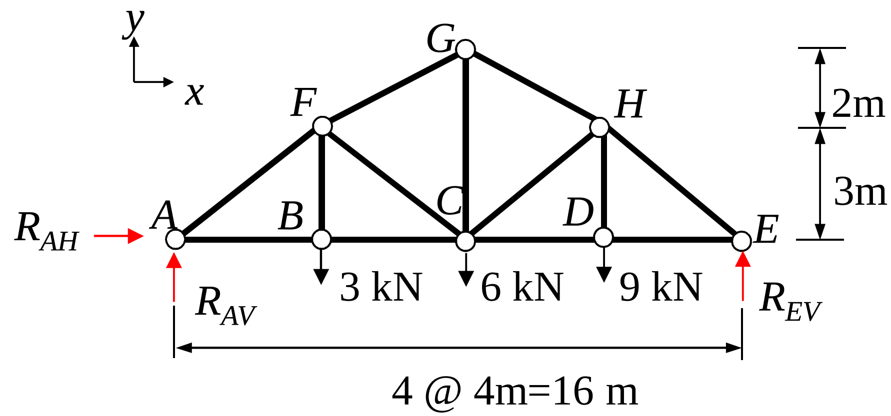
$\Sigma M_A=0 \quad -(16)\,R_{EV}+(4)3+(8)6+(12)9=0,$ $\qquad R_{EV}=10.5$ kN.

$\Sigma M_E=0 \quad (16)\,R_{AV}-(12)3-8(6)-(4)9=0,$ $\qquad R_{AV}=7.5$ kN.

$\Sigma F_x=0$ $\qquad R_{AH}=0$ kN.

(2) **Establish FBD.** We make a cut through the second panel from the left and choose the left portion as the FBD.



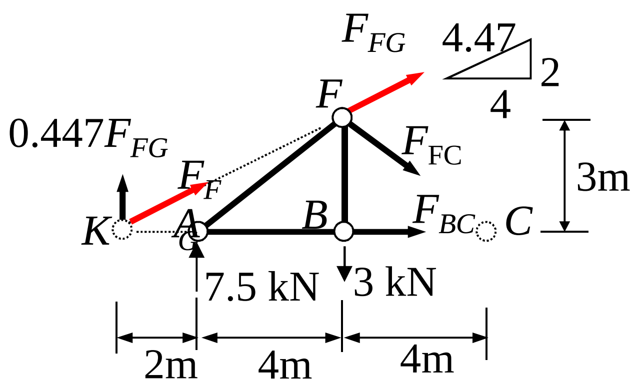


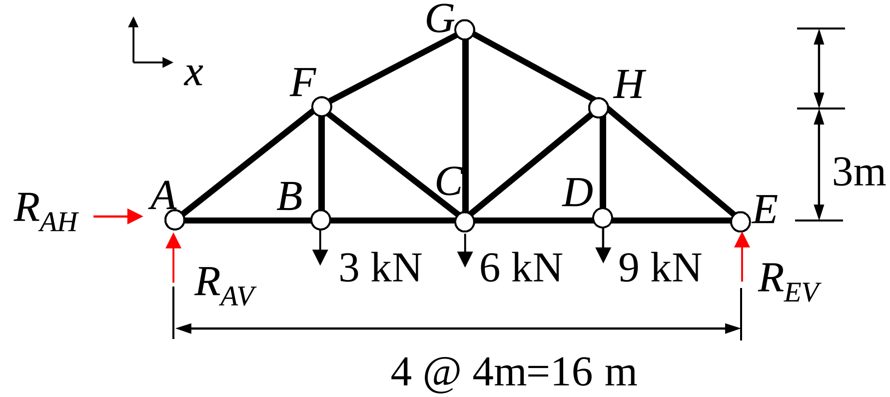
To find $\boldsymbol{F_{BC}}$ we want to find a moment center that is the intersection of the two other unknowns. The intersection point of F_{FG} and F_{FC} is point F.

$\Sigma M_F=0 \quad -(3)\,F_{BC}+(4)7.5=0,$ $\Rightarrow$ $F_{BC}=10.00$ kN.

Similarly, for $\boldsymbol{F_{FG}}$ we take moment about point C so that F_{FG} is the only unknown force in the ensuing equilibrium equation .

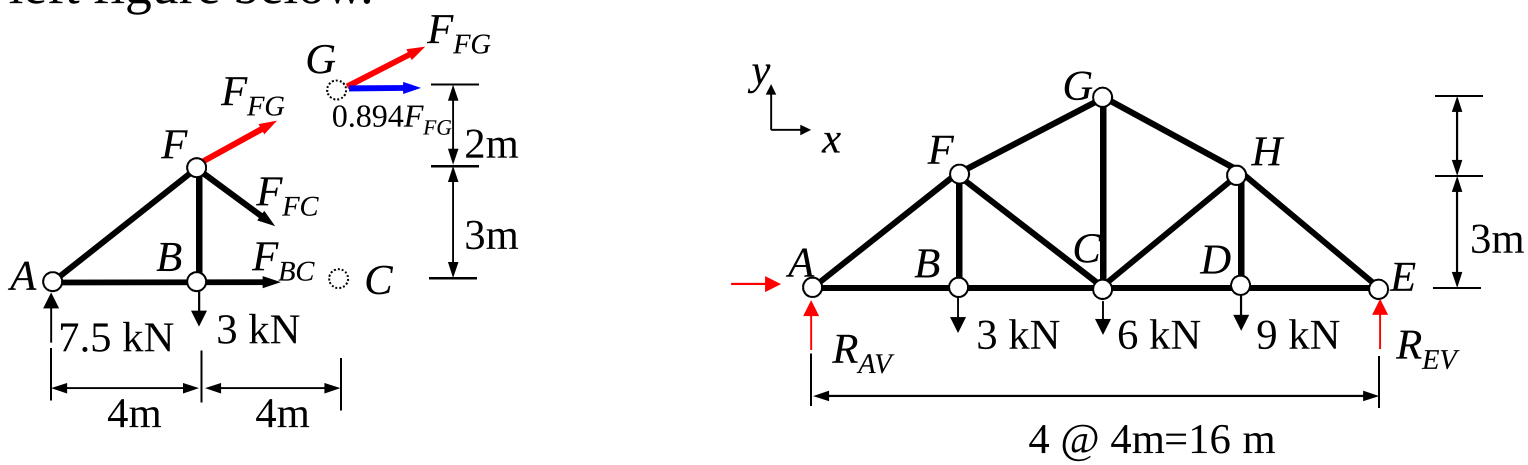
F_{FG} can be transmitted to point K and the horizontal component of F_{FG} at K has no contribution to the equilibrium equation while the vertical component is (2/4.47) F_{FG}=0.447 F_{FG} has, as shown in the left figure below.





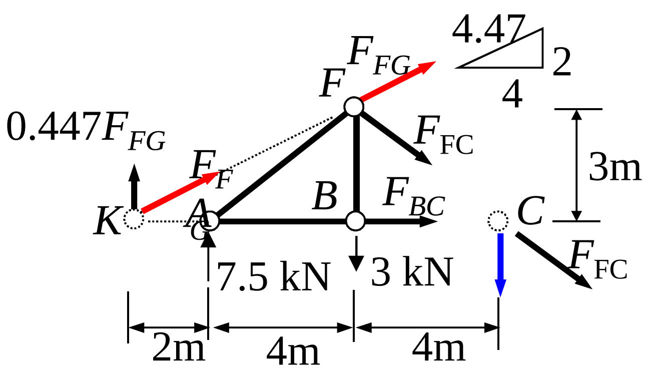
$$\Sigma M_C=0 \quad (10)\, 0.447F_{FG}+(8)7.5-(4)3=0, \quad \Rightarrow \quad F_{FG} = -10.74 \text{ kN}$$

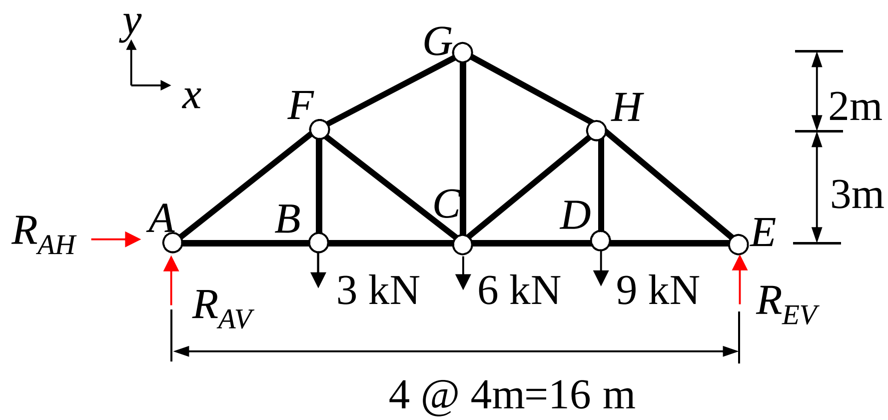
Alternatively we can transmit F_{FG} to point G, and use the horizontal component (4/4.47) F_{FG}=0.894 F_{FG} in the moment equation, as shown in the left figure below.



$$\Sigma M_C=0 \quad (5)\,0.894\mathrm{F}_{\mathrm{FG}} + (8)7.5 - (4)3=0, \quad \Rightarrow \quad F_{FG} = -10.72 \text{ kN.}$$

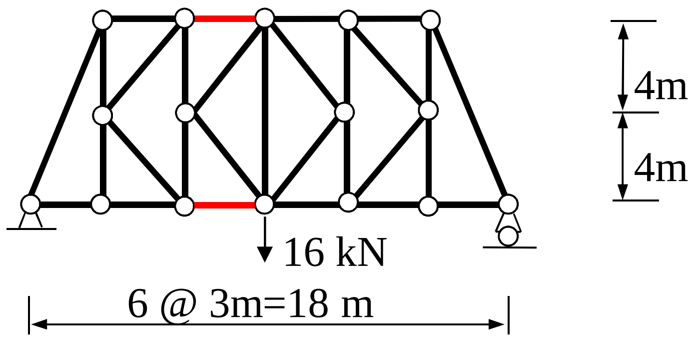
To find $\boldsymbol{F_{FC}}$ *we need to go out of the region of the truss to find the moment center (K) as the shown in the left figure below, and use the vertical component of the transmitted* F_{FC} *at point C.*





$\Sigma M_K=0 \quad (10)\,0.6F_{FC} - (2)7.5+(6)3=0, \quad \Rightarrow \quad F_{FC} = -0.50$ kN.

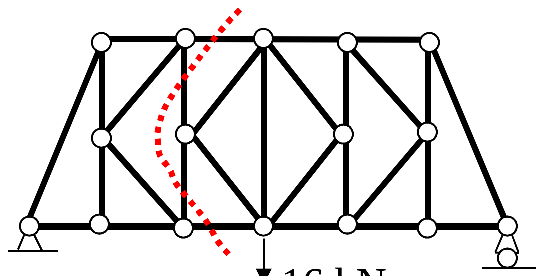
Example 2.7 Find the force in the top and bottom chord members of the third panel from the left of the *K-truss shown.*



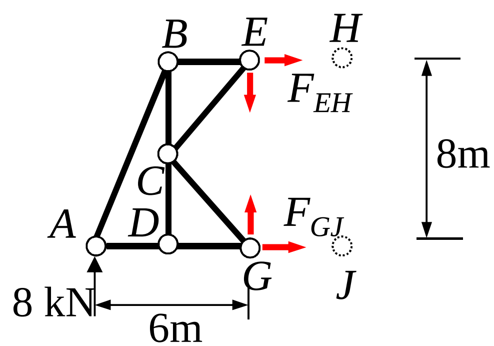
Solution The K-truss is a simple truss that requires a special cut for the solution of top and bottom chord member forces as we shall see shortly. It is stable and determinate.

(1) Find reactions. Since the truss and the loading are **symmetric**, the reactions at both supports are easily found to be 8 kN upward and there is no horizontal reaction at the left support.

(2)**Establish FBD.** The special cut is shown by the dotted line below.



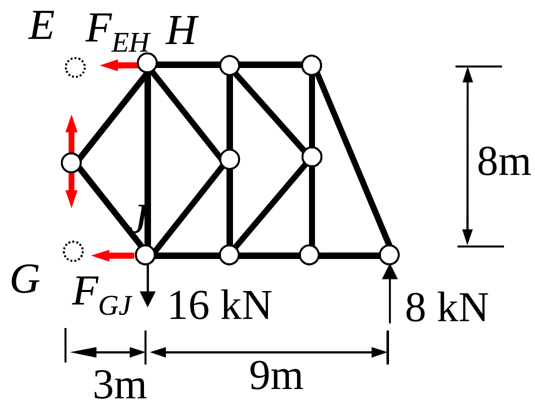
This particular cut separates the truss into two parts. We shall use the left part as the FBD.

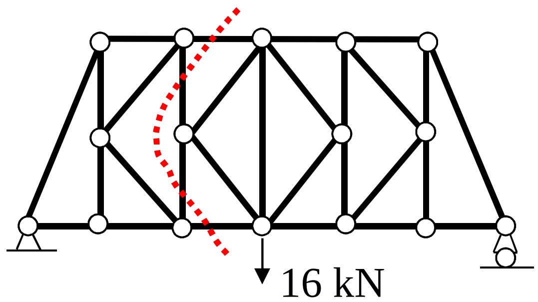


$\Sigma M_E=0$, (6) 8 – (8) F_{GJ} =0 ➡ F_{GJ} = 6 kN.

$\Sigma M_G=0$, (6) 8 + (8) F_{EH} =0 ➡ F_{EH} = –6 kN.

Alternatively, we may choose the right part as the FBD. The same results will follow but the computation is slightly more involved.

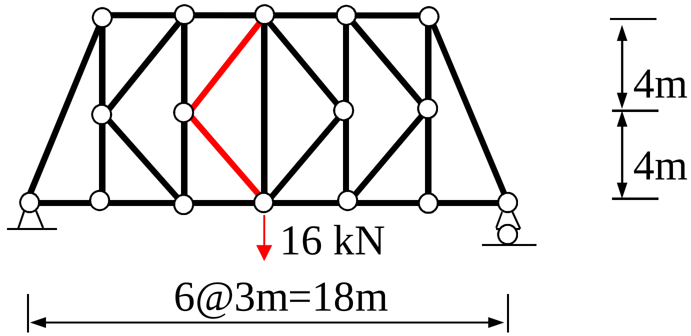




ΣM_E=0, (12) 8 – (3) 16 – (8) F_{GJ} =0 ➡ F_{GJ} = 6 kN.

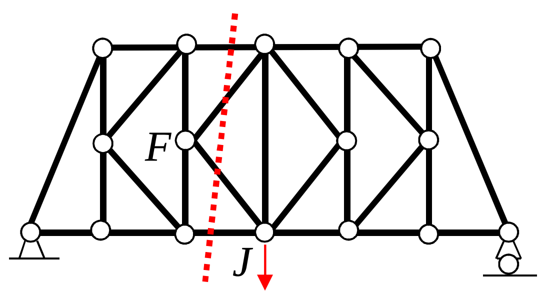
ΣM_G=0, (12) 8 – (3) 16 + (8) F_{EH} =0 ➡ F_{EH} = –6 kN.

Example 2.8 Find the force in the inclined web members of the third panel from the left of the *K-truss shown.*

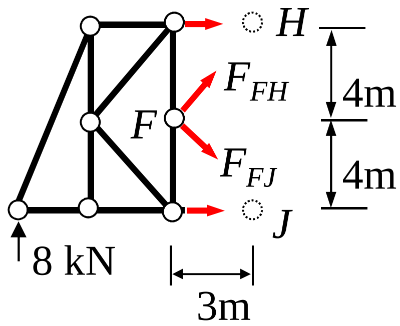


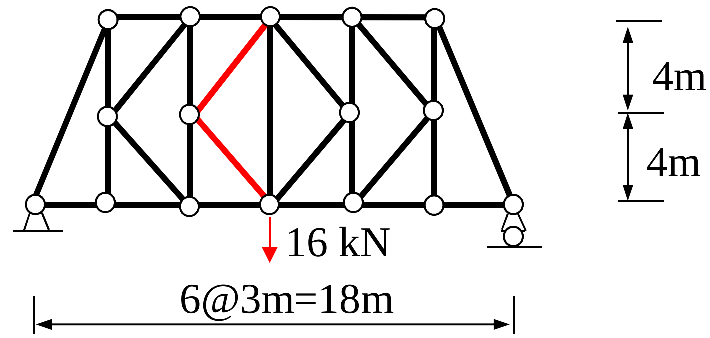
Solution A different cut from that of the last example is needed to expose the web member forces.

(1) Establish FBD. To expose the force in the inclined web members, we may make a cut through the third panel.



This cut exposes four forces, the top and bottom member forces which are known from the last example and the two inclined web member forces, F_{FH} *and* F_{FJ}*, which are unknown.*



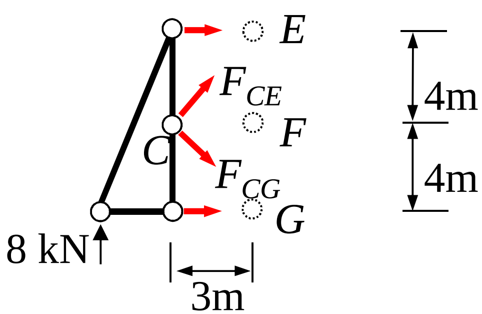


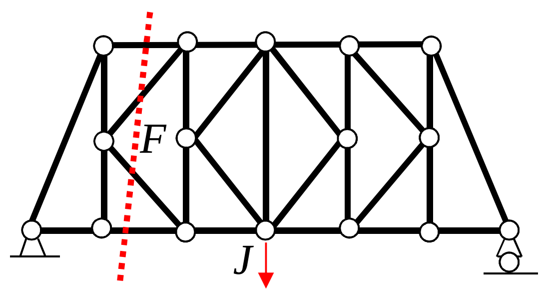
In this case, the application of two force equilibrium equations produces the desired results. In writing the equation for the horizontal forces, we note that the **top and bottom chord member forces cancel each other** and will not appear in the equation. In fact this is a **special feature**, which is useful for the analysis of web member forces.

$$\boxed{\begin{array}{ll} \Sigma F_x = 0 & (0.6)F_{FH} + (0.6)\, F_{FJ} = 0 \\ \Sigma F_y = 0 & (0.8)F_{FH} - (0.8)\, F_{FJ} = 8 \end{array}} \Rightarrow \boxed{\begin{array}{l} F_{FH} = 5 \text{ kN} \\ F_{FJ} = -5 \text{ kN} \end{array}}$$

We observe that not only the top and bottom chord members have the same magnitude forces with opposite signs, the inclined web members are in the same situation. Furthermore, in the present example, the inclined web member forces are the same in the second and third panel, i.e. $F_{CE} = F_{FH} = 5$ kN, $F_{CG} = F_{FJ} = -5$ kN

This is because the FBD for these member forces yields equations identical to those for the third panel.

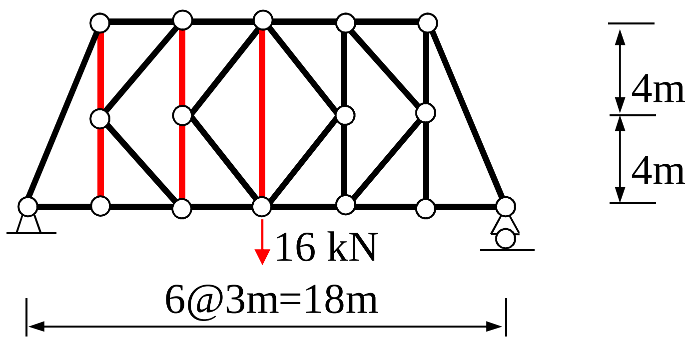




$$\Sigma F_x = 0 \qquad (0.6)F_{CE} + (0.6)F_{CG} = 0$$

$$\Sigma F_y = 0 \qquad (0.8)F_{CE} - (0.8)F_{CG} = 8$$

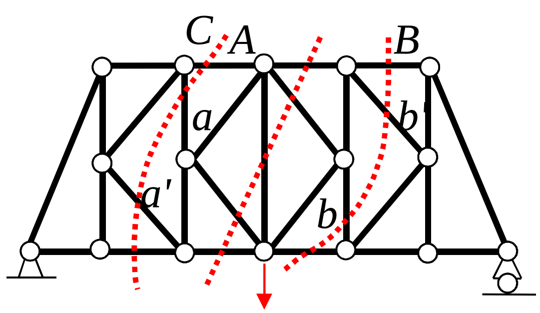
Example 2.9 Discuss methods to find the force in the vertical web members of the *K-truss shown.*

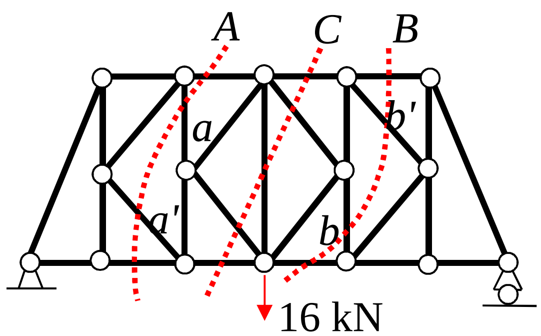


Solution We can use either the method of sections or the method of joints, but the pre-requisite is the same: need to know the force in either the lower inclined web member or the upper inclined web member.

(1) Method of sections.

Cut *A* exposes an upper web member *a*, and a lower web member *a'* . If $F_{a'}$ is known, F_a can be computed from the equilibrium equation for forces in the vertical direction of the FBD to the left of the cut and vice versa.

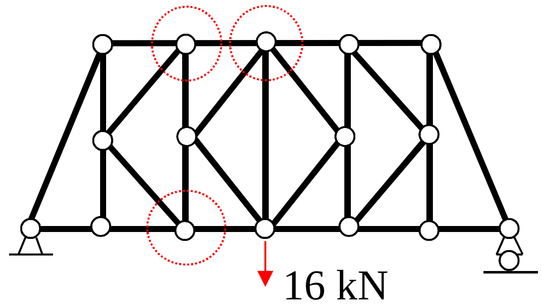




Cut *B* exposes the forces of a lower web member *b*, and an upper web member, *b'*. If $F_{b'}$ is known, F_b can be computed from the equilibrium equation for forces in the vertical direction of the FBD to the right of the cut and vice versa.

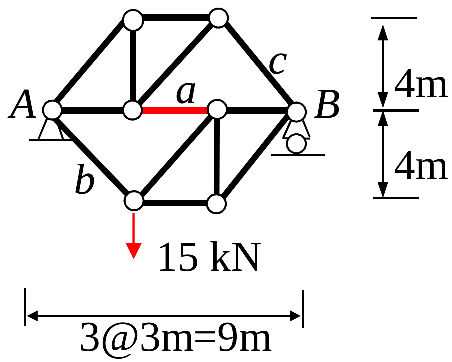
Cut *C* exposes the forces of the central vertical web member and two inclined web members; each force has a vertical component. Once the forces in the two inclined web members are known, the force in the central vertical member can be computed from the equilibrium equation for forces in the vertical direction of the FBD to the left or right of the cut.

(2) Method of joints.



At each of the **circled joints**, the vertical web member forces can be computed if the force of the inclined web member is known.
For the central vertical web member, we need to know the forces of the two joining inclined web members. In the present case, since the load is symmetrical, the two inclined web members have identical forces. As a result, the force in the central vertical web member is zero.

Example 2.10. Find the force in member *a* of the compound truss shown.

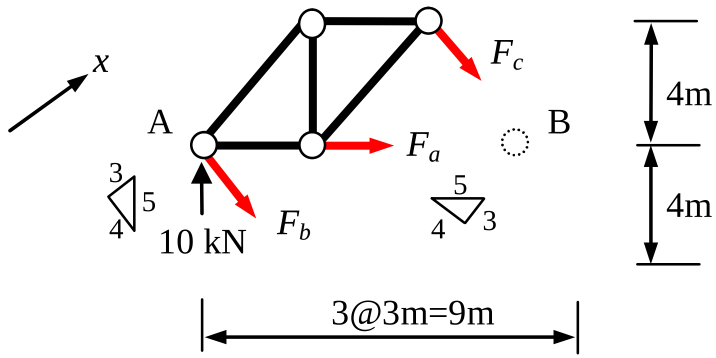


Solution The method of sections is often suitable for compound truss analyses.

(1) Identify truss type. This is a stable and determinate compound truss with three links, *a*, *b* and *c*, linking two simple trusses. Each node has at least three joining members. Thus, the method of joints is not a good option. We need to use the method of section.

(2) Find reactions. Since the geometry is simple enough, we can see that the horizontal reaction at support *A* is zero and the vertical reactions at support *A* and *B* are 10 kN and 5 kN, respectively.

(3) Establish FBD. By cutting through the three links, we obtain two FBDs. We choose the upper-left one because it does not involve the applied force.



To find F_a we note that the other two unknown forces, F_b and F_c, are parallel to each other, making it impossible to take moment about their intersection. Let's examine the force equilibrium in the direction perpendicular to the two parallel forces, denoted in the above figure as the *x*-direction. We can decompose the 10 kN reaction at support *A* and the unknown force F_a into components in the *x*-direction and write the equilibrium equation accordingly.

$\Sigma F_x = 0,$ $(0.6)10 + (0.6)F_a = 0$ ➡ $F_a = -10$ kN.

END OF CH. 2