CHAPTER 2

VELOCITIES AND ACCELERETATIONS

Graphical Method

2.1 A particle moves such that,

$r = b\ (1 - \cos \theta)$
$\theta = 4t$

Find its velocity and acceleration.

Solution

$$\mathbf{r} = r\,e^{i\theta} \qquad (2.1)$$
$$\theta = 4t$$
$$\mathbf{V} = (\dot{r} + i\,r\,\dot{\theta})\,e^{i\theta} \qquad (2.2)$$
$$\mathbf{A} = \left[(\ddot{r} - r\,\dot{\theta}^2) + i(r\ddot{\theta} + 2\dot{r}\dot{\theta})\right]e^{i\theta} \qquad (2.3)$$

But,

$$r = b\ (1 - \cos 4\,t)$$

Applying Equations (2.2) and (2.3), thus,

$$\mathbf{V} = 4\,b\,[\sin 4t + i\,(1 - \cos 4t)]\ e^{i\theta}$$
$$\mathbf{A} = 16\,b\,[(2\cos 4t - 1) + i\,2\sin 4t]\ e^{i\theta}$$

2.2 Write down the components of the velocity and the acceleration for the following cases:

a- A particle moves around a circle with radius 25 cm and center at the origin with a constant angular velocity of 20 rad,/sec and an angular acceleration of 100 rad/s2.
b- A particle moves around a circle with radius 20 cm and center at the origin with a constant angular velocity of 5 rad./s.
c- A particle moves with a speed 10 m/sec and an acceleration of 15 m/sec^2 along a straight line makes 60^o with the x–axis and at a distance 20 cm from it.
d- A particle is moving on an Archimedean spiral.

Solution

(a)

From Eq. (2.2),

$$\mathbf{V} = (\dot{r}\cos\theta - r\,\dot{\theta}\,\sin\theta) + i(r\,\dot{\theta}\,\cos\theta + \dot{r}\sin\theta)$$

Substituting with data, then,

$V_x = -\,500 \sin\theta$ cm/s
$V_y = 500 \cos\theta$ cm/s

For the acceleration, and from Eq. (.3),

$$\mathbf{A} = \left[(\ddot{r} - r\,\dot{\theta}^2)\cos\theta + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\sin\theta\right] + i\left[(\ddot{r} - r\,\dot{\theta}^2)\sin\theta + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\cos\theta\right]$$

$A_x = -\,(10000 \cos\theta + 2500 \sin\theta)$ cm/s^2
$A_y = (-\,10000 \sin\theta + 2500 \cos\theta)$ cm/s^2

(b)

$V_x = -\,100 \sin\theta$ cm/s
$V_y = 100 \cos\theta$ cm/s
$A_x = -\,100 \cos\theta$ cm/s^2
$A_y = -\,100 \sin\theta$ cm/s^2

(c)

$V_x = 5$ m/s
$V_y = 8.66$ m/s
$A_x = 7.5$ m/s^2
$A_y = 15.99$ m/s^2

(d)

$r = 200\ t,\ \ \dot{\theta} = 20$

$$V_x = \dot{r}\cos\theta - r\,\dot{\theta}\,\sin\theta = 200\,\cos\theta - 400\,t\,\dot{\theta}\,\sin\theta$$

$$V_y = r\,\dot{\theta}\,\cos\theta + \dot{r}\sin\theta = 400\,t\cos\theta + 200\,\sin\theta$$

$$A_x = (\ddot{r} - r\,\dot{\theta}^2)\cos\theta + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\sin\theta = -8000\,t\cos\theta + 8000\sin\theta$$

$$\left[(\ddot{r} - r\,\dot{\theta}^2)\sin\theta + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\cos\theta\right] = -8000\,t\sin\theta + 8000\cos\theta$$

2.3 A particle is moving relative to the xOy plane such that,

$x = 3 \cos t$
$y = 2 t$

Find the velocity, the sliding acceleration, and the normal acceleration. Also, find the value of the radius of curvature for t = 0, 1, 2, and 3s.

Solution

The velocity is given by,

$\mathbf{V} = -3 \sin t + i\, 2$

The acceleration is given by,

$\mathbf{A} = -3 \cos t + i\, 0$

The absolute velocity is,

$$V = \sqrt{(-3 \sin t)^2 + 2^2}$$

The velocity is tangent to the path and makes an angle $\theta = \tan^{-1}(2/-3 \sin t)$.

The sliding acceleration is,

$A = -3 \cos t$

It makes an angle $\phi = \tan^{-1}(0/-3 \cos t)$

The normal component of the acceleration is given by,

$A^n = |A \sin(\theta - \phi)|$

The radius of curvature of the is given by,

$\rho = A^n/V^2$

For t = 0 s,

$V = 2$, $\theta = 90^o$, $A = 3$, $\phi = 180^o$, $A^n = 3$, $\rho = 0.75$.

For t = 1 s,

$V = 3.221$, $\theta = 141.612^o$, $A = 1.6$, $\phi = 180^o$, $A^n = 3 = 1.007$, $\rho = 0.097$.

For t = 2 s,

V = 3.383, $\theta = 143.572^o$, A = 1.248, $\phi = 0^o$, A^n = 0.738, ρ = 0.065.

For t = 3 s,

V = 2.044, $\theta = 101.95^o$, A = 2.97, $\phi = 0^o$, A^n =2.906, ρ = 0.69.

2.4 In Problem 2.3, the xOy plane rotates about O with a constant angular velocity of 5 rad/sec, clockwise, find the Coriolis component when t = 4 s, then obtain the relative acceleration between the particle and the plane.

Solution

For t = 4 s

V= 3.025, $\theta = 41.377^o$

Thus,

A^{CR} = 2*3.025*5 = 30.25 with an angle = 41.377 – 90 = – 48.623^o.

The relative acceleration between the particle and the plane has two components, the tangential component, along the tangent of the path (along the velocity),

A^t =1.471 with an angle 41.377^o

The normal component,

$A^n = A^{CR} + A^n$ = 31.553 with an angle = – 48.623^o.

2.5 A particle moves with a constant radial speed of 2 cm/sec away from the center of a disk rotating with a uniform angular velocity of 10 rad/sec, clockwise. Find the acceleration of the particle relative to the disk when t = 5 s, then obtain the value of its absolute acceleration,

Solution

After 5 seconds, the particle is at a distance of 10 cm from the center. The acceleration of the particle relative to the plane is only the Coriolis component

A^{CR} = 2*0.02*10 = 0.4 m/s^2

The acceleration of the point on the disk in contact with the particle is the centripetal component.

$$A^c = -10 * (10)^2 = 10 \text{ m/s}^2$$

The absolute acceleration of the particle is,

$$A = \sqrt{(-10)^2 + 0.4^2} = 10.008 \text{ m/s}^2$$

2.6 The crank of a single cylinder diesel engine is 12 cm long. The length of the connecting rod is 36 cm, and the line of stroke of the piston passes through the crank bearing. The engine runs at 2000 rpm clockwise.

a– Draw the velocity and the acceleration polygons when the crank makes 60^o with the line of stroke. Also, determine the velocity and the acceleration of the piston.

b– Determine the velocity and the acceleration of point C on the connecting rod 15 cm, from the piston pin. Also, find the radius of curvature of the curve traced by this point on a fixed plane at the same position.

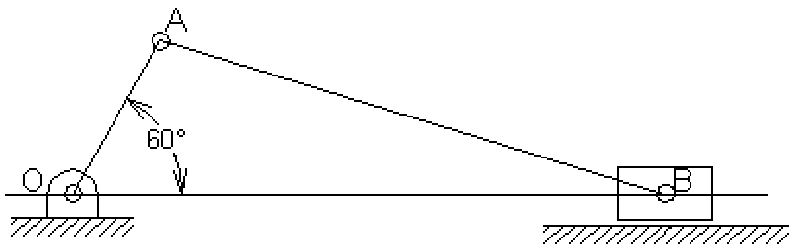
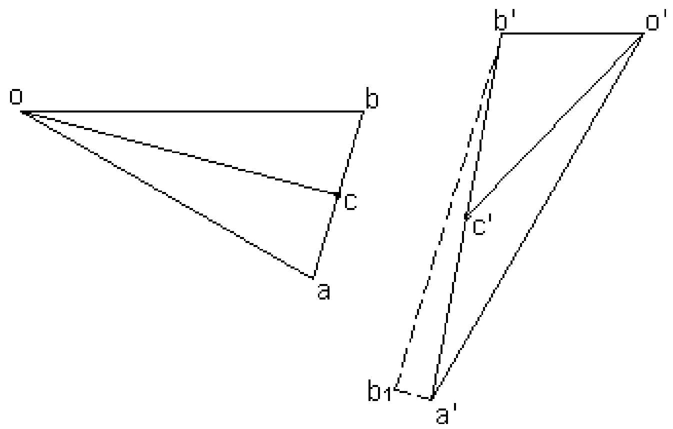


Figure P2.6

Solution

ω = 209.44 rad./s c.w.
V_A = 25.133 m/s
A_A^c = 5264 m/s^2

The velocity and acceleration polygons are shown.



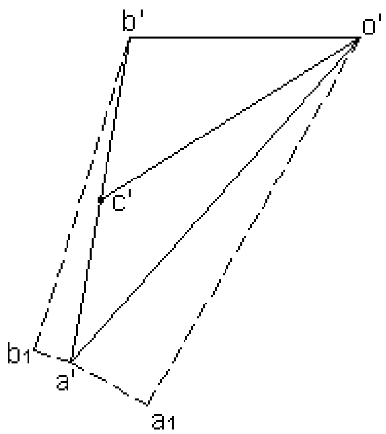
V_{BA} = 13.12 m/s
A^{c}_{BA} = 478.15 m/s^2
V_B = 48.96 m/s
A_B = 1757 m/s^2
V_C = 24.47 m/s
A_C = 3164 m/s^2
2.7 Repeat Problem 2.6 when the crank has an acceleration of 300 rad/s^2.

Solution

The velocity polygon is the same. The value of the transverse acceleration of point A is,

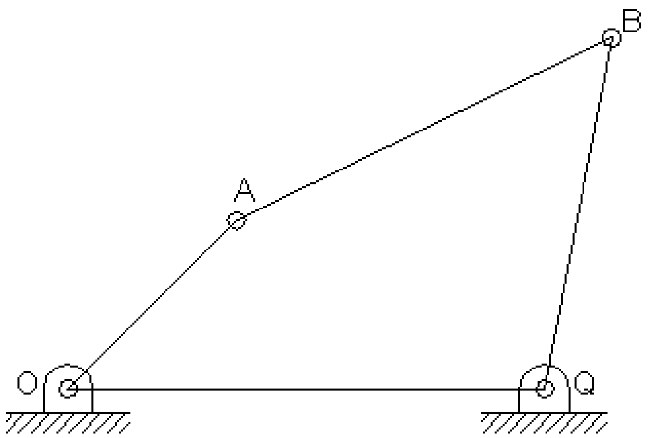
A^{t}_{A} = 1080 m/s^2

The acceleration polygon becomes,



A_B = 2840 m/s^2
A_C = 3787 m/s^2

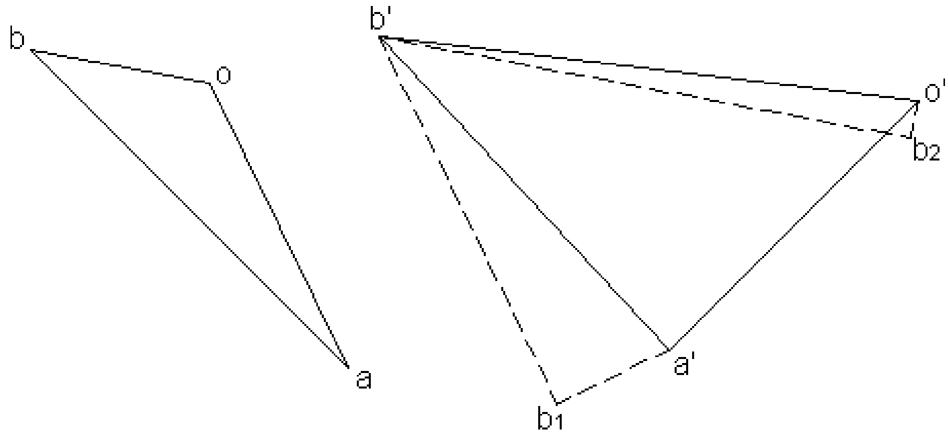
2.8 The lengths of the consequent links in a four–bar mechanism are 8, 4, 7 and 6 cm; the 8 cm link is fixed. If the crank rotates with a uniform speed of 3000 rpm counter clockwise, find the angular acceleration of the rocker when the crank makes 45 degree with the horizontal datum and at the extreme right position of the rocker.



Figure

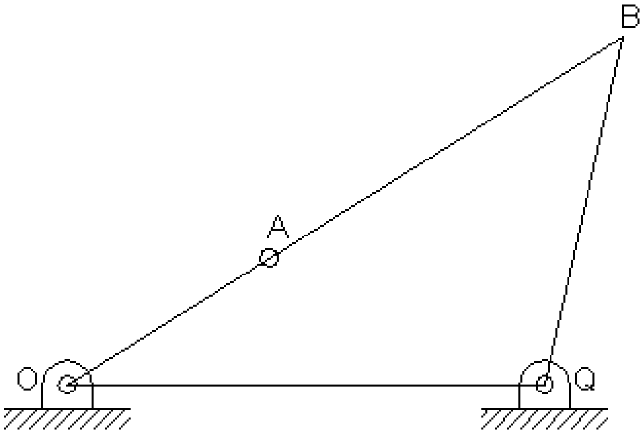
Solution

ω = 314.16 rad./s c. c.w.
V_A = 12.566 m/s
A_A^c = 3948.0 m/s^2

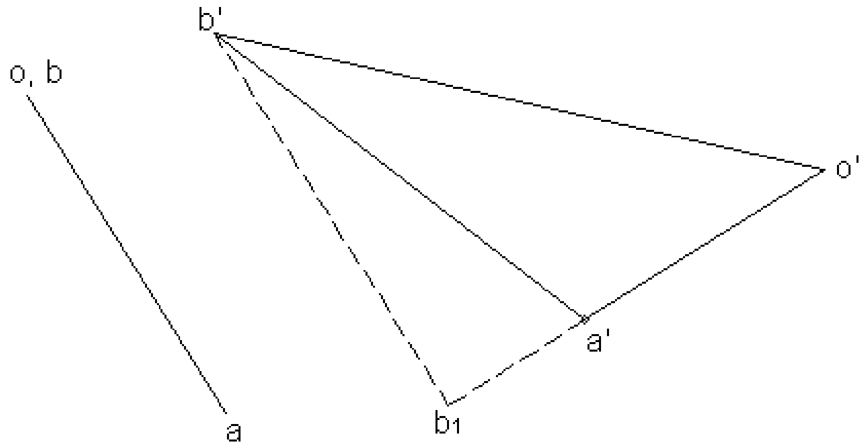


V_{BA} = 8.82 m/s
A_{BA}^c = 1378 m/s^2
V_B = 5.11 m/s
A_{BA}^c = 435.0 m/s^2
A_B^t = 6060 m/s^2

The angular acceleration of the rocker is 101000 rad/s^2. At the extreme right position,



V_A = 12.566 m/s
A_A^c = 3948.0 m/s^2



V_{BA} = 12.566 m/s
A_{BA}^c = 2256 m/s^2
V_B = 0 m/s
A_{BA}^c = 0 m/s^2
A_B^t = 8750 m/s^2

The angular acceleration of the rocker is 142800 rad/s^2

2.9 For the Watt's mechanism shown in Figure P2.9, locate the point P which moves on an approximate straight line. Demonstrate this by obtaining the direction of the velocity and the acceleration of P at several positions. Assume a unit angular velocity for link OA. At the shown position, AB is normal to OA and QB. AP/PB = QB/OA.

OA = 80 mm, AB = 60 mm, QB = 120 mm.

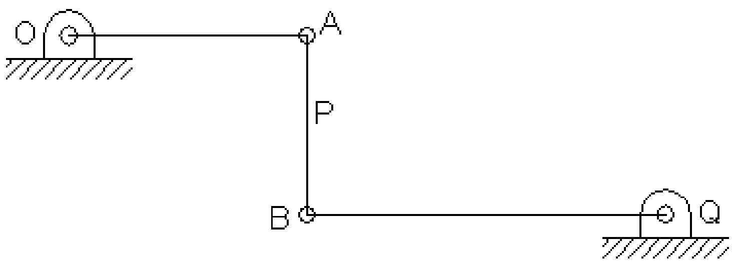
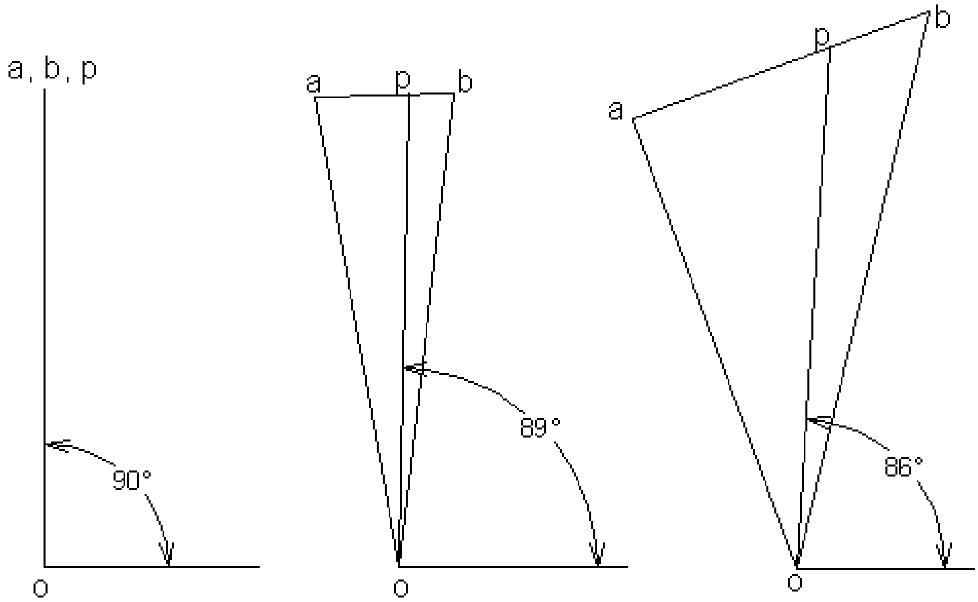


Figure P2.9

Solution

ω = 1 rad./s c.w.



The velocity polygons for link OA makes angles 0°, 10°, and 20°, are shown in Figures (a), (b), and (c), respectively. We can see that the variation in the directions of point P is small.

2.10 For the Peaucellier mechanism shown in Figure P2.10; find the velocity and the acceleration of point P if link (2) rotates with an angular velocity of 5 rad/s and an angular acceleration of 1 rad/s2, both are clockwise. Choose any position for the mechanism.

OQ = QA = 75 mm, OB = OC = 200 mm, AB = BP = AC = CP =75 mm.

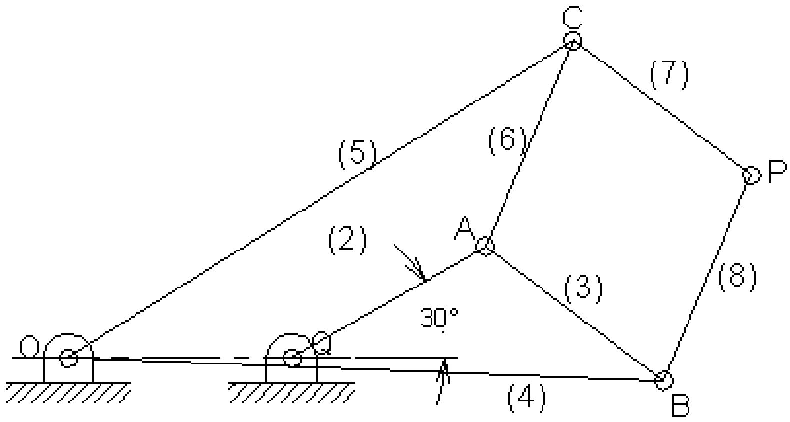
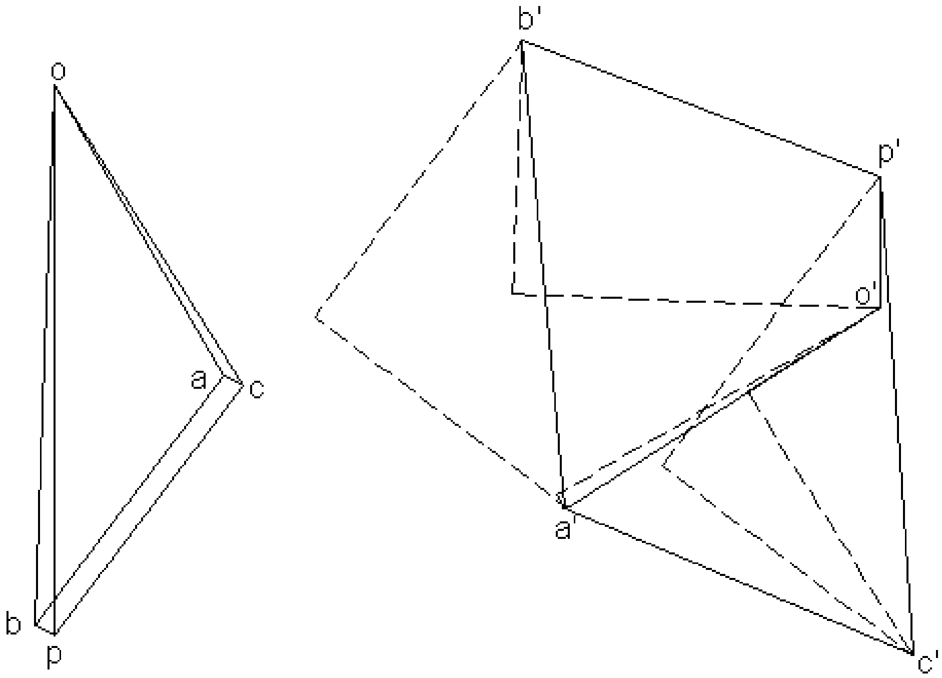


Figure P2.10

Solution

$\omega_2 = 5$ rad./s c.w.
$\alpha_2 = 1$ rad./s^2 c.w.
$V_A = 37.5$ cm/s
$A_A^c = 187.5$ cm/s^2
$A_A^t = 7.5$ m/s^2



The velocity and acceleration of point C are,

V_C = 61.4 cm/s downwards
A_C = 66.76 cm/s^2 upwards

2.11 For the mechanism shown in Figure P2.11, the piston has a vertical velocity downwards of 20 cm/s and an upwards acceleration of 280 cm/s^2 Find the angular velocity and the angular acceleration of the crank OA. Also, find the velocity and acceleration of point D and the angular velocities and the angular accelerations of the consequent links of the mechanism in magnitude and direction.

OA = 50 mm, AB = 150 mm, QB = QC = 80 mm, angle BQC = 90^o,
CE = 150 mm, CD = ED = 80 mm.

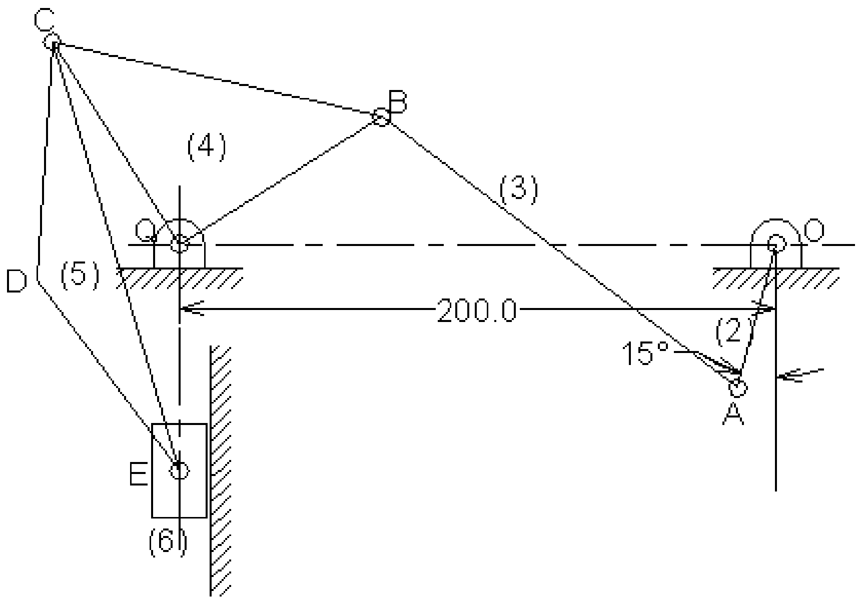
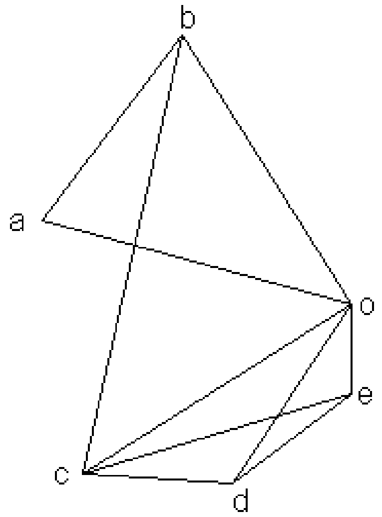
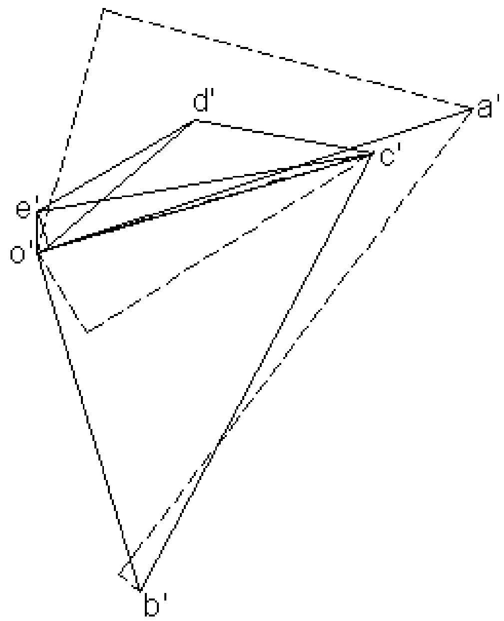


Figure P2.11

Solution

V_E = 20 cm/s
A_E = 280 cm/s





V_D = 48.2 cm/s, ω_5 = 4.18 rad./s, ω_4 = 8.88 rad./s, ω_3 = 3.48 rad./s, ω_2 = 18.38 rad./s
A_D = 138.4 cm/s^2, α_5 = 17.47 rad./s^2, α_4 = 283.75 rad./s^2, α_3 = 261.53 rad./s^2, α_2 = 514.8 rad./s^2

2.12 For the mechanism shown in Figure P2.12, the crank OA rotates with a uniform speed of 42 rad/s counter clockwise. Find the velocity and the acceleration for both sliders B and D when the crank makes 60 degree as shown in the Figure.

OA = 50 mm, AB = 250 mm, AC= 100 mm, CB = 175 mm, CD = 100 mm.

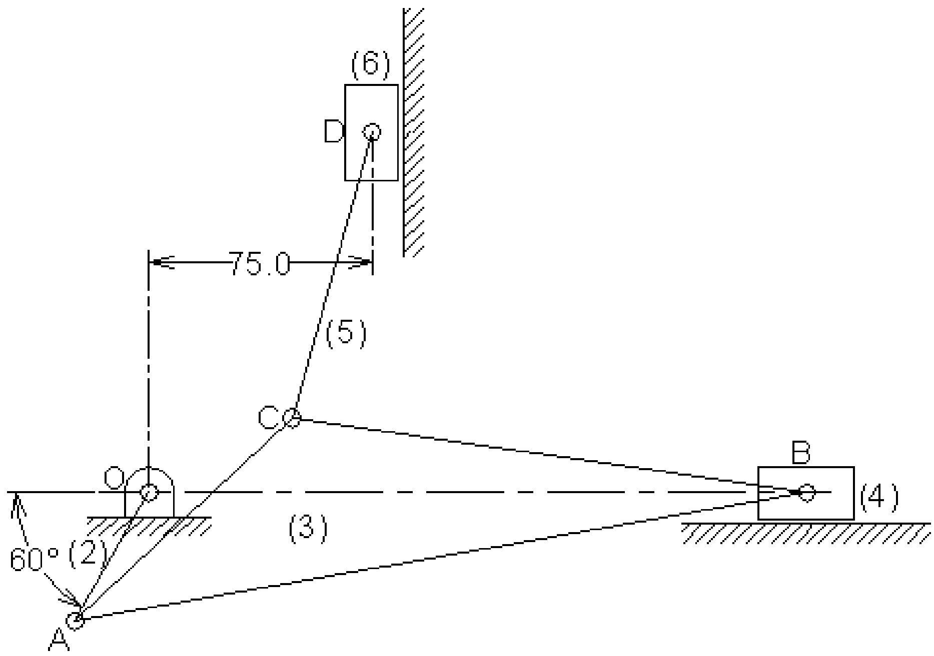
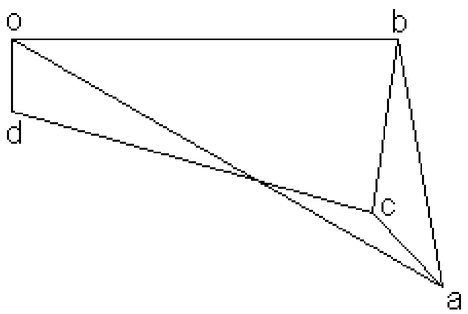
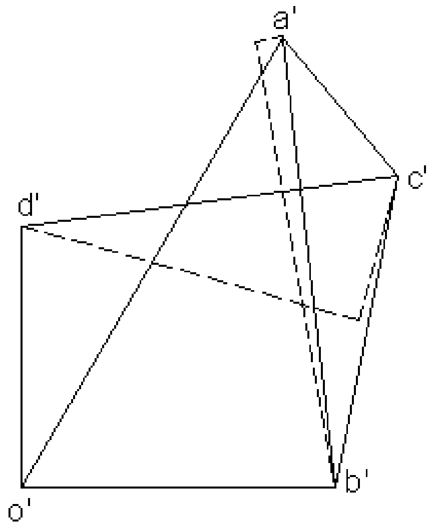


Figure P2.12

Solution

ω_2 = 42.0 rad./s c. c.w.
V_A = 210 cm/s
A_A^c = 88.20 m/s^2





V_B = 163. 4 cm/s

$A_B = 52.5\ m/s^2$
$V_D = -31.1\ cm/s$
$A_D = 44.4\ m/s^2$

Analytical solution

For the crank (2), the data is,

$r_2 = 5$ cm, $\theta_2 = 240^o$, $\omega_2 = 42.0$ rad/s, $\alpha_2 = 0$.

Using equations (2.47) and (2.48), then,

$V_A^x = 181.86$ cm/s, $V_A^y = 51 - 105.0$ cm/s, $A_A^x = 44.1\ m/s^2$ $A_A^y = 86.38\ m/s^2$

Links (3) and (4) form an engine chain with the following data,

$r_3 = 25$ cm, $\alpha = 0$, $h = 0$, $\psi = 33^o$.

Using equations (2.55) to (2.60), equations (2.71) and (2.72), and the output data of the crank, then,

$V_B = 163.4$ cm/s, $A_B = 52.92\ m/s^2$
$V_C^x = 181.86$ cm/s, $V_C^y = -105.0$ cm/s, $A_C^x = 44.1\ m/s^2$ $A_C^y = 76.38\ m/s^2$

Links (3) and (4) form an engine chain with the following data,

$r_3 = 10$ cm, $\alpha = 90^o$, $h = -7.5$ cm.
Using this data and values for point C, we get,

$V_D = -31.23$ cm/s, $A_D = 44.31\ m/s^2$

2.13 The mechanism shown in Figure P2.13 is used in a two cylinder 60° V–engine, the crank OA rotates with a uniform speed of 2000 rpm clockwise. When the crank is horizontal, find the velocity and the acceleration of both sliders B and D. Also find the magnitude and the direction of the angular velocity and the angular acceleration of link CD.

OA = 50 mm, AB = CB = CD =150 mm, AC = 50 mm

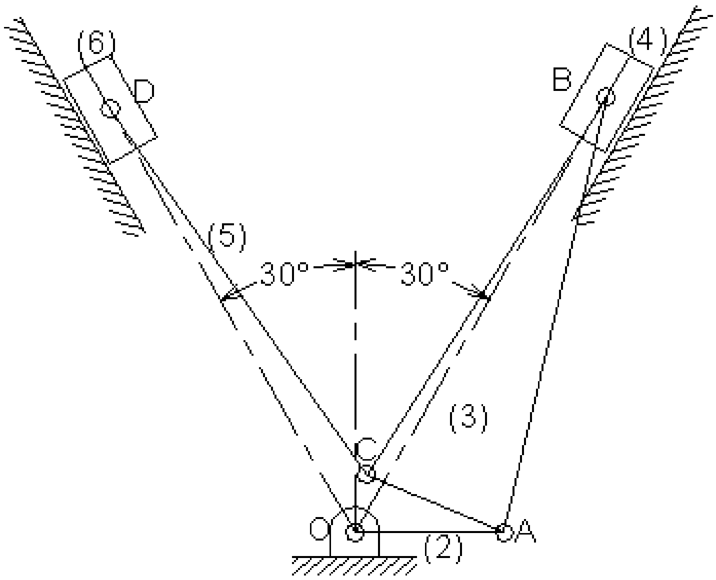
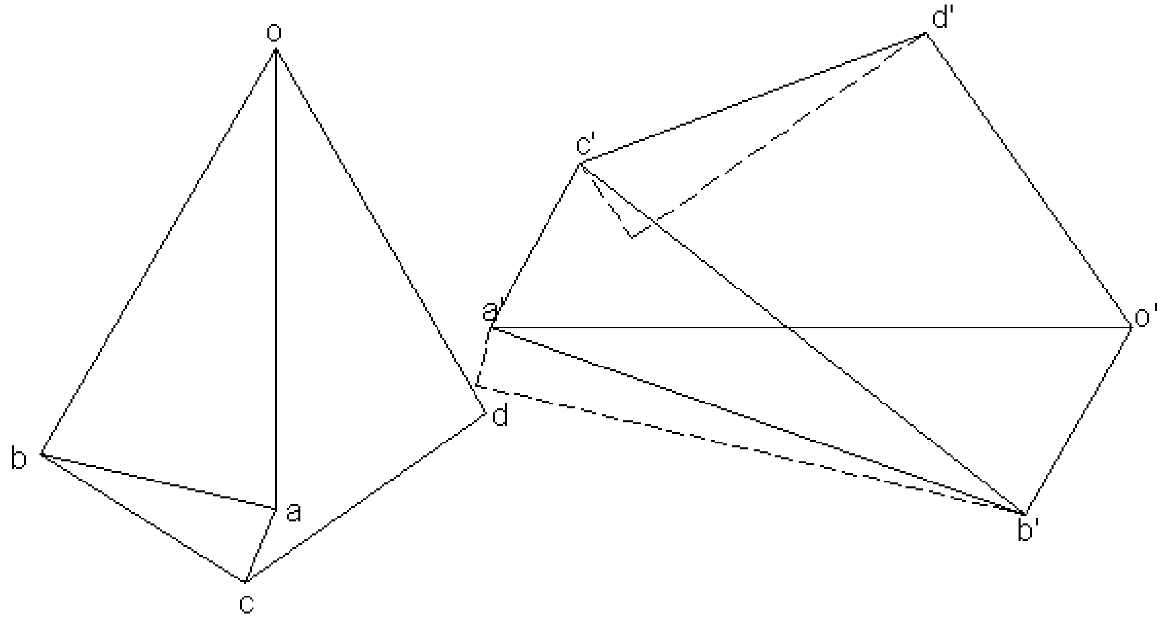


Figure P2.13

Solution

ω_2 = 209.44 rad./s c.w.
V_A = 10.47 m/s
A_A^c = 219.0 m/s^2



V_B = 10.65 m/s, A_B = 732.0 m/s^2.
V_D = – 9.58 m/s, ω_5 = – 50.7 rad./s c.w., A_D = 1231.0 m/s^2, α_5 = – 8153.0 rad./s^2 c.c.w.

Analytical solution

For the crank (2), the data is,

r_2 = 5 cm, $\theta_2 = 0^o$, ω_2 = 209.44 rad./s

Links (3) and (4) form an engine chain with the following data,

r_3 = 15 cm, α = 0, h = 0, ψ = 80^o.

Using equations (2.55) to (2.60), equations (2.71) and (2.72), and the output data of the crank, then,

V_B= –1065 cm/s, A_B = – 732.2 m/s^2
V_C^x = – 71.87 cm/s, V_C^y = – 1215 cm/s, A_C^x = – 1879.0 m/s^2 A_C^y = 563.1 m/s^2

Links (5) and (6) form an engine chain with the following data,

r_3 = 10 cm, α = 90^o, h = – 7.5 cm.

Using this data and the values for point C, we get,

V_D = – 9.56 m/s, A_D = 1245.0 m/s^2, ω_5 = – 44.8 rad./s c.w., α_5 = –8830.0 rad./s^2 c.c.w.

2.14 For the mechanism shown in Figure P2.14, find the velocity and the acceleration of the slider N when the crank makes 126^o degree with the horizontal datum. The crank rotates at a uniform of 300 rpm counter clockwise.

OA = 40 mm, AB = 120 mm, QB = 80 mm, QC = 50 mm, CN = 150 mm.

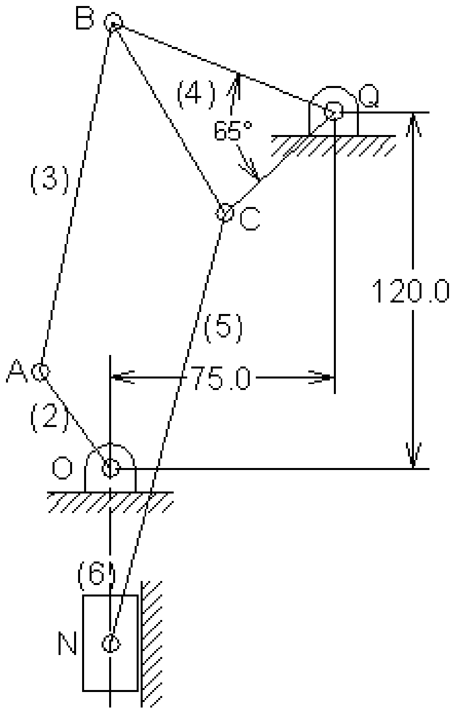
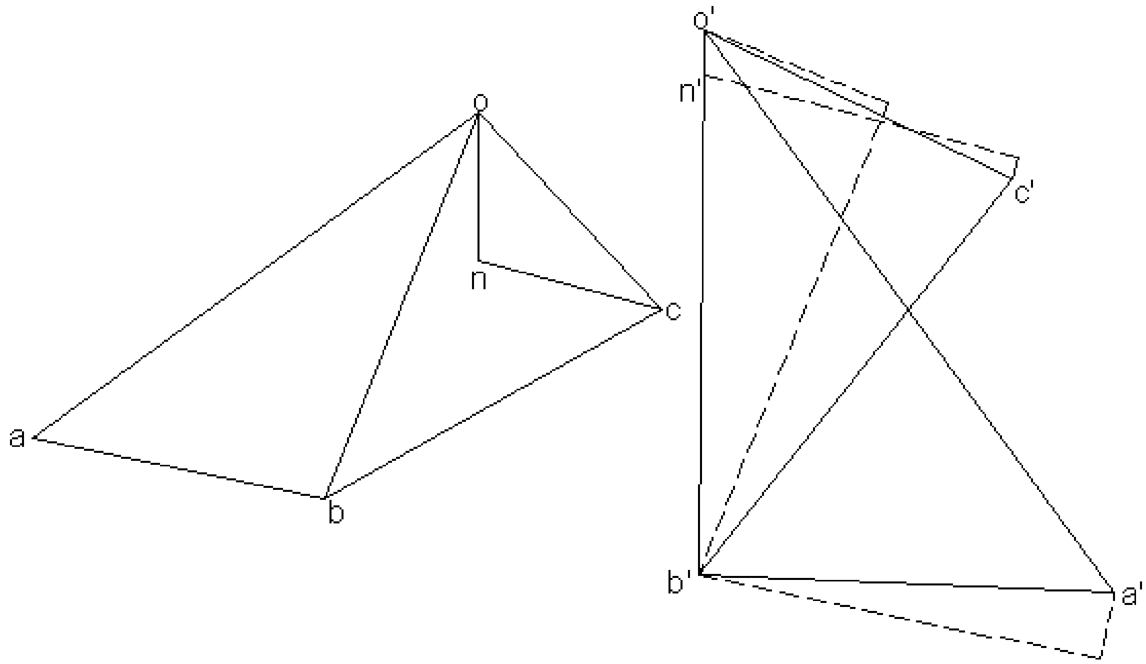


Figure P2.14

Solution

ω_2 = 31.42 rad./s c. c.w.
V_A = 125.67 m/s
A_A^c = 39.48 m/s^2



$V_N = -33.8$ cm/s, $A_N = -3.06$ m/s^2

Analytical solution

For the crank (2), the data is,

$r_2 = 4$ cm, $\theta_2 = 126^o$, $\omega_2 = 31.4$ rad./s

Links (3) and (4) form a four–bar chain with the following data,

$r_3 = 12.0$ cm, $r_4 = 8.0$ cm, $x_Q = 7.5$ cm, $y_Q = 12$ cm, sign for $\theta_3 = +1$

Using equations (2.49) to (2.53), equations (2.71) and (2.72), and the output data of the crank, then,

$V_C^x = 40.3$cm/s, $V_C^y = -43.1$ cm/s, $A_C^x = -17.42$ m/s^2, $A_C^y = -8.43$m/s^2

Links (5) and (6) form an engine chain with the following data,

$r_3 = 15$ cm, $\alpha = 90^o$, h = 0 cm.

Using this data and the values for point C, we get,

$V_D = -32.42$ cm/s, $A_D = 2.61$ m/s^2.

2.15 The crank OA of the crossed link mechanism shown in Figure P2.15 rotates counter clockwise with a uniform speed of 1000 rpm clockwise. Determine the velocity and the acceleration of the block C. Also determine the angular velocity and the angular acceleration of links (3), (4), and (5).

OA = 60 mm, AB = 280 mm, QB = 120 mm, OQ = 270 mm, BC = 300 mm

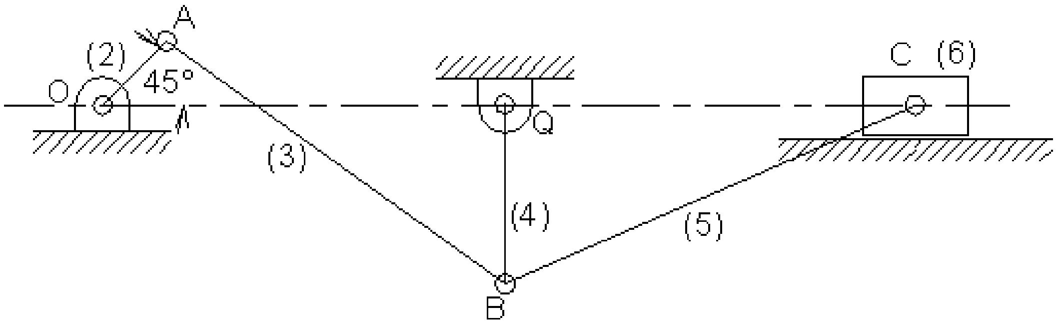
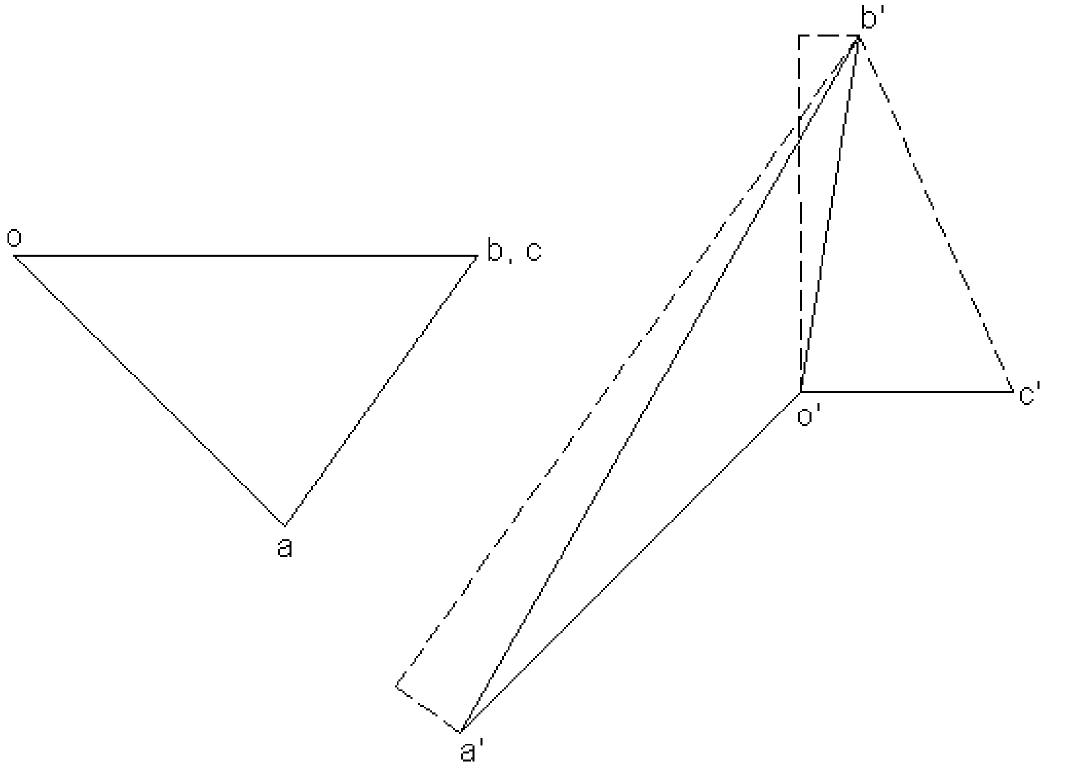


Figure P2.15

Solution

ω_2 = 104.72 rad./s c. c.w.
V_A = 628.32 cm/s
A_A^c = 658.0 m/s^2



V_C = 763.0 cm/s, A_C = 2900 m/s^2
ω_3 = 19.61 rad./s c.c.w., ω_4 = 63.58 c.c.w., rad./s, ω_5 = 0.0 rad./s
α_3 = 3850.0 rad./s^2 c.c.w., α_4 = 675.0 rad./s^2 c.w., α_5 = 1763.3 rad./s^2 c.w.

Analytical solution

This mechanism is a combination of a crank, four–bar, and engine chains. If we follow the same analysis, we get,

ω_3 = 19.62 rad./s c.c.w., ω_4 = 63.58 c.c.w., rad./s, ω_5 = – 0.116 rad./s
α_3 = 3894.0 rad./s^2 c.c.w., α_4 = 679.12 rad./s^2 c.w., α_5 = 1766.0 rad./s^2 c.w.

2.16 For the mechanism shown in Figure P2.16, link (2) rotates clockwise at a constant speed of 600 rpm. Find the angular velocity and the angular acceleration of links (3), (5), and (6).

OA = 80 mm, AC = CB= 120, OQ = 400 mm, QD = 120 mm, DC = 260 mm.

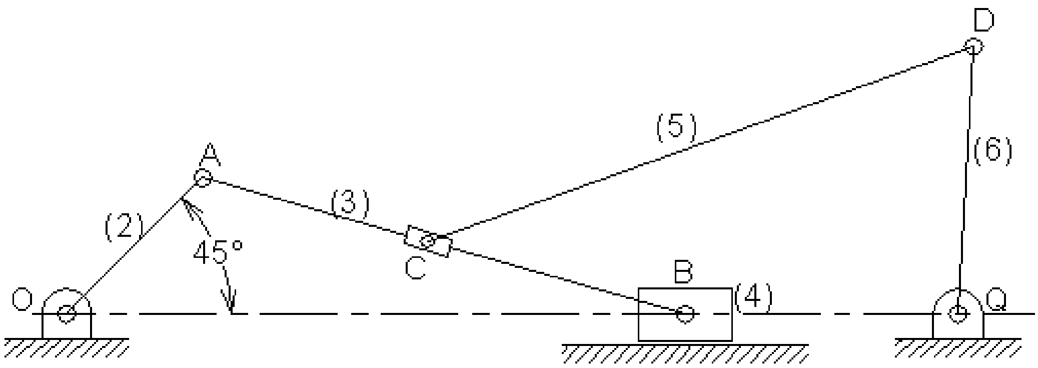
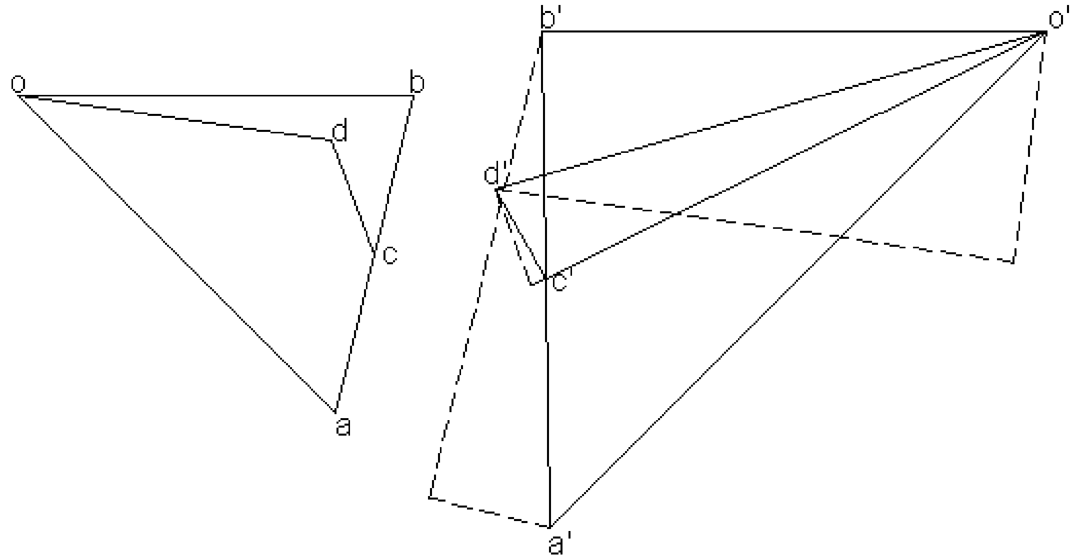


Figure P2.16

Solution

ω_2 = 62.8 rad./s c.w.
V_A = 502.4 cm/s
A_A^c = 315.8 m/s^2



V_B =441 cm/s, A_B = 226.41 m/s^2
ω_3 = 15.21 rad./s c.w., ω_5 = 5.23 rad./s c.c.w., ω_6 = – 29.5 c.w. rad./s
α_3 = 895.83 rad./s^2 c.c.w., α_5 = 176.92 rad./s^2 c.c.w, α_6 = 1950 rad./s^2 c.c.w.

Analytical solution

This mechanism is a combination of a crank, engine, and four–bar chains.

For the crank (2), the data is,

r_2 = 4 cm, $\theta_2 = 60^o$, ω_2 = 62.8 rad./s

This mechanism is a combination of a crank, engine, and four–bar chains.

V_B =441.64 cm/s, A_B = 226.5 m/s^2.
ω_3 = 15.24 rad./s c.w., ω_5 = 5.24 rad./s c.c.w., ω_6 = – 29.55 c.w. rad./s
α_3 = 901.17 rad./s^2 c.c.w., α_5 = 176.76 rad./s^2 c.c.w, α_6 = 1961 rad./s^2 c.c.w.

2.17 Figure P2.17 shows the skeleton outline of an air pump to produce a stroke four times the length of the crank. The crank (link (2)) rotates counter clockwise with a constant speed of 300 rpm. Find the velocity and the acceleration of the piston D.

OA = 40 mm, AB = AC= 40, QB = 130 mm, CD = 150 mm.

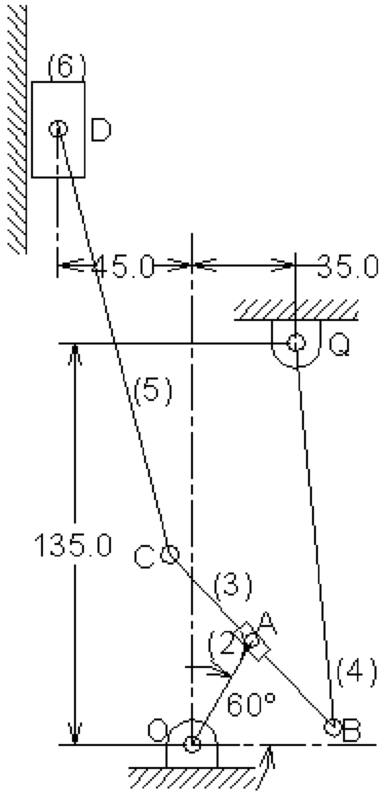
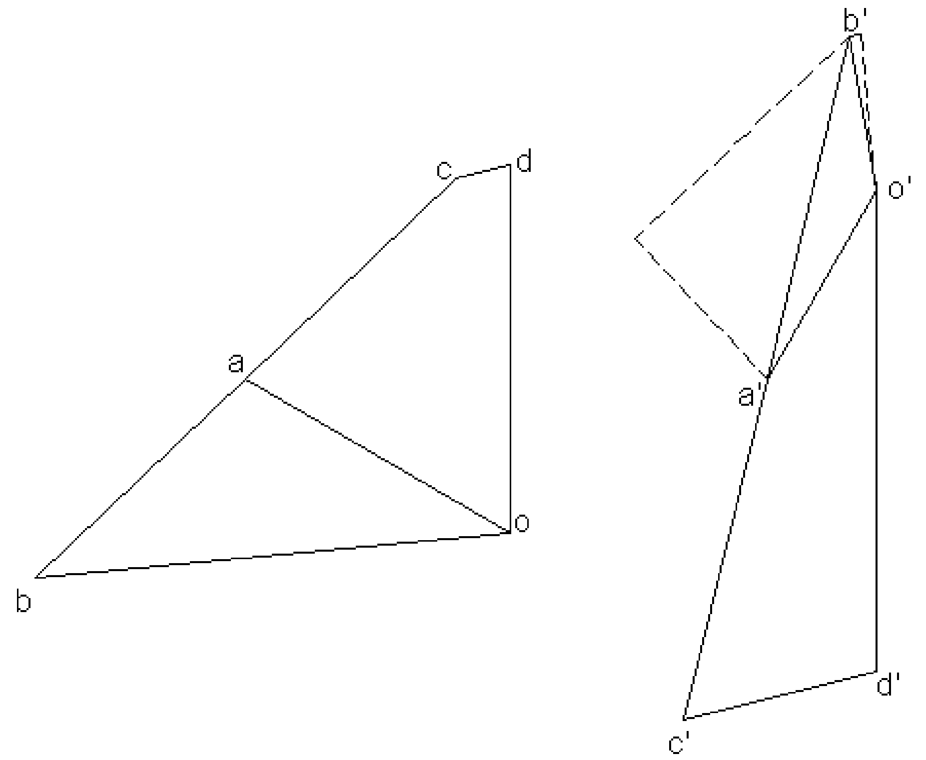


Figure P2.17

Solution

ω_2 = 31.4 rad./s c. c.w.
V_A = 125.6 cm/s
A_A^c = 39.44 m/s^2



V_D = 150.3 cm/s, A_D = – 90.1 m/s^2

Analytical solution

This mechanism is a combination of a crank, four–bar, and engine chains. If we follow the same analysis, we get,

V_D = 150.3 cm/s, A_D = – 8889 m/s^2

2.18 For the crossed link mechanism shown in Figure P2.18 crank OA rotates at 900 rpm clockwise, and with an angular acceleration of 50 rad/sec2 counter clockwise. Determine the velocity and the acceleration of the block D. Also, find the angular velocity and angular acceleration of links (4) and (5).

OA = 60 mm, AB = 200 mm, QB = QC = 100 mm, angle BQC = 60o CD = 200 mm.

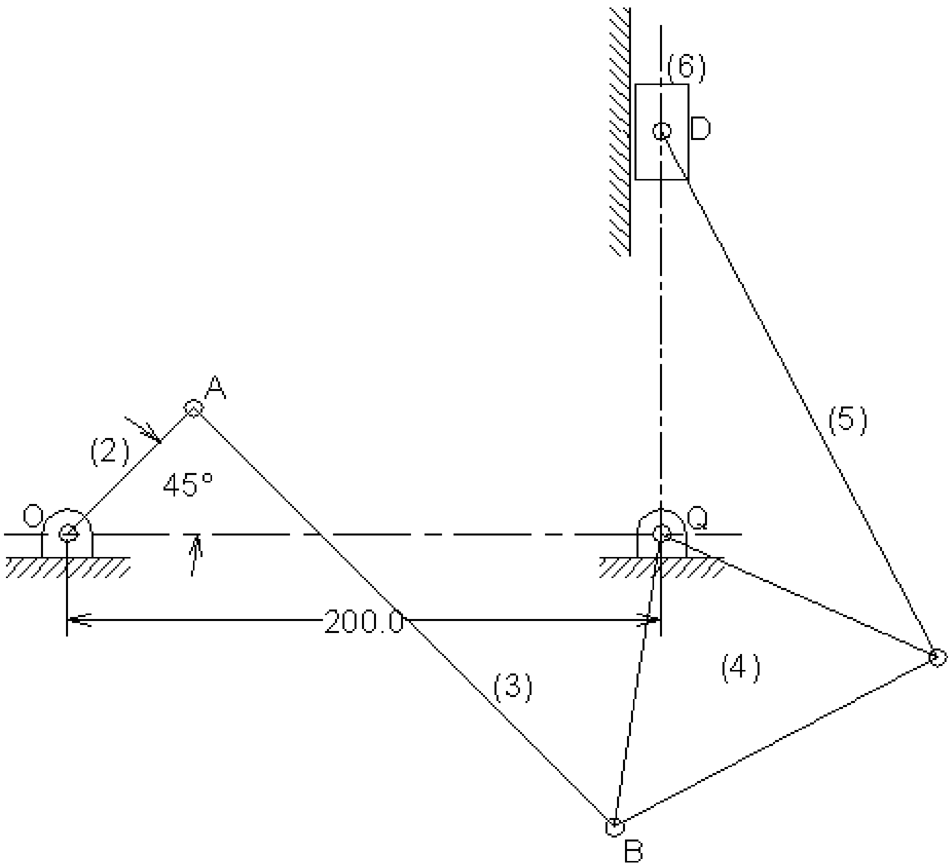
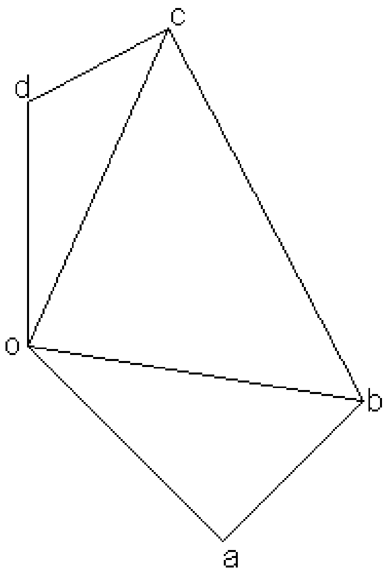
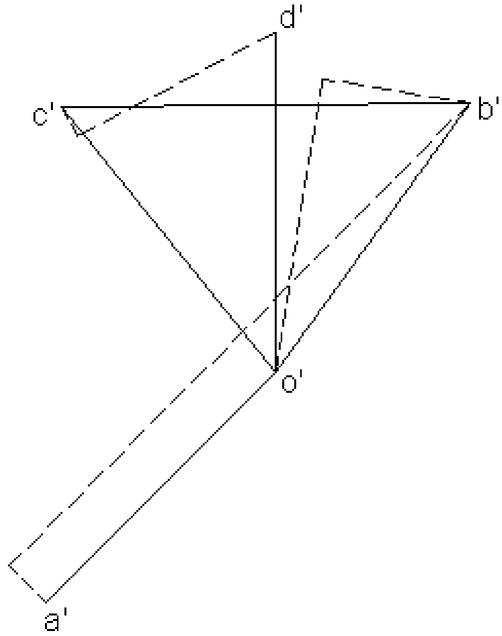


Figure P2.18

Solution

ω_2 = 94.2 rad./s c. c.w.
V_A = 565.2 cm/s
A_A^c = 532.42 m/s^2





V_D = 496.4 cm/s, A_D = 555.4 m/s^2
ω_3 = 20.42 rad./s c.c.w., ω_4 = 69.5 rad./s c.c.w., ω_5 = 3.26 c.w. rad./s
α_3 = 5353.0 rad./s^2 c.c.w., α_4 = 2457.0 rad./s^2 c.c.w, α_5 = 4869.0 rad./s^2 c.c.w.

Analytical solution

This mechanism is a combination of a crank, four–bar, and engine chains. If we follow the same analysis, we get,

V_D = 489.72 cm/s, A_D = 540.8 m/s^2
ω_3 = 20.4 rad./s c.c.w., ω_4 = 69.68 rad./s c.c.w.,
α_3 = 5353.0 rad./s^2 c.c.w., α_4 = 2457.0 rad./s^2 c.c.w,

2.19 Figure P2.19 shows the skeleton outline of the Atkinson gas engine. The crank OA rotates uniformly at 500 rpm counter clockwise. Find the velocity and the acceleration of the block D when the crank makes 30^o.

OA = 60 mm, AB = 180 mm, BC = 20 mm, AC = 180 mm,
QC = 80 mm, CD = 180 mm.

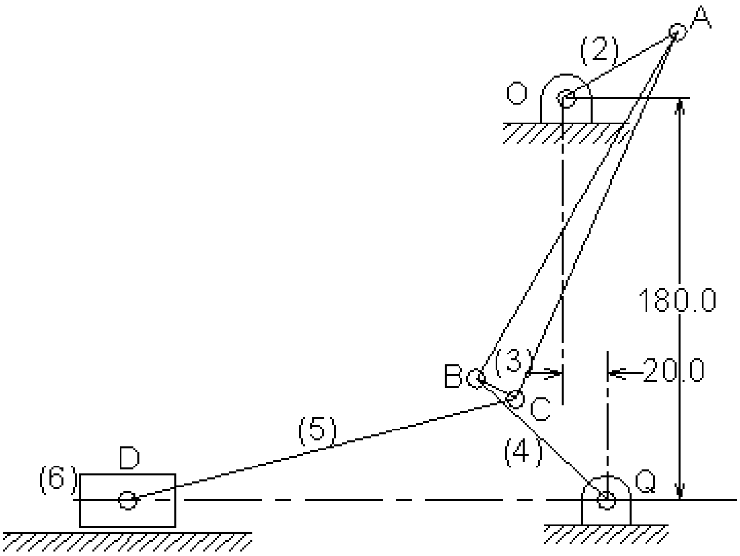
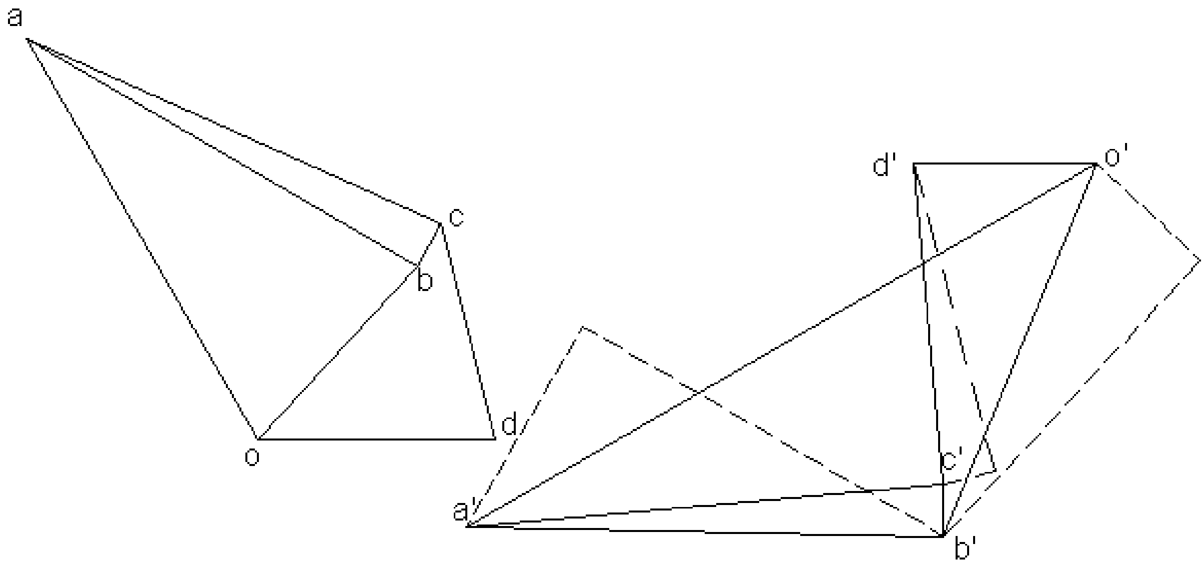


Figure P2.19

Solution

ω_2 = 52.33 rad./s
V_A = 314 cm/s
A_A^c = 164.33 m/s^2



V_D = 161.7 cm/s, A_D = – 41.2 m/s^2

Analytical solution

For the crank (2), the data is,

r_2 = 6 cm, $\theta_2 = 30^o$, ω_2 = 52.33 rad./s

This mechanism is a combination of a crank, four–bar, and engine chains. If we follow the same analysis, we get,

V_D = 162.02 cm/s, A_D = – 41.06 m/s^2

2.20 Find the angular velocity and angular acceleration of link (6), Figure P2.20, if link (2) rotates clockwise with a constant speed of 300 rpm clockwise.

OA = 60 mm, AB = 230 mm, QB = QC = 150 mm, BC = 100 mm,
CD = 270 mm, OD = 180 mm.

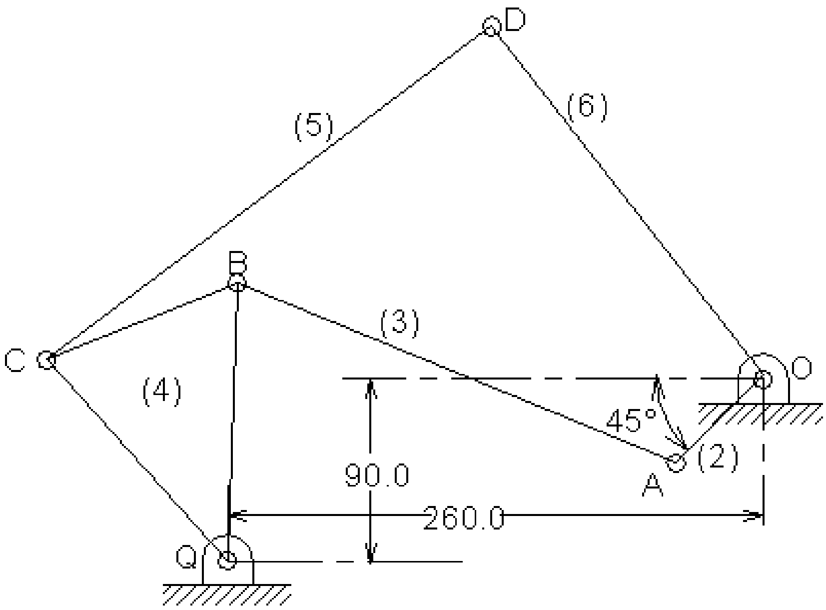
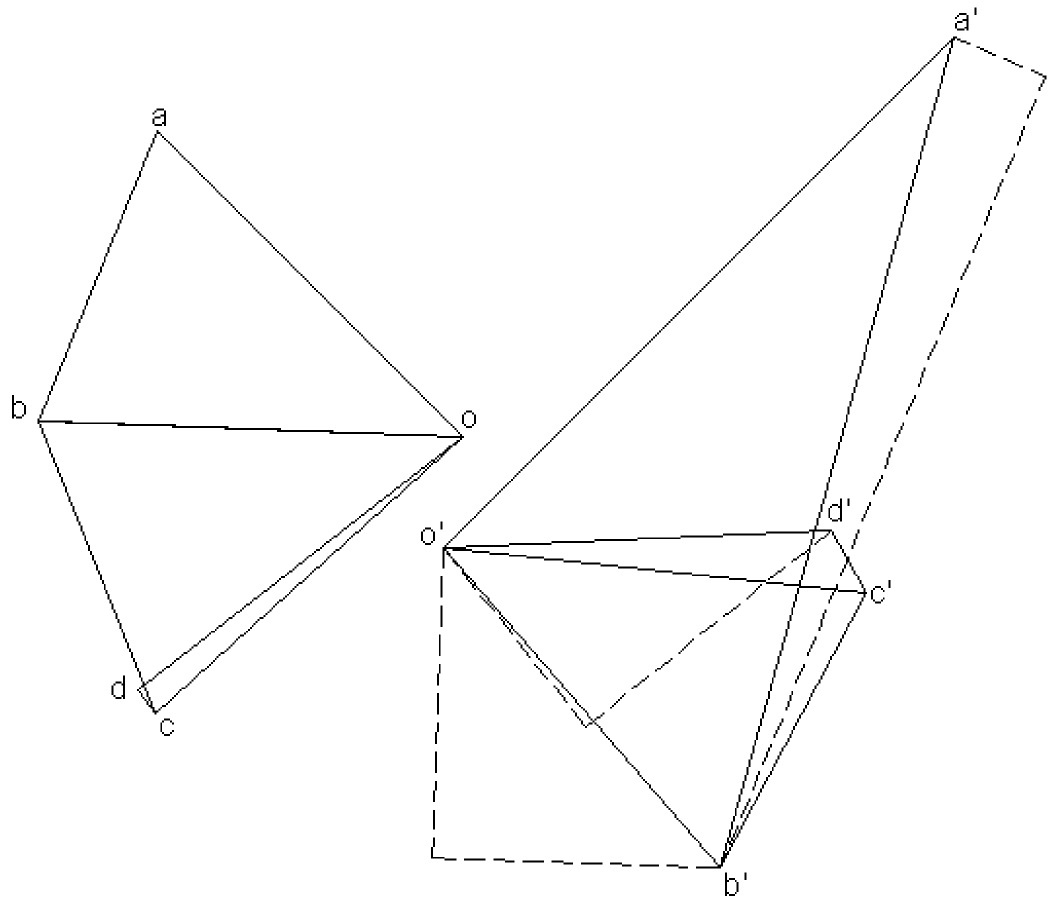


Figure P2.20

Solution

ω_2 = 31.4 rad./s
V_A = 188.4 cm/s
A_A^c = 59.16 m/s^2



ω_6 = 10.17 rad./s c.c.w., α_6 = – 143.56 rad./s^2 c.w.

Analytical solution

For the crank (2), the data is,

r_2 = 6 cm, $\theta_2 = 225^o$, ω_2 = – 31.4rad./s

This mechanism is a combination of a crank, four–bar, and four–bar chains. If we follow the same analysis, we get,

ω_6 = 10.21 rad./s c.c.w, α_6 = – 142.8 rad./s^2 c.c.w.

2.21 Find the angular velocity and the angular acceleration of link (7) of the eight–bar linkage shown in Figure P2.21. Also, find the velocity and the acceleration of point F on the slider (8). The angular velocity of the crank (2) is 1 rad/s counter clockwise. The angular acceleration of the crank is 2 rad/s^2 clockwise. The crank angle is 45^o.

OA = 75 mm, AB = 150 mm, BC = 100 mm, AC = 70 mm, CD = 60 mm, QD =150, DE = 75, EF = 150.

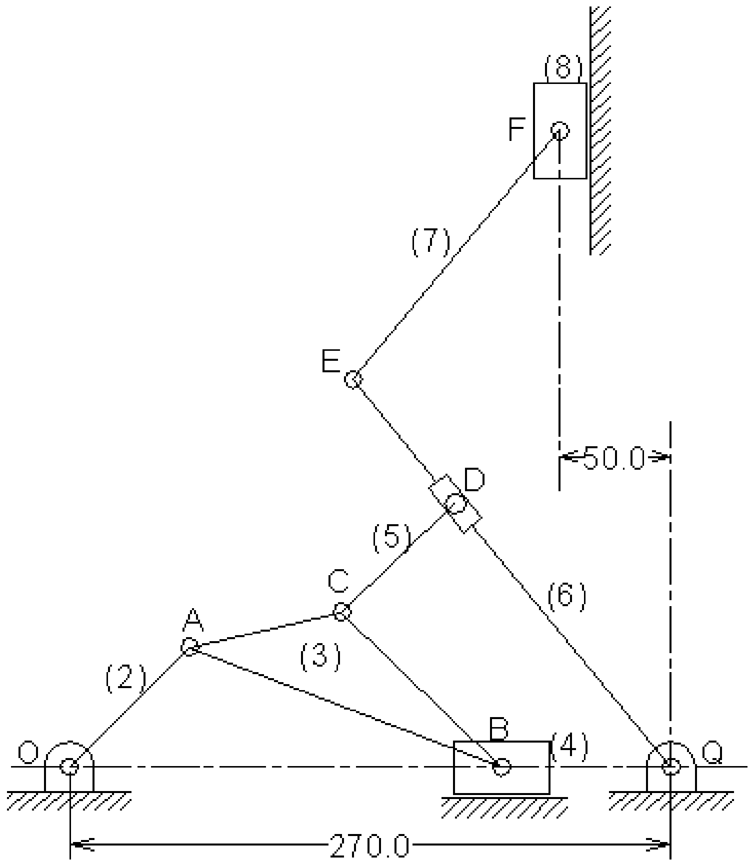
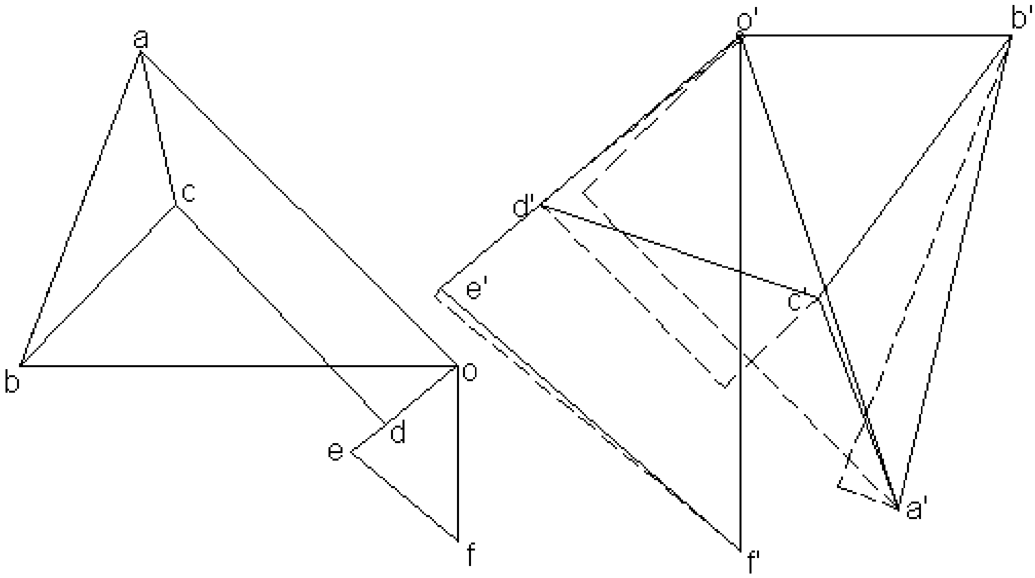


Figure P2.21

Solution

ω_2 = 1 c.c.w. rad./s
α_2 = 2 c.w. rad./s^2
V_A = 7.5 cm/s
A_A^c = 7.5 cm/s^2
A_A^t = 15.0 cm/s^2



V_B = – 7.34 cm/s, A_B = 9.13 m/s^2
V_F = 3.73 cm/s, A_F = 17.25 m/s^2
ω_7 = 0.183 rad./s c.w., α_7 = 0.89 rad./s^2 c.w.

Analytical solution

For the crank (2), the data is,

r_2 = 8.5 cm, $\theta_2 = 45^o$, ω_2 =1.4 rad./s

This mechanism is a combination of a crank, four–bar, engine, and four–bar chains. If we follow the same analysis, we get,

V_B = – 7.34 cm/s, A_B = 9.03 m/s^2
V_F = – 3.73 2.92 cm/s, A_F = – 18.2 m/s^2
ω_7 = – 0.178 rad./s (c.w.), α_7 = – 0.862 rad./s^2 (c.w.)

2.22 Figure P2.22 shows a double slider mechanism. The crank OA rotates clockwise with a uniform speed of 300 rpm clockwise. Find the velocity and the accelerations of the sliders.

OA = 40 mm, AB = 120 mm, AC = 30 mm, angle ACB = 90^o, CD = 12 mm.

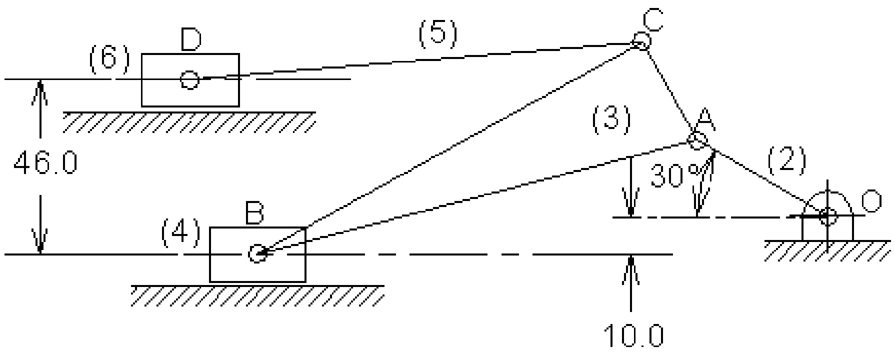
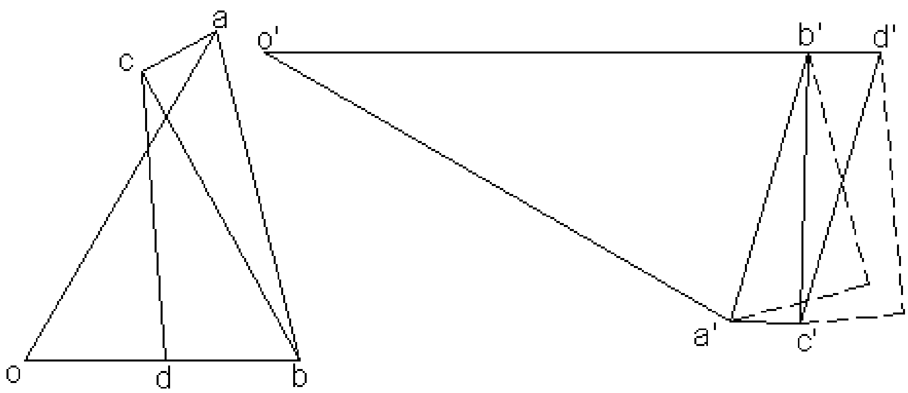


Figure P2.22

Solution

ω_2 = 31.4 rad./s
V_A = 125.6 cm/s
A_A^c = 39.44 m/s^2



V_B = 90.88 cm/s, V_D = 46.38 cm/s
A_B = 39.92 m/s2, A_D = 53.1 m/s^2

Analytical solution

This mechanism is a combination of a crank, four–bar, engine, and four–bar chains. If we follow the same analysis, we get,

V_B = 90.93 cm/s, V_D = 48.17 cm/s
A_B = 39.97 m/s2, A_D = 46.98 m/s^2

2.23 Figure P2.22 shows a toggle mechanism with eight links. The crank OA rotates clockwise with a uniform speed of 200 rpm clockwise. Find the velocity and the accelerations of the slider. Also, find the angular velocities and the angular accelerations of all links.

OA = 25 mm, AB = 80 mm, QB = 30 mm, QC = 35 mm,
CD = 100 mm, UD = UE = DE = 20 mm, EF = 50 mm.

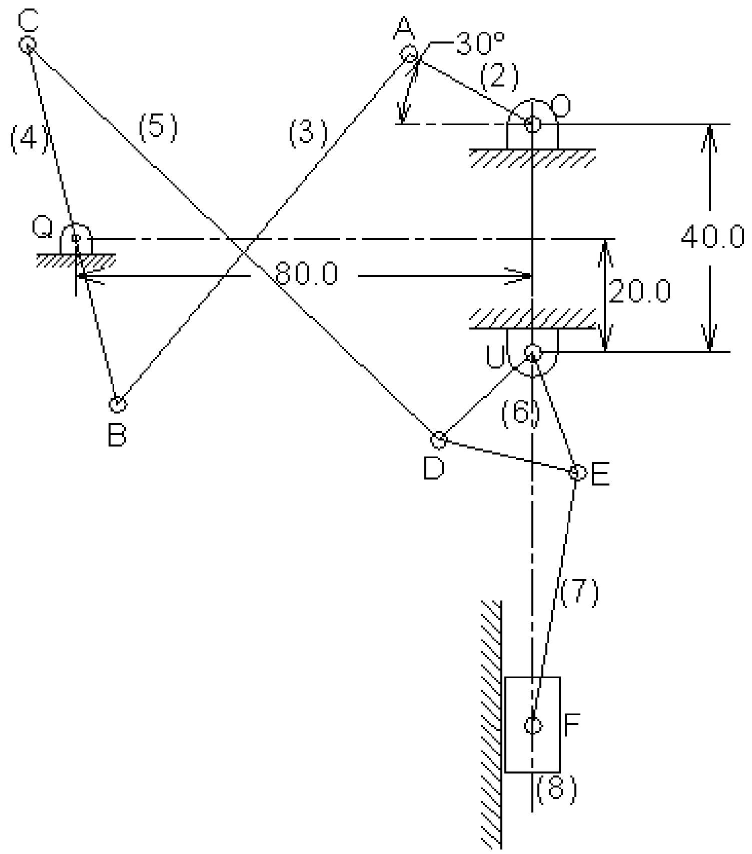
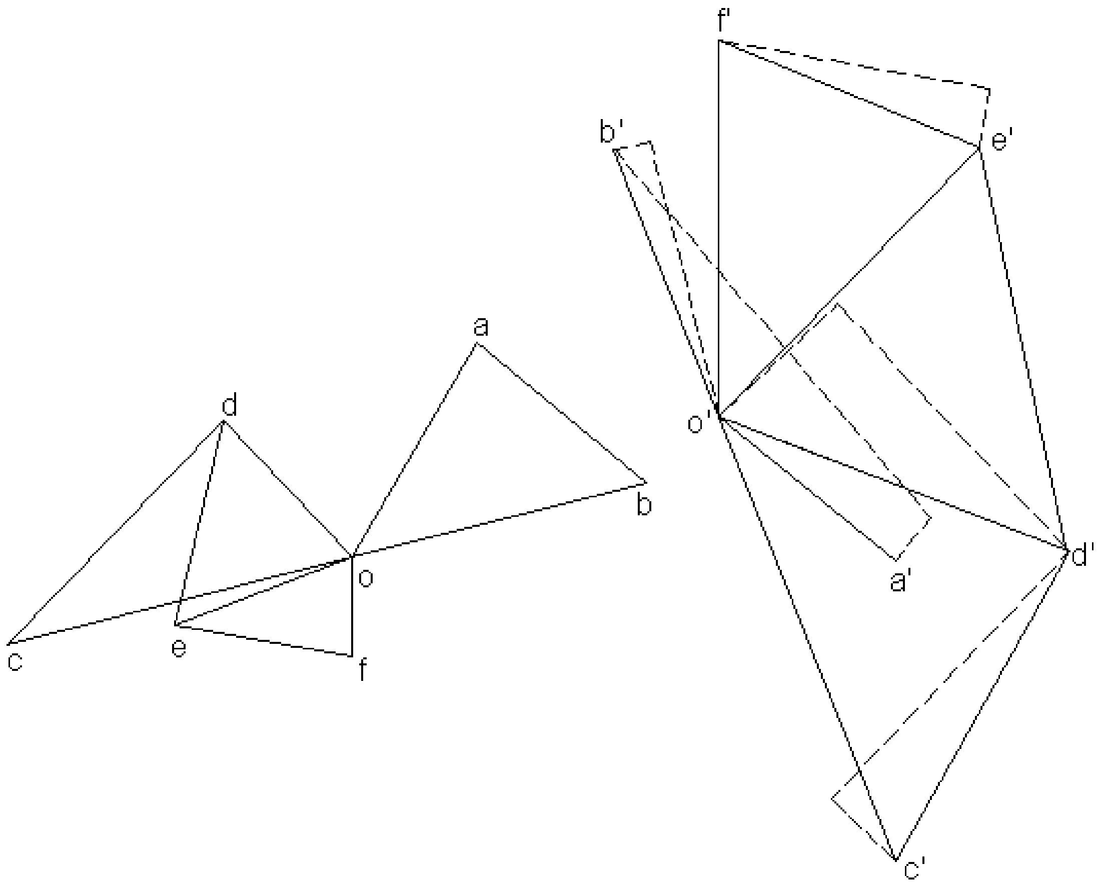


Figure P2.23

Solution

ω_2 = 20.93 rad./s c.w.
V_A = 52.33 cm/s
A_A^c = 10.96 m/s^2



V_F = 20.58 cm/s, A_F = 17.96 m/s^2
ω_3 = 5.82 93.3 rad./s c.c.w., ω_4 = 21.25 rad./s c.c.w., ω_5 = 6.25 c.c.w. rad./s,
ω_6 = 19.82 rad./s c.w., ω_7 = 7.51 c.w. rad./s.
α_3 = 291.88 rad./s^2 c.w., α_4 = 59.67 rad./s^2 c.w, α_5 = 164.3 rad./s^2 c.c.w,
α_6 = 804.5 rad./s^2 c.c.w, α_7 = 264.0 rad./s^2 c.w.

Analytical solution

This mechanism is a combination of a crank, four–bar, four–bar, an engine. If we follow the same analysis, we get,

V_F = 18.08 cm/s, A_F = 47.61 m/s^2
ω_3 = 5.82 93.3 rad./s c.c.w., ω_4 = 21.25 rad./s c.c.w., ω_5 = 3.47 c.c.w. rad./s,
ω_6 = – 27.5 rad./s c.w., ω_7 = 2.71 c.w. rad./s.
α_3 = – 305.13 rad./s^2 c.w., α_4 = – 145.64 rad./s^2 c.w, α_5 = 286.75 rad./s^2 c.c.w,
α_6 = 1026 rad./s^2 c.c.w, α_7 = 184.26 rad./s^2 c.w.

Notice: The difference between the graphical and analytical results is due the inaccuracy of the graphical method. We should rely on the analytical method.

2.24 For the crank shaper mechanism shown in Fig, P2.24, the crank OA rotates at 300 rpm clockwise. Determine the velocity and acceleration of the ram D.

OA = 30 mm, QC = 190 mm, CD =200 mm.

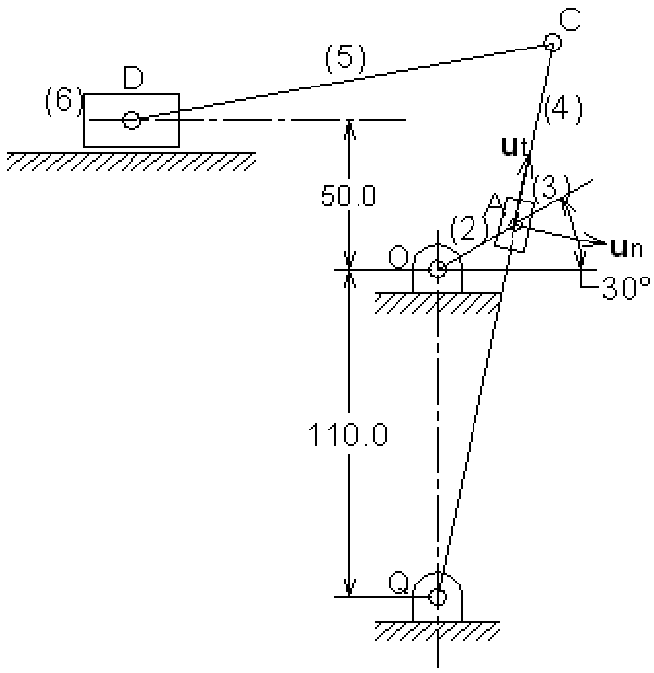
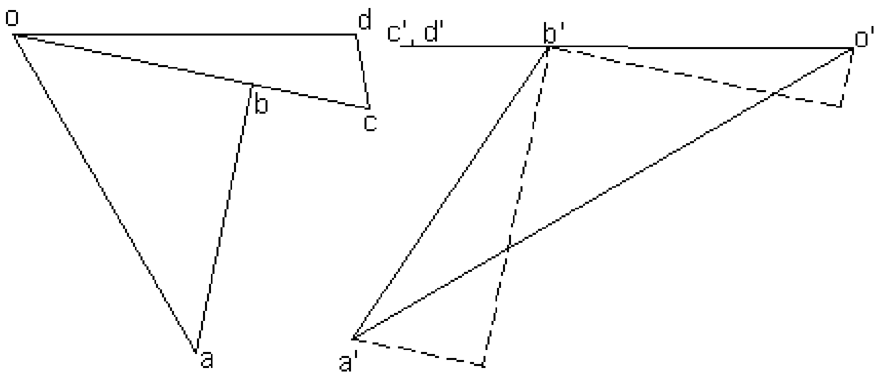


Figure P2.24

Solution

ω_2 = 31.4 rad./s c.w.
V_A = 94.2 cm/s
A_A^c = 29.58 m/s^2



From the velocity polygon, $\mathbf{V}_{BA}$ is in the direction of $\mathbf{u}_t$. Also, $\rho_A = 0$ and $\rho_B = \infty$. Applying Eq. (2.30), then, $\mathbf{A}^n_{CB} = 6.88\ \mathbf{u}_n$ m/s.

$V_D = 87.88$ cm/s, $A_D = 22.91$ m/s^2

Analytical solution

This mechanism is a combination of a crank, shaper, and engine chains. For the shaper chain, we use equations (2.61), (2.62), (2.71), and (2.82) to obtain the input data for the engine chain. Therefore,

$V_D = 88.93$ cm/s, $A_D = 22.85$ m/s^2

2.25The crank of the tilting block mechanism shown in Figure P2.25 rotates at 600 rpm counter clockwise. Find the angular velocity and the angular acceleration of link AC.

OA = 25 mm.

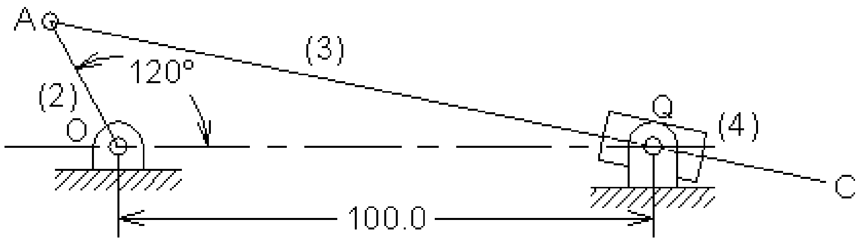
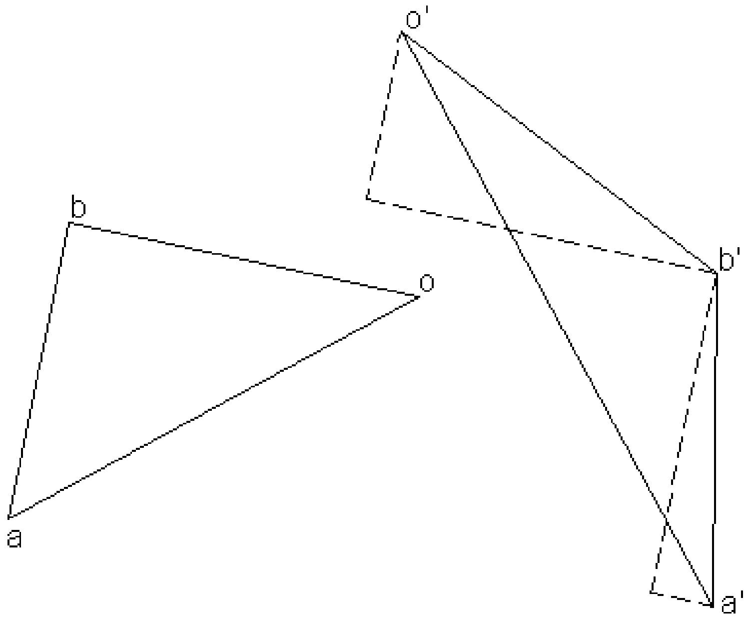


Figure P2.25

Solution

$\omega_2 = 62.8$ rad./s c.w.
$V_A = 157.0$ cm/s
$A^c_A = 298.6$ m/s^2



ω_3 = 9.0 rad./s c.w., α_3 = 464.9 rad./s^2 c.c.w

Analytical solution

This mechanism is a combination of a crank, and a tilting block. For the tilting block chain, we use equations (2.67), and (2.69). Therefore,

ω_3 = – 8.98 rad./s (c.w.), α_3 = 465.1 rad./s^2 (c.c.w)

2.26 The yoke mechanism shown in Figure P2.26 actuates the slider C. The crank rotates counter clockwise at 500 rpm. Find the velocity and the acceleration of C when the crank makes 45^o.

OA = 50 mm, BC = 100 mm, the radius of the yoke is 120 mm.

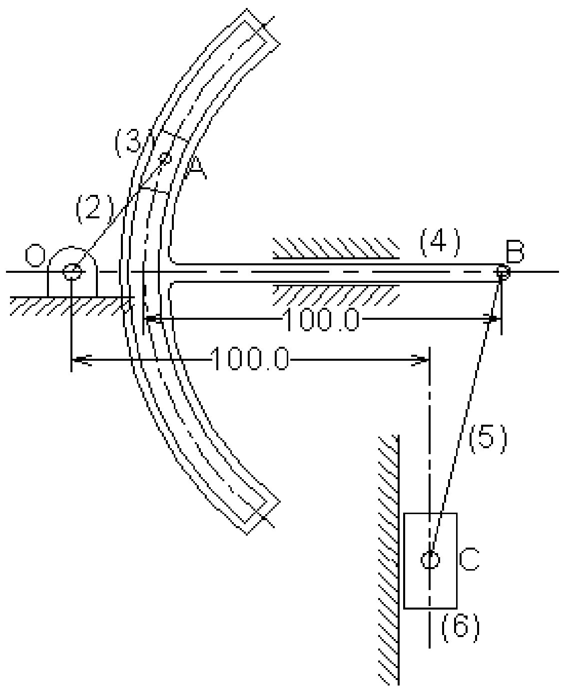
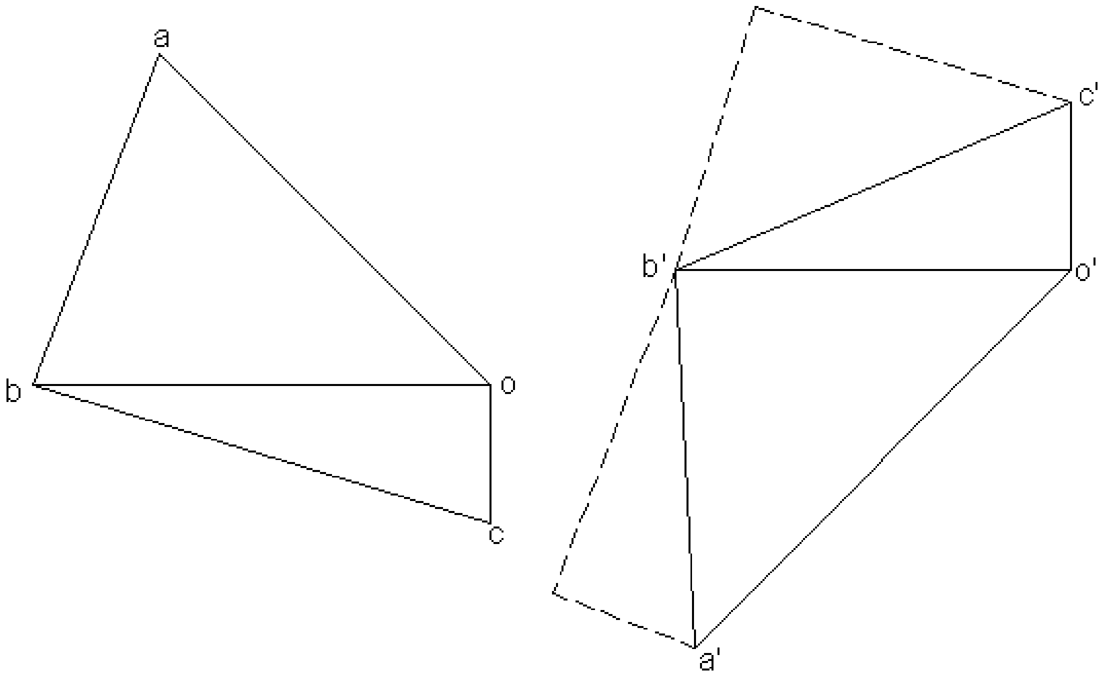


Figure P2.26

Solution

ω_2 = 52.33 rad./s c.w.
V_A = 261.7 cm/s
A_A^c = 136.94 m/s^2



V_C = 77.0 cm/s, A_C = 433.0 m/s^2

Analytical solution

This mechanism is best treated as a combination of a crank, engine, and engine chains. Link (3) for the first engine chain is represented by line AB. For the tilting block chain, we use equations (2.67), and (2.69). Therefore,

V_C = 77.0 cm/s, A_C = 433.3 m/s^2

2.27 Make a complete velocity and acceleration analysis for the mechanism shown in Figure P2.27. The angular velocity of the crank OA is 24 rad/sec clockwise. What is the absolute velocity and the acceleration of point C?

OA = 25 mm, BC = 150 mm, ABC is one link, QD = 15.0 mm, angle ABC = 90^o.

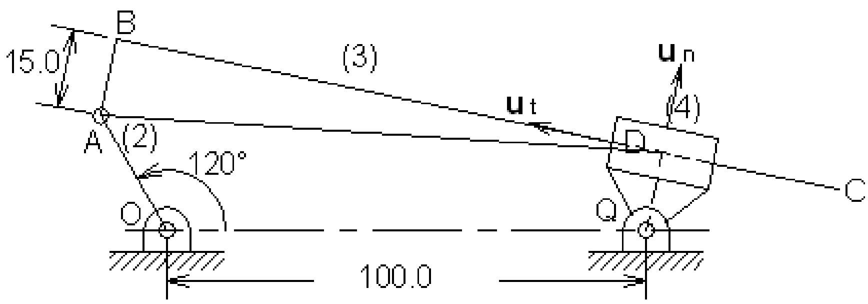


Figure P2.27

Solution

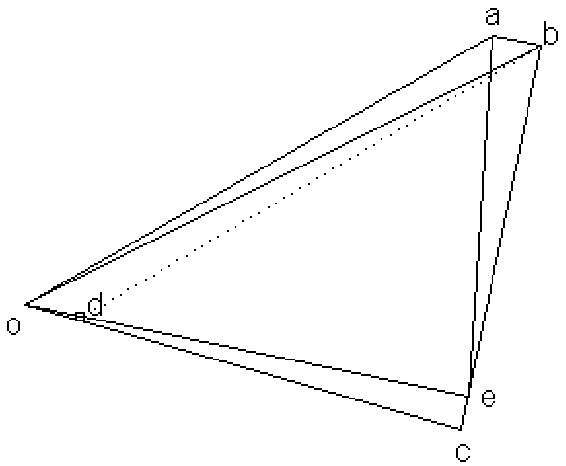
Consider an imaginary link AD. Let point D be on link (3) and point E be on link (4). The angular velocity and the angular acceleration of both links are the same. The sliding velocity and the sliding acceleration between the link and the block are normal to line QD. Consider the imaginary link AD. QD is equal to 15 mm.

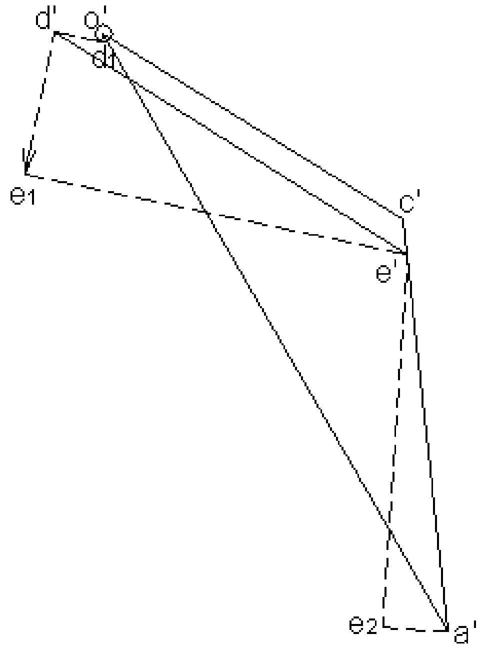
ω_2 = 24.0 rad./s c.w.
V_A = 60.0 cm/s
A_A^c = 14.4 m/s^2

From the velocity polygon, $\mathbf{V}_{DE}$ is 40.35 $\mathbf{u}_t$. Thus,

$A_{DE}^n = -\,3.03\ \mathbf{u}_n$ m/s^2

It is represented by line d'e$_1$. The sliding component of D relative to E is along BD and in the same direction of the transverse component of point D. Thus, we can determine the position of point e'.





The absolute velocity and acceleration of point C are,

V_C = 50.5 cm/s, A_C = 7.36 m/s^2

Analytical solution

Let point B be on the in contact with the block. From the position analysis, BD = 11.46 cm, and the angle of $\mathbf{u}_t$ is $\theta_3 = 169.1^o$.

For the velocity, consider the vector loop,

$$\mathbf{V}_D = \mathbf{V}_A + \mathbf{V}_{BA} + \mathbf{V}_{EA} + \mathbf{V}_{ED}$$

$$1.5\ i\omega_3\ (ie^{i\ \theta 3}) = 2.5\ i\omega_2\ e^{i\ \theta 2} + 1.5\ i\omega_3\ (ie^{i\ \theta 3}) - 11.46\ i\omega_3\ e^{i\ \theta 3} + V_{ED}\ e^{i\ \theta 3}$$

We can eliminate $1.5\ i\omega_3\ (ie^{i\ \theta 3})$ from both sides. The mechanism can be treated as a crank and tilting block with link (3) represented by AQ with length 11.46 cm. The results are,

V_C = 45.36 cm/s, A_C = 7.36 m/s^2

2.28 For the mechanism shown in Figure P 2.28, the velocity of point A is equal 30 cm/s to the right. Find the velocity and the acceleration of C. Also, find the angular velocity and the angular acceleration of link ABC when the crank makes 150^o.

OA = 25 mm, BC = 150 mm, ABC is one link, angle ABC = 90^o.

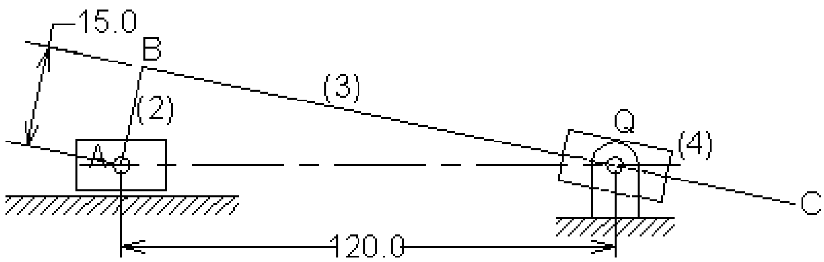
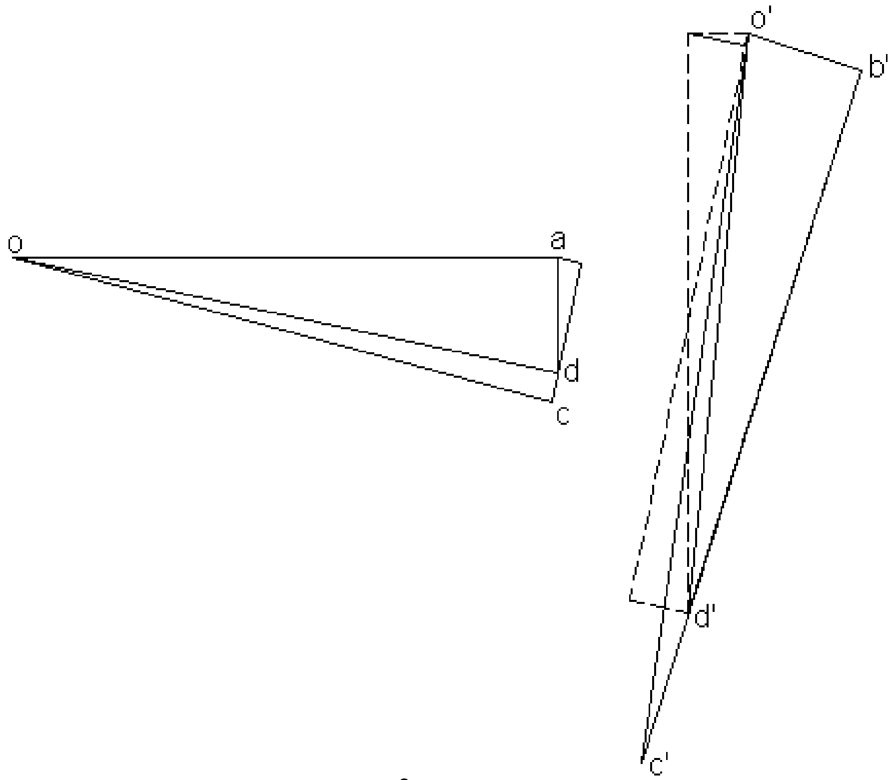


Figure P2.28

Solution

V_A = 30.0 cm/s
A_A = 0.0 cm/s^2



V_C = 30.69 cm/s, A_C = 19.97 cm/s^2

Analytical solution

Let point D be on the link in contact with the block. From the position analysis, θ_3 = 353^o.

Velocity

$$\mathbf{V}_D = \mathbf{V}_A + \mathbf{V}_{DA}$$
$$V_D^{SL} e^{i\,\theta 3} = 30.0 + 12.0\, i\omega_3$$

Equating the real parts, we get V_D^{SL} = 30.22 cm/s

From the imaginary parts, ω_3 = – 0.31

$$\mathbf{V}_C = \mathbf{V}_D + \mathbf{V}_{CD}$$
$$= V_D^{SL} e^{i\,\theta 3} + DC^*\, i\omega_3\, ie^{i\,\theta 3}$$

From this equation,

V_C = 30.23 cm/s,

Acceleration

$\mathbf{A}_D = \mathbf{A}_A + \mathbf{A}_{DA}$
$(A_D^{SL} - i A_D^n)\, e^{i\theta 3} = 12.0\,(-\omega_3^2 + i\alpha_3)\, e^{i\theta 3}$
$V_D^n = 19.05$

From this equation,

$\alpha_3 = -1.588$ rad/s^2, $A_D^{SL} = -1.153$
$\mathbf{A}_C = \mathbf{A}_D + \mathbf{A}_{CD}$
$\mathbf{A}_C = (A_D^{SL} - i A_D^n)\, e^{i\theta 3} + 12.0\,(-\omega_3^2 + i\alpha_3)\, e^{i\theta 3}$

We get $A_C = 19.9$ cm/s^2

2.29 In Figure P2.29, link (4) is guided to move horizontally at a constant speed of 50 cm/s to the left. Determine the angular velocity and the angular acceleration of link (2). Also find the velocity and the acceleration of the block C.

OB = 170 mm, BC = 120 mm.

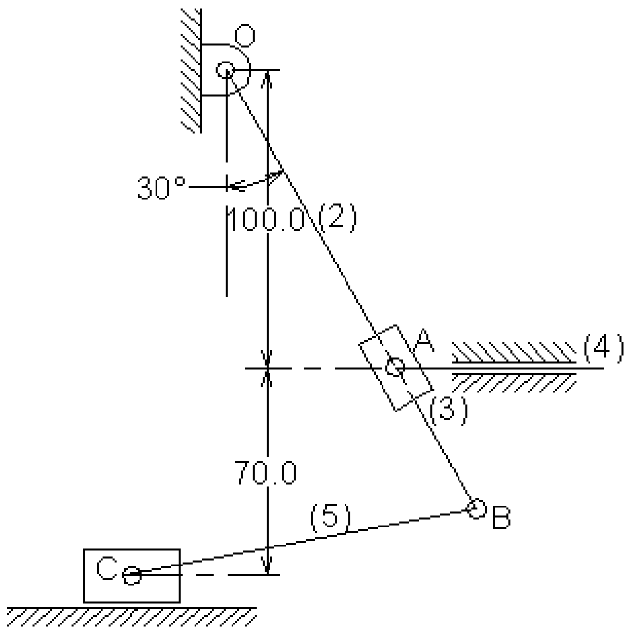
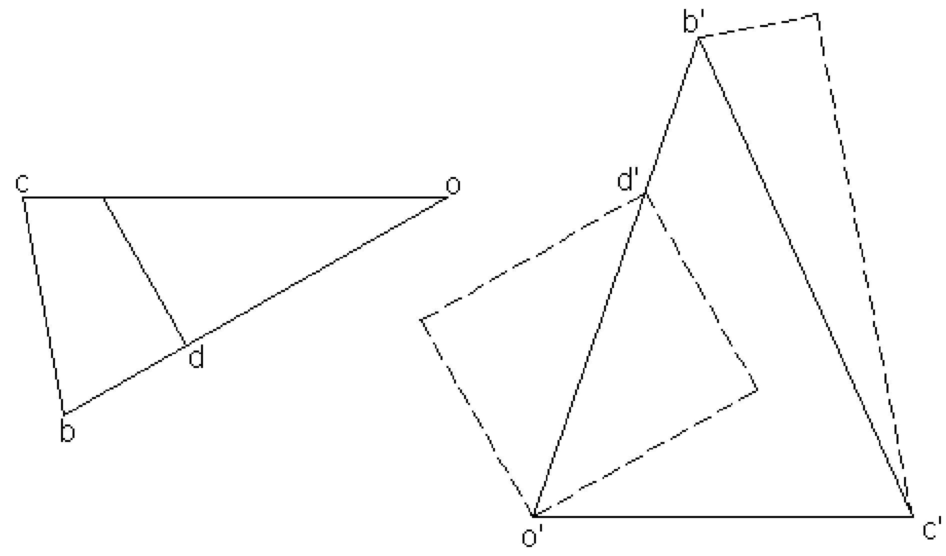


Figure P2.29

Solution

$V_A = 50.0$ cm/s
$A_A = 0.0$ cm/s



ω_2 = 3.77 rad./s c.w., α_2 = 16.22 rad./s^2 c.w
V_C = 61.68 cm/s, A_C = 2.78 m/s^2

Analytical solution

This mechanism is a combination of a crank, shaper, and engine chains. The input data for the shaper chain is,

$$V_A^x = -50,\ V_A^y = 0,\ A_A^x = 0,\ A_A^y = 0$$

Thus,

ω_2 = – 3.92 rad./s (c.w.), α_2 = – 16.12 rad./s^2 (c.w)

For the engine chain,

V_C = – 64.2 cm/s, A_C = – 2.64 m/s^2

2.30 Determine the velocity and the acceleration of the ram B of the mechanism shown in Figure P2.30. The crank is 30 mm long and rotates at 300 rpm clockwise. At the shown position, the crank makes 30^o.

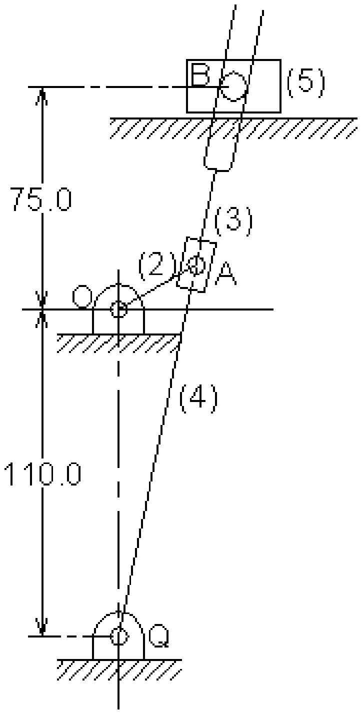
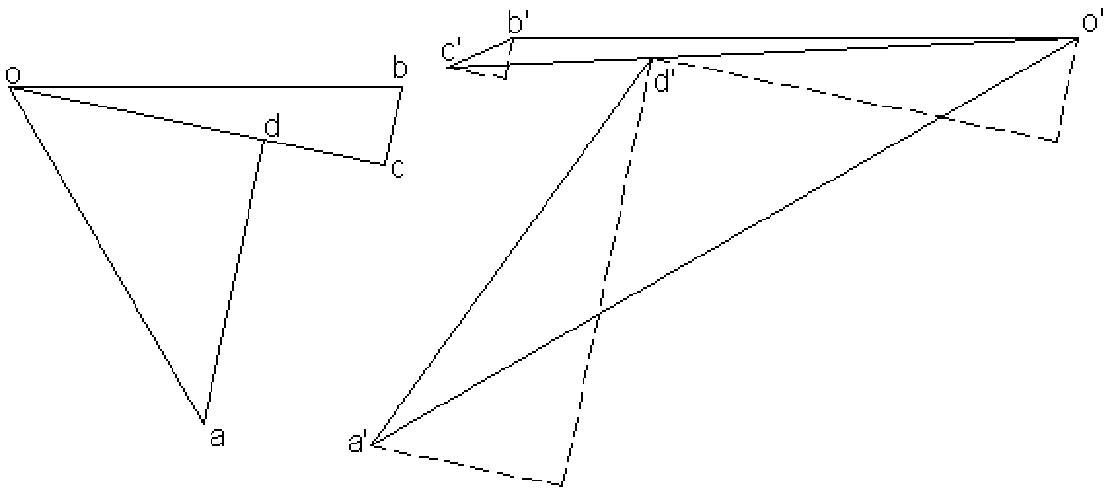


Figure P2.30

Solution

ω_2 = 52.33 rad./s c.w.
V_A = 261.7 cm/s
A_A^c = 136.94 m/s^2

Let points D and C be on link (4) in contact with points A and B, respectively.



V_C = 94.88 cm/s, A_C = 22.86 m/s^2

Analytical solution

For the crank (2), the data is

r_2 = 2 cm, $\theta_2 = 45^o$, ω_2 = 62.8rad./s c.c.w

This mechanism is treated as a shaper chain. Let point C be on link (4) in contact with point B.

For the shaper chain, we obtain the kinematic values for point C, θ_4 = 78.26^o.

V_C^x = 90.95 cm/s, V_C^y = – 18.90 cm/s, A_C^x = – 22.93 m/s^2, A_C^y = 10.33 m/s^2

These values are used as inputs for the slider B. The vector loops for this slider are,

$$V_B = (90.95 - i\ 18.9) + V_{BC}^{SL} e^{i\,\theta 4}$$
$$A_B = (22.93 - i10.33) + (A_{BC}^{SL} + i\,A_{BC}^{n})e^{i\,\theta 4}$$

Solving these equations we get,

V_C = 94.88 cm/s, A_C = 21.86 m/s^2

2.31 Crank (2) of the single slider crank inversion shown in Figure P2.31 rotates counter clockwise at 600 rpm. Find the sliding velocity and acceleration of the piston, and the angular velocity and angular acceleration of the cylinder. The crank is 45 mm and makes an angle 45^o. The piston rod AB is 60 mm.

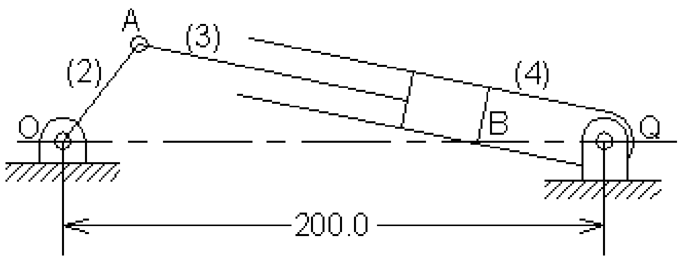
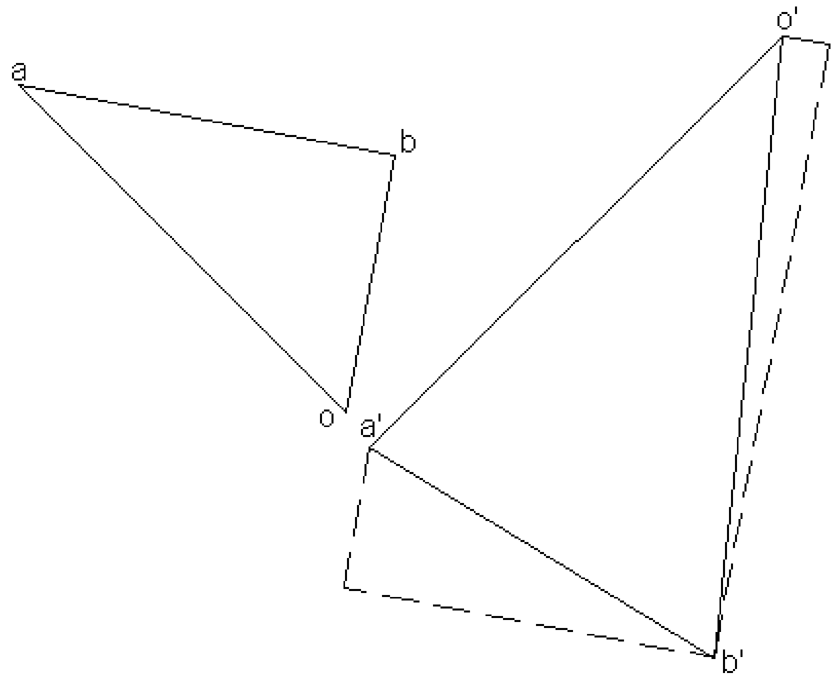


Figure P2.31

Solution

ω_2 = 62.8 rad./s c.w.
V_A = 282.6 cm/s
A_A^c = 177.47m/s^2

There is no loss of generality if we consider that the center of the block to be at the end of the crank. The problem is treated then as a shaper mechanism.



ω_4 = 9.26 rad./s c.w., α_4 = 1103.0 rad./s^2 c.c.w 670
$V_{Sliding}$ = 233.0 cm/s, $A_{Sliding}$ = 114.7 m/s^2

Analytical solution

The mechanism is treated as a shaper mechanism. For the crank (2), the data is

r_2 = 4.5 cm, $\theta_2 = 45^o$, ω_2 = 62.8 rad./s c.c.w

For the shaper chain,

ω_4 = – 9.3 rad./s c.w., α_4 = 1112.0 rad./s^2 c.c.w
$V_{Sliding}$ = 233.6 cm/s, $A_{Sliding}$ = 114.7 m/s^2

2.32 Link (4), Figure P2.32 is rotating at 30 rad/s, counter clockwise. Determine the velocity and the acceleration of point C, and the angular velocity and the angular acceleration of link (2).

OA = 120 mm, QB = 180 mm, BC = 160 mm.

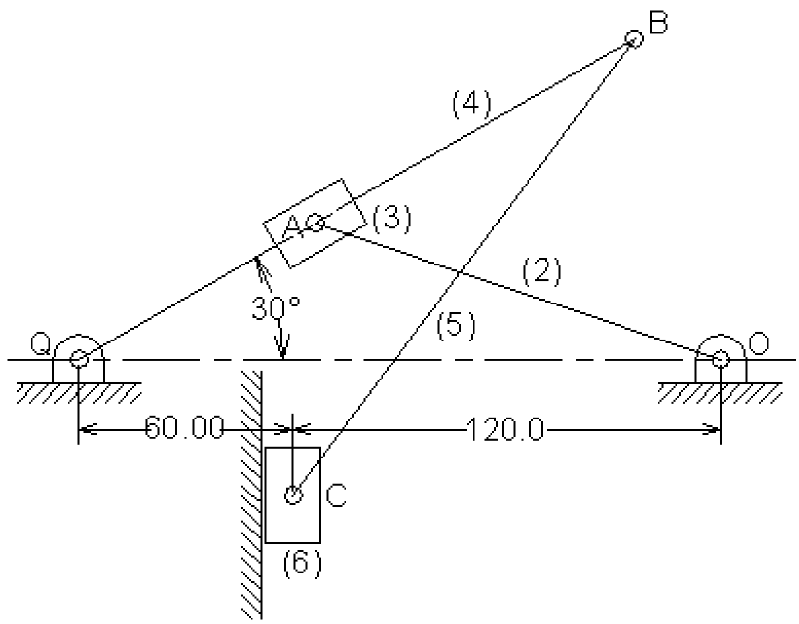
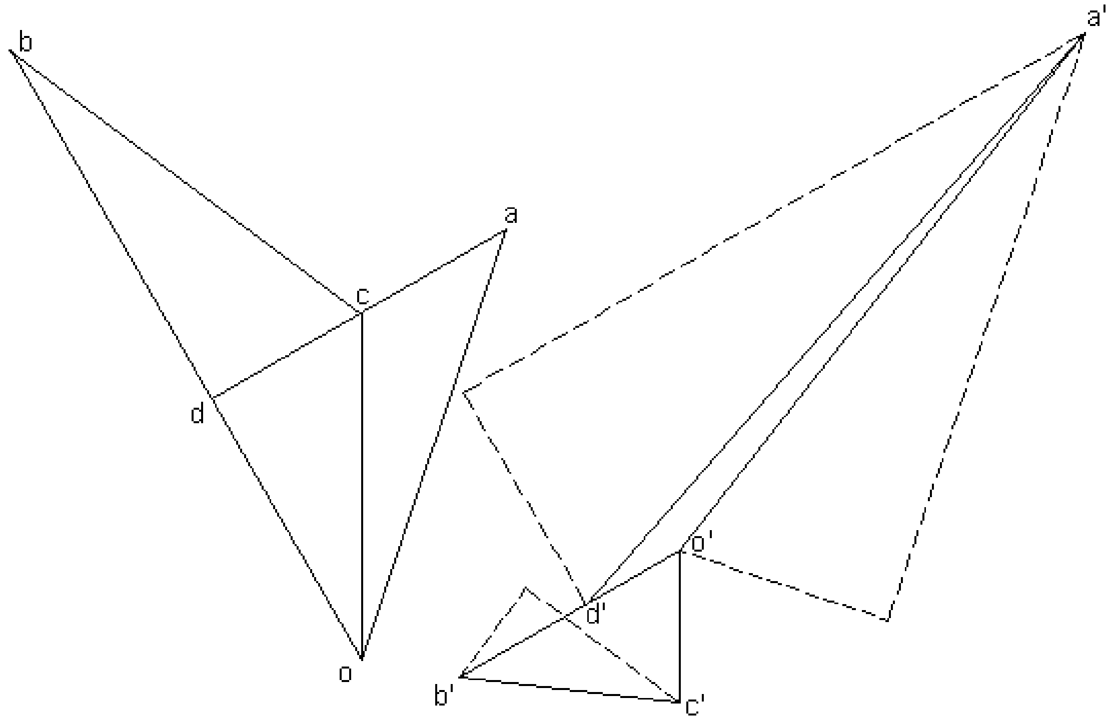


Figure P2.32

Solution

ω_2 = 30 rad./s c.c.w.
V_A = 230.4 cm/s
A_A^c = 69.12 m/s^2

Let point D be on link (4) in contact with point A.



ω_2 = 29.1 rad./s c.w., α_2 = 3302 rad./s^2 c.w
V_C = 264.6 cm/s, A_C = 97.77 m/s^2

Analytical solution

Links (4) and (5) form an engine mechanism with QB as the crank. We apply the equations for the crank–engine chains we obtain,

V_C = 265.53 cm/s, A_C = – 97.22 m/s^2

For links (2) and (3) we form the vector loops for the velocity and acceleration. From position analysis, we obtain θ_2 = 161.0^o and QA = 7.65 cm. Thus

$$i\ 12\ \omega_2\ e^{i\,\theta 2} = (V_{AD}^{SL} + i\ 76.5\ \omega_4)\ e^{i\,\theta 4}$$

$$V_{AD}^{SL} = 264 \text{ cm/s}$$

$$\omega_2 = -\ 29.15 \text{ (c.w.)}$$

$$12\ (-\ \omega_2{}^2 + i\alpha_2)\ e^{i\,\theta 2} = 76.5(-\ 30\ {}^2)\ e^{i\,\theta 4} + (A_{AD}^{SL} + i\,A_{AD}^{n})e^{i\,\theta 4}$$

$$A_{AD}^{n} = 158.4 \text{ m/s}^2$$

Multiplying by $e^{-\,i\,\theta 4}$ and equating the real parts we obtain A_{AD}^{SL}. From the imaginary parts we get,

$\alpha_2 = -\ 2967$ rad./s^2 (c.w.)

2.33 Block (5) is hinged at the end of link (6) and slides on link (3) of the four–bar mechanism as shown in Figure P2.33. If link (2) rotates clockwise at 200 rpm, determine the angular velocity and the angular acceleration of link (6). The crank angle is 135^o.

OA = 80 mm, AB = 240 mm, QB = 160 mm, UC = 200 mm.

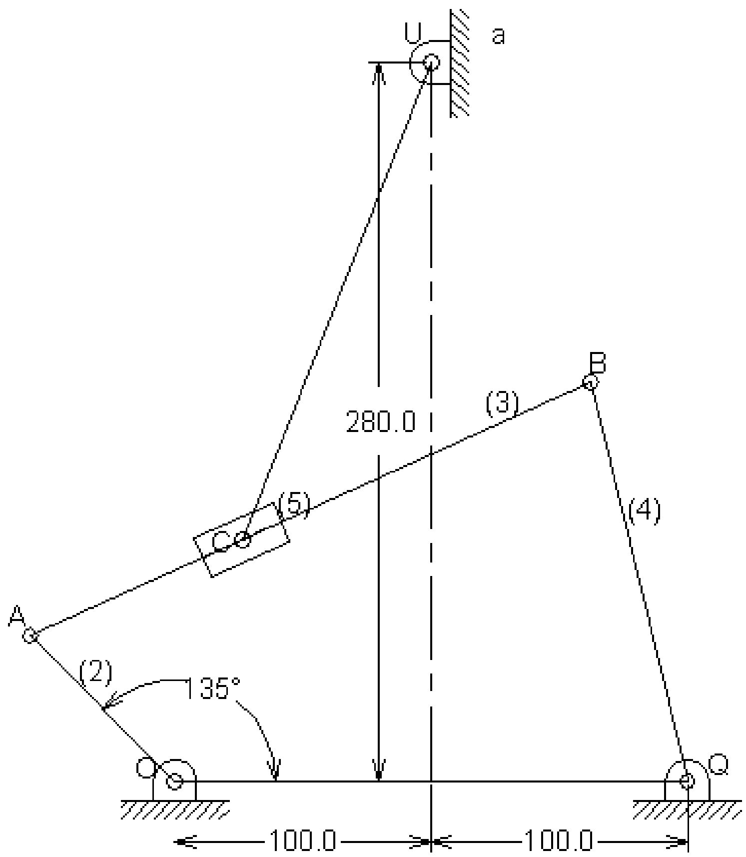
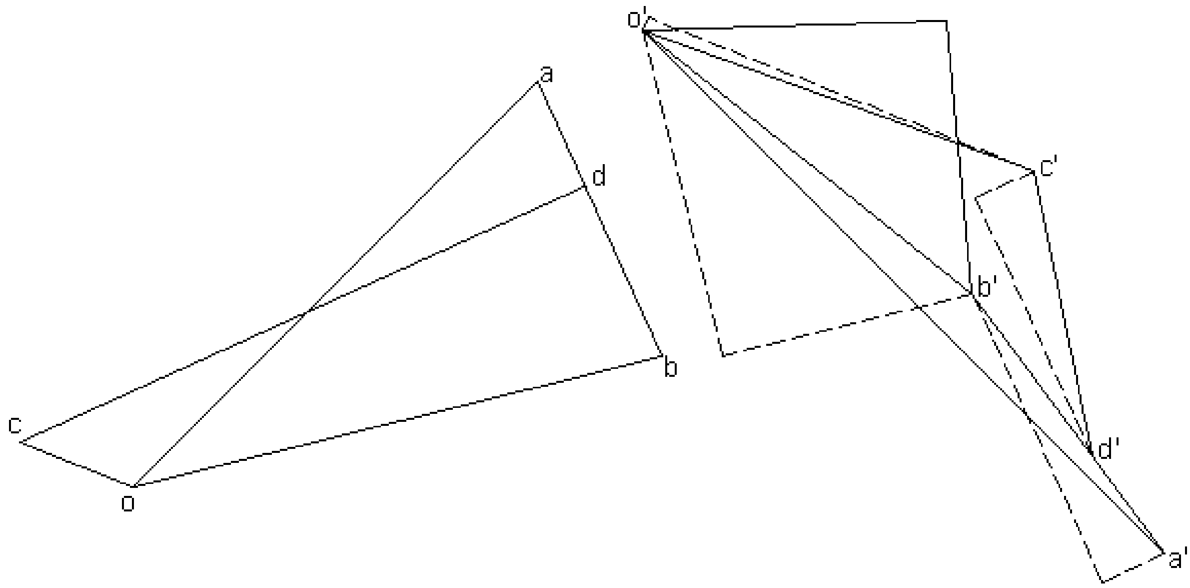


Figure P2.33

Solution

ω_2 = 20.9 rad./s c.c.w.
V_A = 176.47 cm/s
A_A^c = 35.06 m/s^2

Let point D be on link (3) in contact with point A.



ω_2 = 1.78 rad./s c.w., α_2 = 98.8 rad./s^2 c.c.w

Analytical solution

Links (2), (3) and (4) form a four–bar mechanism. We can obtain the kinematic data for point C, then use vector loops for the velocity and acceleration analysis for links (5) and (6) as in the previous problem.

2.34 In the mechanism shown in Figure P2.34, link (2) rotates clockwise at 100 rpm. Find the velocity and acceleration of the block 5. The crank OA is 120 mm.

Solution

ω_2 = 10.5 rad./s c.w.
V_A = 125.6 cm/s
A_A^c = 13.15 m/s^2

Let points C and D to be on link (3) in contact with points A and B, respectively.

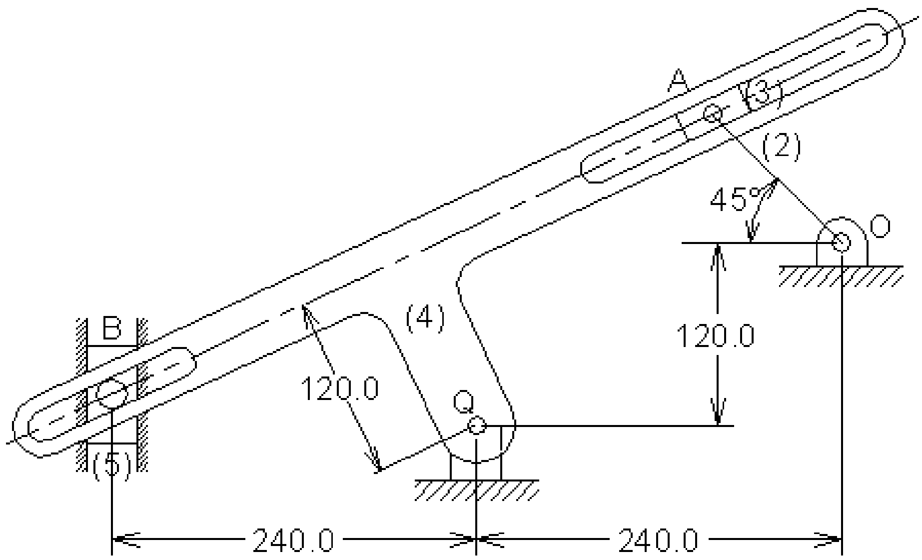
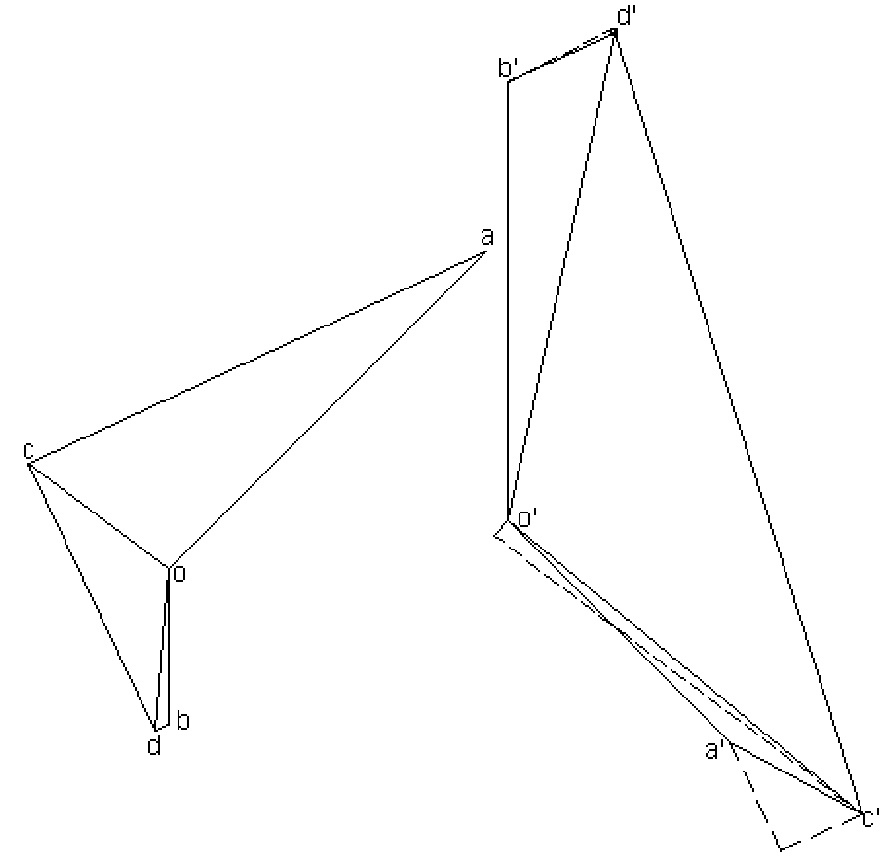


Figure P2.34



V_B = 43.3 cm/s, A_B = 182.1 m/s^2

Analytical solution

Links (2), (3) and (4) form a shaper mechanism. We can obtain the kinematic data for point D, then use vector loops for the velocity and acceleration analysis for links (5) and (6) as in the previous problem.

2.35 In the mechanism shown in Figure P2.35 link (2) rotates clockwise at 100 rpm. Find the velocity and the acceleration of block (8).

OA = 60 mm, AB = 240 mm, AC = 80 mm, CD = 240 mm, DE = 140 mm.

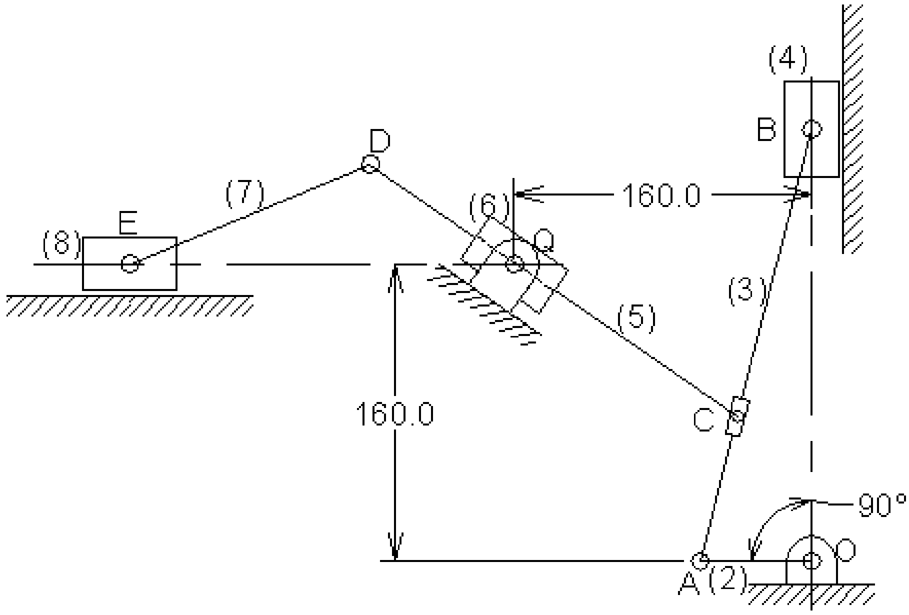
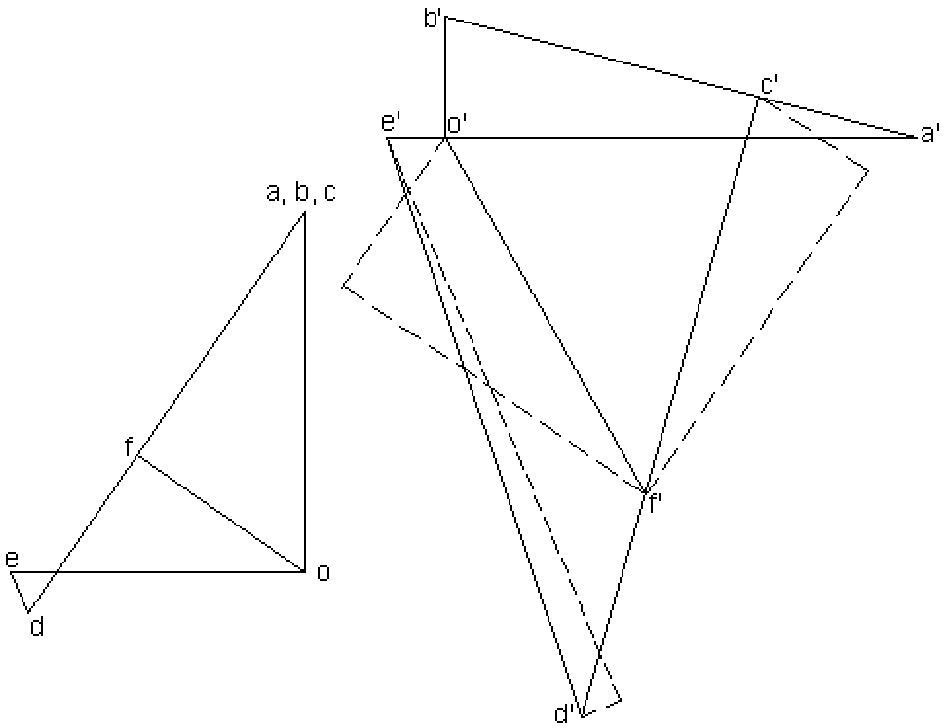


Figure P2.35

Solution

ω_2 = 10.5rad./s c.w.
V_A = 125.6 cm/s
A_A^c = 13.15 m/s^2

Let point E be on link (5) in contact with point Q.



V_B = 102.6 cm/s, A_B = 16.2 cm/s^2

Analytical solution

Links (2), (3) and (4) form an engine mechanism. We can obtain the kinematic data for point C. Links (5) and (6) form a tilting block chain. We can obtain the kinematic data for point D which is used as an input data for the engine chain formed by links (7) and (8).

2.36 For the mechanism shown in Figure P2.36, the angular velocity of the crank is 72 rad/s clockwise. Calculate the angular velocity and the angular acceleration of the link UE when the crank is horizontal.

OA = 40 mm, AB = 250 mm, AF = 170 mm, QB = 120 mm,
FC = 50 mm, CD = 30 mm, UE = 180 mm.

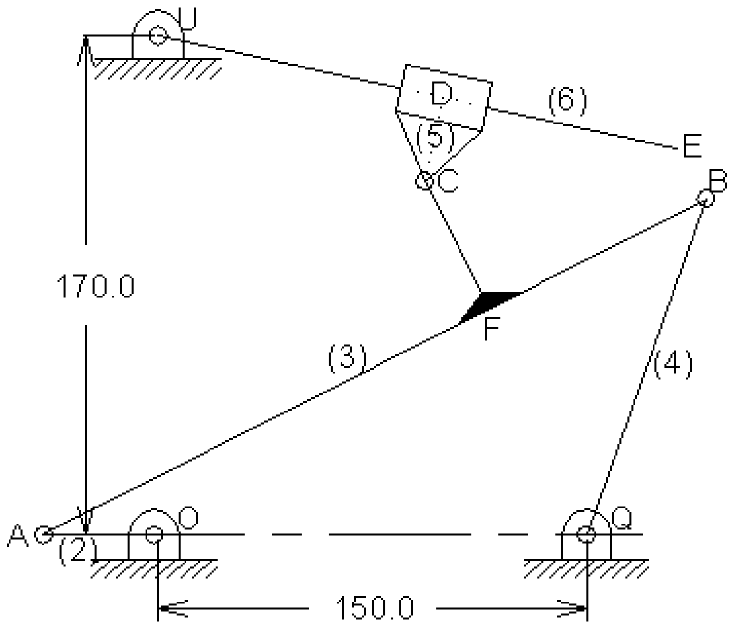
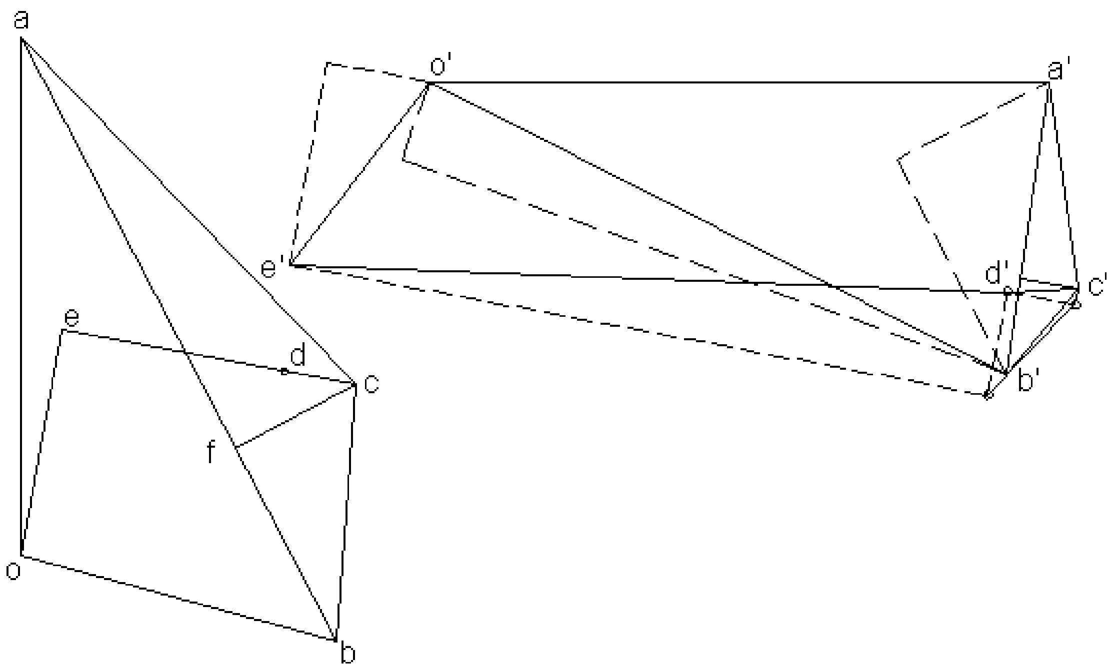


Figure P2.36

Solution

ω_2 = 10.5 rad./s c.w.
V_A = 125.6 cm/s
A_A^c = 13.15 m/s^2

Let point E be on link (6) in contact with point D.



ω_6 = 13.4 rad./s c.c.w., α_6 = 724.7 rad./s^2 c.c.w.

Analytical solution

Links (2), (3) and (4) form a four–bar mechanism. We can obtain the kinematic data for point C. Links (5) and (6) form a shaper chain.

2.37 Figure P2.37 shows two slotted links OA and QB, each of which are driven independently, the angular speed of OA is 30 rad/s clockwise and the angular speed of QB is 20 rad/sec clockwise. Find the absolute velocity and the absolute acceleration of the pin P.

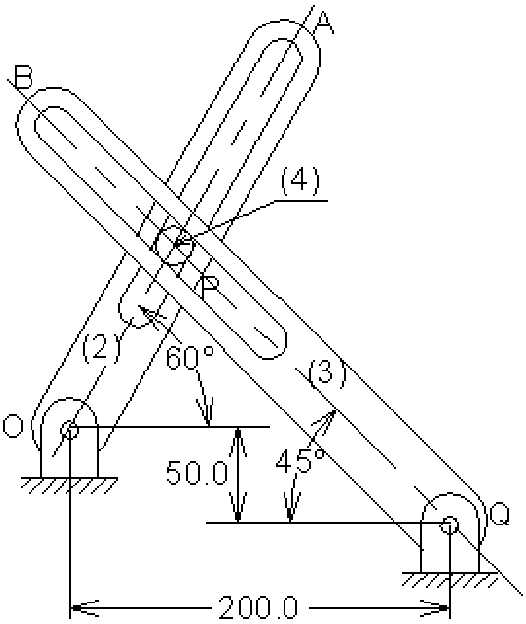
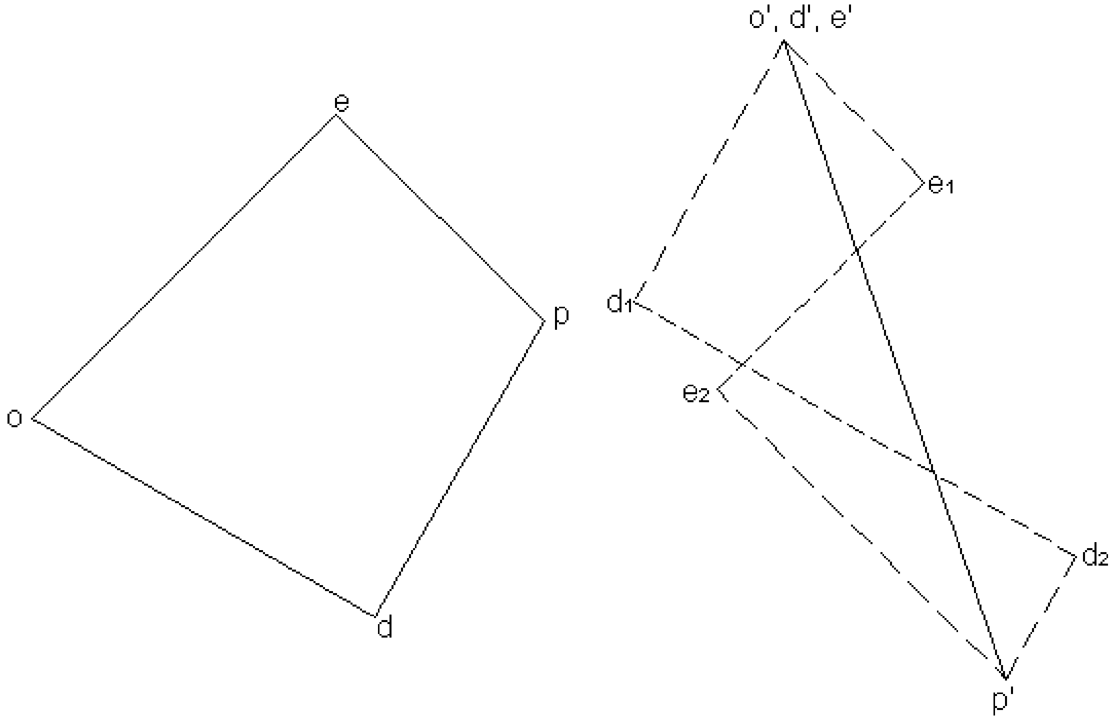


Figure P2.37

Solution

ω_2 = 30.0 rad./s c.w.
ω_3= 20.0 rad./s c.w.



o'd$_1$ represents A_D^c, d$_1$d$_2$ represents A_{PD}^n, and d$_2$p' represents A_{PD}^{SL}
o'd$_1$ represents A_D^c, e$_1$e$_2$ represents A_{PE}^n, and e$_2$p' represents A_{PE}^{SL}

Let points D and E be on links (2) and (3), in contact with point P, respectively.

V_P = 462 cm/s, A_P = 237.3 m/s^2

Analytical solution

From position analysis, OD = 11.16 cm and QE = 20.52 cm. Also, $\theta_2 = 60^o$ and $\theta_3 = 135^o$.

Velocity

The vector loop for point P is

$$\mathbf{V}_P = \mathbf{V}_D + \mathbf{V}_{PD} = \mathbf{V}_E + \mathbf{V}_{PE}$$
$$(i\ 11.16\ \omega_2 + V_{PD}^{SL})e^{i\,\theta 2} = (i\ 20.52\ \omega_3 + V_{PE}^{SL})\ e^{i\,\theta 3}$$

The solution of this equation gives V_{PD}^{SL} and V_{PE}^{SL}. From these values, V_P = 460.0 cm/s.

Acceleration

$(-11.16\ \omega_2^2 + A_{PD}^{SL} + i\,A_{PD}^{n})\ e^{i\,\theta 2} = (-20.5\ \omega_3^2 + A_{PE}^{SL} + i\,A_{PE}^{n})\ e^{i\,\theta 3}$

$A_{PD}^{n} = -179.52$ m/s^2 and $A_{PE}^{n} = 1103.3$ m/s^2

Therefore, $A_P = 235.5$ m/s^2

The solution of this equation gives A_{PD}^{SL} and A_{PE}^{SL}. Therefore, $A_P = 235.5$ m/s^2

2.38 In the mechanism shown in Figure P2.38, crank OA rotates uniformly at 120 rpm clockwise. Find the angular velocity and angular acceleration of link DE and the sliding velocity and the sliding acceleration of the block C.

OA = 30 mm, AB = 95 mm, QB = 90 mm, BC = 80 mm,

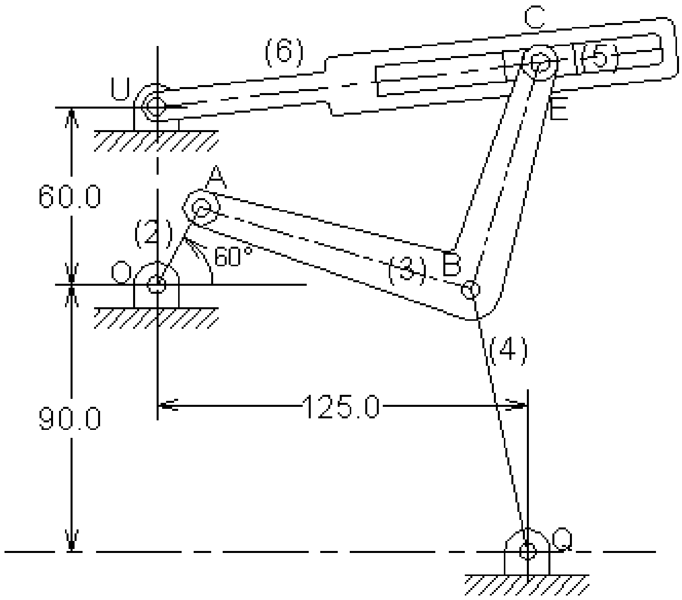
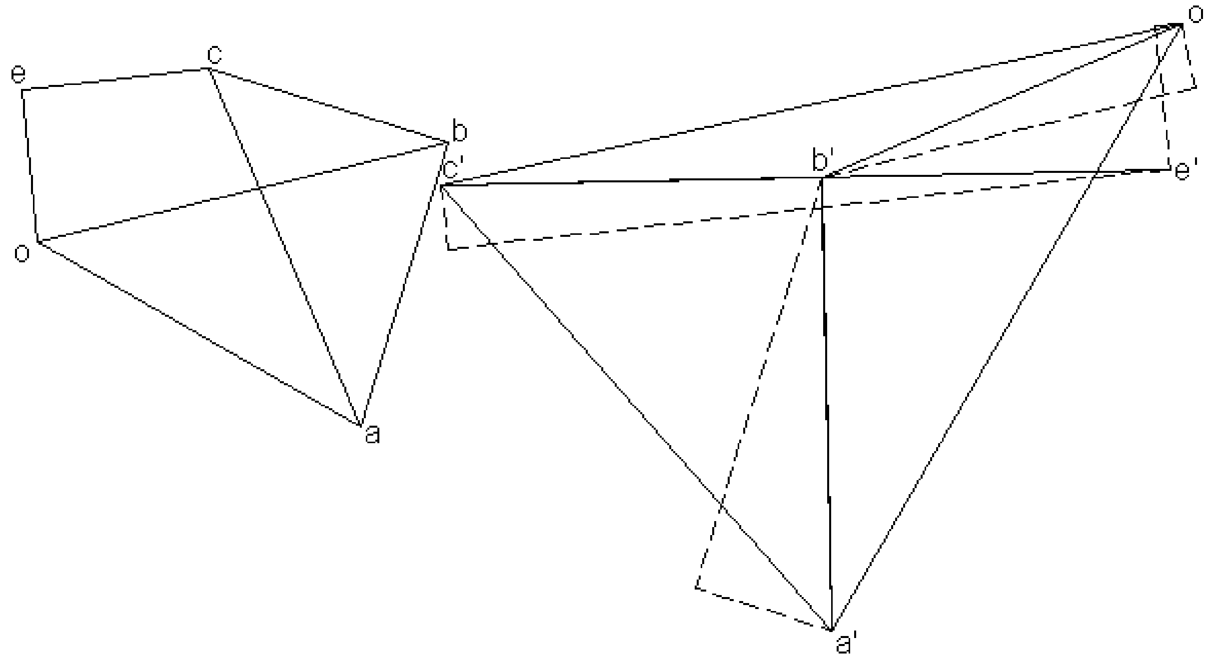


Figure P2.38

Solution

$\omega_2 = 12.6$ rad./s c.w.
$V_A = 125.6$ cm/s
$A_A^c = 5.73$ m/s^2

Let point E be on link (6) in contact with point C.



V_C^{SL} = 19.08 cm/s, A_C^{SL} =4.91 m/s^2

2.39 In Figure P2.39, link OA is 100 mm long, rotates at a uniform speed of 15 rad./s counter clockwise. Find the angular velocity and the angular acceleration of link (4).

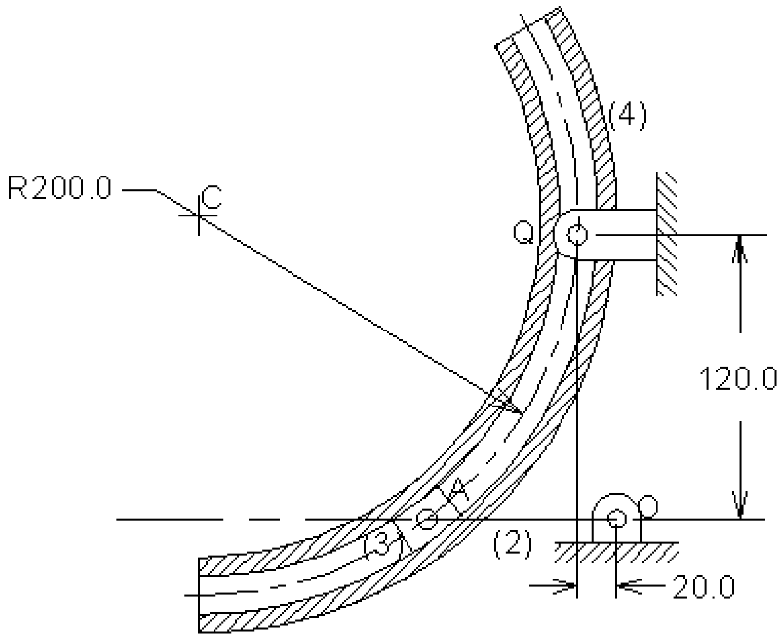
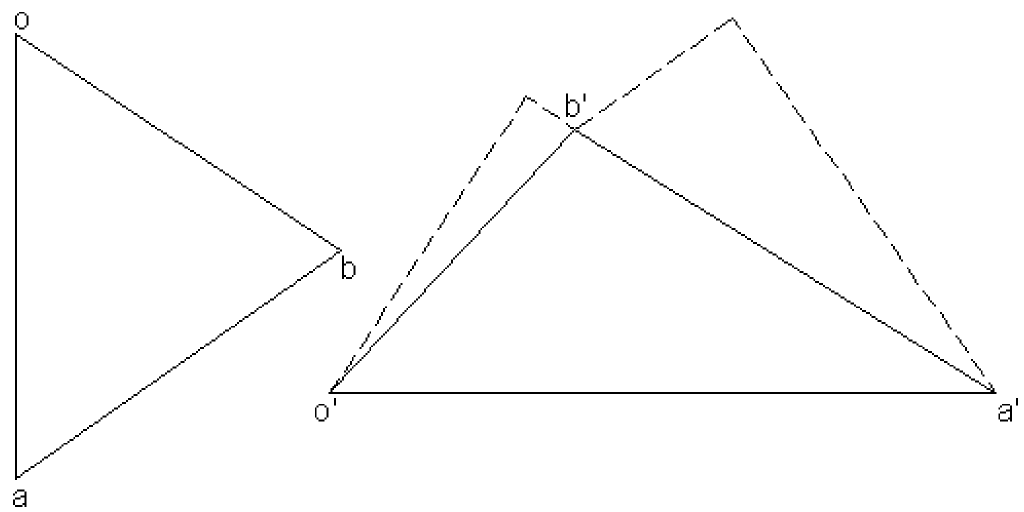


Figure P2.39

Solution

ω_2 = 15.0 rad./s c.c.w.
V_A = 150 cm/s
A_A^c = 5.73 m/s^2

Let point B be on link (4) in contact with point A, point C be at the center of link (4), $\mathbf{u}_n$ is a unit vector along line CA, and $\mathbf{u}_t$ is a unit vector is leading $\mathbf{u}_n$.



ω_4 = 9.12 rad./s c.c.w., α_4 = 13.66 rad./s^2 c.c.w.

Analytical solution

This mechanism is can be simply treated as an equivalent four–bar chain with AC as link (3) with length 200 mm and QC as link (4) with length 200 mm. From the position analysis, we can obtain θ_3 = 125.18^o and θ_4 = 176.44^o. The input data for the four–bar chain is,

V_A^x = 0, V_A^y = – 150 cm/s, A_A^x = 5.73 m/s^2, A_A^y = 0.

Applying equations (2–50) and (2–53), we get,

ω_4 = 9.12 rad./s c.c.w., α_4 = 13.66 rad./s^2 c.c.w

2.40 The cam shown in Figure P2.40 moves to the left with a constant speed of 10 cm/s. Find the angular velocity and the angular acceleration of the follower. Dimensions are in millimeters.

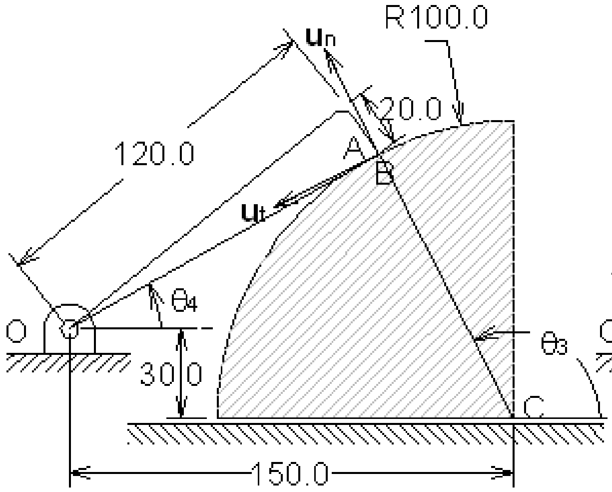


Figure P2.40

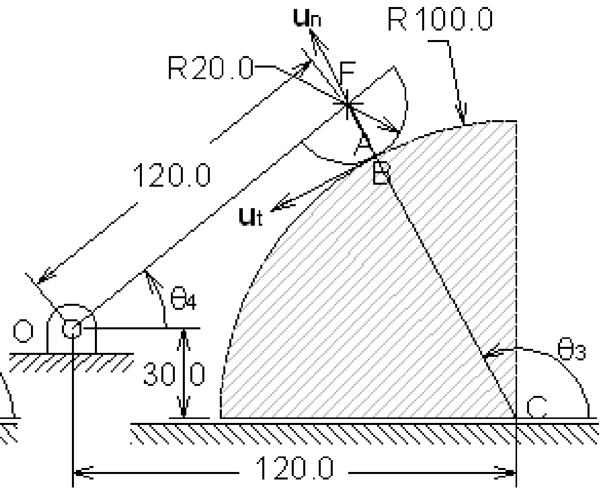
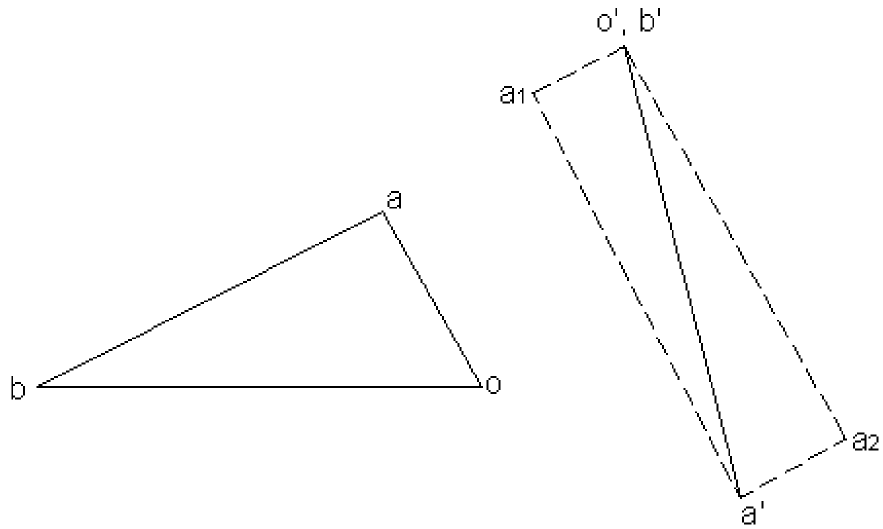


Figure P2.41

Solution

V_B = 10 cm/s
A_B = 0 cm/s^2



ω_{OA} = .0.38 rad./s c.c.w., α_{OA} = 0.642 rad./s^2 c.w
b'a_2 = A^n_{AB}

Analytical solution

$\mathbf{u}_n = e^{i\,\theta 3}$

From the position analysis, $\theta_3 = 117^o$, $\theta_4 = 29^o$
For the velocity,

$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$
$-10 = iV_{BA}\, e^{i\,\theta 3} + 12\, \omega_4\, e^{i\,\theta 4}$

From this equation we get,

$V_{BA} = 8.75$ cm/s
$\omega_4 = 0.38$

For the acceleration, $\rho_A = 0$ and $\rho_B = 10$. Applying Eqn. (2.30), then

$A^n_{AB} = -8.656$ cm/s^2

The loop equation is

$0 = (A^n_{AB} + i\,A^{SL}_{AB})e^{i\,\theta 3} + 12\,(-\omega_4^2 + i\,\alpha_4)\, e^{i\,\theta 4}$

Solving this equation we get,

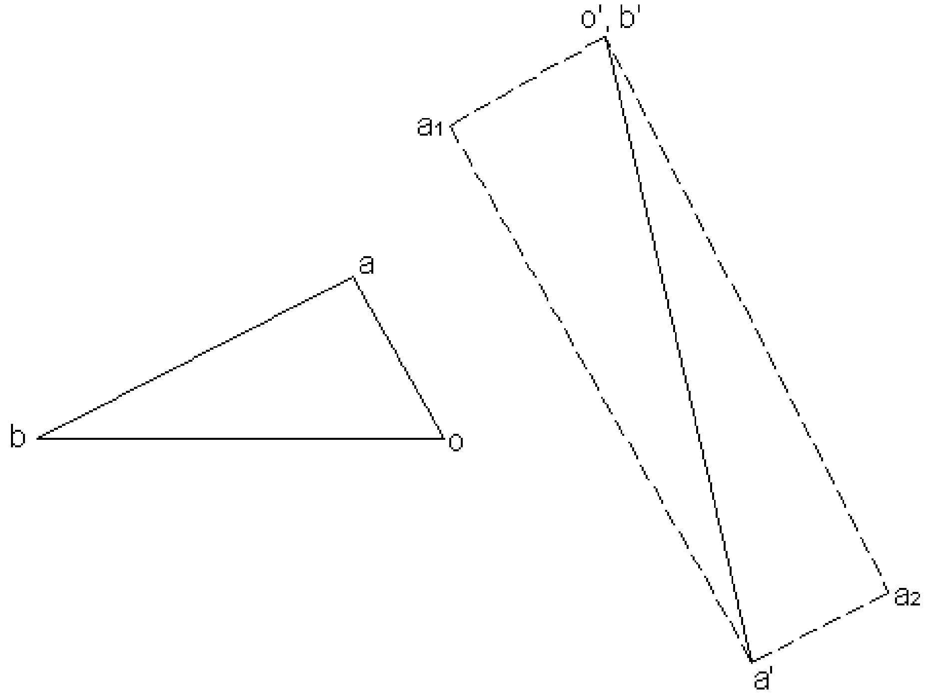
$\alpha_4 = 0.642$ rad./s^2 c.w

The mechanism can be treated as a four–bar chain and we obtain the same result.

2.41 Repeat Problem 2.40 when the tip of the follower is spherical, Figure P2.41.

Solution

$V_B = 10$ cm/s
$A_B = 0$ cm/s^2



ω_{OA} = .0.39 rad./s c.c.w., α_{OA} = 0.38 rad./s^2 c.w

b'a$_2$ = A^n_{AB}

Analytical solution

As stated in the previous problem, this mechanism can be treated as a four–bar chain with AC as link (3) with length 120 mm and QC as link (4) with length 120 mm. From the position analysis, we can obtain $\theta_3 = 117^o$ and $\theta_4 = 39^o$. The input data for the four–bar chain is,

$V^x_A = 0$, $V^y_A = -10$ cm/s, $A^x_A = 0$, $A^y_A = 0$.

Applying equations (2.50) and (2.53), we get,

ω_4 = 0.40 rad./s c.c.w., α_4 = – 0.41 rad./s^2 (c.w.)

2.42 The straight sided cam shown in Figure P2.42 rotates clockwise at 600 rpm and actuates an oscillating follower. Find the angular velocity and angular acceleration of the follower and the roller.

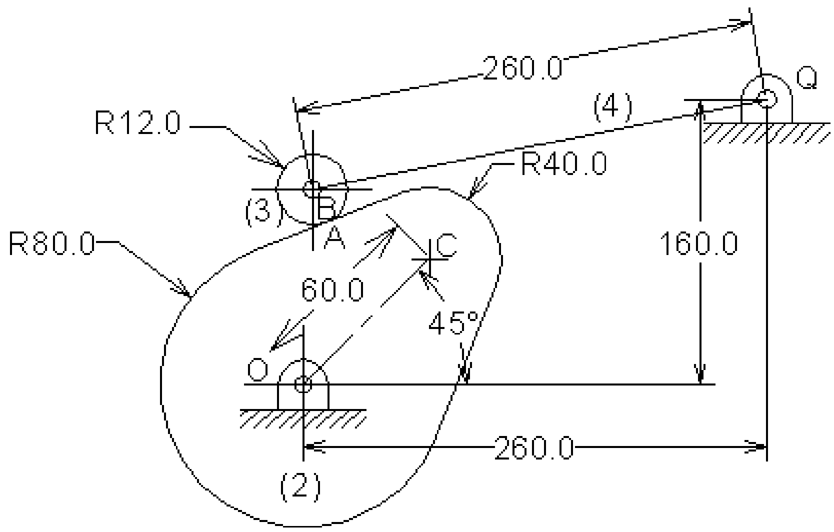
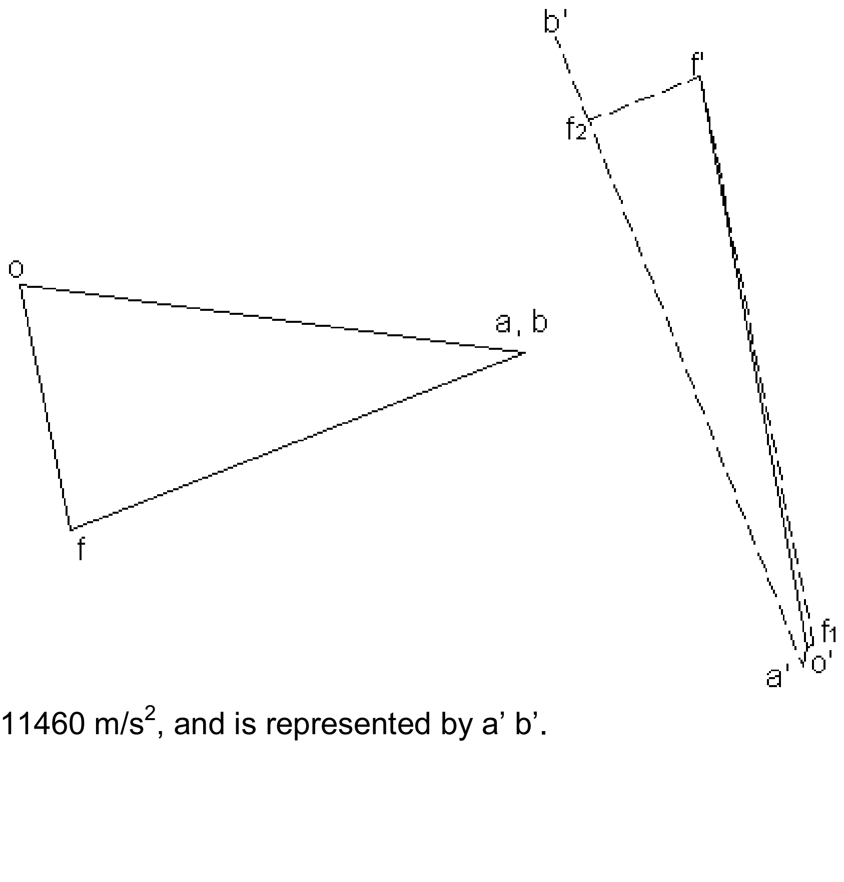


Figure P2.42

Solution

Let $\mathbf{u}_n$ be fro A to B.

ω_2 = 62.8 rad./s c.c.w.
V_A = 574.0 cm/s
A_A^c = 360.47 m/s^2



A_{BA}^n = 11460 m/s^2, and is represented by a' b'.

A^{c}_{FB} = 1520.76 m/s^2, and is represented by b' f_2.

ω_R = 275.75 rad./s c.c.w., α_R = 95250 rad./s^2 c.w.
ω_F = 10.92 rad./s c.c.w., α_F = 37615 rad./s^2 c.w.

Analytical solution

For the Analytical solution, vector loops should be used. Referring to the schematic diagram of the system, shown in the figure, and using position analysis, we get,

θ_2 = 111°, θ_4 = 191°, DF = 4.18 cm

For the velocity,

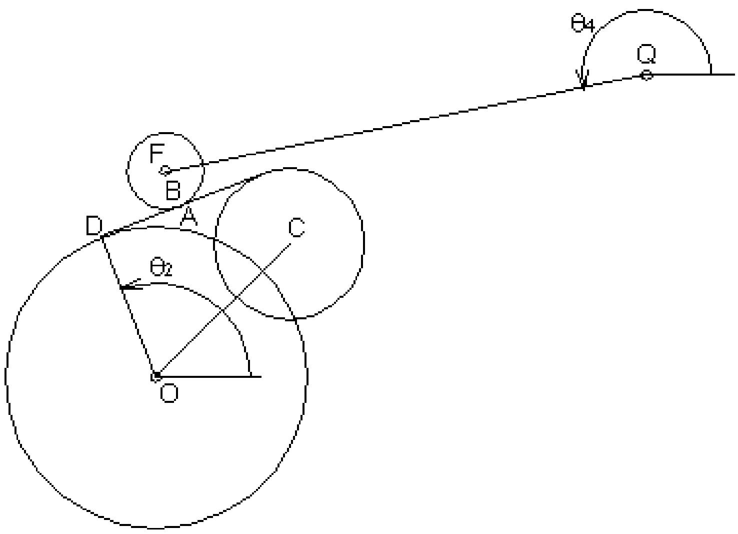
$\mathbf{V}_F = \mathbf{V}_A + \mathbf{V}_{BA} + \mathbf{V}_{FB}$
$(8 - i\,4.18)\, i\, 62.8\, e^{i\,\theta 2} + 0 + 2\, i\, \omega_R\, e^{i\,\theta 2} = 27.0\, i\, \omega_F\, e^{i\,\theta 4}$

Multiplying both sides by $e^{-i\,\theta 4}$ and equating the real parts, we get,

ω_R = 276.392 rad./s

From the imaginary parts,

ω_F = 11.16 rad./s



For the acceleration, the vector loop is,

$$-(8 + i\,4.18)\, 62.8^2\, e^{i\,\theta 2} + A^{n}_{BA}\, e^{i\,\theta 2} + 2\,(-\omega_R^2 + i\,\alpha_R)\, e^{i\,\theta 2} = 27.0\,(-\omega_F^2 + i\,\alpha_F)\, e^{i\,\theta 4}$$

Multiplying both sides by e$^{-i\,\theta 4}$ and equating th real parts, we get,

α_R = 93770 rad./s^2 c.c.w., α_F = – 35050 rad./s^2 (c.w.)

2.43 The large roller of the mechanism shown in Figure P2.43 rotates about center O with a speed of 150 rpm clockwise. The arm OQ rotates at 200 rpm clockwise and carries a small roller which rotates freely about center Q. The contact between the two rollers is pure rolling (no sliding). There is a slip between the small roller and the follower. Determine the magnitude and the direction of the angular velocities and the angular accelerations of the follower and the small roller.

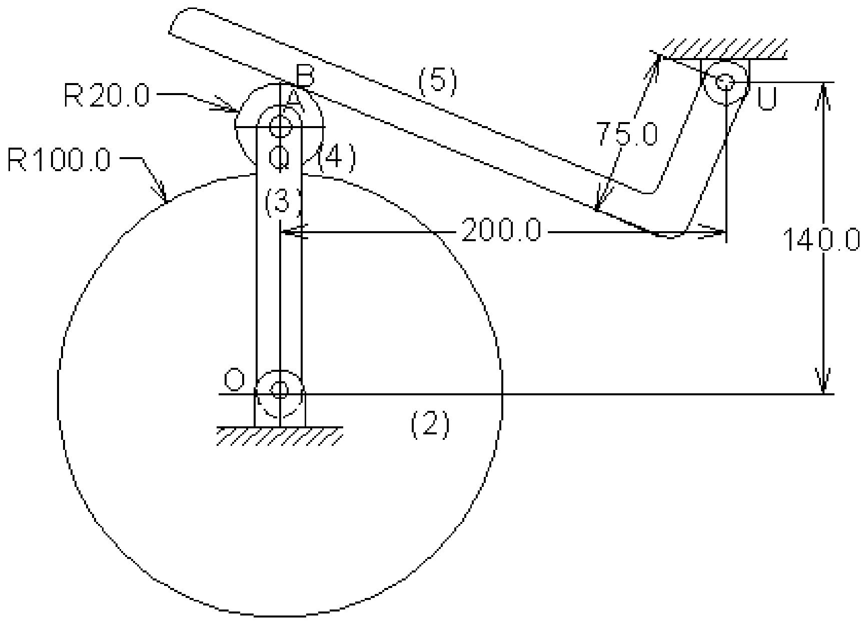
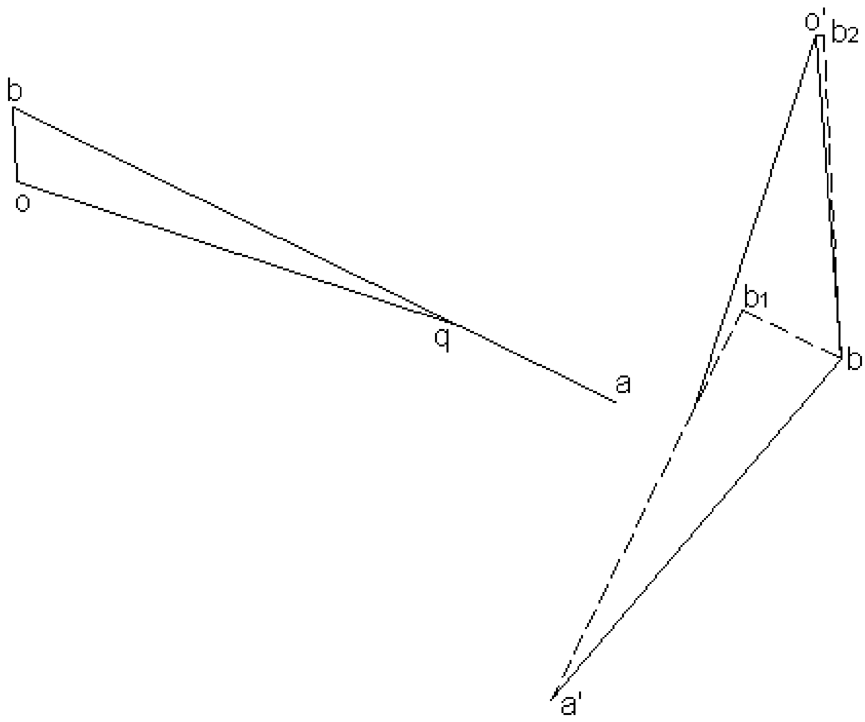


Figure P2.43

Solution

Let point P be the contact point between the two rollers..

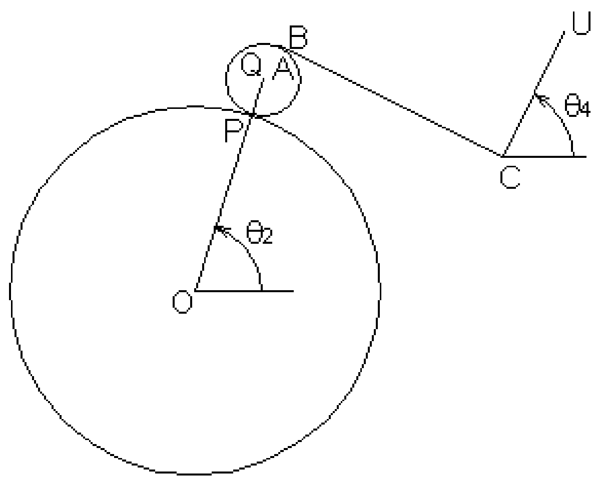
ω_2 = 15.7 rad./s c.w.
ω_3 = 20.93 rad./s c.w.
V_P = 157.0 cm/s
V_Q = 251.2 cm/s
A_Q^c = 52.58 m/s^2



ω_R = 47.1 rad./s c.w., α_R =0.
ω_5 = 3.63 rad./s c.w., α_5 = 284.2 rad./s^2 c.w.

Analytical solution

For the Analytical solution, vector loops should be used. Referring to the schematic diagram of the system, shown in the figure, and using position analysis, we get,



CB = 134.9 mm, θ_2 = 72^o, θ_4 = 64^o

$\mathbf{V}_Q = \mathbf{V}_P + \mathbf{V}_{QP}$
$12 \times i251.2\, e^{i\,\theta 2} = 10 \times i157\, e^{i\,\theta 2} + 2\, i\, \omega_R\, e^{i\,\theta 2}$

Thus,

$\omega_R = 47.1$ (c.w.)

We can conclude that $\alpha_R = 0$. For the follower,

$\mathbf{V}_B = \mathbf{V}_Q + \mathbf{V}_{AQ} + \mathbf{V}_{BA}$
$(-\,7.5 + i13.49)\, i\omega_5\, e^{i\,\theta 4} = -12\, i\, 20.93\, e^{i\,\theta 2} + 2 \times i47.1\, e^{i\,\theta 4} + i\, V_{BA}^{SL}\, e^{i\,\theta 4}$

Multiplying both sides by $e^{-i\,\theta 4}$ and equating the real parts, we get,

$\omega_5 = -\,2.59$ rad./s

From the imaginary parts,

$V_{BA}^{SL} = 154.52$ cm/s

For the acceleration, the vector loop is,

$(-\,7.5 + i13.49)\,(-\,\omega_F^2 + i\,\alpha_F)\, e^{i\,\theta 4} = -12\,(20.93)^2\, e^{i\,\theta 2} + (-\,(47.1)^2 + i\, A_{BA}^{SL})\, e^{i\,\theta 4}$

Multiplying both sides by $e^{-i\,\theta 4}$ and equating th real parts, we get,

$\alpha_5 = 284.2$ rad./s^2 c.c.w.

2.44 The mechanism shown in Figure P2.44 is driven through the crank OA at a speed of 10 rad/s counter clockwise. There is a pure rolling contact between the roller and the follower. Find the magnitude and the direction of angular velocity and the angular acceleration of the follower and the roller.

OA = 100 mm, AB = 400 mm, QB = 200 mm, AC = BC = 225 mm.

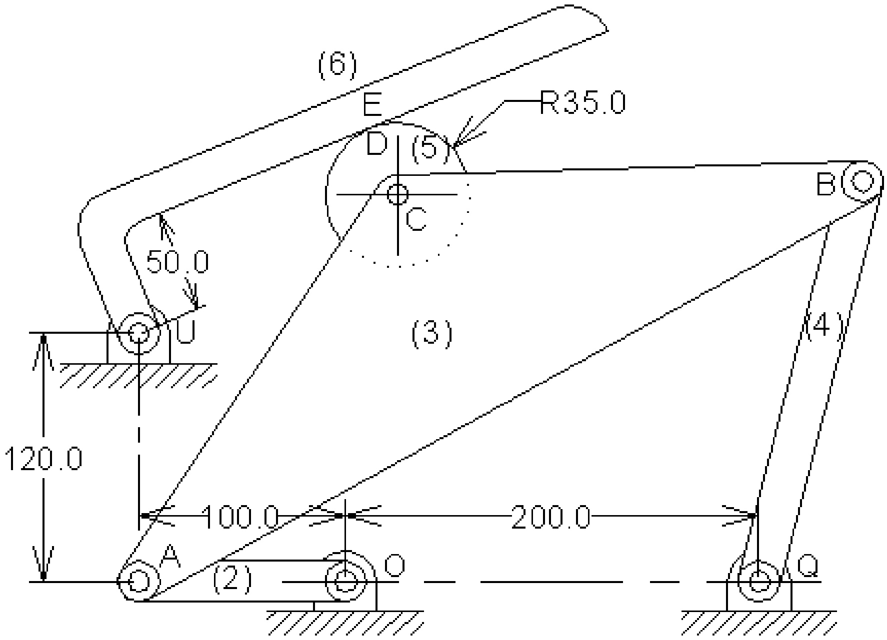
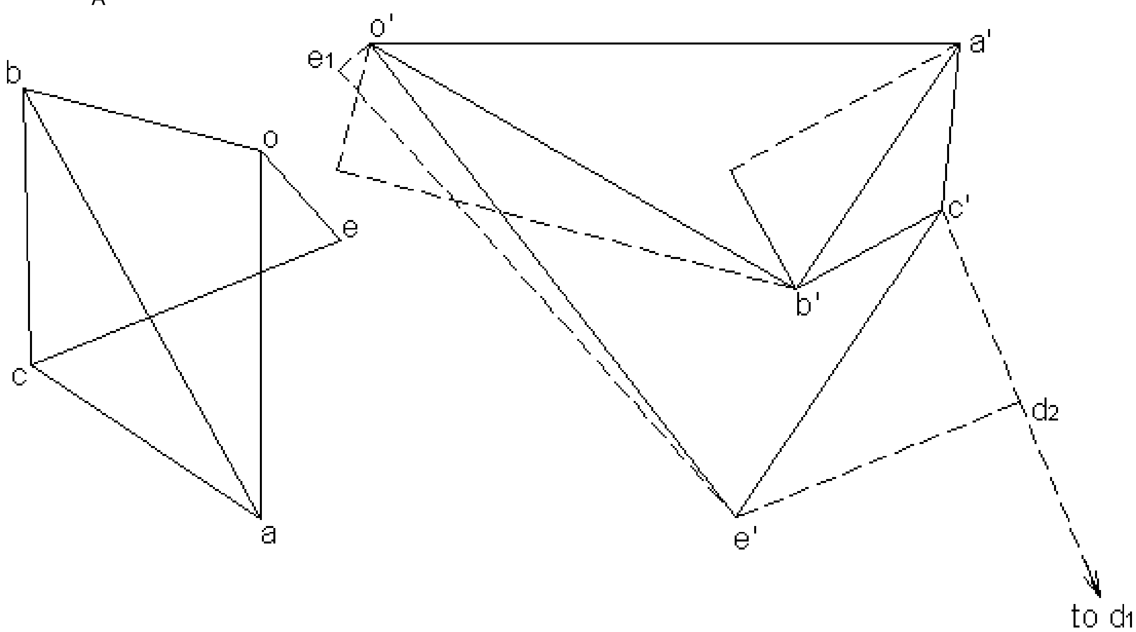


Figure P2.44

Solution

ω_2 = 10 rad./s c.c.w.
V_A = 100 cm/s
A_A^c = 10 m/s^2



c'd_1 represents A_{DC}^c , d_1 d_2 represents A_{ED}^n , d_2 e' represents A_{DC}^t .

ω_R = 25.87 rad./s c.w., α_R = 137.1 rad./s^2 c.w.
ω_6 = 2.16 rad./s c.w., α_6 = 67.34 rad./s^2 c.w.

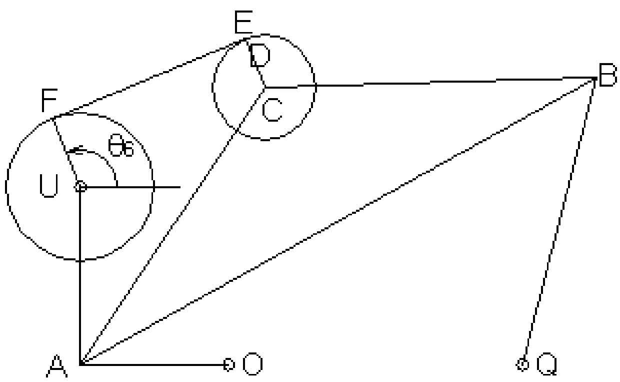
Analytical solution

For the Analytical solution, links (2), (3), and (4) are analyzed using the crank–four–bar chain to obtain,

V_C^x = – 62.14 cm/s, V_C^y = –58.01 cm/s, A_C^x = 10.65 m/s^2, and A_C^y = –2.79 m/s2.

For links ((5) and (6), we use vector loops. From the position analysis, and according the figure shown below we get,

θ_6 = 284.2^o, FE = 14.11 cm



For the velocity,

$$\mathbf{V}_E = \mathbf{V}_C + \mathbf{V}_{DC} + \mathbf{V}_{ED}$$

For pure rolling, $\mathbf{V}_{ED}$ = 0. Then,

$$(5.0 - i14.1)\, i\omega_6\, e^{i\,\theta 6} = (-\,62.14 - i\,58.01) + i\,3.5\,\omega_R\, e^{i\,\theta 6}$$

Multiplying by $e^{-i\,\theta 6}$ and equating the real parts,

ω_6 = – 2.16 rad/s

From the imaginary parts,

ω_R = – 25.61

For the acceleration,

$\mathbf{A}_E = \mathbf{A}_C + \mathbf{A}_{DC} + \mathbf{A}_{ED}$

$(5.0 - i14.1)(-\omega_6^2 + i\,\alpha_6)\,e^{i\theta_6} = (1065.0 - i\,279.4) + 3.5\,(-\omega_R^2 + i\,\alpha_R)e^{i\theta_6} + A_{ED}^n\,e^{i\theta_6}$

From Eq. (2.30), $A_{ED}^n = 1949$. Equating the real parts, we get,

$\alpha_6 = -70.05$ rad./s^2 (c.w.)

From the imaginary parts,

$\alpha_R = -133.34$ rad./s^2 (c.w.)

2.45 Repeat Problem 2.44 if pure sliding occurs between the roller and the follower (the roller is fixed with link (3)).

Solution

In this case, $\omega_R = \omega_3 = 3.333$ c.c.w. and $\alpha_R = \alpha_3 = -5.738$ rad./s^2 (c.w.).

For the acceleration, $V_{ED}^n = -3.53$ m/s^2. For link (6), we obtain the same results as in the previous

Notice: For the analytical solution, the position analysis should be performed first. The software used in this analysis in this manual linked the position and velocity and acceleration together as indicated in the Appendix of Chapter 7.

2.46 The crank OA of the mechanism shown in Figure P2.46 with a uniform speed of 10 rad/s clockwise. Find the angular velocity and the angular acceleration of link (6) analytically.

r2 = 60 mm, r3 = 200 mm, r5 = 150 mm, r6 = 150 mm.

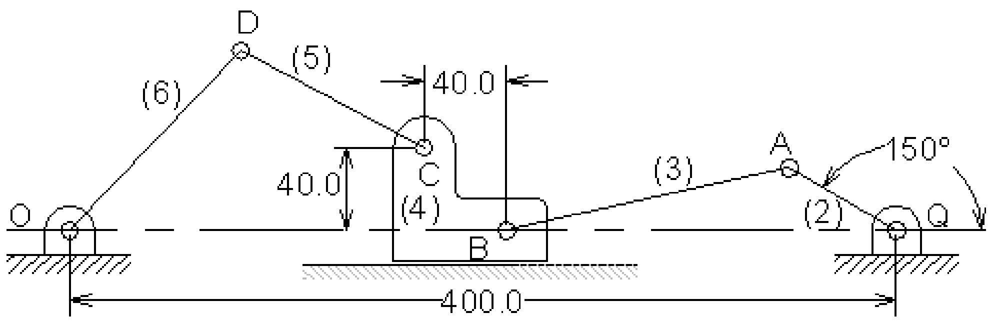


Figure P2.46

Solution

For the crank (2), the data is

$r_2 = 6$ cm, $\theta_2 = 150^o$, $\omega\theta_2 = -10$ rad/s, $\alpha_2 = 0$.

Using equations (2.47) and (2.48), then

$V_A^x = 30$ cm/s, $V_A^y = 51.96$ cm/s, $A_A^x = 5.2$ m/s^2 $A_A^y = -3$ m/s^2

Links (3) and (4) form an engine chain. Using equations (2.56) and (2.60), then

$V_B^x = 37.88$ cm/s, $V_B^y = 0$ cm/s, $A_B^x = 6.14$ m/s^2 $A_A^y = 0$ m/s^2

Links (5) and form a four–bar chain. The velocity and acceleration components of point B are the same for point C, and are used an input data for this chain. Using equations (2.50) and (2.52), then,

$\omega_4 = -2.39$ (c.w.)
$\alpha_3 = -33.27$ rad./s^2 (c.w.).

2.47 Solve Problems 2.12 to 2.46 analytically.

The analytical solution for each problem is presented after the graphical solution.