
2 First Law- the $E = Mc^2$ of thermodynamics

Problem 2.1 Consider the melting of 100 gms of ice at 273 K and 1 atm. of pressure. The densities of ice and water under these conditions are, respectively, 0.92 gms/cc and 1.0 gms/cc. The latent heat of fusion of ice is 80 cal/gm. Apply the first law to determine the heat absorbed, the work done and the change in internal energy. Do you expect the internal energy per unit mass of water to be greater or lesser than that of ice, and why?

In this problem 100 gms of ice is melting at its melting point, which has been taken to be 273 K. The actual melting point is close to this. The amount of heat that needs to be added to ice is given by $\Delta Q = m \cdot l = 80 \cdot 100 = 8000$ calories. Before answering the rest of the question, let us settle a point of principle. One sometimes comes by the query as to whether there is any *pressure dependence* to latent heats, or whether latent heats only depend on T. It should be kept in mind that latent heat is the amount of heat required to change certain amount of the system in one phase to another, and is measured (or calculated) at the phase coexistence point. But such a point lies on the phase coexistence curve which relates the pressure to the temperature and vice versa. Therefore, it makes sense to only specify the temperature dependence of the latent heat without any loss of information.

Next, we calculate the work done in the melting process above. The volume of 80 gms of ice is $V_s = 80/0.92 = 86.96$ cc, while the volume of water is $80/1.0 = 80$ cc. Hence the work done is $P\Delta V = -6.96 \cdot 10^{-6} m^3 \cdot 1.0/9.87 \cdot 10^6 N/m^2 = -6.96/9.87 J = -6.96/(4.18 \cdot 9.87) = -0.17$ cal. Here we have used various properties of SI units: the unit of volume is m^3 which is 10^6 cc, 1 atm = $1/9.87$ MPa (mega pascal), 1 Pa = $1 N/m^2$, 1 J (oule) = 1 N.m and finally 1 cal = 4.18 J. Thus we see that the work done is a negligible fraction of the heat transferred!

Applying the first law to this process, the change ΔU in the internal energy is $\Delta U = \Delta Q - P\delta V = 8000.17$ cal. The internal energy has increased in the process. Water being in liquid form is expected to have a higher internal energy than the frozen ice which is in solid form.

Problem 2.2 Show that the conditions for adiabatic changes of an ideal gas are governed by $\frac{dP}{P} + \gamma(T) \frac{dV}{V} = 0$ $\frac{dP}{P} - \frac{\gamma(T)}{\gamma(T)-1} \frac{dT}{T} = 0$, and that they can be integrated to $PV^\gamma = const.$ and $P = const \cdot T^{\frac{\gamma}{\gamma-1}}$ when the specific heats are constant.

We need the ideal gas law and the first law to solve this problem. Let us keep things general and assume that there are n moles of the ideal gas under

consideration. The ideal gas law is then $PV = nRT$. Since the changes are adiabatic, the first law says

$$0 = dQ = dU + PdV = nC_V(T)dT + PdV \quad (2.1)$$

Here $C_V(T)$ is the *molar* specific heat at *constant volume*. For an ideal gas this is a function of T only. Note that we have considered the general situation, and not assumed C_V to be a constant. On using $T = PV/nR$, this becomes

$$0 = nC_V(T) \left(\frac{P}{nR} dV + \frac{V}{nR} da \right) P + PdV = \frac{C_P(T)}{R} dV + \frac{C_V(T)}{R} V dP \quad (2.2)$$

We have used the relation $C_P(T) = C_V(T) + R$ for ideal gases. Note that both specific heats are still functions of T, and not assumed to be constants. Consequently, the adiabaticity condition becomes

$$\gamma(T) \frac{dV}{V} + \frac{dP}{P} = 0 \quad (2.3)$$

Where $\gamma(T) = C_P(T)/C_V(T)$ is the *temperature dependent* ratio of the specific heats. One can eliminate V in favor of T i.e use $V = nRT/V$, to also write this condition as

$$\gamma(T) \left(\frac{dT}{T} - \frac{dP}{P} \right) + \frac{dP}{P} = 0 \rightarrow \frac{dP}{P} - \frac{\gamma(T)}{\gamma(T) - 1} \frac{dT}{T} = 0 \quad (2.4)$$

This can not be integrated in general. However, when C_V , and hence C_P and γ are independent of T, these equations can be readily integrated to give

$$PV^\gamma = \text{const} \quad P = \text{const} \cdot T^{-\frac{\gamma}{\gamma-1}} \quad (2.5)$$

Problem 2.3 Show that for an atmosphere in hydrostatic equilibrium, the heat Q is a state function at each height i.e $Q(V(z), T(z), z)$. Also show that $H+gh-Q=\text{const}$. Apply this to the problem of the adiabatic atmosphere. The quantity $H+gh$ is sometimes referred to as the *dry static energy*.

Even though the heat differential dQ can not be represented as an exact differential in general, it can be so expressed if the transformations are along a fixed path. The reader should verify that this is indeed so for simple situations like isothermal, isobaric changes etc.

In the case of the atmosphere in *hydrostatic equilibrium*, this equilibrium condition is essentially like fixing a path. The equation for hydrostatic equilibrium is given by

$$\frac{dP}{dz} = -g\rho \quad vdP = -gdz \quad (2.6)$$

Here z is the height as measured from the ground. This equilibrium equation results from the physical consideration that a *negative* pressure gradient is

necessary to balance the downward pull of gravity. In the second form, we have traded the density ρ for the *specific volume* v . Now let us consider the first law for a unit mass of the atmosphere by treating T, P as the independent thermodynamic parameters:

$$dq = c_p dT - v dP = c_p dT + g dz = d(z + gh) = dq(P(z), T(z), z) \quad (2.7)$$

Here h is the enthalpy per unit mass. We have taken the specific heats of the atmosphere to be independent of T . Thus we see that in this case heat at any particular height is indeed a state function.

If the atmosphere satisfies adiabaticity i.e $dq = 0$, it immediately follows that $h + gz$ is a constant. This latter quantity is sometimes referred to as *Dry static Energy*.

In the case of an *adiabatic atmosphere*, pressure P is related to density as $P = a\rho^\gamma$. This case was examined in Example 2.5 where it was shown that the temperature falls off linearly with height. The precise relation was

$$T(z) = T_0 - \frac{\gamma - 1}{\gamma} \frac{gM}{R} z \quad (2.8)$$

This indicates that the atmosphere can not extend beyond $z_c = (RT_0/gM)(\gamma - 1)/\gamma$, where T_0 is the ground temperature. As shown there this is roughly 27.85 km. But it is also worthwhile to examine the density profile. Then the condition eqn.(2.6) combined with adiabaticity condition yields

$$\frac{d\rho}{dz} = -\frac{g}{a\gamma} \rho^{2-\gamma} \quad (2.9)$$

This can easily be solved to give

$$\rho(z) = \left(\rho_0^{\gamma-1} - \frac{g}{a} \frac{\gamma-1}{\gamma} z \right)^{\frac{1}{\gamma-1}} \quad (2.10)$$

Here ρ_0 is the density at the ground. Again the atmosphere is seen to extend only upto $z' = (a\gamma\rho_0^{\gamma-1})/(g(\gamma-1))$. But the ideal gas equation $P = (\rho/M) \cdot RT$ means $RT_0 = aM\rho_0^{\gamma-1}$ and therefore $z' = z_c$.

Problem 2.4 An empty container is filled adiabatically at temperature T_0 at pressure P_0 with dry air. A volume V_0 is transferred from outside. Calculate the final temperature of the air inside the container. Give a physical reasoning for the rise in temperature.

This is the problem of *filling* of vessels and is strictly speaking belongs to the domain of *thermodynamics of Open systems*. The reader is encouraged to consult J. Kestin's book on *A Course in Thermodynamics* for a lucid discussion. Let v_0 be the volume of unit mass of the gas that is filled into a rigid vessel. If the outside pressure is P_0 , the work done *on* the system is $P_0 v_0$.

Since the filling in question is adiabatic this leads to a change in specific internal energy (the internal energy per unit mass) of $\Delta u = u_f - u_i = P_0 v_0$.

If T_f is the final temperature after filling, $u_f = c_v T_f$. Hence $u_f = u_i + P_0 v_0 = h_i$, where h is the enthalpy per unit mass. If T_i is the initial temperature, the gas being ideal, its initial enthalpy is given by $h_i = c_p T_i$. Hence the final temperature is $T_f = (c_p/c_v) T_i > T_i$.

Essentially the kinetic energy of the initial flow is converted into internal energy after filling. This is the explanation for the increase in temperature. Interestingly, this is another way of measuring the ratio of specific heats. For air, this ratio being 1.4, there is substantial heating upon filling i.e a 40% rise in temperature.

Problem 2.5 Chemists find the so called *enthalpy diagrams* very useful. These diagrams show various products of formation like H_2O from H_2 and O_2 etc. along with their enthalpies, called *enthalpy of formation*, at, say, atmospheric pressure. Draw such a diagram for H_2 and O_2 taking the enthalpy of the uncombined constituents to be 0 when enthalpy of formation are as follows: $H_2O_2(-188)$, $H^+(0)$, $OH^-(230)$, $H_2O(\text{vapor})(-242)$ and $H_2O(\text{liquid})(-285)$. Calculate the heat released when these are transformed into each other. The enthalpies are in kJ/mole, and the reactions are assumed to take place at 1 atm.

First let us briefly review the thermodynamic basis behind the concept of *heat of reactions*. The processes to which this applies occur at *constant pressure*. Consequently $dQ = dH$ where H is the enthalpy. Again, since the transformations take place along a fixed path i.e constant pressure, heat itself is a state function. Thus $\Delta_{fi}Q = H_f - H_i$.

It is also important to remember the *sign convention* used: if the heat of reaction is negative, the reaction is *exothermic* i.e it releases heat to the surroundings, and if it is positive, the reaction is *endothermic* i.e the reaction needs heat to be supplied from outside. Thus all the products listed above are the result of exothermic reactions of H_2 and O_2 . It is also obvious that oxygen and hydrogen themselves are assigned zero enthalpies of formation.

It is then clear that the product with the most *negative* enthalpy of formation is also the most stable.

Such a diagram in this case would consist of a horizontal line for each product with the enthalpy of formation for that product arranged in the order of these enthalpies of formation. So the top line would be for the uncombined constituents " $nH_2 + mO_2$ " (i.e n moles of H_2 reacting with m moles of O_2) with 0, and the bottom most line would be for H_2O in liquid state with -285.

Let us illustrate the idea with the reaction $2H_2 + O_2 \rightarrow 2H_2O$. We take the final state to be the liquid state of H_2O . The initial enthalpy H_i is 0 while the final enthalpy is $H_f = -470\text{kJ}$ (because 2 moles of H_2O are produced). The change in enthalpy $H_f - H_i$, which is also the *heat of the reaction*, is *negative* meaning that in this reaction 470 kJ of heat is produced. But the catch is that

H_2 and O_2 do not react on their own at 1 atm and room temperature! So they have to be made to react somehow and the enthalpy of formation *inferred*.

Now let us look at the reverse reaction $2H_2O(l) \rightarrow 2H_2 + 2O_2$. Now the change in enthalpy, and hence the heat of the reaction, is 470 KJ meaning that this amount of heat has to be supplied to convert two moles of liquid water into oxygen and hydrogen. This example makes the sign convention clearer as clearly heat has to be supplied to dissociate water into oxygen and hydrogen!

We can consider reactions between any two levels of a enthalpy diagram. But the number of moles of the reactants as well as the products must be appropriately chosen. Let us for example consider conversion of liquid water into steam as such a reaction i.e $H_2O(l) \rightarrow H_2O(g)$. The entropy change is 43 kJ, according to the data given. We already know that heat has to be supplied for this process. The latent heat of 540 cal/g actually translates to about 41 kJ if we use 18 as the molecular weight of water.

Let us consider Hydrogen Peroxide(H_2O_2) transforming to water i.e $2H_2O_2 \rightarrow 2H_2O(l) + O_2$. This also illustrates the importance of correctly choosing the number of moles for a balanced reaction. The initial enthalpy is -376 kJ while the final enthalpy is -470 kJ(note O_2 in the final state contributes zero enthalpy). Hence the heat of the reaction is -94 kJ which means for every mole of hydrogen peroxide the reaction gives out 47 kJ of heat.

Finally, let us consider reactions involving the ion OH^- . Clearly a reaction of the type $H_2 + O_2 \rightarrow 2OH^-$ is nonsensical as electrical charge will not be conserved in such a reaction. What is a correct reaction is $2H_2 + O_2 \rightarrow 2OH^- + H_2$. This reaction is exothermic with 230 kJ of heat given out for every mole of OH^- .

Problem 2.6 Consider a Carnot cycle operating with an ideal gas of constant specific heats C_V, C_P . The cycle starts at P_0, V_0 and goes through the following stages:an isothermal expansion to $2V_0$, a subsequent adiabatic expansion to $4V_0$, an isothermal compression to such a volume that an adiabat can connect this third state to the original state, and finally an adiabatic compression to the original state. Separately calculate the total heat given out, and the total work done during the cycle. Are they equal? Why?

This may appear to be a somewhat contrived problem, but it is designed to show the workings of a Carnot cycle in a nontrivial way. Let the starting configuration A be (P_0, V_0, T_0) . Assuming there are n moles of the ideal gas, we should have $T_0 = P_0V_0/nR$. Specifying the specific heats is just a red herring to mislead the reader. As long as we can directly calculate various $\Delta Q, \Delta W$, as we can indeed do in this example, there is no need to know the specific heats individually. However, their ratio $\gamma = C_P/C_V$ is relevant.

First stage: in this stage, which is an *isothermal change*, the temperature remains at T_0 but volume doubles to $2V_0$. Consequently pressure drops to $P_0/2$ leading to the configuration B with parameters $(P_0/2, 2V_0, T_0)$. Since

the transformation is isothermal, and the internal energy U for ideal gases only depends on T , there is no change of internal energy during this stage i.e. $\Delta_1 U = 0$. Therefore, work done and heat absorbed are related by $\Delta_1 W = \Delta_1 Q$. The work done is given by

$$\Delta_1 W = \int_{V_0}^{2V_0} P dV = nRT_0 \int_{V_0}^{2V_0} \frac{dV}{V} = nRT_0 \ln 2 > 0 \quad (2.11)$$

From what we said above, $\Delta_1 Q = nRT_0 \ln 2 > 0$.

Second stage: This is an adiabatic change and hence $\Delta_2 Q = 0$. Neither the internal energy change $\Delta_2 U$ nor the work done $\Delta_2 W$ vanish (in fact they are equal and opposite). But we are required only to calculate $\Delta_2 W$. Because of adiabaticity, $PV^\gamma = K$ where K is a constant which in this case takes the value $K = (2V_0)^\gamma P_0/2$. Since the final volume is $4V_0$, it is easy to see that the pressure at the end of this stage is $2^{-1-\gamma}P_0$. The final temperature can be worked out using the ideal gas law and turns out to be $2^{1-\gamma}T_0$. Thus the end point of the second stage is the state C parametrised by $2^{-1-\gamma}P_0, 4V_0, 2^{1-\gamma}T_0$. The work done during this stage is easily calculated:

$$\Delta_2 W = \int_{2V_0}^{4V_0} K \frac{dV}{V^\gamma} = \frac{K}{1-\gamma} ((4V_0)^{1-\gamma} - (2V_0)^{1-\gamma}) = \frac{nRT_0}{1-\gamma} (2^{1-\gamma} - 1) < 0 \quad (2.12)$$

The last step follows on substituting the value of K and using the ideal gas law.

Third stage: This stage is again isothermal, but now at the temperature $2^{1-\gamma}T_0$. As before, there is no change of internal energy during this stage i.e. $\Delta_3 U = 0$ and consequently $\Delta_3 Q = \Delta_3 W$. But the volume V_3 at the end of this stage needs to be carefully evaluated. The pressure P_3 at the end of this stage follows from ideal gas law to be $P_3 = (2^{1-\gamma}P_0V_0)/V_3$. But this state with $(P_3, V_3, 2^{1-\gamma}T_0)$ is stated to lie on the adiabat passing through the initial state P_0, V_0, T_0 . Hence

$$P_0V_0^\gamma = P_3V_3^\gamma = \frac{P_0V_02^{1-\gamma}}{V_3}V_3^\gamma \rightarrow V_3 = 2V_0 \quad (2.13)$$

Thus the state of the system after the third stage is D with parameters $(2^{-\gamma}P_0, 2V_0, 2^{1-\gamma}T_0)$. The work done during this stage can be calculated as before and the result is

$$\Delta_3 W = -nRT_0 2^{1-\gamma} \ln 2 < 0 \quad (2.14)$$

Also, $\Delta_3 Q = \Delta_3 W$.

Last stage: In this stage the system adiabatically returns from the state D to its initial state. Obviously $\Delta_4 Q = 0$, and $\Delta_4 W$ is calculated, exactly as in stage 2, to be

$$\Delta_4 W = \frac{nRT_0}{1-\gamma} (1 - 2^{1-\gamma}) > 0 \quad (2.15)$$

It is clearly seen that $\Delta_2 W + \Delta_4 W = 0 = \Delta_2 Q + \Delta_4 Q$. Since heat and work are equal to each other during the stages 1 and 3, the total work and total heat absorbed must match. Explicitly

$$\Delta_{tot} W = \Delta_1 W + \Delta_2 W + \Delta_3 W + \Delta_4 W = nRT_0(1 - 2^{1-\gamma}) \ln 2 = \Delta_{tot} Q \quad (2.16)$$

As the totality of changes represents a cycle i.e the system returns to its original state after all the stages, $\Delta_{tot} U = 0$, and first law demands the equality of total work done and total heat absorbed.

Problem 2.7 In the Clement-Desormes for measuring the ratio γ of specific heats for an ideal gas, one starts with the gas at some initial pressure P_1 and temperature T_1 in a container which is then allowed to adiabatically decompress to a pressure P_2 and temperature T_2 by quickly opening and closing a valve. The gas is then heated at constant volume till it reaches the original temperature T_1 , but at a different pressure P_3 . Show how γ can be determined from a knowledge of P_1, P_2, P_3 . What fraction of the gas was lost to outside during the adiabatic decompression?

Let the volume of the vessel containing the gas at the initial pressure P_1 at temperature T_1 be V_1 . Let us consider the gas to be ideal with a constant ratio γ of specific heats. Under the adiabatic decompression, or equivalently rearefaction, its pressure drops to $P_2 < P_1$. The adiabaticity condition being $PV^\gamma = \text{const.}$, the volume of the gas immediately after decompression is $V_2 = (P_1/P_2)^{1/\gamma} V_1 > V_1$. The temperature of the gas after this step is $T_2 = (P_1/P_2)^{(1-\gamma)/\gamma} < T_1$.

In the original Clement-Desormes method, the gas is sealed inside the container after letting $V_2 - V_1$ to escape to the outside. Therefore the fraction of the gas lost to the outside is

$$\frac{V_2 - V_1}{V_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} - 1 \quad (2.17)$$

After this first step of adiabatic decompression, the system characterised by (P_2, V_1, T_2) is heated at constant volume till it reaches the original temperature T_1 . Thus the final pressure P_3 is given by

$$P_3 = P_2 \frac{T_1}{T_2} = P_2 \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}} \rightarrow \frac{P_3}{P_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} \quad (2.18)$$

Thus γ can be determined as

$$\gamma = \frac{\ln \frac{P_2}{P_1}}{\ln \frac{P_3}{P_1}} \quad (2.19)$$

From a knowledge of the three pressures P_1, P_2, P_3 , the ratio of specific heats γ can be determined.

Problem 2.8 Show that any two points on the P-V plane can be connected by a combination of an isochore and an adiabat of the type PV^γ (note that the system need not be an ideal gas). If the heat Q discharged by a system during isochoric compression from P_i to P_f is given by $A(P_f - P_i)$, calculate the internal energy difference $U(P, V) - U(P_0, V_0)$ for arbitrary values of P_0, V_0, P, V (Callen)

This problem can be handled in two different ways. Let the point P_0, V_0 in the P-V plane be called A, and the point P, V by B. In the first method, connect the point A to an intermediate point C whose coordinates are (P', V) i.e a point whose volume is the same as that of B, along an *adiabat*. Therefore $P' = P_0(V_0/V)^\gamma = P_0 r^{-\gamma}$. The equation for the adiabat is given to be $PV^\gamma = K$ with K as a constant. Such an equation is also obeyed by black body radiation, for example. In fact, in the text the reader can find more examples. Since for an adiabat $\Delta Q = 0$, it follows from first law that $\Delta_{adia}U = -\Delta_{adia}W$. Hence

$$\Delta_{idea}U = -\Delta_{idea}W = -\int_{V_i}^{V_f} \frac{K}{V^\gamma} dV = -\frac{K}{1-\gamma}(V_f^{1-\gamma} - V_i^{1-\gamma}) \quad (2.20)$$

The constant K can be replaced by either $P_i V_i^\gamma$ or $P_f V_f^\gamma$ depending on the context. This will be explicitly illustrated in this problem. Using this for the change from A to C, and using $K = P_0 V_0^\gamma$, one finds

$$U(P', V) - U(P_0, V_0) = -\frac{P_0 V_0^\gamma}{1-\gamma}(V^{1-\gamma} - V_0^{1-\gamma}) = -\frac{P_0 V_0}{1-\gamma}(r^{1-\gamma} - 1) \quad (2.21)$$

Here $r = V/V_0$. Now let us connect C to B by an isochore (same volume). We chose the volume of C to be the same as that of B precisely so we can do this. The change in U during this step is given to be

$$U(P, V) - U(P', V) = A(V)(P - P') = A(V)(P - P_0 r^{-\gamma}) \quad (2.22)$$

Adding the last two equations one gets

$$U(P, V) - U(P_0, V_0) = A(V)(P - P_0 r^{-\gamma}) - \frac{P_0 V_0}{1-\gamma}(r^{1-\gamma} - 1) \quad (2.23)$$

The second way to solve this problem, connect the state A to an intermediate state D along an *isochore* instead of by an adiabat as in the first method. The coordinates of D are then (P^*, V_0) where P^* is the pressure required to put D along an adiabat passing through B i.e $P^* = Pr^\gamma$. Same reasonings as before give

$$U(P, V) - U(P_0, V_0) = A(V_0)(Pr^\gamma - P_0) - \frac{PV}{1-\gamma}(1 - r^{\gamma-1}) \quad (2.24)$$

Superficially this answer looks different from the previous one. The reader is encouraged to show their equivalence.

Problem 2.9 Consider the air in a room, which is not airtight, being isobarically heated to a higher temperature. If air is treated as an ideal gas, show that the total internal energy of the air within the room does not change despite the heating. Since the air escaping from the room goes to merely heat the outside and hence that amount of heat is wasted, is there still any benefit to this way of heating?

First let us consider an infinitesimal amount of heating. Let the initial parameters be (P, T, V) , and let there be n moles of the ideal gas initially. Actually it is necessary to further assume that C_V is constant (remember that even for an ideal gas $C_V(T)$ can be a function of T). As a result of raising the temperature by ΔT due to heating, the volume increases by $\Delta V = (nR\Delta T)/P$. The fact that pressure does not change during heating has been made use of.

If the room is airtight, the internal energy has to increase as a result of heating. Since the room is not airtight, as the volume tends to increase due to heating, the excess volume leaks out. This has the effect of *reducing* the number of moles n . If Δn is such a decrease, then $\Delta n = (n\Delta T)/T$. The last remark simply follows from the fact that to this approximation $(\delta n/n) = (\Delta V/V)$.

The increase in U due to the change in temperature by ΔT is $nC_V\Delta T$ while the decrease in U due to reduction in number of moles is ΔnC_VT , and it is easy to see that the two effects exactly cancel. This means that the internal energy of the *enclosed* gas does not change at all!

Let us now look at the same problem without restricting to small changes. Let the parameters *before* heating be P, V, T, n and the parameters *after* heating be P, V', T', n' . The total internal energy before heating is $U = nC_VT$ while that after heating is $U' = n'C_VT'$. We shall assume that the volume before heating was just the volume of the enclosure. Let the number of moles left after leakage to outside is n^* . Then the internal energy of the air enclosed after heating is $U^* = n^*C_VT'$.

Applying ideal gas to the *enclosed* gas before and after, $PV = nRT$ and $PV = n^*RT'$. From this it follows that $U = U^*$. It is also clear that this would not have been the case if C_V were T -dependent.

Even though the leakage causes heat to be wasted to outside, the temperature of the interior has indeed increased and therefore this type of heating is still sensible.

Problem 2.10 A medium size iceberg weighs about 100,000 metric tons. If the energy received from the Sun is 2 cal/sq.cm in a minute, how long will it take to completely melt such an iceberg if all the solar energy incident on 1 square km is used for it? The latent heat of fusion of ice is 80 cal/g. Considering that the cross-sectional area of such icebergs is about 1000 square metres, how long will an iceberg last in its journey?

The mass of the iceberg in kg's is 10^8 kgs. The latent heat being 80 cal/g, it would take $80 \cdot 10^8$ kcal to completely melt all of the iceberg. The solar constant being 2 cal/sqcm/min means that in one minute the heat received is $2 \cdot 10^4 \text{ cal/m}^2$ or 20 kcal per square meters. The total heat received in one square km is therefore $10^6 \cdot 20 = 2 \cdot 10^7$ kcal in one minute. This is much less than the total heat of $8 \cdot 10^9$ kcals required to completely melt the iceberg. Therefore in one minute the fraction of the iceberg melted by solar heat collected over 1 square kilometer is 1/400. In other words it would take such a heat 400 minutes or 6h 40m to melt the iceberg.

Since the cross-sectional area is only 1000 sq.m, the complete melting of the iceberg will take 1000 times longer, which works out to be approximately 278 days.

Problem 2.11 What fraction of ice will still remain after 1 Kg of ice has been supplied with 200 kJ of heat(all at 273 K)?

This problem is exactlt in the same spirit as the previous problem. Only note the units used! Since one cal is 4.18 J, the latent heat is $80 \cdot 4.18 \text{ kJ/kg}$. This works out to approximately 335 kJ/kg. Hence the supplied heat of 200 kJ is not enough to melt 1 kg of ice. Instead the fraction that is melted is 200/335. The fraction of ice remaining is therefore 135/335.

Problem 2.12 The specific heat of a solid substance near absolute zero has been found to vary with temperature as $C_V(T) = 2.0 T e^{-3.0T}$ J/mol.K. How much heat will be needed to raise the temperature of 1 mol of this substance from 0 K to 10 K?

Since the substance is a solid, effects of changes in volume can be neglected to a good approximation. The amount of heat required to raise the temperature by dT is $dQ = nC_V(T)dT$ (in this case it is legitimate to represent the heat change as a perfect differential). Thereore, the heat required in Joules(J) is

$$\begin{aligned} \Delta Q &= \int_0^{10} 2.0 T e^{-3.0T} dT = 2.0 \left[\left(-\frac{T}{3} - 1/9 \right) e^{-3T} \right]_0^{10} \\ &= 2.0 \left[-\frac{31}{9} e^{-30} + \frac{1}{9} \right] \approx 0.22 \quad (2.25) \end{aligned}$$