QUESTIONS AND SOLUTIONS: CHAPTER 2

The shearbox test

Q2.1 Describe with the aid of a diagram the essential features of the conventional shearbox apparatus. Stating clearly the assumptions you need to make, show how the quantities measured during the test are related to the stresses and strains in the soil sample.

Q2.1 Solution
Diagram of shear box: See main text Figure 2.14

Assume that the stresses and strains are uniform and continuous, and that the actual deformation in shear (main text Figure 2.15a) is idealised as indicated in main text Figure 2.15b.

The known or measured quantities are

A	*the sample area on plan, assumed to remain constant during the test)*
H	*the initial height of the sample*
N	*the normal (hanger) load*
F	*the shear force*
x	*the relative horizontal displacement between the upper and lower halves of the shearbox*
y	*the upward movement of the shearbox lid.*

Consideration of main text Figure 2.15b gives strains
<u>*shear strain* $\gamma = x/H$</u>
<u>*volumetric strain* $\varepsilon_{vol} = -y/H$</u>

In terms of stresses,
<u>*shear stress on central horizontal plane* $\tau = F/A$</u>
<u>*normal stress on central horizontal plane* $\sigma = N/A$</u>

If it is further assumed that the pore water pressure u is zero (so that $\sigma' = \sigma$) and the central horizontal plane is the plane of maximum stress obliquity $(\tau/\sigma')_{max}$, a Mohr circle of stress may be drawn (eg main text Figure 2.30), and the mobilised effective angle of friction is

<u>$\phi'_{mob} = \tan^{-1}\{(\tau/\sigma')_{max}\}$</u>

Q2.2 With the aid of sketches, describe, explain and contrast the results you would expect to obtain from conventional shearbox tests on samples of dry sand which were (a) initially loose, and (b) initially dense. What factors would you take into account in selecting a soil strength parameter for use in design?

Q2.2 Solution

Typical graphs of (a) shear stress τ against shear strain γ; (b) volumetric strain ε_{vol} against shear strain γ; and (c) specific volume v against shear strain γ are as shown in main text Figure 2.21.

In the test carried out on the initially dense sample, the shear stress gradually increases with shear strain to a peak at P, before falling to a steady value at C which is maintained as the shear strain is increased. The sample may undergo a very small compression at the start of shear, but then begins to dilate. The curve of ε_{vol} vs γ becomes steeper, indicating that the rate of dilation $-d\varepsilon_{vol}/d\gamma$ is increasing. The slope of the curve reaches a maximum at p, but with continued shear strain the curve becomes less steep until at c it is horizontal. When the curve is horizontal $d\varepsilon_{vol}/d\gamma$ is zero, indicating that dilation has ceased. The peak shear stress at P coincides with the maximum rate of dilation at p. The steady state shear stress at C corresponds to the achievement of the critical specific volume at c.

The initially loose sample displays no peak strength, but eventually reaches the same critical shear stress as the first sample. The second sample does not dilate, but gradually compresses during shear until the same critical specific volume is reached (i.e. the volumetric strain remains constant).

In both cases, a critical state, is reached in which the soil continues to shear at constant specific volume, constant shear stress and constant normal effective stress.

A dense sample displays a peak strength because additional work has to be done to overcome the effect of the initially high degree of interlocking – high, that is, relative to the equilibrium specific volume for continued shear at the vertical effective stress at which the test is carried out. The initial dense packing means that the particles are forced to "ride up" over each other ($\Rightarrow$ dilation) for deformation to occur (see the "saw blades analogy", Figure 2.24).

In design, it may be safer to use the critical state strength ϕ'_{crit} than the peak strength ϕ'_{peak}, because

- *the peak strength depends on the extent to which the soil is dense in relation to the critical state under the effective stress conditions at failure. It is not a soil constant, and is unlikely to be the same throughout the mass of soil involved in a potential failure mechanism*
- *it is unlikely that the peak strength will be mobilised simultaneously throughout the soil mass; instead, progressive failure at an average strength rather lower than the peak may occur.*

However, the factors of safety used in many traditional methods of design may well allow for these possibilities, and their use in connection with the critical state strength could lead to overconservatism.

Development of a critical state model

Q2.3 Mining operations frequently generate large quantities of fine, particulate waste known as tailings. Tailings are generally transported as slurries, and stored in reservoirs impounded by embankments or dams made up from the material itself. In order to investigate the geotechnical behaviour of a particular tailings material (G_s=2.70), an engineer carried out three slow, drained shear tests - each over a period of one day - and three fast, undrained shear tests - each over a period of two minutes - in a conventional 60mm × 60 mm shearbox apparatus.

The three samples in each group were initially allowed to come into drained equilibrium under the application of vertical hanger loads of 100 N, 200 N and 300 N. During each shear test, the hanger load was kept constant and the ultimate shear force F_{ult} recorded. Immediately after each test, a water content sample was taken from the centre of the rupture zone. All of the samples were initially saturated, and all of the tests were carried out with the sample under water in the shearbox.

The test results are summarised in table 2.8. Use the results of the drained tests to construct a critical state model in terms of the normal effective stress σ' and shear stress τ on the shear plane, and the specific volume v. Give the values of ϕ'_{crit}, v_o and λ. Deduce a relationship between the undrained shear strength τ_u and the normal effective stress at the start of the test, and compare its predictions with the experimental data from the undrained tests.

Table 2.8: Shearbox test data, Q2.3

Test type	Vertical load V, N	Shear load F_{ult}, N	Water content w, %
slow, drained	100	53	35.1
	200	105	31.3
	300	156	29.5
fast, undrained	100	42	36.0
	200	80	32.6
	300	120	30.6

Q2.3 Solution

The critical state model must be constructed using the drained test data only, because only in these tests do we know that the pore water pressure u = 0 and that the vertical effective stress σ' is equal to the normal load divided by the sample area. We must assume that the data given for the slow tests were measured at true critical states.

For each sample,
the normal effective stress $\sigma' = V\ (kN)/A\ (m^2)$
the ultimate shear stress $t_{ult} = F_{ult}\ (kN)/A\ (m^2)$

and the specific volume v may be calculated from the water content w using main text Equation 1.10 with S_r*=1,*

$v = 1 + w.G_s$ *(main text Equation 2.12)*

Vertical load V, N	***normal effective stress σ', kPa***	***ln(σ')***	*Shear load F_{ult}, N*	***Shear stress τ_{ult}, kPa***	*Water content w, %*	***Specific volume v***
100	***27.8***	***3.325***	*53*	***14.7***	*35.1*	***1.95***
200	***55.6***	***4.018***	*105*	***29.2***	*31.3*	***1.85***
300	***83.3***	***4.422***	*156*	***43.3***	*29.5*	***1.80***

Plot graphs of τ_{ult} against σ' and v against lnσ' to determine the critical state parameters, as in main text Figure 2.28 (Example 2.2).

$\phi'_{crit} \approx 28°$; $v_o \approx 2.43$; $\lambda \approx 0.14$

During the undrained tests, there is no overall volume change. Assuming that the specific volume is uniform throughout the sample, it must remain constant during the test. The critical state eventually reached therefore depends on the as-tested specific volume. Our model predicts that, at the critical state, the vertical effective stress σ' is related to the specific volume by the expression

$v = v_o - \lambda.\ln\sigma'$ *(main text Equation 2.11)*

or

$\sigma' = \exp\{(v_o-v)/\lambda\}$

The normal effective stress at the critical state is related to the shear stress τ_{ult} by the expression

$\tau_{ult} = \sigma'.\tan\phi'_{crit}$ *(main text Equation 2.10)*

Hence

$\tau_{ult} = \exp\{(v_o-v)/\lambda\}.\tan\phi'_{crit}$

where $v = 1 + w.G_s$. The calculated and measured values of τ_{ult} for the undrained tests are compared below:

Vertical load V, N	***normal effective stress σ', kPa***	*Shear load F_{ult}, N*	***Measured shear stress τ_{ult}, kPa***	*Water content w, %*	***Specific volume v***	***Calculated shear stress, τ_{ult} kPa***
100	***27.8***	*42*	***11.7***	*36.0*	***1.972***	***14.0***
200	***55.6***	*80*	***22.2***	*32.6*	***1.880***	***27.0***
300	***83.3***	*120*	***33.3***	*20.6*	***1.826***	***39.8***

The measured values are smaller than the theoretical values by about 16%. This is probably due to internal drainage and discontinuous sample behaviour.

Determination of peak strengths

Q2.4 Table 2.9 gives results obtained from a shearbox test on a 60 mm × 60 mm sample of dry sand of unit weight 18 kN/m^3.

Table 2.9: Shearbox test data, Q2.4

	Reading on proving ring deflexion dial gauge (divisions)
Zero force	91
Peak shear force for a hanger load of 3kg	128
Peak shear force for a hanger load of 10kg	162
Peak shear force for a hanger load of 20kg	210

One division on the proving ring dial gauge corresponds to a force of 1.1N across the proving ring.

(a) Plot the data on a graph of shear stress against normal effective stress, and sketch the peak strength failure envelope.

(b) What is the peak resistance to shear on a horizontal plane at a depth of 3 m below the top of a dry embankment made from this soil?

(c) A model of the embankment is constructed from the same sand at a scale of 1:10. What is the peak resistance to shear on a horizontal plane at a depth of 300mm below the top of the model?

(d) Would you expect the model to behave in the same way as the real embankment?

Q2.4 Solution
(a) The normal stress on the sample is given by the hanger load (kg) × 9.81 (N/kg) ÷ the sample area, 0.06m × 0.06m = 3.6×10^{-3}m^2, ÷ 1000 to convert from Pa to kPa.

The shear force on the sample is given by 1.1 × (the number of proving ring dial divisions - the number of divisions at zero load), i.e. 1.1 × (n - 91). To convert this to the shear stress, it is necessary to divide the shear force by the area of the sample, 0.06m × 0.06m = 3.6×10^{-3}m^2, and divide by 1000 to convert from Pa to kPa.

Hanger load, kg	*Normal stress, kPa*	*Peak shear load, N*	*Peak shear stress, kPa*
3	*8.175*	*40.7*	*11.31*
10	*27.25*	*78.1*	*21.69*
20	*54.5*	*130.9*	*36.36*

These data are plotted on a graph of τ against σ' in Figure Q2.4. The peak strength failure envelope is highly non-linear, with $\phi'_{peak} = 55°$ at $\sigma' \approx 8$ kPa, falling to $\phi'_{peak} = 34°$ at $\sigma' \approx$ 55 kPa

(b) At a depth of 3m below the top of a dry embankment made of this sand, the vertical effective stress is 3m×18kN/m^3 = 54kPa. This corresponds to a hanger load of 20kg, at which the peak shear stress is approximately 36.4 kPa

(c) In the 1:10 scale model, the vertical effective stress at a depth of 300mm is about 0.3m × 18kN/m3 = 5.4 kPa. From Figure Q2.4, this gives a peak shear resistance of approximately 7.7kPa

(d) The model would not be expected to behave in the same way as the real embankment, because the operational values of ϕ'_{peak} at corresponding depths in the model and the real embankment are quite different.

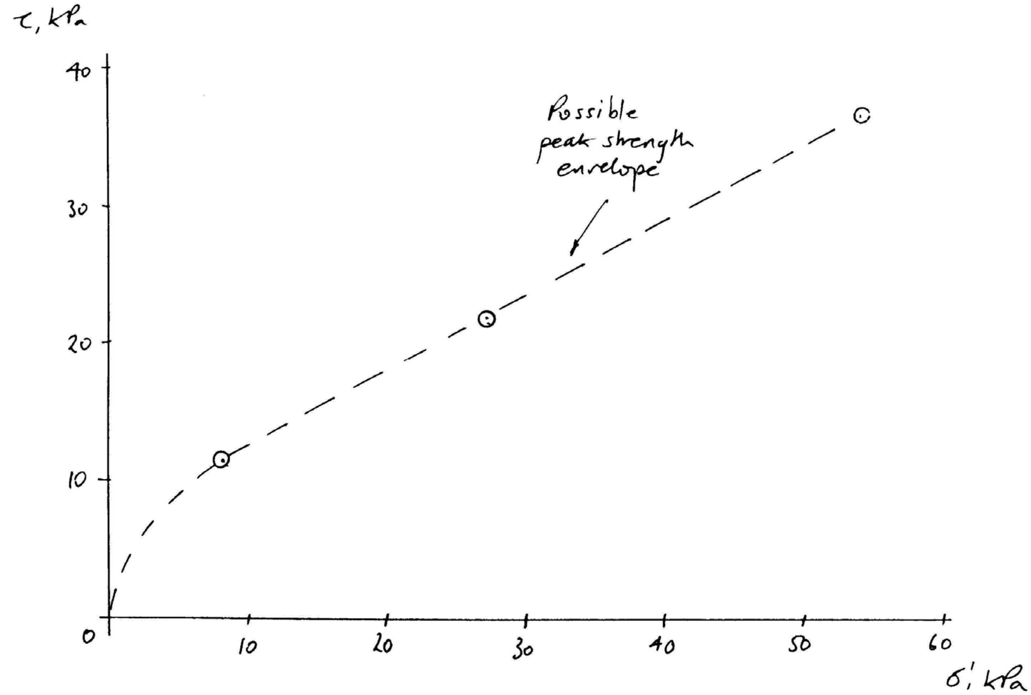

Figure Q2.4: Shear stress against normal effective stress at peak, Q2.4

Use of strength data to calculate friction pile load capacity

Q2.5 A friction pile, 300 mm in diameter, is driven to a depth of 25 m in dense sand of unit weight 19 kN/m^3. The ratio of horizontal to vertical effective stresses is 0.5. The angle of friction between the pile and the sand is 26° and the resistance offered at the base of the pile may be ignored. The natural water table, below which the pore water pressures are hydrostatic, is 5m below ground level. During construction works, the water table is temporarily lowered to a depth of 16m by pumping from wells. A load test on the pile is carried out while pumping to lower the groundwater level is still in progress. Calculate the ultimate load capacity of the pile (a) observed in the test, and (b) after pumping from the wells has stopped, and the water table has recovered to its natural level.

Q2.5 Solution

The vertical total stress σ_v, the pore water pressure u and the vertical (σ'_v) and horizontal (σ'_h) effective stresses all vary linearly with depth between the soil surface and the water table, and between the water table and the base of the pile.

In general at depth z, with the water table at a depth h,

$\sigma_v = \gamma . z;$

u = 0 above the water table ($z \le h$)

$u = \gamma_w .(z - h)$ below the water table ($z \ge h$)

$\sigma'_v = \sigma_v - u$

$\sigma'_h = 0.5 \times \sigma'_v$

shear stress on pile $\tau = \sigma'_h \times \tan 26°$

(a) With the water table depth h = 16m. $\gamma = 19\ kN/m^3$ and $\gamma_w = 9.81\ kN/m^3$, the following relationship between shear stress τ and depth z is calculated:

	z, m	σ_v, *kPa*	*u, kPa*	σ'_v, *kPa*	σ'_h, *kPa*	τ, *kPa*
At the soil surface	*0*	*0*	*0*	*0*	*0*	*0*
At the water table	*16*	*304*	*0*	*304*	*152*	*74.14*
At the base of the pile	*25*	*475*	*88.29*	*386.71*	*193.36*	*94.31*

The frictional resistance to pile movement is given by integrating the shear stress τ over the surface area of the pile. The surface area of the upper 16m of the pile is $(\pi \times 0.3)m \times 16m = 15.08m^2$, and the average shear stress over this area is $74.14kPa \div 2 = 37.07kPa$. The surface area of the lower 9m of the pile is $(\pi \times 0.3)m \times 9m = 8.48m^2$, and the average shear stress over this area is $(74.14kPa + 94.31kPa) \div 2 = 84.23kPa$. Thus the overall frictional resistance is

$(15.08m^2 \times 37.07kPa) + (8.48m^2 \times 84.23kPa) = \underline{1273kN}$

(b) With the water table depth h = 5m. $\gamma = 19\ kN/m^3$ and $\gamma_w = 9.81\ kN/m^3$:

	z, m	σ_v, kPa	u, kPa	σ'_v, kPa	σ'_h, kPa	τ, kPa
At the soil surface	0	0	0	0	0	0
At the water table	5	95	0	95	47.5	23.17
At the base of the pile	25	475	196.2	278.8	139.4	67.99

The surface area of the upper 5m of the pile is $(\pi \times 0.3)\text{m} \times 5\text{m} = 4.71\text{m}^2$, *and the average shear stress over this area is* $23.17\text{kPa} \div 2 = 11.59\text{kPa}$. *The surface area of the lower 20m of the pile is* $(\pi \times 0.3)\text{m} \times 9\text{m} = 18.85\text{m}^2$, *and the average shear stress over this area is* $(23.17\text{kPa} + 67.99\text{kPa}) \div 2 = 45.58\text{kPa}$. *Thus the overall frictional resistance is*

$$(4.71\text{m}^2 \times 11.59\text{kPa}) + (18.85\text{m}^2 \times 45.58\text{kPa}) = \underline{914\text{kN}}$$

Q2.6 The depth of the friction uplift pile described in main text Example 2.4 is increased to 20m, where the undrained shear strength of the clay is 40 kPa. Calculate the short- and long-term uplift resistance of the 20m pile.

Q2.6 Solution

The total shear resistance of the clay/pile interface is given by

$$T = \text{average shear stress} \times \text{surface area of pile}$$

(a) In the short term, the average shear stress is the average undrained shear strength on the interface, so that

$$T = [(0 + 40\text{kPa}) \div 2] \times [(\pi \times 0.5\text{m}) \times 20\text{m}] = \underline{628\text{ kN}}$$

(b) In the long term, the ultimate shear stress on the interface is given by

$$\tau_{ult} = \sigma'_h . \tan\delta$$

where $\sigma'_h = 0.5 \times \sigma'_v$ *is the horizontal effective stress and* δ *is the angle of friction between the clay and the pile*

At a depth z,

$$\sigma_v\ (\text{kPa}) = \{\gamma (\text{kN/m}^3) \times z\ (\text{m})\} = \{18\ (\text{kN/m}^3) \times z\ (\text{m})\}$$

$$u\ (\text{kPa}) = \{\gamma_w\ (\text{kN/m}^3) \times z\ (\text{m})\} = \{9.81\ (\text{kN/m}^3) \times z\ (\text{m})\}, \text{ and}$$

$$\sigma'_v = \sigma_v - u$$

As in (a), $T = \text{average shear stress} \times \text{surface area of pile}$

The shear stress τ *on the soil/pile interface is now*

$0.5 \times \sigma'_v.\tan\delta$

which increases linearly from zero at the top of the pile to

$0.5 \times [(18\ kN/m^3 \times 20\ m) - (9.81\ kN/m^3 \times 20\ m)] \times \tan 20° = 29.81\ kPa$ at the base

Hence

$T = [(0 + 29.81\ kPa) \div 2] \times [(\pi \times 0.5\ m) \times 20\ m] = \underline{468\ kN}$

Stress analysis and interpretation of shearbox test data

Q2.7 A drained shearbox test was carried out on a sample of saturated sand. The normal effective stress of 41.67 kPa was constant throughout the test, and the initial sample dimensions were 60 mm × 60 mm on plan × 30 mm deep). In the vicinity of the peak shear stress, the data recorded were as shown in table 2.10.

Table 2.10: Shearbox test data, Q2.7

Shear stress τ, kPa	42.5	43.1	42.8
relative horizontal displacement x, mm	0.30	0.40	0.80
upward movement of shearbox lid y, mm	0.05	0.075	0.105

(a) Draw the Mohr circle of stress for the soil sample when the shear stress is a maximum, stating the assumption that you need to make. Determine ϕ'_{peak}, and the orientations of the planes of maximum stress ratio $(\tau/\sigma')_{max}$. Draw the Mohr circle of strain increment leading to the peak, and hence determine the maximum angle of dilation, ψ_{max}. Use an empirical relationship between ϕ'_{peak},ψ_{max} and ϕ'_{crit} to estimate the critical state friction angle, ϕ'_{crit}.

(b) Three further drained tests on similar samples of the same soil were carried out, at different normal effective stresses. The peak and critical state shear stresses were:

Normal effective stress, kPa	20	100	200
Peak shear stress, kPa	23.8	83.9	132.0
Critical state shear stress, kPa	12.6	63.2	126.4

For all four tests, plot the peak and critical state shear stresses τ_{peak} and τ_{crit} as a function of the normal effective stress σ'. Sketch failure envelopes for both peak and critical states, and comment briefly on their shapes. Which would you use for design, and why?

Q2.7 Solution
(a) At τ_{max} (= 43.1 kPa), $\phi'_{peak} = \tan^{-1}\{(\tau/\sigma')_{max}\} = \tan^{-1}(43.1/41.67) = \underline{46°}$

assuming that the central horizontal plane is a plane of maximum stress ratio. The Mohr circle of stress is shown in Figure Q2.7a.

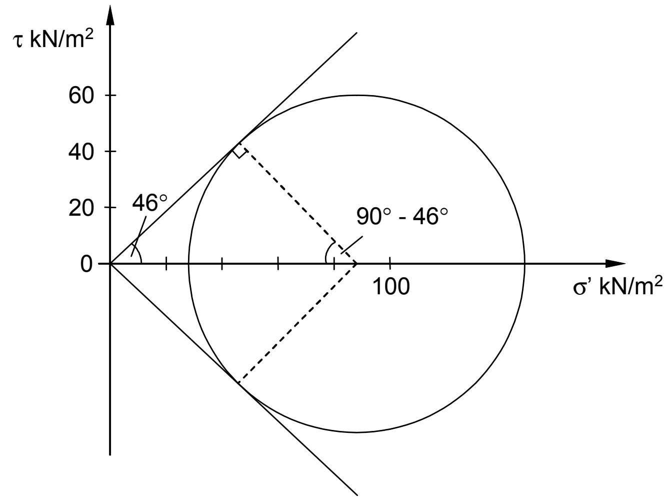

Figure Q2.7a: Mohr circle of stress, Q2.7

The first plane of maximum stress ratio is horizontal (this is an assumption that has to be made to draw the Mohr circle of stress). From Figure Q2.7a, the second plane of maximum stress ratio is at $(90° - \phi'_{peak}) = (90° - 46°) = 44°$ to the horizontal, either clockwise or anticlockwise depending on whether the shear stress on the horizontal plane plots positive or negative. (Note: the answer given in the main text is slightly ambiguous here. The planes of maximum stress ratio are horizontal and either + or - 44° to the horizontal and not, as might be interpreted from the answer given in the main text, + and - 44° to the horizontal).

The increments of shear ($\Delta\gamma$) and vertical ($\Delta\varepsilon_v$) strain leading up to peak are given by

$\Delta\varepsilon_v = \Delta y/H = 0.025/30 = 0.083\%$, *and*

$\Delta\gamma = \Delta x/H = 0.1/30 = 0.333\%$

where Δx and Δy are the incremental relative horizontal displacement of the two halves of the shearbox and the upward displacement of the shearbox lid respectively, and H = 30 mm ins the initial sample height. The increment of horizontal strain $\Delta\varepsilon_h = 0$. The Mohr circle of strain increment is shown in Figure Q2.7b, and is plotted with coordinates $(\Delta\varepsilon, \Delta\gamma/2) = (0.083\%, 0.167\%)$ for the strains associated with (normal to) the horizontal plane and (0, -0.167%) for the strains associated with (normal to) the vertical plane.

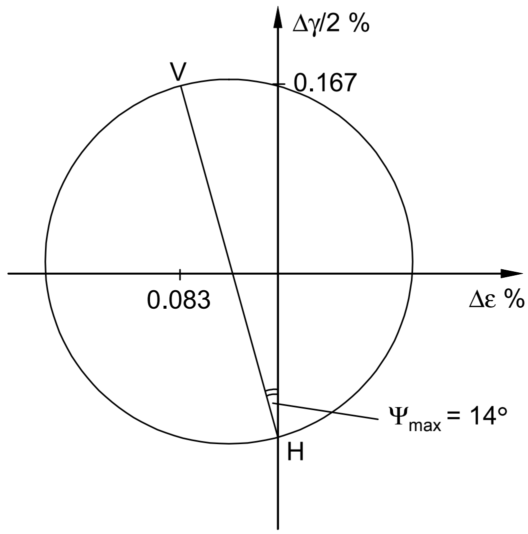

Figure Q2.7b: Mohr circle of strain increment leading up to peak, Q2.7

From Figure Q2.7b, the angle of dilation at peak is given by

$\psi_{max} = \Delta y/\Delta x = 2.5/10 \Rightarrow \underline{\psi_{max} = 14°}$

We might expect $\phi'_{crit} \sim \phi'_{peak} - 0.8 \times \psi_{max}$ *(main text Equation 2.14), giving*

$\phi'_{crit} \sim 46° - 11°$ *or* $\underline{\phi'_{crit} \sim 35°}$

(b) The data are plotted as τ_{peak} *and* τ_{crit} *against* σ' *in Figure Q2.7c.*

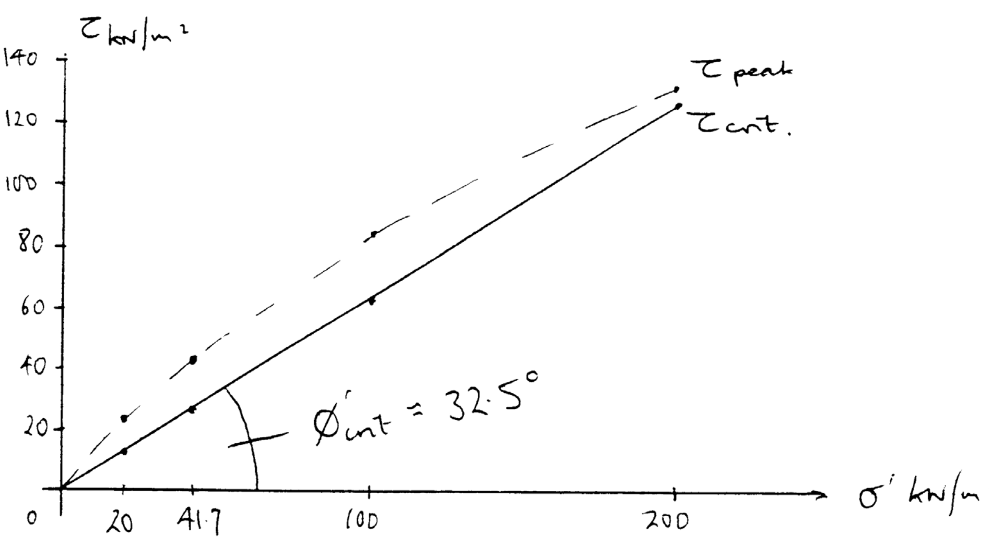

Figure Q2.7c: Failure envelopes in terms of peak and critical state strengths, Q2.7

The failure envelopes sketched in Figure Q2.7c show that

- ϕ'_{crit} *is constant (= 32.5°, closer to* $\phi'_{peak} - \psi_{max}$ *= 32° than the estimate of 35° based on* $\phi'_{peak} - 0.8 \times \psi_{max}$*) because there is no dilation at the critical state*
- ϕ'_{peak} *reduces as the normal effective stress* σ' *increases, because the amount of dilation needed to reach the appropriate (critical) specific volume is reduced.*

In design, it may be safer to use the critical state strength ϕ'_{crit} *than the peak strength* ϕ'*peak, because*

- *the peak strength depends on the extent to which the soil is dense in relation to the critical state under the effective stress conditions at failure. It is not a soil constant, and is unlikely to be the same throughout the mass of soil involved in a potential failure mechanism*
- *it is unlikely that the peak strength will be mobilised simultaneously throughout the soil mass; instead, progressive failure at an average strength rather lower than the peak may occur.*

However, the factors of safety used in may traditional methods of design may well allow for these possibilities, and their use in connection with the critical state strength could lead to overconservative design.

Q2.8 To investigate the drained strength of a natural silt containing thin clay laminations at a spacing of approximately 6 mm, an engineer carried out a series of shearbox tests. The clay laminations were inclined at various angles θ to the horizontal. With the laminations horizontal (θ = 0), the rupture formed entirely in the clay and the apparent angle of shearing resistance was 18°. With the laminations at an angle θ = 60°, the rupture formed entirely in the silt and the apparent angle of shearing resistance was 30°. Stating clearly the assumptions you need to make, construct Mohr circles of stress at failure for various values of apparent angle of shearing resistance, marking on each the stress state corresponding to the clay laminations. (Hint: the mobilized strength on the clay laminations must never exceed 18°). Plot a graph showing the relationship between the angle θ and the apparent angle of shearing resistance of the soil.

Q2.8 Solution

When θ *= 0, the shear plane forms in the clay so* ϕ'_{crit} *= 18° for the clay. When* θ *= 60°, the shear plane forms in the silt so* ϕ'_{crit} *= 30° for the silt.*

Assume that the sample behaves as a continuum up to rupture, and that the central horizontal plane of the shearbox is the plane of maximum and apparent stress ratio $(\tau/\sigma') = \tan\phi'_{apparent}$*. The easiest procedure is to construct Mohr circles of stress for apparent* ϕ' *values of 21°, 24°, 27° and 30° and deduce the corresponding orientation of the clay laminations such that the stress ratio on the laminations is* $(\tau/\sigma') = \tan 18°$ *. Each value of* $\phi'_{apparent}$ *will give four possible orientations of the clay laminations (*θ *measured clockwise from the horizontal), as indicated in Figure Q2.8a.*

Figure Q2.8a shows a general Mohr circle from which algebraic expressions for the orientations θ *(measured clockwise from the horizontal) of the yellow clay laminations to give the given value of* $\phi'_{apparent}$*. Remember that the rotation on the Mohr circle must be divided by 2 to give the actual rotation in the physical plane.*

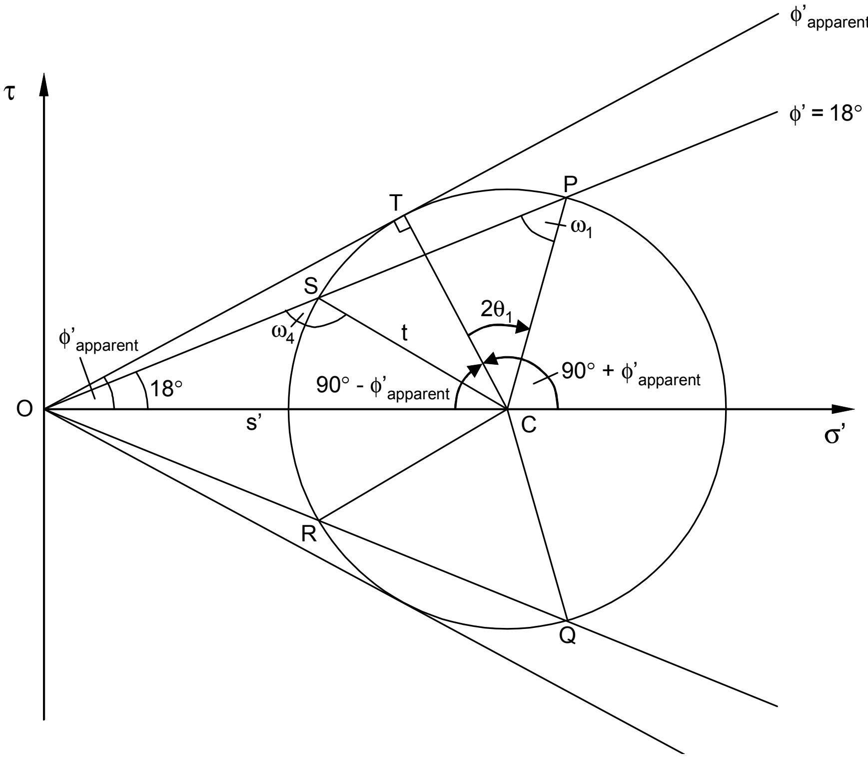

Figure Q2.8a: Mohr circle of stress, Q2.8

The orientations θ of the clay laminations are given by the angles clockwise from the horizontal plane θ_1, θ_2, θ_3 and θ_4, corresponding to the points P, Q, R and S respectively on Figure Q2.8a.

From triangle OTC, $t/s' = \sin\phi'_{apparent}$

From triangle OPC, angle OCP $= 180° - \omega_1 - 18°$ and angle OCP $= 2\theta_1 + (90° - \phi'_{apparent})$

Applying the sine rule to triangle OPC,

$s'/\sin\omega_1 = t/\sin 18° \Rightarrow \sin\omega_1 = \sin 18°/(t/s')$ or $\sin\omega_1 = \sin 18°/\sin\phi'_{apparent}$ (note ω_1 is acute, ie less than 90°)

Applying the sine rule to triangle OSC,

$s'/\sin\omega_4 = t/\sin 18° \Rightarrow \sin\omega_4 = \sin 18°/(t/s')$ or $\sin\omega_4 = \sin 18°/\sin\phi'_{apparent}$ (note ω_4 is obtuse, ie greater than 90°)

By considering the geometry of the Mohr circle shown in Figure Q2.8a, the values of θ_1 to θ_4 may be determined as follows.

$2\theta_1 = (90° + \phi'_{apparent}) - (\omega_1 + 18°) \Rightarrow \theta_1 = 0.5 \times (72° - \omega_1 + \phi'_{apparent})$

$2\theta_2 = (90° + \phi'_{apparent}) + (\omega_1 + 18°) \Rightarrow \theta_2 = 0.5 \times (108° + \omega_1 + \phi'_{apparent})$

$2\theta_3 = (90° + \phi'_{apparent}) + (\omega_4 + 18°) \Rightarrow \theta_3 = 0.5 \times (108° + \omega_4 + \phi'_{apparent})$

$2\theta_4 = (90° + \phi'_{apparent}) + (\omega_4 + 18°) + 2(180° - 18° - \omega_4) \Rightarrow \theta_4 = 0.5 \times (432° - \omega_4 + \phi'_{apparent})$

The values of ω_1, ω_4 and θ_1 to θ_4 for $\phi'_{apparent}$ = 21°, 24°, 27° and 30° are detailed below.

$\phi'_{apparent}$	ω_1	ω_4	θ_1	θ_2	θ_3	θ_4
21	59.57	120.43	16.72	94.29	124.72	166.28
24	49.44	130.56	23.28	90.72	131.28	162.72
27	42.90	137.10	28.05	88.98	136.05	160.95
30	38.12	141.83	31.94	88.06	139.92	160.01

These values are used to construct the graph of apparent angle of shearing resistance $\phi'_{apparent}$ against orientation of the clay laminations θ shown in Figure Q2.8b: note that for orientations of the laminations θ between 32° and 88°, and between 140° and 160°, the value of $\phi'_{apparent}$ is equal to ϕ' for the silt, 30°.

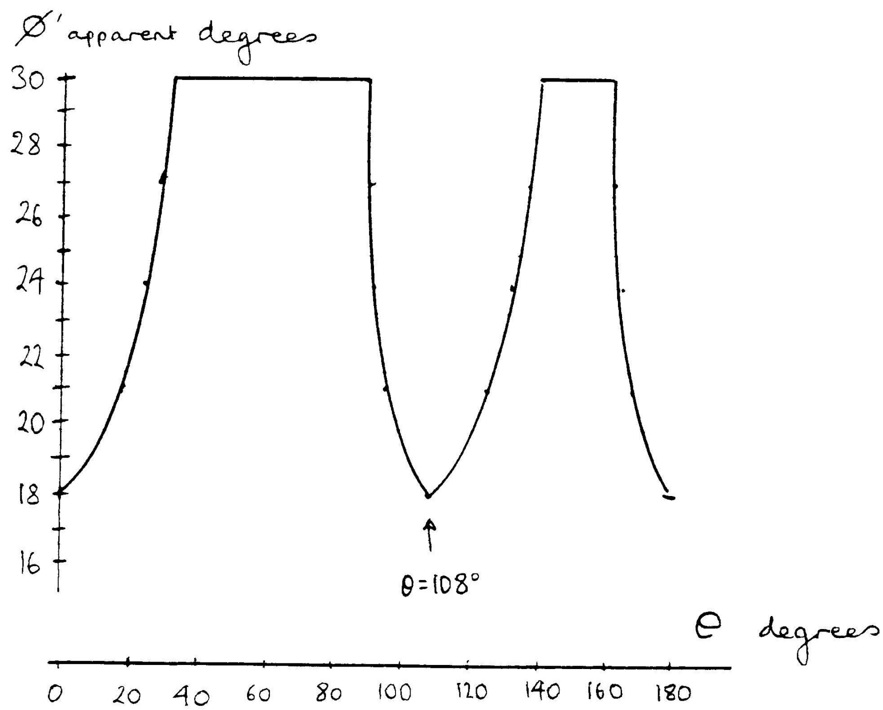

Figure Q2.8b: apparent effective angle of friction against angle of lamination inclination, Q2.8

Note that unless you are very confident with geometry and trigonometry, this problem is probably much more easily addressed by drawing out the four individual Mohr circles to scale and measuring off the angles θ_1 to θ_4. The principles, and hopefully the answers, are however the same.

QUESTIONS AND SOLUTIONS: CHAPTER 3

Laboratory measurement of permeability; fluidization; layered soils

Q3.1 Describe by means of an annotated diagram the principal features of a constant head permeameter. Give three reasons why this laboratory test might not lead to an accurate determination of the effective permeability of a large volume of soil in the ground. Suggest how each of these problems might be overcome.

Q3.1 Solution
Diagram of constant head permeameter: see main text Figure 3.8

Inaccurate determination of the in situ *permeability might result from*

a) *sample disturbance – unrepresentative void ratio of a uniform soil*
b) *sample disturbance – destruction of soil fabric e.g. in a soil with a layered structure*
c) *large scale inhomogeneities e.g. fissures and high permeability lenses, which cannot be represented in the small scale laboratory sample*
d) *low permeability of a soil with fine particles leads to inaccurate determination of flowrate due to evaporation losses and general measurement errors*

These can be overcome by

a) *testing recompacted samples at maximum and minimum achievable void ratio to give possible limits to the* in situ *permeability*
b) *& c) carrying out field pumping tests*
c) *using a falling head permeameter*

Q3.2 Describe by means of an annotated diagram the principal features of a falling head permeameter.

Show that the water level in the top tube h would be expected to change with time t according to the following equation

$$\ln(h/h_o) = -(kA_1/A_2L).t$$

where h_o is the initial water level in the top tube, A_1 is the cross sectional area of the sample and L is its length, k is the soil permeability and A_2 is the cross sectional area of the top tube.

Give two reasons why this laboratory test might not lead to an accurate determination of the effective permeability of a large volume of soil in the ground.