Test Solutions

Practical Linear Algebra: A Geometry Toolbox, 3rd edition

Chapter 2 Here and There: Points and Vectors in 2D

1. The operations have the following results.

(a)	vector	(b)	point
(c)	point	(d)	vector
(e)	vector	(f)	point

2. The midpoint between $\mathbf{p}$ and $\mathbf{q}$ is

$$\mathbf{m} = \frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}.$$

3. The barycentric coordinates of the point $\mathbf{q} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ with respect to the triangle with vertices

$$\mathbf{p}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \mathbf{p}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \qquad \mathbf{p}_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

are $(1/2, 1/2, 0)$. Check: $\mathbf{q} = \frac{1}{2}\mathbf{p}_1 + \frac{1}{2}\mathbf{p}_2 + 0\mathbf{p}_3$.

4. The vector $\mathbf{v} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$ is length $\sqrt{(-4)^2 + (2)^2} = \sqrt{20}$.

5. The length of $-3\mathbf{v}$ is $|3|\|\mathbf{v}\| = 3 \times 5 = 15$.

6. The distance between the points

$$\mathbf{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{q} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

is

$$\|\mathbf{p}-\mathbf{q}\| = \| \begin{bmatrix} -2 \\ -3 \end{bmatrix} \| = \sqrt{(-2)^2+(-3)^2} = \sqrt{13}$$

7. A unit vector has length one.

8. To normalize the vector $\mathbf{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, first find the length, which is $\sqrt{4^2+2^2} = 2\sqrt{5}$. Then the normalized vector is

$$\frac{\mathbf{v}}{2\sqrt{5}} = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}.$$

9. No, they are linearly dependent.

10. Yes, $\mathbf{v}_1$ and $\mathbf{v}_2$ form a basis for $\mathbb{R}^2$ because they are linearly independent.

11. The angle between the vectors $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$ is $90°$ by inspection. The dot product of the given vectors is zero, confirming this result.

12. The angles fall into the following categories: θ_1 is obtuse, θ_2 is a right angle, and θ_3 is acute.

13. The orthogonal projection $\mathbf{u}$ of $\mathbf{w}$ onto $\mathbf{v}$ is clear from a sketch; however, the calculations result in

$$\mathbf{u} = \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 5 \end{bmatrix}}{1^2} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}.$$

Therefore, the $\mathbf{u}^{\perp}$ that completes the orthogonal decomposition of $\mathbf{w}$ is

$$\mathbf{u}^{\perp} = \mathbf{w} - \mathbf{u} = \begin{bmatrix} -5 \\ 5 \end{bmatrix} - \begin{bmatrix} -5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}.$$