CHAP 2

GROUP REPRESENTATION

22. Problem

Show that the set of real numbers forms a field.

Solution

The set of real numbers

1. has more than two elements
2. forms an Abelian group under addition
3. zero excluded, the real numbers also form an Abelian group under multiplication

Hence it forms a field.

23. Problem

Show that the set of complex numbers forms a field.

Solution

This problem can be solved proceeding exactly in the same manner.

24. Problem

Show that the two matrix representatives

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

of the operator A are connected by a similarity transformation.

Solution

Let C be the required connecting matrix. Then

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = C^{-1} \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} C$$

where C is a 2 x 2 matrix. This yields

$$C = \begin{bmatrix} 2 & 3 \\ 2 & 6 \end{bmatrix}.$$

25. Problem

Show that if a vector space L has two complementary invariant subspaces under a matrix representation D(G) of a group G, then D(G) must be fully reducible.

Solution

The two complimentary invariant subspaces of the vector space L are L_3 and L_{12}. These can be represented by

$$L_3 = \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix} \qquad L_{12} = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}.$$

Let us first consider L_3. Then the matrix D (R) of D (G) must be given by

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix} = \begin{bmatrix} cx_1 \\ fx_2 \\ ix_3 \end{bmatrix}.$$

This yields c = 0, f = 0. The matrix D(R) becomes

$$\begin{bmatrix} a & b & 0 \\ d & e & 0 \\ g & h & i \end{bmatrix}.$$

Let us next consider L_{12}. Then we have

$$\begin{bmatrix} a & b & 0 \\ d & e & 0 \\ g & h & i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 \\ dx_1 + ex_2 \\ gx_1 + hx_2 \end{bmatrix}.$$

This yields

g = 0 and h = 0.

Consequently:

$$\begin{bmatrix} a & b & 0 \\ d & e & 0 \\ 0 & 0 & i \end{bmatrix}.$$

This shows that D(G) is fully reducible.