CHAPTER 2

2.1 (c)

$$\bar{S}_{12} = \bar{V}_1\left(\frac{\bar{V}_1^* - \bar{V}_2}{\bar{Z}^*}\right) = (7560/\underline{10^\circ})\left(\frac{7560/\underline{-10^\circ} - 7200/\underline{0^\circ}}{50/\underline{-90^\circ}}\right)$$

$$= (7560/\underline{10^\circ})\left(\frac{1335.5/\underline{-79.4^\circ}}{50/\underline{-90^\circ}}\right) = (7560/\underline{10^\circ})(26.71/\underline{10.58^\circ})$$

$$= 201{,}927.6/\underline{20.58^\circ} = 189{,}016.3 + j71{,}046.5 \text{ VA}$$

(a) $P_{12} = 189{,}016.3$ W

(b) $Q_{12} = 71{,}046.5$ vars

2.2 (a) From Eq. (2.7),

$$P_{12} = \frac{1}{R^2 + X^2}(R|V_1|^2 - R|V_1||V_2|\cos\gamma + X|V_1||V_2|\sin\gamma)$$

where

$$Z = 50/\underline{26^\circ} = 50(\cos 26^\circ + j\sin 26^\circ) = 44.94 + j21.92 \ \Omega/\Phi$$

$$P_{12} = \frac{1}{44.94^2 + 21.92^2}(44.94|7560|^2 - 44.94|7560||7200|\cos 10^\circ$$

$$+ 21.92|7560||7200|\sin 10^\circ) = 146{,}658.6 \text{ W}$$

(b) From Eq. (2.8),

$$Q_{12} = \frac{1}{R^2 + X^2}(X|V_1|^2 - X|V_1||V_2|\cos\gamma - R|V_1||V_2|\sin\gamma)$$

$$= \frac{1}{44.94^2 + 21.92^2}(21.92|7560|^2 - 21.92||7200|\cos 10^\circ$$

$$- 44.94|7560||7200|\sin 10^\circ = -138{,}790.8 \text{ vars}$$

(c) $\bar{S}_{12} = P_{12} + jQ_{12} = 146{,}658.6 - j138{,}790.8$ VA

Alternatively, from Eq. (2.4),

$$\bar{S}_{12} = \bar{V}_1\left(\frac{\bar{V}_1^* - \bar{V}_2^*}{\bar{Z}^*}\right) = (7560/\underline{10^\circ})\left(\frac{7560/\underline{-10^\circ} - 7200/\underline{0^\circ}}{50/\underline{-26^\circ}}\right)$$

$$= (7560/\underline{10^\circ})(26.71/\underline{-53.4^\circ}) = 201{,}927.6/\underline{-43.4^\circ}$$

$$= 146{,}715.5 - j138{,}741.9 \text{ VA}$$

2.3 (a) $V_{B(L-N)} = \dfrac{V_{B(L-L)}}{\sqrt{3}}$

$V_{pu(L-L)} = \dfrac{V_{(L-L)}}{V_{B(L-L)}}$ and $V_{pu(L-N)} = \dfrac{V_{(L-N)}}{V_{B(L-N)}}$

However, $V_{(L-N)} = \dfrac{V_{(L-L)}}{\sqrt{3}}$

therefore, $V_{pu(L-N)} = \dfrac{V_{(L-N)}}{V_{B(L-N)}} = \dfrac{V_{(L-L)}/\sqrt{3}}{V_{B(L-L)}/\sqrt{3}}$

$$= \frac{V_{(L-L)}}{V_{B(L-L)}} = V_{pu(L-L)}$$

(b) $VA_{B(1\Phi)} = \dfrac{VA_{B(3\Phi)}}{3}$

However, $VA_{3\Phi} = 3VA_{1\Phi}$

therefore, $VA_{pu(1\Phi)} = \dfrac{VA_{1\Phi}}{VA_{B(1\Phi)}} = \dfrac{VA_{3\Phi}/3}{VA_{B(3\Phi)}/3}$

$$= VA_{pu(3\Phi)}$$

(c) $$Z_{B(Y)} = \frac{[V_{B(L-N)}]^2}{VA_{B(1\Phi)}} = \frac{[V_{B(L-L)}/\sqrt{3}]^2}{VA_{B(3\Phi)}/3} = \frac{[V_{B(L-L)}]^2}{VA_{B(3\Phi)}}$$

However, $3Z_{B(Y)} \triangleq Z_{B(\Delta)}$

therefore, $Z_{pu(Y)} = Z_{pu(\Delta)}$

2.4 (a) For 1Φ system,

$$Z_B = \frac{V_B}{I_B} = \frac{1000(kV_B)}{I_B} = \frac{1000(kV_B)}{kVA_B/kV_B}$$

therefore,

$$Z_B = \frac{1000(kV_B)^2}{kVA_B} = \frac{1000(kV_B)^2}{1000(MVA_B)} = \frac{(kV_B)^2}{MVA_B}$$

(b) Since $kVA_B = \sqrt{3}(kV_B)I_{B(L)}$

$$I_{B(L)} = \frac{kVA_B}{\sqrt{3}\ kV_B}$$

Also, $V_{B(L-L)} = \sqrt{3}\ I_{B(L)}Z_{B(3\Phi)}$ from which

$$Z_{B(3\Phi)} = \frac{V_{B(L-L)}}{\sqrt{3}\ I_{B(L)}}$$

or

$$Z_{B(3\Phi)} = \frac{1000[kV_{B(L-L)}]}{\sqrt{3}\ I_{B(L)}} = \frac{1000[kV_{B(L-L)}]}{kVA_{B(3\Phi)}/kV_{B(L-L)}}$$

$$= \frac{1000[kV_{B(L-L)}]^2}{kVA_{B(3\Phi)}} = \frac{1000[kV_{B(L-L)}]^2}{1000[MVA_{B(3\Phi)}]}$$

$$= \frac{[kV_{B(L-L)}]^2}{MVA_{B(3\Phi)}}$$

2.5 Since $V_{B(L-L)} = \sqrt{3}\, V_{B(L-N)}$ (1)

and $I_{B(\Delta/\Phi)} = \frac{1}{\sqrt{3}} I_{B(Y/\Phi)}$ (2)

dividing Eq. (1) by Eq. (2) side by side,

$Z_{B(\Delta)} = 3Z_{B(Y)}$

2.6 In terms of line #1,

$$X_{pu(m)1} = \frac{X_{m(physical),\Omega}}{X_B} = \frac{X_{m(physical)}}{V_{B(1)}/I_{B(2)}} \quad (1)$$

Using the base current in line #2, since I_2 is the current that induces mutual voltage into line #1,

$$X_{pu(m)2} = (\text{Physical } X_m)[\frac{I_{B(2)}}{V_{B(1)}}] \quad (2)$$

Multiplying right side of Eq. (2) by $(V_{B(2)}/V_{B(2)})$,

$$X_{pu(m)1} = (\text{Physical } X_m)[\frac{I_{B(2)}}{I_{B(1)}}][\frac{V_{B(2)}}{V_{B(2)}}]$$

or

$$X_{pu(m)} = (\text{Physical } X_m)\frac{VA_B}{[V_{B(1)}][V_{B(2)}]}$$

since $VA_B = VA_{B(1)} = VA_{B(2)}$

$$= V_{B(1)}I_{B(1)} = V_{B(2)}I_{B(2)}$$

Therefore, $X_{pu(m)} = (\text{Physical } X_m)\dfrac{MVA_B}{[kV_{B(1)}][kV_{B(2)}]}$

2.7 (a) 34.5 kV

(b) $Z_{B(LV)} = \dfrac{[kV_{B(LV)}]^2}{MVA_B} = \dfrac{34.5^2}{20} = 59.5125\,\Omega$

(c) $n = \dfrac{345/\sqrt{3}}{34.5/\sqrt{3}} = 10$

(d) $X_{LV} = \dfrac{X_{HV}}{n^2} = \dfrac{714.15\,\Omega}{10^2} = 7.1415\,\Omega$

(e) $X_{pu} = \dfrac{X_{LV}}{X_{B(LV)}} = \dfrac{7.1415}{59.5125} = 0.12\text{ pu}$

2.8 (a) $\bar{I} = \dfrac{\bar{V}}{R_s + jX_s}$ from which $\bar{I}^* = \dfrac{\bar{V}^*}{R_s - jX_s}$

therefore, $P + jQ = \bar{V}\bar{I}^* = \dfrac{\bar{V}\bar{V}^*}{R_s - jX_s}$

$$= \frac{|V|^2}{R_s - jX_s} \tag{1}$$

Multiplying Eq. (1) by its conjugate,

$$P^2 + Q^2 = \frac{|V|^4}{R_s^2 + X_s^2}$$ from which

$$R_s^2 + X_s^2 = \frac{|V|^4}{P^2 + Q^2}$$

From Eq. (1),

$$P + jQ = \frac{|V|^2}{R_s - jX_s} \left(\frac{R_s + jX_s}{R_s + jX_s}\right) = \frac{|V|^2(R_s + jX_s)}{R_s^2 + X_s^2}$$

$$= \frac{P^2 + Q^2(R_s + jX_s)}{|V|^2}$$

or $$R_s + jX_s = \frac{|V|^2}{P^2 + Q^2}(P + jQ)$$

therefore, $$R_s = \frac{|V|^2}{P^2 + Q^2}(P)$$

(b) Thus, $$X_s = \frac{|V|^2}{P^2 + Q^2}(Q)$$

(c) Since $$Z_{pu} = (Z_{physical})\frac{S_B}{V_B^2} \text{ pu}$$

$$R_{pu(s)} = (P_{physical})\frac{S_B(V_{pu})^2}{P^2 + Q^2} \text{ pu}$$

(d) Therefore, $X_{pu(s)} = (Q_{physical}) \dfrac{S_B(V_{pu})^2}{P^2 + Q^2}$ pu

2.9 (a) Since the real power absorbed depends upon the applied voltage,

$$P = \frac{V^2}{R_P} \qquad \text{from which } R_P = \frac{V^2}{P}$$

$$\text{Since } Z_{pu} = (Z_{physical}) \frac{S_B}{V_B^2} \text{ pu}$$

$$R_{P(pu)} = (\frac{S_B}{P})(\frac{V}{V_B})^2 = \frac{V_{pu}^2}{P_{pu}} \text{ pu}$$

(b) Similarly $Q = \dfrac{V^2}{X_p}$ from which $X_{pu} = \dfrac{V^2}{Q}$

$$\text{Therefore, } X_{P(pu)} = (\frac{S_B}{Q})(\frac{V}{V_B})^2 = \frac{V_{pu}^2}{Q_{pu}} \text{ pu}$$

2.10 (a) Nominally 69 kV Circuits:

$$I_{B(L)} = \frac{kVA_{B(3\Phi)}}{3kV_{B(L-N)}} = \frac{5000}{3x39.84} = 41.8 \text{ A}$$

$$I_{B(\Phi)} = \frac{I_{B(L)}}{\sqrt{3}} = \frac{41.8}{\sqrt{3}} = 24.1 \text{ A}$$

$$Z_B = \frac{kV_{B(L-N)}}{I_{B(L)}} = \frac{39.84x10^3}{41.8} = 954 \ \Omega$$

$$Y_B = \frac{1}{Z_B} = \frac{1}{954} = 0.00105\ S$$

<u>Nominally 13 kV Circuits:</u>

$$I_{B(L)} = \frac{5000}{3x7.97} = 209\ A$$

$$I_{B(\Phi)} = \frac{209}{\sqrt{3}} = 121\ A$$

$$Z_B = \frac{7.97x10^3}{209} = 38.2\ \Omega$$

$$Y_B = \frac{1}{38.2} = 0.0262\ S$$

<u>Nominally 480 V Circuits:</u>

$$kV_{B(L-L)} = 13.8(\frac{0.480}{13.2}) = 0.502\ kV$$

$$kV_{B(L-N)} = \frac{0.502\ kV}{\sqrt{3}} = 0.290\ kV$$

$$I_{B(L)} = \frac{5000}{3x0.290} = 5750\ A$$

$$I_{B(\Phi)} = \frac{5750}{\sqrt{3}} = 3320\ A$$

$$Z_B = \frac{290}{5750} = 0.0504\ \Omega$$

$$Y_B = \frac{1}{0.0504} = 19.83\ S$$

Table P 2.10 Table for Problem 2.10

Quantity	Nominally 69kV Circuits	Nominally 13kV Circuits	Nominally 480V Circuits
$kVA_{B(3\Phi)}$	5,000 kVA	5,000 kVA	5,000 kVA
$kV_{B(L-L)}$	69 kV	13.8 kV	0.502 kV
$kV_{B(L-N)}$	39.84 kV	7.97 kV	0.290 kV
$I_{B(L)}$	41.8 A	209 A	5760 A
$I_{B(\Phi)}$	24.1 A	121 A	3320 A
Z_B	954 Ω	38.2 Ω	0.0504 Ω
Y_B	0.00105 S	0.0262 S	19.83 S

(b) Transformer T_1:

$$\text{Rating} = 15{,}000 \text{ kVA} = \frac{15{,}000}{5{,}000} = 3 \text{ pu}$$

$$Z_{T_1} = 0.01 + j0.08 \text{ pu based on its ratings}$$

$$= (0.01 + j0.08)\left(\frac{5{,}000}{15{,}000}\right)\left(\frac{13.8}{13.8}\right)^2 = 0.0033 + j0.0267 \text{ pu}$$

Transformer T_2:

$$\text{Rating} = 1{,}500 \text{ kVA} = \frac{1{,}500}{5{,}000} = 0.3 \text{ pu}$$

$$Z_{T_2} = 0.01 + j0.05 \text{ pu based on its ratings}$$

$$= (0.01 + j0.05)\left(\frac{5{,}000}{1{,}500}\right)\left(\frac{13.2}{13.8}\right)^2 \text{ pu}$$

$$= 0.0305 + j0.1525 \text{ pu}$$

Generator G_1:

$$\text{Rating} = 10/12.5 \text{ MW/MVA} = \frac{12{,}500}{5{,}000} = 2.5 \text{ pu}$$

$$Z_{G_1} = j1.1\left(\frac{5,000}{12,500}\right)\left(\frac{13.8}{13.8}\right)^2 = j0.44 \text{ pu}$$

Generator G_2:

$$\text{Rating} = 4/5 \text{ MW/MVA} = \frac{5,000}{5,000} = 1 \text{ pu}$$

$$Z_{G_2} = j0.9\left(\frac{5,000}{5,000}\right)\left(\frac{13.2}{13.8}\right)^2 = j0.8235 \text{ pu}$$

Load 1:

$$8,000 + j6,000 \text{ kW/kvar} = \frac{8,000}{5,000} + j\,\frac{6,000}{5,000} = 1.6 + j1.2 \text{ pu}$$

Load 5:

$$\bar{I}_1 = |I_1|(\cos\Theta + j\sin\Theta) = 52.3(0.707 + j0.707)$$

$$= \frac{37 + j37}{209} = 0.177 + j0.177 \text{ pu}$$

Since it is not being solved for the circuit, assume that V_5 = 1.0 pu

69 kV Circuit:

$$Z = R + jX = 50(0.445 + j0.976)\ \Omega$$

$$= \frac{1}{954}\,[50(0.445 + j0.976)] = 0.0222 + j0.0512 \text{ pu}$$

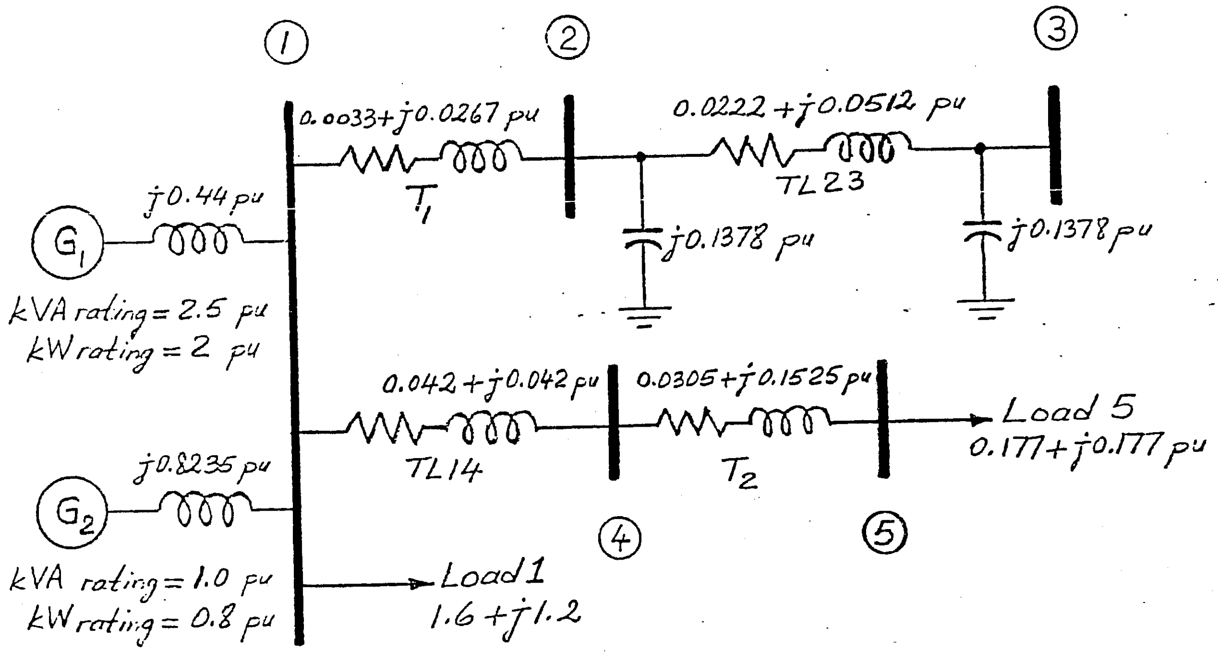
$$\frac{B}{2} = \frac{1}{2}(50)(5.78 \times 10^{-6} \text{ S})] = \frac{25 \times 5.78 \times 10^{-6} \text{ S}}{0.00105 \text{ S}}$$

$$= 0.1378 \text{ pu}$$

13 kV Circuit:

$$Z = R + jX = 2(0.8 + j0.8)$$

$$= \frac{1}{38.2}[2(0.8 + j0.8)] = 0.042 + j0.042 \text{ pu}$$



2.11 (a) $$S_{load,pu} = \frac{500}{1000} + j\,\frac{200}{1000} = 0.5 + j0.2 \text{ pu}$$

where $P_{pu} = 0.5$ pu and $Q_{pu} = 0.2$ pu

Therefore, $$R_{pu} = \frac{V^2_{pu}}{P_{pu}} = \frac{1.0^2}{0.5} = 2\text{pu}$$

$$X_{pu} = \frac{V_{pu}^2}{Q_{pu}} = \frac{1.0^2}{0.2} = 5 \text{ pu}$$

(b) $$R_{pu} = \frac{V_{pu}^2 \cdot S_B \cdot P}{P^2 + Q^2} = \frac{1.0^2(1000)500}{500^2 + 200^2} = 1.7241 \text{ pu}$$

$$X_{pu} = \frac{V_{pu}^2 \cdot S_B \cdot Q}{P^2 + Q^2} = \frac{1.0^2(1000)500}{500^2 + 200^2} = 0.6896 \text{ pu}$$