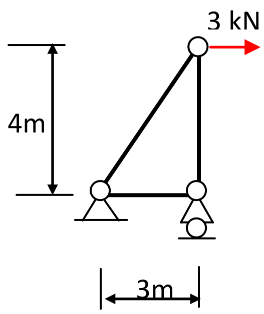
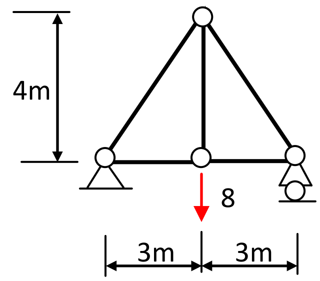
2. Truss Analysis: Force Method, Part I

Problem 2.1 Use the method of joints to find all reaction and member forces in the trusses shown.

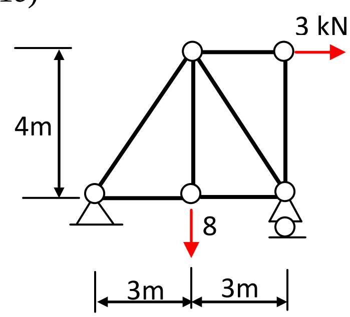
(1*a*)



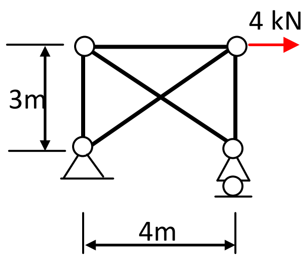
(1*b*)



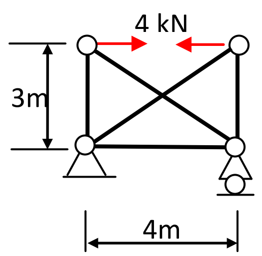
(1*c*)



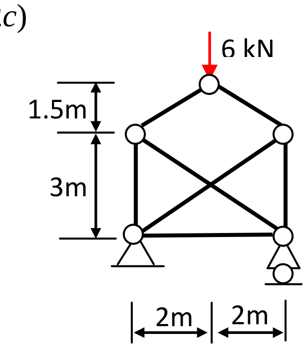
(2*a*)



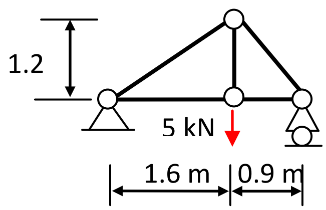
(2*b*)



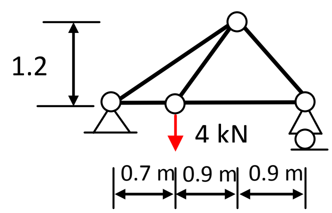
(2*c*)



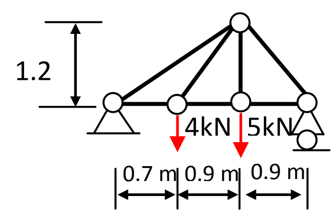
(3*a*)



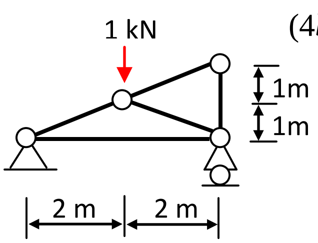
(3*b*)



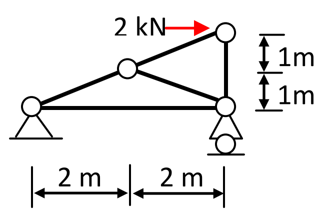
(3*c*)



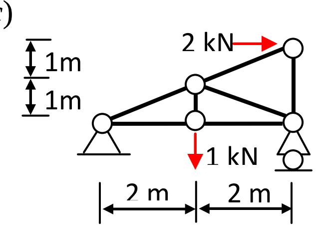
(4*a*)



(4*b*)



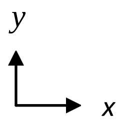
(4*c*)



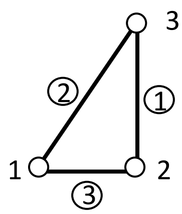
Solution

In all the solutions, the horizontal direction is designated as the *x*-axis and the vertical direction as the *y*-axis.

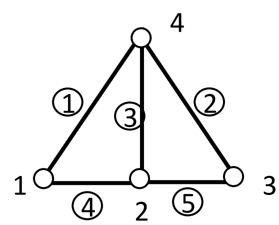
All the joints and members are then numbered.



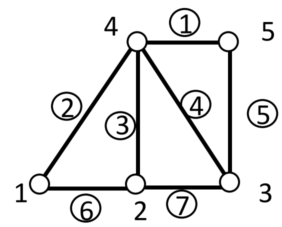
(1-*a*)



(1-*b*)

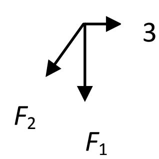


(1-*c*)



Solution (1*a*) The inclination of member 2 is represented by a 3-4-5 triangle. Reactions are R_{x1},R_{y1},R_{y2}.

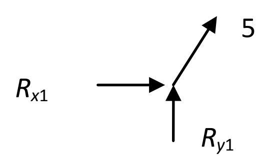
Joint 3.



$\Sigma F_x = 0$, $-F_2\,(3/5)+3=0$, $F_2 = 5$ kN

$\Sigma F_y = 0$, $-F_2\,(4/5) - F_1 = 0$, $F_1 = -4$ kN

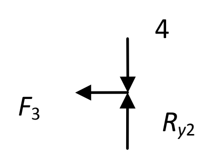
Joint 1.



$\Sigma F_x = 0$, $5\,(3/5)+R_{x1}=0$, $R_{x1} = -3$ kN

$\Sigma F_y = 0$, $5\,(4/5)+R_{y1}=0$, $R_{y1} = 4$ kN

Joint 2.



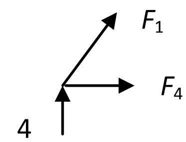
$\Sigma F_x = 0$, $F_3 = 0$ kN

$\Sigma F_y = 0$, $-4+R_{y2}=0$, $R_{y2} = 4$ kN

Solution (1*b*) The inclination of members 1 and 2 are represented by a 3-4-5 triangle. Reactions are R_{x1},R_{y1},R_{y3}.

By symmetry, $R_{y1}=R_{y3}=4$ kN. At joint 2, $F_3=8$ kN by observation.

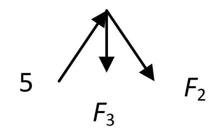
Joint 1.



$\Sigma\, F_y = 0,$ $F_1\,(4/5) + 4 = 0,\ F_1 = \text{-}5$ kN

$\Sigma\, F_x = 0,$ $F_1\,(3/5) + F_4 = 0,\ F_4 = 3$ kN

Joint 4.



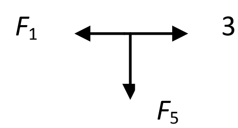
$\Sigma\, F_x = 0,$ $-5(3/5) + F_2\,(3/5) = 0,\ F_2 = \text{-}5$ kN

$\Sigma\, F_y = 0,$ $2*[5(4/5)] - F_3 = 0,\ F_3 = 8$ kN, Check!

Solution (1*c*) The inclination of members 2 and 4 are represented by a 3-4-5 triangle. Reactions are R_{x1}, R_{y1}, R_{y3}.

At joint 2, F_3=8 kN, $F_6 = F_7$ by observation. At joint 5 F_6=0 by observation.

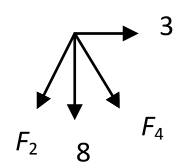
Joint 5.



$\Sigma\, F_x = 0,$ $-F_1 + 3 = 0,\ F_1 = 3$ kN

$\Sigma\, F_y = 0,$ $F_5 = 0,\ F_5 = 0$ kN Check!

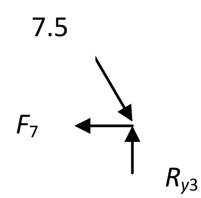
Joint 4.



$\Sigma\, F_x = 0,$ $-F_2\,(3/5) + F_4\,(3/5) + 3 = 0,\ F_2 = F_4 + 5,$

$\Sigma\, F_y = 0,$ $-F_2\,(4/5) - F_4\,(4/5) - 8 = 0,\quad F_2 = -F_4 - 10,$ Solve to get $F_2 = -2.5$ kN, $F_4 = -7.5$ kN

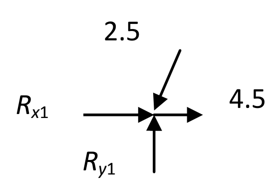
Joint 3.



$\Sigma\, F_x = 0,$ $-F_7 + 7.5(3/5) = 0,\ F_7 = 4.5\text{ kN} = F_6$

$\Sigma\, F_y = 0,$ $-7.5(4/5) + R_{y3} = 0,\ R_{y3} = 6$ kN

Joint 1.

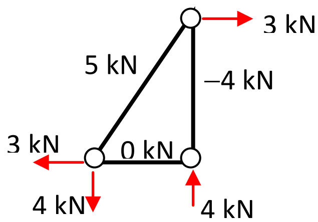


$\Sigma\, F_y = 0,$ $R_{y1} - 2.5\,(4/5) = 0,\ R_{y1} = 2$ kN

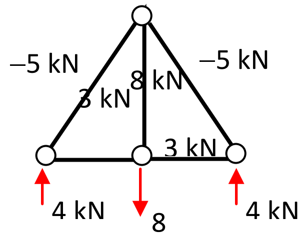
$\Sigma\, F_x = 0,$ $R_{x1} - 2.5\,(3/5) + 4.5 = 0,\ R_{x1} = -3$ kN

Solution Presentation:

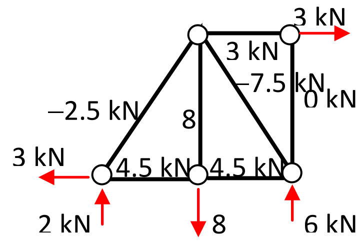
(1*a*)



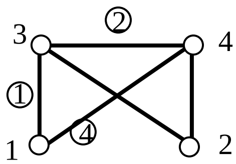
(1*b*)



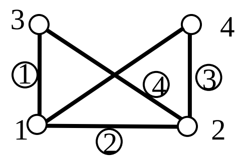
(1*c*)



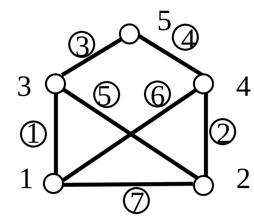
(2*a*)



(2*b*)

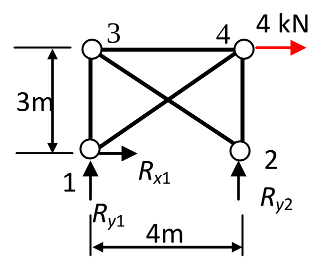


(2*c*)



Solution (2*a*) The inclination of member 2 is represented by a 3-4-5 triangle. Reactions are R_{x1},R_{y1},R_{y2}.

In this case, it is easier to find the reactions first before going from joint to joint.

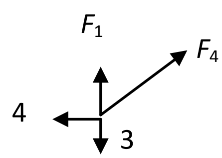


ΣM_1=0, (4) R_{y2} – (4)(3) = 0, R_{y2} = 3 kN.

ΣF_x=0, R_{x1} = –4 kN

ΣM_2=0, (4) R_{y1} + (4)(3) = 0, R_{y1} = –3 kN.

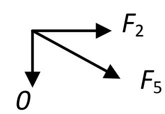
Joint 1.



$\Sigma F_x = 0$, F_4 (4/5) –4 = 0, F_4 = 5 kN

$\Sigma F_y = 0$, F_4 (3/5) + F_1–3 = 0, F_1 = 0 kN

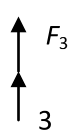
Joint 3.



$\Sigma F_y = 0$, F_5 (3/5) = 0, F_5 = 0 kN

$\Sigma F_x = 0$, F_5 (4/5) + F_2 = 0, F_2 = 0 kN

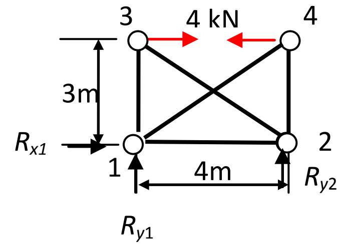
Joint 2.



$\Sigma F_y = 0$, F_3 +3 = 0, F_3 = -3 kN

Solution (2*b*) The inclination of members 4 and 5 are represented by a 3-4-5 triangle. Reactions are R_{x1},R_{y1},R_{y2}.

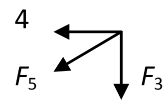
In this case, it is easier to find the reactions first before going from joint to joint.



$\Sigma F_x = 0$, R_{x1}=0.

$\Sigma F_y = 0$, $R_{y1}+R_{y2}=0$, $R_{y1}=R_{y2}$

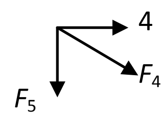
Joint 3.



$\Sigma F_x = 0$, $-F_5\,(4/5) - 4 = 0$, $F_5 = -5$ kN

$\Sigma F_y = 0$, $-F_5\,(3/5) - F_3 = 0$, $F_3 = 3$ kN

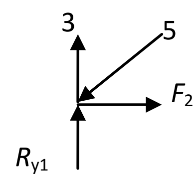
Joint 4.



$\Sigma F_x = 0$, $F_4\,(4/5) + 4 = 0$, $F_4 = -5$ kN

$\Sigma F_y = 0$, $-F_4\,(3/5) - F_1 = 0$, $F_1 = 3$ kN

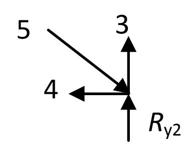
Joint 1.



$\Sigma F_x = 0$, $-5\,(4/5) + F_2 = 0$, $F_2 = 4$ kN

$\Sigma F_y = 0$, $-5\,(3/5) + 3 + R_{y1} = 0$, $R_{y1} = 0$ kN

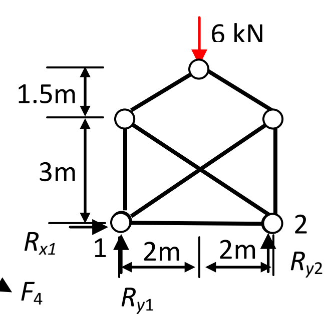
Joint 2.



$\Sigma F_x = 0$, $5\,(4/5) - 4 = 0$, Check!

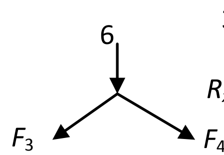
$\Sigma F_y = 0$, $-5\,(3/5) + 3 + R_{y2} = 0$, $R_{y2} = 0$ kN$= R_{y1}$

Solution (2*c*) The inclination of members 3~6 is represented by a 3-4-5 triangle. Reactions are R_{x1},R_{y1},R_{y2}.



By observation, R_{x1}=0

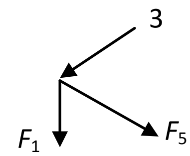
Joint 4.



$\Sigma F_x = 0$, $-F_3\,(4/5) + F_4\,(4/5) = 0$, $F_3 = F_4$

$\Sigma F_y = 0$, $-F_3\,(3/5) - F_4(3/5) - 6 = 0$, $F_3 = F_4 = -5$ kN

Joint 3.

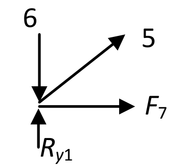


$\Sigma F_x = 0$, $\quad -3\ (4/5) + F_5\ (4/5) = 0$, $F_5 = 5$ kN

$\Sigma F_y = 0$, $\quad -F_1 - F_5(3/5) - 3 = 0$, $F_1 = -6$ KN

Joint 5. From symmetry with joint 3, $F_1 = F_2 = -6$ kN

Joint 1.



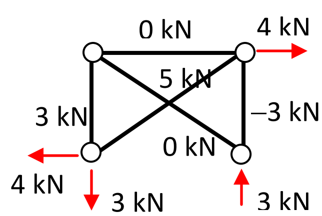
$\Sigma F_x = 0$, $\quad 5(4/5) + F_7 = 0$, $F_7 = -4$ kN

$\Sigma F_y = 0$, $\quad 5(3/5) - R_{y1} = 0$, $R_{y1} = 3$ kN

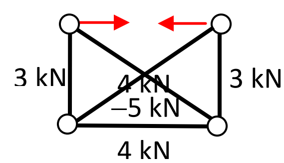
Joint 2. From symmetry, $R_{y2} = 3$ kN

Solution presentation.

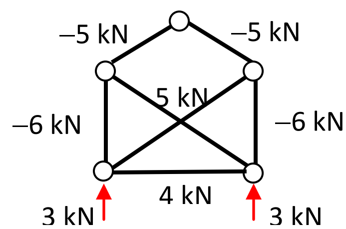
(2*a*)



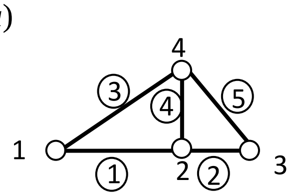
(2*b*)



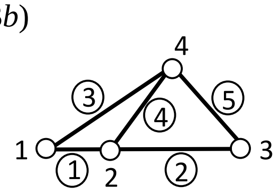
(2*c*)



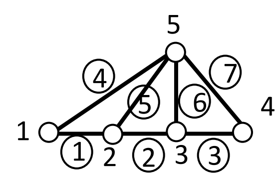
(3*a*)



(3*b*)

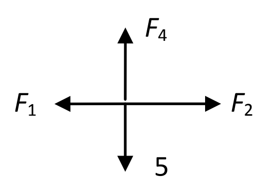


(3*c*)



Solution (3*a*) The inclination of members 3,5 is represented by a 3-4-5 triangle. Reactions are R_{x1}, R_{y1}, R_{y3}.

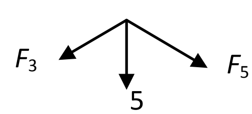
Joint 2.



$\Sigma F_x = 0$, $\quad -F_1 + F_2 = 0$, $F_2 = F_1$

$\Sigma F_y = 0$, $\quad -5 + F_4 = 0$, $F_4 = 5$ kN

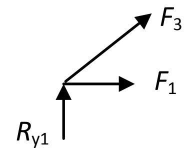
Joint 4.



$\Sigma F_x = 0$, $\quad -F_3\ (4/5) + F_5\ (3/5) = 0$,

$\Sigma F_y = 0$, $\quad -F_3\ (3/5) - F_5\ (4/5) - 5 = 0$, $F_5 = -4$ kN, $F_3 = -3$ kN

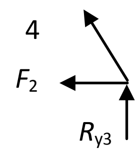
Joint 1.



Σ $F_y = 0$, $R_{y1} + F_3$ (3/5) =0, R_{y1} = 1.8 kN

Σ $F_x = 0$, $F_1 + F_3$ (4/5) =0, F_1=2.4 kN= F_2

Joint 3.



Σ $F_y = 0$, $R_{y3} + F_5$ (4/5) =0, R_{y3} = 3.2 kN

Solution (3*b*) The inclination of members 3~5 is represented by a 3-4-5 triangle. Reactions are R_{x1},R_{y1}, R_{y3}.

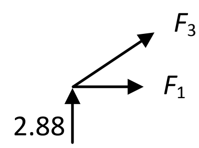
It is easier to get the reactions from the FBD of the whole truss through one moment equation and two force equations.

Σ $F_x = 0$, R_{x1}=0,

Σ M_3 = 0, 4(1.8) –R_{y1}(2.5)=0, R_{y1}= 2.88 kN

Σ $F_y = 0$, $R_{y1} + R_{y3}$ -4 =0, R_{y3}= 1.12 kN

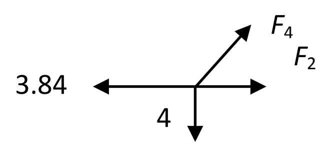
Joint 1.



Σ $F_y = 0$, 2.88 + F_3 (3/5) =0, F_3= –4.8 kN

Σ $F_x = 0$, $F_1 + F_3$ (4/5) =0, F_1= –3.84 kN

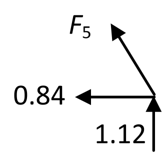
Joint 2.



Σ $F_y = 0$, F_4 (4/5) –4 =0, F_4= 5kN

Σ $F_x = 0$, F_2 + 5 (3/5) –3.84=0, F_2=0.84 kN

Joint 3.



Σ $F_y = 0$, 1.12 + F_5 (4/5) =0, F_5 = –1.4 kN

Σ $F_x = 0$, *-0.84*- F_5 (3/5) =0, Check!

Solution (3*c*) The inclination of members 4,5,7 is represented by a 3-4-5 triangle. Reactions are R_{x1},R_{y1}, R_{y4}.

It is easier to get the reactions from the FBD of the whole truss through one moment equation and two force equations.

$\Sigma\ F_x = 0,\qquad R_{x1}=0,$

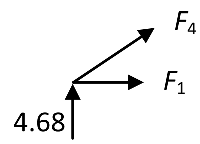
$\Sigma\ M_4 = 0,\ -4(1.8)+R_{y1}(2.5)\ -5(0.9)=0,\ R_{y1}= 4.68$ kN

$\Sigma\ F_y = 0,\qquad R_{y1} + R_{y4}\ -4-5 =0,\ R_{y4}= 4.32$ kN

Joint 1.

$\Sigma\ F_y = 0,\qquad 4.68 + F_4\ (3/5) =0,\ F_4 =-7.8$ kN

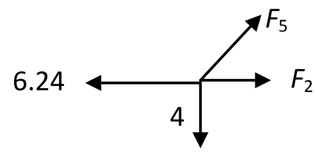
$\Sigma\ F_x = 0,\qquad F_1+ F_4\ (4/5) =0,\ F_1=6.24$ kN



Joint 2.

$\Sigma\ F_y = 0,\qquad -4 + F_5\ (4/5) =0,\ F_5 =5$ kN

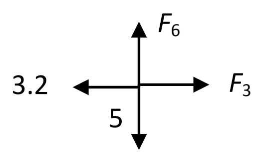
$\Sigma\ F_x = 0,\qquad F_2+ F_5\ (3/5)\ -6.24=0,\ F_2=3.24$ kN



Joint 3.

$\Sigma\ F_y = 0,\qquad -5 + F_6 =0,\ F_6 =5$ kN

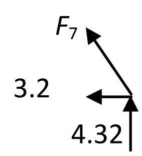
$\Sigma\ F_x = 0,\qquad F_3-3.24=0,\ F_3=3.24$ kN



Joint 4.

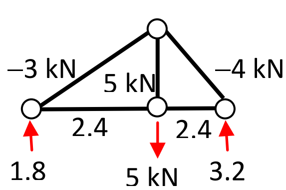
$F_x = 0,\ \ -F_7\ (3/5)\ -3.24=0,\ F_7 = -5.4$ kN

$\Sigma\ F_y = 0,\qquad 4.32 + F_7\ (4/5) =0,$ Check!

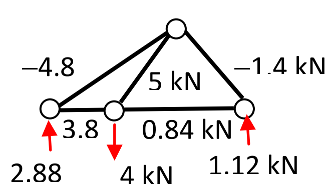


Solution presentation.

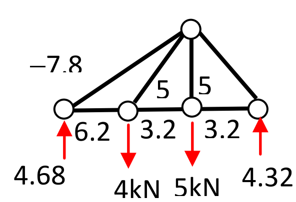
(3*a*)



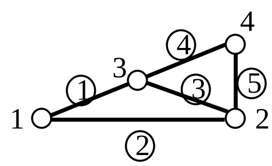
(3*b*)

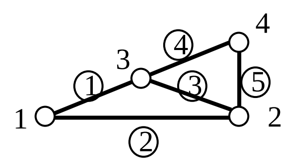


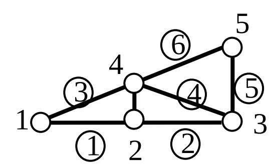
(3*c*)



(4*a*) (4*b*) (4*c*)







Solution (4*a*) All inclined members are represented by a 1-2-2.236 triangle. Reactions are R_{x1},R_{y1}, R_{y2}.

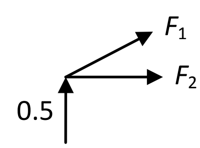
From the FBD of the whole truss,

$\Sigma F_x = 0$, $R_{x1}=0$,

$\Sigma M_2 = 0$, $-1(2)+R_{y1}(4)=0$, $R_{y1}= 0.5$ kN

$\Sigma F_y = 0$, $R_{y1} + R_{y2} -1 =0$, $R_{y2}= 0.5$ kN

Joint. 1.

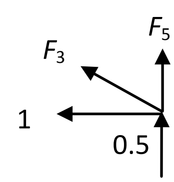


$\Sigma F_y = 0$, $0.5 + F_1 (1/2.236) =0$, $F_1 = -1.12$ kN

$\Sigma F_x = 0$, $F_2+ F_1 (2/2.236) =0$, $F_2=1.0$ kN

Joint 4. $F_4= F_5 =0$, Zero Force members.

Joint 2.



$\Sigma F_x = 0$, $-1- F_3 (2/2.236) =0$, $F_3= -1.12$ kN

$\Sigma F_y = 0$, $F_5+ F_3 (1/2.236)+0.5 =0$, $F_5=0$ kN, Check!

Solution (4*b*) All inclined members are represented by a 1-2-2.236 triangle. Reactions are R_{x1},R_{y1}, R_{y2}.

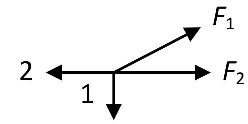
From the FBD of the whole truss,

$\Sigma F_x = 0$, $2+R_{x1}=0$, $R_{x1}= -2$ kN

$\Sigma M_2 = 0$, $2(2)+R_{y1}(4)=0$, $R_{y1}= -1$ kN

$\Sigma F_y = 0$, $R_{y1} + R_{y2} =0$, $R_{y2}= 1$ kN

Joint 1.

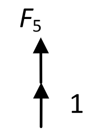


$\Sigma\ F_y = 0$, $\quad -1 + F_1\ (1/2.236) = 0$, $F_1 = 2.24$ kN

$\Sigma\ F_x = 0$, $\quad F_2 + F_1\ (2/2.236) - 2 = 0$, $F_2 = 0$ kN

Joint 3. Zero force member. $F_3 = 0$ kN, $F_4 = F_1$

Joint 2.



$\Sigma\ F_y = 0$, $\quad 1 + F_5 = 0$, $F_5 = -1$ kN

Solution (4*c*) All inclined members are represented by a 1-2-2.236 triangle. Reactions are R_{x1},R_{y1}, R_{y3}.

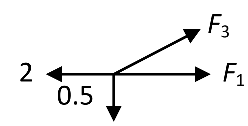
From the FBD of the whole truss,

$\Sigma\ F_x = 0$, $\quad 2 + R_{x1} = 0$, $R_{x1} = -2$ kN

$\Sigma\ M_3 = 0$, $\quad -1(2) + R_{y1}(4) + 2(2) = 0$, $R_{y1} = -0.5$ kN

$\Sigma\ F_y = 0$, $\quad R_{y1} + R_{y3} - 1 = 0$, $R_{y3} = 1.5$ kN

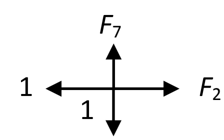
Joint 1.



$\Sigma\ F_y = 0$, $\quad -0.5 + F_3\ (1/2.236) = 0$, $F_3 = 1.12$ kN

$\Sigma\ F_x = 0$, $\quad F_1 + F_3\ (2/2.236) - 2 = 0$, $F_1 = 1$ kN

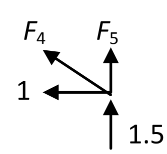
Joint 2.



$\Sigma\ F_y = 0$, $\quad -1 + F_2 = 0$, $F_2 = 1$ kN

$\Sigma\ F_x = 0$, $\quad F_7 - 1 = 0$, $F_7 = 1$ kN

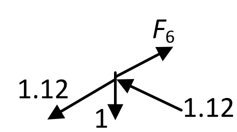
Joint 3.



$\Sigma\ F_x = 0$, $\quad -F_4\ (2/2.236) - 1 = 0$, $F_4 = -1.12$ kN

$\Sigma\ F_y = 0$, $\quad F_5 + F_4\ (1/2.236) + 1.5 = 0$, $F_5 = -1$ kN

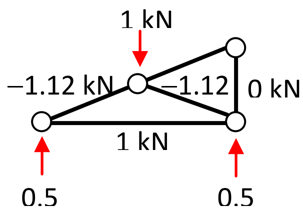
Joint 4.



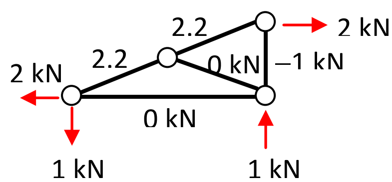
$\Sigma\ F_y = 0$, $\quad -1 + F_6\ (1/2.236) - 1.12(1/2.236) + 1.12(1/2.236) = 0$, $F_6 = 2.24$ kN

Solution presentation.

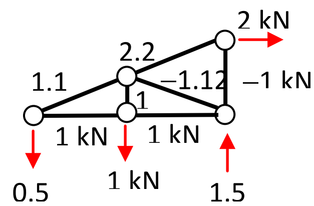
(4*a*)



4*b*)

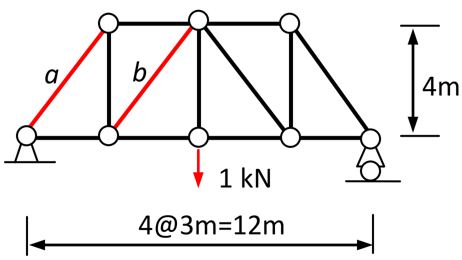


(4*c*)

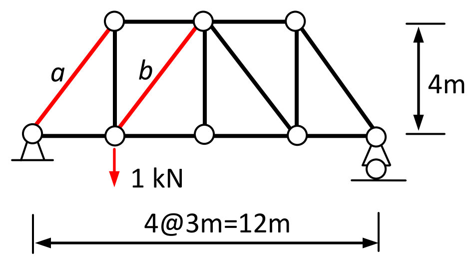


Problem 2.2 Solve for the force in the marked members in each truss shown.

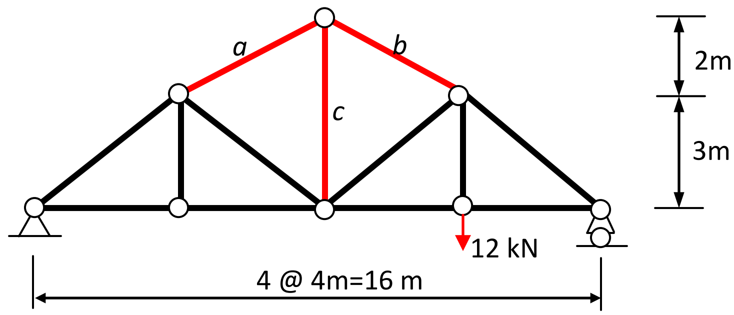
(1-*a*)



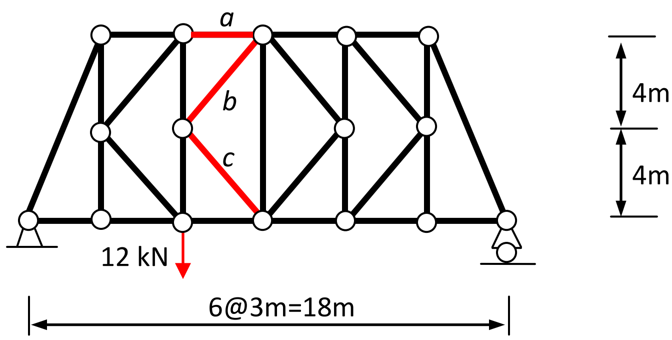
(1-*b*)



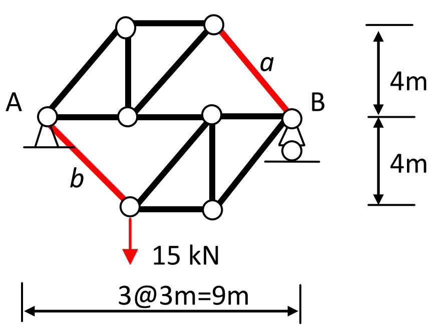
(2)



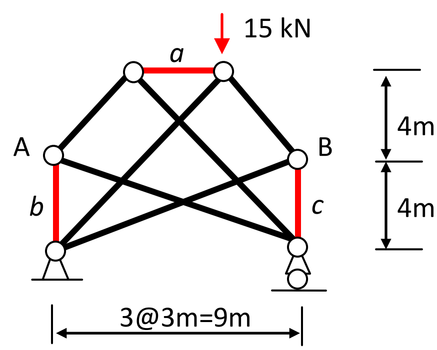
(3)



(4*a*)

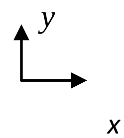


(4*b*)

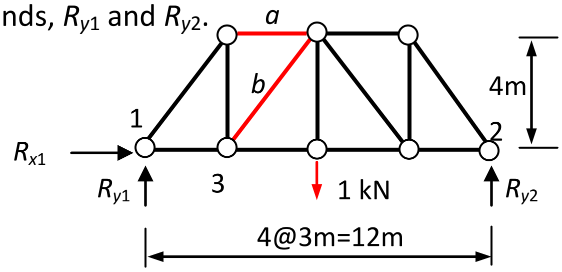


Solution

In all the solutions, the horizontal direction is designated as the *x*-axis and the vertical direction as the *y*-axis.



Solution (1-*a*) The reactions are vertical at the two ends, R_{y1} and R_{y2}.

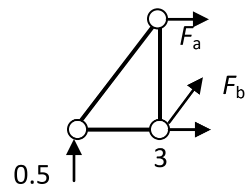


From the FBD of the whole truss,

$\Sigma M_2 = 0$, $-1(6)+R_{y1}(12)=0$, $R_{y1}= 0.5$ kN

$\Sigma F_y = 0$, $R_{y1} + R_{y2} -1=0$, $R_{y3}= 0.5$ kN

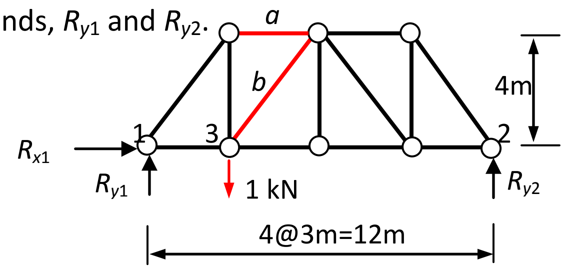
From the left portion of the truss.



$\Sigma M_3 = 0$, $0.5(3)+F_a(4)=0$, $F_a = -0.375$ kN

$\Sigma F_y = 0$, $F_b\ (4/5)+ +0.5=0$, $F_b = -0.625$ kN

Solution (1-*b*) The reactions are vertical at the two ends, R_{y1} and R_{y2}.

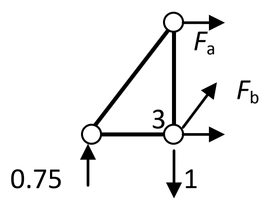


From the FBD of the whole truss,

$\Sigma M_2 = 0$, $-1(9)+R_{y1}(12)=0$, $R_{y1}= 0.75$ kN

$\Sigma F_y = 0$, $R_{y1} + R_{y2} -1=0$, $R_{y2}= 0.25$ kN

From the left portion of the truss.



$\Sigma M_3 = 0$, $0.75(3)+F_a(4)=0$, $F_a = -0.375$ kN

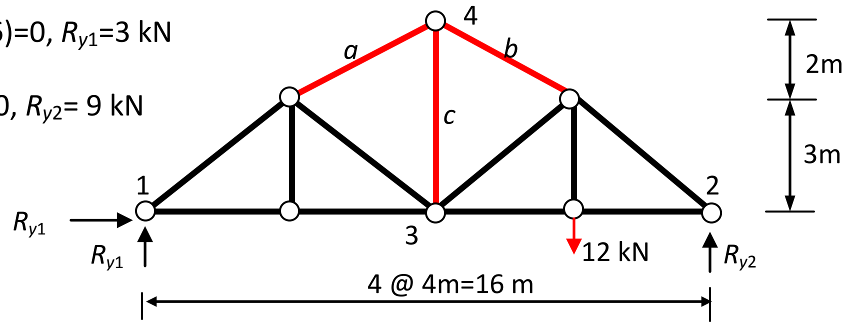
$\Sigma F_y = 0$, $F_b\ (4/5)+ +0.5=0$, $F_b = -0.625$ kN

Solution (2) The reactions are vertical at the two ends.

The inclination of members *a* and *b* is represented by a 2-4-4.472 triangle.

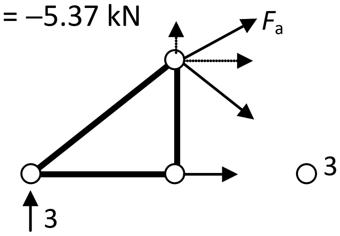
$\Sigma M_2 = 0$, $\quad -12(4)+R_{y1}(16)=0,\ R_{y1}=3$ kN

$\Sigma F_y = 0$, $\quad R_{y1} + R_{y2} - 12=0,\ R_{y2}= 9$ kN



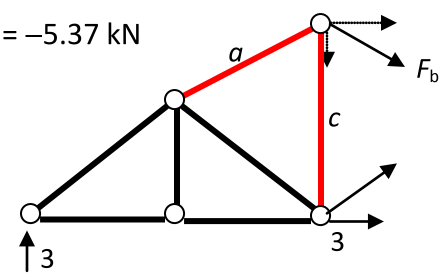
For F_a

$\Sigma M_3 = 0$, $\quad 3(8)+F_a(4/4.472)(5)+ F_a(2/4.472)(4)=0,\ F_a = -5.37$ kN



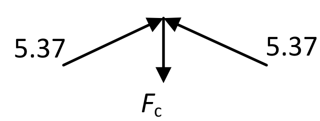
For F_b

$\Sigma M_3 = 0$, $\quad 3(8)+F_b(4/4.472)(5)+ F_b(2/4.472)(0)=0,\ F_b = -5.37$ kN



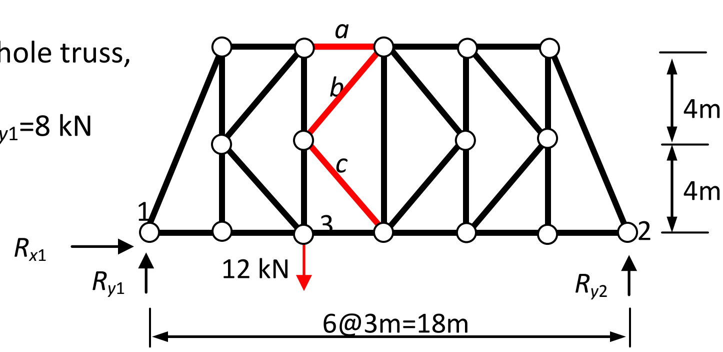
For F_c, look at Joint 4.

$\Sigma F_y = 0$, $\quad 5.37\,(2/4.472)+5.37(2/4.472) - Fc=0,\ F_c = 4.80$ kN

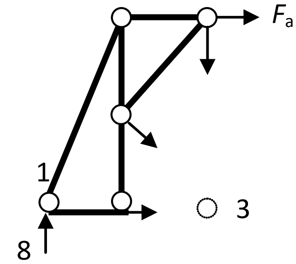


Solution (3) From the FBD of the whole truss,

$\Sigma M_2 = 0$, $\quad -12(12)+R_{y1}(18)=0,\ R_{y1}=8$ kN

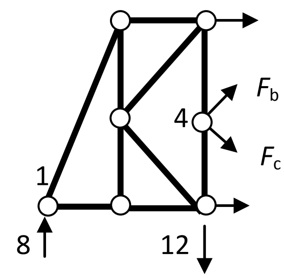


For F_a, choose the left portion of the truss.



$\Sigma M_3 = 0$, $8(6)+F_a(8)=0$, $F_a= -6$ kN

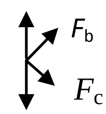
For F_b and F_c ,look at another left portion.



$\Sigma F_y = 0$, $F_b\,(4/5) - F_c(4/5)+8-12=0$

$F_b - F_c=5$

Joint 4.



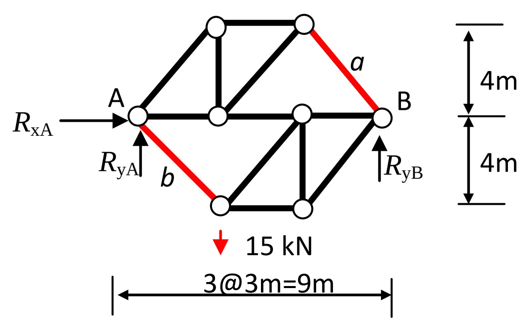
$\Sigma F_x = 0$, $F_b\,(3/5)+F_c(3/5=0$

$F_b+F_c=0$

From the above two equations, F_b =2.5 kN, F_c = –2.5 kN

Solution (4*a*)

From the FBD of the whole truss,

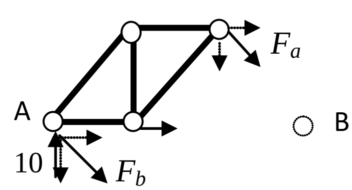


$\Sigma M_B = 0$, $-15(6)+R_{yA}(9)=0$, R_{yA}=10 kN

$\Sigma F_y = 0$, $R_{yA} + R_{yB} - 15=0$, R_{yB} =5 kN

$\Sigma F_x = 0$, $R_{xA} = 0$

From the selected FBD,



$\Sigma M_B = 0$, $10(9) - F_b(4/5)(9)=0$, F_b=12.5 kN

$\Sigma M_A = 0$, $F_a(4/5)(6)+ F_a(3/5)(4)=0$, F_a=0 kN

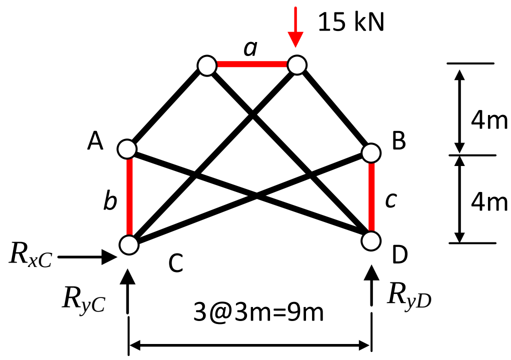
Solution (4-*b*)

From the FBD of the whole truss,

$\Sigma\, M_D = 0,\quad R_{yC}(9) - 15(3) = 0,\ R_{yc} = 5$ kN

$\Sigma\, F_y = 0,\quad R_{yC} + R_{yD} - 15 = 0,\ R_{yD} = 10$ kN

$\Sigma\, F_x = 0,\quad R_{xC} = 0$

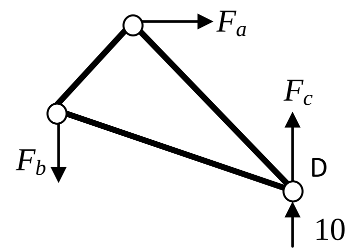


From the selected FBD,

$\Sigma\, F_x = 0,\quad F_a = 0$ kN

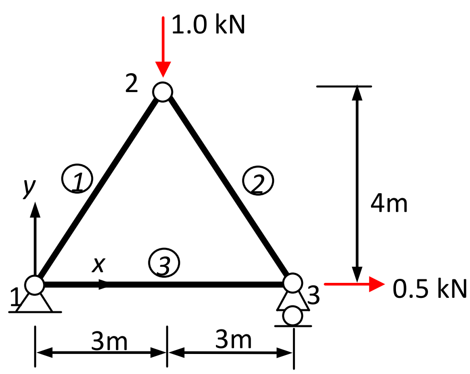
$\Sigma\, M_D = 0,\quad F_b = 0$

$\Sigma\, F_y = 0,\quad F_c + 10 = 0,\ F_c = -10$ kN



Problems

2.3 The loaded truss shown is different from that in Example 2-11 only in the externally applied loads. Modify the results of Example 2-11 to establish the matrix equilibrium equation for this problem.



Solution

Node 1: Externally applied forces are 0 in *x*-direction, 0 in *y*-direction.

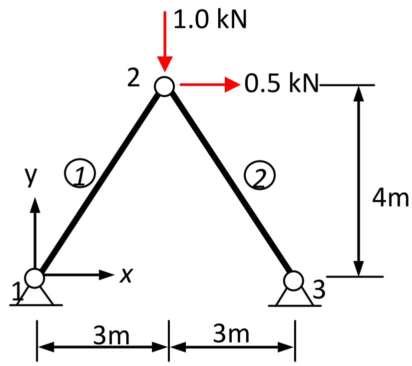
Node 2: Externally applied forces are 0 in *x*-direction, –1 in *y*-direction.

Node 3: Externally applied forces are 0.5 in *x*-direction, 0 in *y*-direction.

As a result, the matrix equilibrium equation becomes:

$$\begin{bmatrix} -0.6 & 0 & -1.0 & -1.0 & 0.0 & 0 \\ -0.8 & 0 & 0.0 & 0.0 & -1.0 & 0 \\ 0.6 & -0.6 & 0 & 0 & 0 & 0 \\ 0.8 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 1.0 & 0 & 0 & 0.0 \\ 0 & -0.8 & 0.0 & 0 & 0 & -1.0 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -1.0 \\ 0.5 \\ 0 \end{Bmatrix}$$

2.4 Establish the matrix equilibrium equation for the loaded truss shown.



Solution

Member 3 of Example 2-11 no longer exists, but both node 1 and 3 have two reactions. The unknown forces are the two member forces followed by the four reaction forces.

As a result, the contribution of member forces table changes:

Contribution of Member Forces

Member Number	Force Number	Equation Number and Value of Entry							
		2*i*-1	Coeff.	2*i*	Coeff.	2*j*-1	Coeff.	2*j*	Coeff.
1	1	1	–0.6	2	–0.8	3	0.6	4	0.8
2	2	3	–0.6	4	0.8	5	0.6	6	□0.8

The contribution of reaction forces table becomes:

Contribution of Reaction Forces

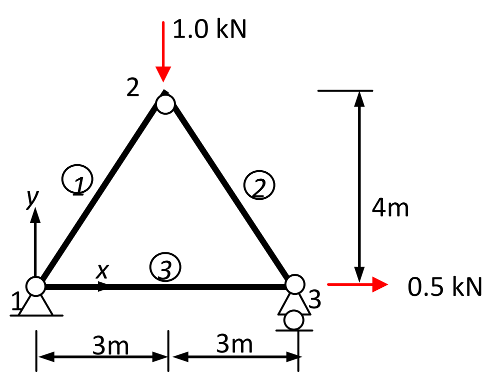
Reaction Number	Force Number	Equation Number and Value of Entry			
		2*i*-1	Coeff.	2*i*	Coeff.
1	3	1	−1.0	2	0.0
2	4	1	0.0	2	−1.0
3	5	5	−1.0	6	0.0
4	6	5	0.0	6	−1.0

There is no change in the contribution of externally applied forces. The resulting matrix equilibrium equation becomes

$$\begin{bmatrix} -0.6 & 0 & -1.0 & 0.0 & 0 & 0 \\ -0.8 & 0 & 0.0 & -1.0 & 0 & 0 \\ 0.6 & -0.6 & 0 & 0 & 0 & 0 \\ 0.8 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & -1.0 & 0.0 \\ 0 & -0.8 & 0 & 0 & 0.0 & -1.0 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.5 \\ -1.0 \\ 0 \\ 0 \end{Bmatrix}$$

Problems

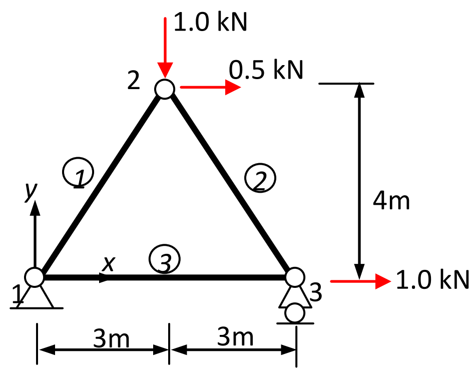
2.5 The loaded truss shown is different from that in Example 2-11 only in the externally applied loads. Use the force transfer matrix of Eq. 2-6 to find the solution.



Solution Use the force transfer matrix multiplied by the nodal forces to get the member and reaction forces.

$$\begin{bmatrix} 0.0 & 0.0 & 0.83 & 0.63 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.83 & 0.63 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & -0.38 & 1.0 & 0.0 \\ -1.0 & 0.0 & -1.0 & 0.0 & -1.0 & 0.0 \\ 0.0 & -1.0 & -0.67 & -0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.67 & -0.5 & 0.0 & -1.0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -1.0 \\ 0.5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0.63 \\ -0.63 \\ 0.88 \\ -0.5 \\ 0.5 \\ 0.5 \end{Bmatrix}$$

2.6 The loaded truss shown is different from that in Example 2-11 only in the externally applied loads. Use the force transfer matrix of *Eq*. 2-6 to find the solution.



Solution The force transfer matrix is the same as in the last problem. Multiplying it with the externally applied forces vector gives:

$$\begin{bmatrix} 0.0 & 0.0 & 0.83 & 0.63 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.83 & 0.63 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & -0.38 & 1.0 & 0.0 \\ -1.0 & 0.0 & -1.0 & 0.0 & -1.0 & 0.0 \\ 0.0 & -1.0 & -0.67 & -0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.67 & -0.5 & 0.0 & -1.0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.5 \\ -1.0 \\ 1.0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -0.21 \\ -1.04 \\ 1.62 \\ -1.5 \\ 0.17 \\ 0.83 \end{Bmatrix}$$