

$$\frac{M_{He}}{M_s} = p_0 \left(1 - \frac{39}{68} \alpha R \right) = 0.15 \quad (1)$$

The proportion of helium at the surface is given by

$$p_0(1 - \alpha R) = 0.033 \quad (2)$$

Eliminating p_0 from (1) and (2)

$$0.033 \left(1 - \frac{39}{68} \alpha R \right) = 0.15(1 - \alpha R)$$

giving $\alpha R = 0.893$. Hence, from (2),

$$p_0(1 - 0.893) = 0.033 \quad \text{or} \quad p_0 = 0.31.$$

CHAPTER 2

2.1 The mass of the galaxy ISM is

$$M_{ISM} = 4 \times 10^{13} \times (9.461 \times 10^{15})^3 \times 2 \times 10^{-21} = 6.77 \times 10^{40} \text{ kg}$$

The mass of stars in the galaxy is

$$M_{stars} = 10^{11} \times 10^{30} = 10^{41} \text{ kg}$$

$$\frac{\text{mass of ISM}}{\text{mass of observable galaxy}} = \frac{M_{ISM}}{M_{ISM} + M_{stars}} = \frac{6.77 \times 10^{40}}{6.77 \times 10^{40} + 10^{41}} = 0.404$$

Taking dark matter into account the mass of the galaxy is

$$M_{total} = 20 \times 1.677 \times 10^{41} = 3.35 \times 10^{42}$$

$$\text{Then } \frac{\text{mass of ISM}}{\text{total mass of galaxy}} = \frac{6.77 \times 10^{40}}{3.35 \times 10^{42}} = 0.0202$$

2.2 (i) Consider N representative bodies. The total number with masses between M_1 and M_2 is

$$Q = NC \int_{M_1}^{M_2} M^{-2.35} dM = \frac{NC}{1.35} \left(\frac{1}{M_1^{1.35}} - \frac{1}{M_2^{1.35}} \right). \quad (1)$$

With $M_2 = \infty$ and $M_1 = 0.075 M_{\odot}$ this gives the number of main sequence stars,

$$Q_{ms} = \frac{NC}{1.35(0.075M_{\odot})^{1.35}} = 24.5 \frac{NC}{M_{\odot}^{1.35}}.$$

The total mass of these stars is

$$M_{ms} = NC \int_{M_1}^{M_2} M \times M^{-2.35} dM = \frac{NC}{0.35} \left(\frac{1}{M_1^{0.35}} - \frac{1}{M_2^{1.35}} \right)$$

and with the same limits of mass this gives $M_{ms} = \frac{7.07 NC}{M_{\odot}^{0.35}}$.

The average mass of a main sequence star is thus

$$\overline{M} = \frac{M_{ms}}{Q_{ms}} = 0.29 M_{\odot}.$$

(ii) From (1) the number of brown dwarfs is

$$Q_{bd} = \frac{NC}{1.35} \left\{ \frac{1}{(0.013M_{\odot})^{1.35}} - \frac{1}{(0.075M_{\odot})^{1.35}} \right\} = 236.1 \frac{NC}{M_{\odot}^{1.35}}$$

The ratio of the number of brown dwarfs to main sequence stars is

$$\square \quad \frac{Q_{bd}}{Q_{ms}} = \frac{236.1}{24.5} = 9.6.$$

CHAPTER 3

3.1 The relationship is equivalent to $\log r_n = n \log b + \log a$.

| Planet | $n(x)$ | r_n | $\log r_n(y)$ | Formula |
|---------|--------|-------|---------------|---------|
| Mercury | 0 | 0.387 | -0.412 | 0.361 |
| Venus | 1 | 0.723 | -0.141 | 0.626 |
| Earth | 2 | 1.000 | 0.000 | 1.085 |
| Mars | 3 | 1.524 | 0.188 | 1.882 |
| Jupite2 | 5 | 5.203 | 0.716 | 5.659 |
| Saturn | 6 | 9.539 | 0.980 | 9.813 |
| Uranus | 7 | 19.19 | 1.283 | 17.02 |