Chapter 2

Solution: 1) An aircraft is flying at $M = 0.8$ and at $h = 12{,}000m$.
i) What are the air density and the speed of sound at sea level?
Density, $\rho_0 = 1.22505$ kg/m^3, Speed of sound, $a_0 = 340.3$ m/s.
ii) What are the air density and the speed of sound at that altitude?
At an altitude of 11 km,

$$p_{20}/p_0 = 0.2234,\ \rho_{20}/\rho_0 = 0.2971,$$

and at 12 km

$$p_2/p_{20} = \ \rho_2/\rho_{20} = \exp[-0.15769].=0.8541$$

Hence,

$$p_2/p_0 = 0.1908,\ \rho_2/\rho_0 = 0.2538,$$

Hence $\rho_2 = 0.3109$ kg/m^3.
Over this entire altitude range, ratio of the speed of sound, $a_{20}/a_0 = 0.8671$.
Hence at 12 km, $a_2 =$ 295.0741 m/s.

iii) What is the air density at an altitude, $h = 2{,}000m$?
temperature ratio=1-13/288.15=0.9549; density ratio=0.9549^4.256=0.8217
Air density=1.0066 kg/m^3.

Solution: 2) An American aircraft is flying at $M = 0.8$ at $h = 30{,}000$ ft.
i) What are the air density and the speed of sound at sea level?
Air density= 23.77*10^{-4} slugs/ft^3. Speed of sound= 761 mph=1116.1ft/s
ii) What are the air density and the speed of sound at that altitude?
Air density= 8.91*10^{-4} slugs/ft^3. Speed of sound= 678.1 mph=994.5467 ft/s

3) **Solution:**

$$C_{Lw} = \frac{2s}{S}\int_0^1 C_L c(\eta)d\eta = \frac{2s}{S}C_{L0}c_0\int_0^1\sqrt{1-\eta^2}\,d\eta .$$

$$\int_0^1\sqrt{1-\eta^2}\,d\eta = \frac{\pi}{4};$$

$$C_{Lw} = \frac{2s}{S}\int_0^1 C_L c(\eta)d\eta = \frac{2s}{S}C_{L0}c_0\int_0^1\sqrt{1-\eta^2}\,d\eta = \frac{2sc_0}{S}C_{L0}\frac{\pi}{4};$$

Solution:

$$\bar{y}_{CL} = \frac{2s}{S}\frac{\int_0^1 C_L c(\eta)\eta d\eta}{C_{Lw}} = \frac{2s}{SC_{Lw}}\int_0^1 C_L c(\eta)\eta d\eta = \frac{2s}{SC_{Lw}}C_{L0}c_0\int_0^1\eta\sqrt{1-\eta^2}\,d\eta .$$

$$\int_0^1\eta\sqrt{1-\eta^2}\,d\eta = \frac{1}{3}.$$

Hence, $\bar{y}_{CL} = \dfrac{4s}{3\pi}$.

Solution:

$$C_{Lw} = \frac{2s}{S}\int_0^1 C_L c_0(1-\eta(1-\lambda))d\eta = \frac{2sC_{L0}c_0}{S}\int_0^1\sqrt{1-\eta^2}\,(1-\eta(1-\lambda))d\eta;$$

Using the integrals from the previous solution,

$$C_{Lw} = \frac{2sC_{L0}c_0}{S}\int_0^1\sqrt{1-\eta^2}\,(1-\eta(1-\lambda))d\eta = \frac{2sc_0}{S}C_{L0}\left(\frac{\pi}{4}-\frac{(1-\lambda)}{3}\right).$$

4) **Solution**

Given: Airspeed, $V = 30\ m/s$,

Atmosphere: $\rho = 1.0\ kg/m^3$, sound speed: $a = 332.5\ m/s$, viscosity: $\mu = 1.73 \times 10^{-5}\ kg/m\text{-}s$,

$q = 0.5\rho V^2 = 450\, N/m^2$.

Wing Geometry: span $b = 7.2\ m$, root chord $c_0 = 1.2\ m$,

$$S = sc_0\frac{\pi}{2} = 3.6 * 1.2 * 1.5708 = 6.7858 m^2,\quad AR = \frac{8s}{\pi c_0} = \frac{24}{\pi} = 7.6395$$

$$C_{L\alpha}\big|_{AR=\infty} = 0.112/\deg = 0.112\times 180/\pi/rad = 6.4171/rad$$

a) Wing lift curve slope: $C_{L\alpha} = \dfrac{C_{L\alpha}\big|_{AR=\infty}}{1+\dfrac{C_{L\alpha}\big|_{AR=\infty}}{\pi AR}} = \dfrac{0.112}{1+\dfrac{6.4171}{24}}\dfrac{1}{\deg} = 0.0884\dfrac{1}{\deg}$,

$C_{L\alpha} = 0.0884/\deg$.

b) $\bar{\bar{c}} = \dfrac{8c_0}{3\pi} = \dfrac{3.2}{\pi} = 1.0186m$, $C_L = \dfrac{dC_L}{d\alpha}\bigg|_{\alpha=0}(\alpha-\alpha_0) = 3.2C_{L\alpha} = 0.2829$,

$$C_D = C_{D0} + C_L^2/\pi AR = 0.01 + 0.2829\times 0.2829/(\pi * 7.6395) = 0.0133.$$

$C_L = 0.2829$, $C_D = 0.0133$

c) $L = qC_L S = 450 * 0.2829 * 6.7858 = 863.8663N$,

$D = qC_D S = 450\times 0.0133\times 6.7858 = 40.6130N$,

$M_{ac} = qC_{mac}S\bar{\bar{c}} = -450 * 0.04 * 6.7858 * 1.0186 = 124.4163Nm$,

$L = 863.8663N$, $D = 40.6130N$, $M_{ac} = 124.4163Nm$.

d) $M = V/a = 30/332.5 = 0.09$,

$\mathrm{Re} = \rho V\bar{c}/\mu = 30 * 1.02 * 10^5/1.73 = 30.6 * 10^5/1.73 = 1.8\times 10^6$.

$M = 0.09$, $\mathrm{Re} = 1.8\times 10^6$.

5) **Solution:**

(a) Taper ratio = $\lambda = 1.2/2.2 = 0.5455$.

$$S = s\times c_0\times(1+\lambda) = 4\times 2.2\times 1.5455 = 12.9822m^2,\ \bar{c} = \frac{S}{b} = \frac{12.9822}{8} = 1.6228m,$$

$$AR = \frac{s}{c_0}\left(\frac{4}{1+\lambda}\right) = \frac{4}{2.2}\times\frac{4}{1.5455} = 4.7057,$$

$$\bar{\bar{c}} = \frac{2c_0}{3}\left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right) = \frac{4.4}{3}\times\frac{1.5455+0.5455^2}{1.5455} = 1.7491m.$$

$\bar{c} = 1.6228m$, $S = 12.9822m^2$, $AR = 4.7057$, $\lambda = 0.5455$, $\bar{\bar{c}} = 1.7491m$.

(b) Estimate the wing lift curve slope at a Reynolds number = 3×10^6.

$$a = \frac{\pi a_\infty AR}{a_\infty + \pi\sqrt{AR^2+4}} = \frac{\pi * 0.1 * 4.7057}{0.1 * (180/\pi) + \pi\times 5.1131} = 0.0678/\deg.$$

$a = 0.0678/\deg$

(c)

(d) Using tabulated data, find the maximum lift-to-drag ratio and the corresponding angle of attack for maximum C_L/C_D.

$(L/D)_{\max} = 99$, $\alpha_m = 4^o$.

6) **Solution**: Weight: $W = 10{,}000\ N$, Centre of Gravity location: $h_{cg} = 0.5$, Wing Area: $S = 20\ m^2$, Wing Lift Curve Slope: $C_{L\alpha} = 0.06/deg$, Aerodynamic Centre: $h_{ac} = 0.25$, Wing moment coefficient: $C_{M_{ac}} = -0.05$,

Downwash at zero α: $\varepsilon_0 = 0$, $S_T = 2m^2$

Horizontal Tail Volume Ratio: $\overline{V}_H = 0.6$, Tail lift curve slope: $C_{l\alpha t} = 0.04/deg$,

Tail downwash gradient: $d\varepsilon/d\alpha = 0.3$, the dynamic pressure at the wing $= 480\,N/m^2$,

Ratio of dynamic pressure ratio at tail to that at wing: $\eta_{pr} = 1$.

(a) Find the aircraft aerodynamic centre in terms of the aerodynamic mean chord.

$$\frac{a_1}{a} = \frac{0.04}{0.06} = 0.6667\,,\ \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) = 0.7\,,$$

$$h_n = h_{ac} + \overline{V}_T \frac{a_1}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) = 0.25 + 0.6 \times 0.6667 \times 0.7 = 0.53\,,\ h_n = 0.53.$$

(b) Find $C_{M\alpha}$ ($= dC_M/d\alpha$) and C_{M0}.

$$dC_{M_{cg}}/d\alpha = (dC_L/d\alpha)(h_{cg} - h_n) - (dC_L/d\alpha)_{tail}\overline{V}_T\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right).$$

$$= -0.06 \times 0.03 - 0.04 \times 0.6 \times 0.7 = -0.0186/\deg$$

$$C_{M_0} = C_{M_{ac}} = -0.05.$$

$$dC_{M_{cg}}/d\alpha = -0.0186/\deg\,,\ C_{M_0} = -0.05.$$

(c) Find the absolute angle of attack (from zero lift) and lift coefficient for trim conditions.

$$\frac{\partial C_L}{\partial \alpha} = a + \frac{S_T}{S}a_1\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) = 0.06 + 0.1 * 0.7 = 0.13\,,$$

The coefficient of lift at trim=500/480=1.0417.

$$\alpha = 1.0417/0.13 = 8.0131^o.$$

$$C_L = 1.0417\,,\ \alpha = 8.0131^o$$

(d) Find the trim airspeed at sea level ($\rho = 1.225\ kg/m^3$).

$$V_{trim} = \sqrt{\frac{2 \times 480}{1.225}} = 27.099 \approx 28m/s\,,\ V_{trim} \approx 28m/s.$$

7 **Solution**:

area=0.25 m^2, flap area=0.06 m^2, flap mean aerodynamic chord is 5 cm, the tail plane lift and flap hinge moment coefficients are $a_1 = 3.5/rad$, $a_2 = 1.75/rad$, $a_3 = 0.35/rad$, $b_1 = 0.075/rad$, $b_2 = -0.015/rad$, $b_3 = -0.03/rad$. The wind tunnel wind speed is 60 *m/s*. The wing incidence angle is 3° and the tab is set at 5°.

i) if the flap angle is -4°, what is the tail plane zero lift angle? What are the corresponding lift and hinge moment coefficients? What are the lift force and flap hinge moment?

$C_L = a_1(\alpha - \alpha_0) + a_2\eta + a_3\beta$, $\alpha_0 = 0^o$

$$C_L = a_1(\alpha - \alpha_0) + a_2\eta + a_3\beta = (3.5\times 3 - 1.75\times 4 + 0.35\times 5)\times\frac{\pi}{180}$$

$$= \frac{5.25\times\pi}{180} = 5.25\times 0.0174532935 = 0.09163$$

$$C_H = b_1(\alpha - \alpha_0) + b_2\eta + b_3\beta = (0.075\times 3 + 0.015\times 4 - 0.03\times 5)\times\frac{\pi}{180}$$

$$= \frac{0.135\times\pi}{180} = 0.00075\pi = 0.002356\,.$$

$$L = 0.5\rho V^2 SC_L = 0.5\times 1.225\times 3600\times 0.25\times C_L = 2205\times 0.25\times 0.09163 = 50.511N$$

$$H = 0.5\rho V^2 S_f \bar{\bar{c}}_f C_H = 0.5\times 1.225\times 3600\times 0.06\times 0.05\times C_H$$

$$= 2205\times 0.003\times 0.002356 = 0.01558Nm\,.$$

$\alpha_0 = 0^o$, $C_L = 0.09163$, $C_H = 0.002356$, $L = 50.511N$, $H = 0.01558Nm$.

ii) At flap angle can the flap be expected to move freely?

$$C_H = b_1(\alpha - \alpha_0) + b_2\eta + b_3\beta = 0$$

$$\Rightarrow \eta = -\frac{b_1(\alpha - \alpha_0) + b_3\beta}{b_2} = \frac{0.075\times 3 - 0.03\times 5}{0.015} = 5^o,\ \eta = 5^o.$$

iii) What is the lift at this flap angle?

$$C_L = a_1(\alpha - \alpha_0) + a_2\eta + a_3\beta = (3.5\times 3 + 1.75\times 5 + 0.35\times 5)\times\frac{\pi}{180} = \frac{21\times\pi}{180} = 0.3665$$

$$L = 0.5\rho V^2 SC_L = 0.5\times 1.225\times 3600\times 0.25\times C_L = 202.03N,\ L = 202.03N.$$

8) **Solution:** The aircraft properties are: wing area=40 m^2, aerodynamic mean chord = 2.5 m, tail plane area=5 m^2, tail moment arm = 10m, the wing lift, tail plane lift and elevator hinge moment coefficients are $a = 4.5/rad$, $a_1 = 2.8/rad$, $a_2 = 1.2/rad$, $a_3 = 0.3/rad$, $b_1 = 0.01/rad$, $b_2 = -0.012/rad$, $b_3 = -0.03/rad$. The flight conditions are, $d\varepsilon/dC_L = 0.1$, $C_{M0} = -0.1$, $\eta_T = -3^\circ$, $h_{ac} = 0.1$, $h_{cg} = 0.2$ and $L_T = 0.01mg$.

i) Determine the wing and tail plane lift coefficients.

$$C_{M_{cg}} = C_{M_0} + C_L(h_{cg} - h_{ac}) - \bar{V}_T C_{L_T} = 0$$

Hence, $C_L = -\dfrac{C_{M_0}}{(h_{cg} - h_{ac}) - \bar{V}_T\, C_{L_T}/C_L}$, $\bar{V}_T = \dfrac{S_T}{S}\times\dfrac{l_t}{\bar{\bar{c}}} = 0.5$.

$C_{L_T}/C_L = 0.01\times S_{ref}/S_T = 0.08$. $h_{cg} - h_{ac} = 0.1$.

$$C_L = -\frac{C_{M_0}}{(h_{cg} - h_{ac}) - \bar{V}_T\, C_{L_T}/C_L} = \frac{0.1}{0.06} = 1.6667.$$

$C_{L_T} = 0.08\times 1.6667 = 0.13336$, $C_L = 1.6667$, $C_{L_T} = 0.13336$.

ii) Determine the downwash angle and the tail plane angle of attack α^T.

$\varepsilon = 0.1 \times C_L = 0.16667 = 9.5495^o$,

$$\alpha^T = \alpha + \eta_T - \varepsilon = \frac{1.66667}{4.5} \times \frac{180}{\pi} - 3 - 9.5495 = 8.67^o, \; \varepsilon = 9.5495^o, \; \alpha^T = 8.67^o.$$

iii) Determine elevator setting η, and if the tab angle β, is set to zero.

$$C_{L_T} = a_1\alpha^T + a_2\eta + a_3\beta, \; \beta = 0, \; \Rightarrow \; \eta = \frac{C_{L_T} - a_1\alpha^T}{a_2}$$

$$\eta = \frac{0.13336 - 2.8 \times 8.67 \times \pi / 180}{1.2} = -0.242 rad = -13.86^o, \; \eta = -13.86^o.$$

iv) Calculate η and β when $C_H = 0$.

When, $C_H = b_1\alpha^T + b_2\eta + b_3\beta = 0$, $\eta = -\dfrac{b_1\alpha^T + b_3\beta}{b_2}$.

Hence,

$$C_{L_T} = \left(a_1 - a_2\frac{b_1}{b_2}\right)\alpha^T + \left(a_3 - a_2\frac{b_3}{b_2}\right)\beta = \bar{a}_1\alpha^T + \bar{a}_3\beta.$$

$$\beta = \frac{C_{L_T} - \bar{a}_1\alpha^T}{\bar{a}_3}, \; \frac{a_2}{b_2} = -100, \; a_1 - b_1\frac{a_2}{b_2} = \bar{a}_1 = 3.8, \; a_3 - b_3\frac{a_2}{b_2} = \bar{a}_3 = -2.7.$$

$$\beta = \frac{C_{L_T} - \bar{a}_1\alpha^T}{\bar{a}_3} = \frac{0.13336 - 3.8 \times 8.67 \times \pi / 180}{-2.7} = 0.1636 = 9.37^o.$$

$$\eta = -\frac{b_1\alpha^T + b_3\beta}{b_2} = -\frac{b_1}{b_2}\alpha^T - \frac{b_3}{b_2}\beta = \frac{1}{1.2}8.67 - \frac{3}{1.2}9.37 = -16.2^o.$$

$\beta = 9.37^o$, $\eta = -16.2^o$.

9) Repeat 8) if in addition the wing flaps are down providing 25% of the lift and if $C_{M0} = -0.2$.

$$\text{i) } C_L = -\frac{C_{M_0}}{(h_{cg} - h_{ac}) - \bar{V}_T C_{L_T} / C_L} = \frac{0.2}{0.06} = 3.3334. \; C_{L_T} = 0.08 \times 3.3334 = 0.26667.$$

$C_L = 3.3334$. $C_{L_T} = 0.26667$.

ii) $\varepsilon = 0.1 \times C_L = 0.33334 = 19.0990^o$.

Since the flaps are providing 25% of the lift,

$$\alpha^T = \alpha + \eta_T - \varepsilon = 0.75\frac{3.33334}{4.5} \times \frac{180}{\pi} - 3 - 19.0990 = 9.73^o, \; \alpha^T = 9.73^o.$$

iii) Determine elevator setting η, and if the tab angle β, is set to zero.

$$\eta = \frac{C_{L_T} - a_1\alpha^T}{a_2} = \frac{0.26667 - 2.8 \times 9.73 \times \pi / 180}{1.2} = -0.209 rad = -11.96^o,$$

$\eta = -11.96^o$.

iv) $\beta = \frac{C_{L_T} - \bar{a}_1 \alpha^T}{\bar{a}_3} = \frac{0.26667 - 3.8 \times 9.73 \times \pi / 180}{-2.7} = 0.140 = 8.02^o$.

$\eta = -\frac{b_1 \alpha^T + b_3 \beta}{b_2} = -\frac{b_1}{b_2} \alpha^T - \frac{b_3}{b_2} \beta = \frac{1}{1.2} 9.73 - \frac{3}{1.2} 8.02 = -11.94^o$.

$\beta = 8.02^o$, $\eta = -11.94^o$