Test

Practical Linear Algebra: A Geometry Toolbox, 3rd edition

Chapter 2 Here and There: Points and Vectors in 2D

1. Suppose we are given $\mathbf{p}, \mathbf{q} \in \mathbb{E}^2$ and $\mathbf{v}, \mathbf{w} \in \mathbb{R}^2$. Do the following operations result in a point or a vector?

(a) $\mathbf{p} - \mathbf{q}$	(b) $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$
(c) $\mathbf{p} + \mathbf{v}$	(d) $3\mathbf{v}$
(e) $\mathbf{v} + \mathbf{w}$	(f) $\mathbf{p} + \frac{1}{2}\mathbf{w}$

2. What barycentric combination of the points $\mathbf{p}$ and $\mathbf{q}$ results in the midpoint of the line through these two points?

3. What are the barycentric coordinates of the point $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ with respect to the triangle with vertices

$$\mathbf{p}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \mathbf{p}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \qquad \mathbf{p}_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}?$$

4. What is the length of the vector

$$\mathbf{v} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}?$$

5. If a vector $\mathbf{v}$ is of length 5, then what is the length of the vector $-3\mathbf{v}$?

6. Find the distance between the points

$$\mathbf{p} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{q} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

7. What is the length of a unit vector $\mathbf{u}$?

8. Normalize the vector $\mathbf{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

9. If $\mathbf{v} = 3\mathbf{w}$, are $\mathbf{v}$ and $\mathbf{w}$ linearly independent?

10. Do the vectors
$$\mathbf{v}_1 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
form a basis for $\mathbb{R}^2$? Explain.

11. Compute the angle (in degrees) formed by the vectors
$$\mathbf{v} = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}.$$

12. Are the following angles acute, obtuse, or right?
$$\cos\theta_1 = -0.3 \qquad \cos\theta_2 = 0 \qquad \cos\theta_3 = 0.3$$

13. Given the vectors
$$\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} -5 \\ 5 \end{bmatrix},$$
find the orthogonal projection $\mathbf{u}$ of $\mathbf{w}$ onto $\mathbf{v}$. Decompose $\mathbf{w}$ into components $\mathbf{u}$ and $\mathbf{u}^{\perp}$.