Chapter 2: Energy – Solutions

2-1. You are given the depth upstream (y_1) and downstream (y_2) of a sluice gate. Using conservation of energy at the sluice gate, find an equation for the unit discharge, q, in terms of y_1 and y_2.

Solution:

$$E_1 = y_1 + \frac{q^2}{2gy_1^2} = E_2 = y_2 + \frac{q^2}{2gy_2^2}$$

Re-arranging, we get

$$y_1 - y_2 = \frac{q^2}{2g}\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right)$$

Some modest algebra gets us to,

$$y_1 - y_2 = \frac{q^2}{2g}\left(\frac{y_1^2 - y_2^2}{y_1 y_2}\right)$$

Factoring the numerator on the right hand side, we get,

$$y_1 - y_2 = \frac{q^2}{2g}\left(\frac{(y_1 - y_2)(y_1 + y_2)}{y_1^2 y_2^2}\right)$$

Simplifying,

$$1 = \frac{q^2}{2g}\left(\frac{(y_1 + y_2)}{y_1^2 y_2^2}\right)$$

Re-arranging and solving for q,

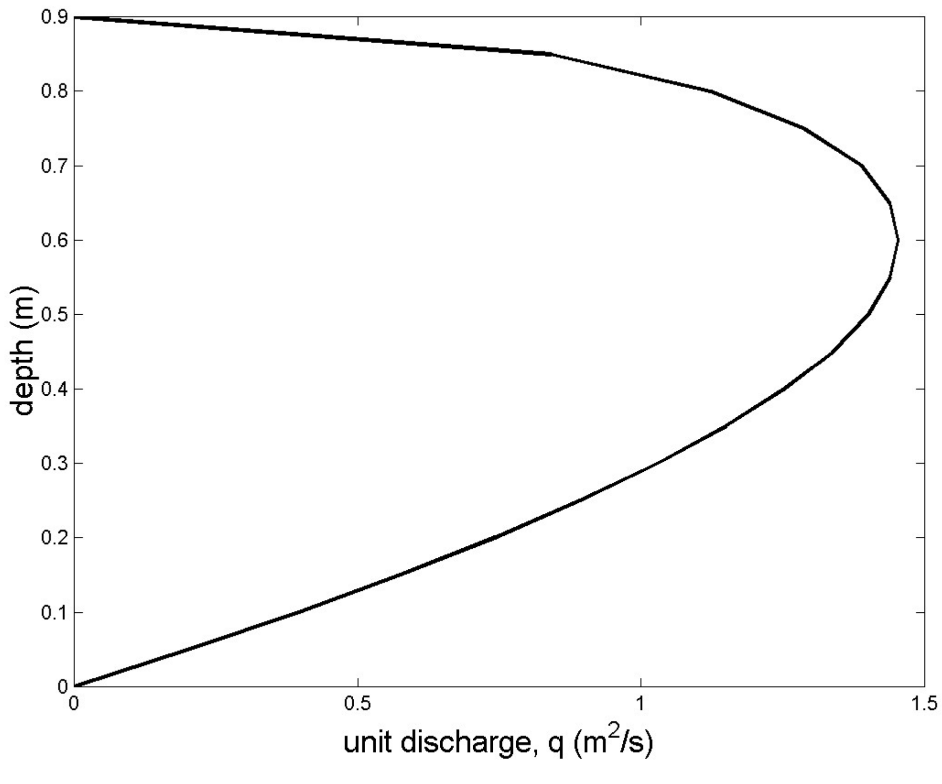
$$q = y_1 y_2 \sqrt{\frac{2g}{y_1 + y_2}}$$

2-2. For a fixed specific energy of E^*=0.9 m and in a rectangular channel, vary depth from 0.0 m to 0.9m in increments of 0.01 or 0.05 meters and determine the unit discharge, q, at each depth. Plot depth vs. q over the range of calculated values. Verify that the maximum specific discharge occurs at $y = 2/3E^*$.

Solution:
Using the equation,

$$E^* = y + \frac{q^2}{2gy^2} = 0.9 \text{ m}$$

We let $y = \{0.0, 0.05, 0.1, 0.15, \ldots 0.90\}$ m. and use the Excel goal seek function to determine the value of q that solves the equality above. For instance, when $y = 0.60$ m, we find that $q = 1.45$ m^2/s, which is the largest q that is observed for this fixed amount of energy and corresponds to a depth of y=2/3E^*. The graph over the requested range of depths is shown below:



Visually, it should be clear that the unit discharge, q, is maximized at 0.6 m. This is 2/3rds of the energy provided, $E^* = 0.9$ m

2-3. Given a rectangular channel 3.50 feet wide. The flow depth upstream of a sluice gate is 2.15 feet. Downstream of the sluice gate the depth is 1.25 feet.

a. Find the unit discharge, q, and the discharge, Q.

b. What specific energy, E, does the flow have?

Solution:

a. Using the equation from problem 2-1, we have:

$$q = y_1 y_2 \sqrt{\frac{2g}{y_1 + y_2}} = (2.15)\cdot(1.25)\sqrt{\frac{2(32.2)}{2.15+1.25}} = 11.7\,\frac{\text{ft}^2}{\text{s}}$$

$$Q = q\cdot w = (11.7)\cdot(3.50) = 40.9\,\frac{\text{ft}^3}{\text{s}}$$

b. Specific energy for either depth, y, is the same:

$$E = y + \frac{q^2}{2gy^2}$$

Use y_1:

$$E = 2.15 + \frac{(11.7)^2}{2(32.2)(2.15)^2} = 2.61\,\text{ft}$$

2-4. The flow depth upstream of a sluice gate is 0.60 meters. The velocity is 0.9 m/s.

a. What is the minimum allowable gate opening for the upstream flow to be possible as specified?

b. If the gate opening is instantaneously set to 0.15 meters:

i. What is the initial unit discharge, q, under the gate?

ii. What is the final depth at the upstream side of the gate.

Solution:

a. The minimum gate opening will be the alternate depth to the depth upstream of the gate:

$$y_2 = \frac{2y_1}{-1+\sqrt{1+\frac{8gy_1^3}{q^2}}} = \frac{2(0.60)}{-1+\sqrt{1+\frac{8(9.81)(0.60)^3}{[(0.9)(0.6)]^2}}} = 0.18\,\text{m}$$

b.

i. The gate is a choke, so the flow will be the value of q_{init} that has the same energy as the flow initially possesses:

$$E = y + \frac{q^2}{2gy^2} = 0.60 + \frac{[(0.9)(0.6)]^2}{2(9.81)(0.6)^2} = 0.64\,\text{m}$$

$$0.64 = 0.15 + \frac{q_{init}^2}{2(9.81)(0.15)^2}$$

By iteration, the value of q_{init} that satisfies the above equation is approximately 0.47 m^2/s.

ii. Final depth on the upstream side of the gate will be the alternate depth to 0.15 m for a unit discharge of (0.60)(0.9)=0.54 m^2/s. This is:

$$y_1 = \frac{2y_2}{-1+\sqrt{1+\frac{8gy_2^3}{q^2}}} = \frac{2(0.15)}{-1+\sqrt{1+\frac{8(9.81)(0.15)^3}{[0.54]^2}}} = 0.79\,\text{m}$$

2-5. Water is flowing at a velocity of 2.6 ft/s and a depth of 1.1 feet in a rectangular channel.

a. The flow encounters a smooth upward step of 0.2 feet.

i. What is the depth of flow on the step?

ii. What is the absolute change in water level compared to the channel bottom before the step?

iii. What are the Froude numbers upstream and at the step?

b. Find the maximum allowable size of upward step for the upstream flow to be possible as specified.

Solution:

a. $$E = 1.1 + \frac{[(1.1)(2.6)]^2}{2(32.2)(1.1)^2} = 1.2\text{ ft}$$

$$E_{\min} = 1.5 \cdot \left(\frac{q^2}{g}\right)^{1/3} = 1.5 \cdot \left(\frac{[(1.1)(2.6)]^2}{32.2}\right)^{1/3} = 0.95\text{ ft}$$

Since E-$E_{\min}$ = 1.2 – 0.95 > 0.2 feet, the step is ***not*** a choke.

i. $$E - \Delta z = y_2 + \frac{q^2}{2gy_2^2}$$

$$1.2 - 0.2 = 1.0 = y_2 + \frac{[(1.1)(2.6)]^2}{2(32.2)(y_2^2)}$$

Solve for y_2. There are two roots, 0.81 feet or 0.50 feet. Because incoming flow is subcritical and step is not a choke, use same flow regime as incoming flow, so use subcritical root. $y_2 = 0.81$ feet.

ii. The absolute change in water level is calculated by considering change in water level at location 1 vs. location 2. At location, the water level is at $y_1 = 1.1$ feet. At location 2, the water level is at $y_2 + \Delta z = 0.81 + 0.2 = 1.01$ feet. Thus the change in water level from location 1 to location 2 is 1.1 – 1.01 = 0.09 feet. Location 2 is 0.09 feet lower than at location 1.

iii. The Froude numbers at locations 1 and 2 are:

$$F_{r,1} = \frac{v_1}{\sqrt{gy_1}} = \frac{2.6}{\sqrt{(32.2)(1.1)}} = 0.43\text{ (subcritical)}$$

$$F_{r,1} = \frac{q}{y_2\sqrt{gy_2}} = \frac{(2.6)\cdot(1.1)}{(0.81)\cdot\sqrt{(32.2)(0.81)}} = 0.69\text{ (subcritical)}$$

b. Maximum upward step is calculated by observing that,

$$E - E_c = \Delta z_{\max}$$

$$1.2 - 0.95 = 0.25\text{ feet}$$

2-6. Consider a system with a specific discharge of 3.0 m^2/s. The depth, y_1, upstream of the step is 0.9 meters. The downward step height is 0.2 meters. Determine:

a) The downstream specific energy, E_2.
b) The downstream depth, y_2.
c) The absolute change in the water surface from location 1 to location 2.
d) The downstream Froude number, $F_{r,2}$.

Solution:

Before making any other calculations, let's first determine the Froude number.

$$F_r = \frac{v}{\sqrt{gy}} = \frac{q}{y\sqrt{gy}} = \frac{3.0}{0.9\sqrt{(9.81)\cdot(0.9)}} = 1.12$$

Therefore, the flow is supercritical.

a) First we calculate the upstream energy:

$$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 0.90 + \frac{(3.0)^2}{2(9.81)(0.90)^2} = 1.47 \text{ m}$$

The downward step adds energy to the flow equal to the step height so:

$$E_2 = E_1 + \Delta z = 1.47 + 0.2 = 1.67 \text{ m}$$

b) The downstream depth will satisfy:

$$E_2 = y_2 + \frac{(3.0)^2}{2gy_2^2} == 1.67 \text{ m}$$

Since the step adds energy, the downstream depth must also be supercritical (subcritical flow is not accessible for this problem). Thus, by iteration: $y_2 = 0.68$ m.

c) The channel bottom is reduced by 0.2 m, and the depth decreased from 0.90 m to 0.68 m. Thus the absolute change in depth is the water surface level is 0.2 + (0.90 – 0.68) = 0.42 m (downwards), that is the water surface is 0.42 m lower downstream compared to upstream.

d) Downstream Froude number is:

$$F_r == \frac{q}{y\sqrt{gy}} = \frac{3.0}{0.68\sqrt{(9.81)\cdot(0.68)}} = 1.71$$

Thus the flow has become more supercritical, owing to the 0.2 meter increase in specific energy.

2-7. Consider a system with a discharge of 9.0 m^3/s. The channel is rectangular. The width at location 1 is w_1=4.5 m. A constriction is encountered at location 2 downstream such that the width w_2=3.0 m. The depth of flow y_2 at downstream location 2 is 0.7 meters. Determine:

a. The specific discharges at locations 1 and 2.
b. The downstream specific energy, E_2.
c. The downstream Froude number, $F_{r,2}$.
d. The upstream specific energy, E_1.
e. The upstream depth, y_1.
f. The upstream Froude number, $F_{r,1}$.
g. The absolute change in the water surface from location 1 to location 2.
h. Sketch the transition of the system from location 1 through the flow constriction to location 2 on an E-y diagram.

Solution:

a. Specific discharge at 1:

$$q_1 = \frac{Q}{w_1} = \frac{9.0}{4.5} = 2.0\,\frac{\text{m}^2}{\text{s}}$$

$$q_2 = \frac{9.0}{3.0} = 3.0\,\frac{\text{m}^2}{\text{s}}$$

b. The downstream specific energy is:

$$E_2 = y_2 + \frac{q_2^2}{2gy_2^2} = 0.7 + \frac{(3.0)^2}{2(9.81)(0.7)^2} = 1.64\text{ m}$$

c. $$F_{r,2} = \frac{q_2}{y_2\sqrt{gy_2}} = \frac{3.0}{0.7\cdot\sqrt{(9.81)\cdot(0.7)}} = 1.64$$

d. $E_1 = E_2 = 1.64$ m

e. Looking for the value of y_1 that satisfies,

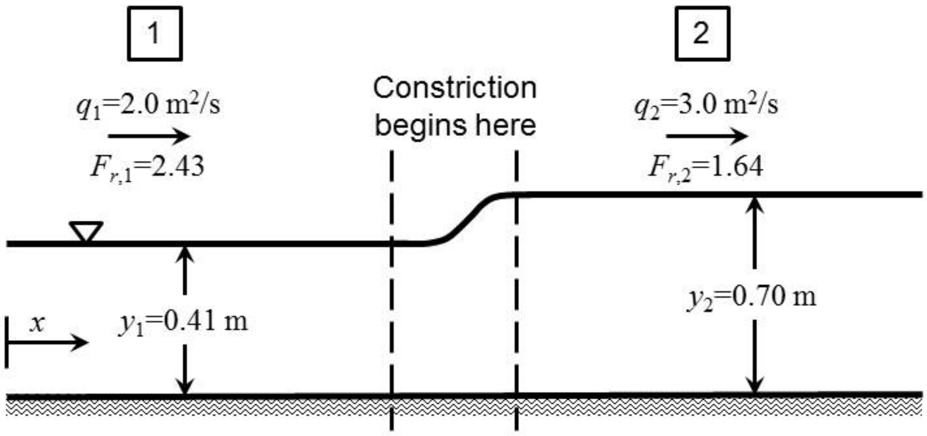
$$E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = y_1 + \frac{(2.0)^2}{2(9.81)(y_1)^2} = 1.64\text{ m}$$

There are two solutions: y_1 = 0.41 m or 1.64 m. Because the downstream flow supercritical and because the constriction is not a choke, we choose the root that corresponds to the same flow regime, supercritical in this case. Thus y_1 = 0.41 m.

f. $$F_{r,1} = \frac{q_1}{y_1\sqrt{gy_1}} = \frac{2.0}{0.41\cdot\sqrt{(9.81)\cdot(0.41)}} = 2.43$$

g. At location 1, the water surface is at 0.41 m. At location 2, the water level is at y_2 = 0.7 m. Thus the change in water level from location 1 to location 2 is 0.7 – 0.41 = 0.29 m (the flow has become deeper by 0.29 m).

h. The sketch for this system is shown below:



2-8. Re-Solve Problem 2-7 except w_2 = 6.0 m (so the flow encounters an expansion, not a constriction) and y_2 = 1.0 m.

Solution:

a. Specific discharge at 1:

$$q_1 = \frac{Q}{w_1} = \frac{9.0}{4.5} = 2.0\,\frac{\text{m}^2}{\text{s}}$$

$$q_2 = \frac{9.0}{6.0} = 1.5\,\frac{\text{m}^2}{\text{s}}$$

b. The downstream specific energy is:

$$E_2 = y_2 + \frac{q_2^2}{2gy_2^2} = 1.0 + \frac{(1.5)^2}{2(9.81)(1.0)^2} = 1.11\,\text{m}$$

c. $F_{r,2} = \dfrac{q_2}{y_2\sqrt{gy_2}} = \dfrac{1.5}{1.0\cdot\sqrt{(9.81)\cdot(1.0)}} = 0.47$

d. $E_1 = E_2 = 1.11\,\text{m}$

e. Looking for the value of y_1 that satisfies,

$$E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = y_1 + \frac{(2.0)^2}{2(9.81)(y_1)^2} = 1.11\,\text{m}$$

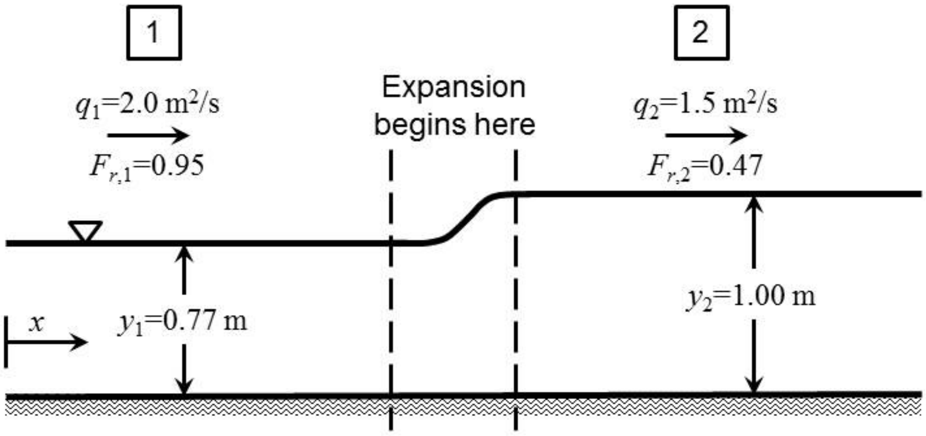
There are two solutions: y_1 = 0.71 m or 0.77 m. (Note: the flow is very close to critical conditions, so it becomes necessary to carry more precision in the energy.) The specific energy of 1.11 m reported above is more precisely 1.1147 m. Using this value, it is possible to determine the two alternate depths reported above. Because the downstream flow is subcritical and because the expansion will not make supercritical flow accessible, we choose the root that corresponds to the same flow regime, subcritical in this case. Thus y_1 = 0.77 m.

f. $F_{r,1} = \dfrac{q_1}{y_1\sqrt{gy_1}} = \dfrac{2.0}{0.77\cdot\sqrt{(9.81)\cdot(0.77)}} = 0.95$

Note that although the upstream flow is subcritical, it is only barely below 1.0.

g. At location 1, the water surface is at 0.77 m. At location 2, the water level is at y_2 = 1.0 m. Thus the change in water level from location 1 to location 2 is 1.0 – 0.77 = 0.23 m (the flow has become deeper by 0.23 m).

h. The sketch for this system is shown below:



2-9. Re-Solve Problem 2-7 except w_2 = 6.0 m (so the flow encounters an expansion, not a constriction) and y_2 = 0.3 m.

Solution:

a. Specific discharge at 1:

$$q_1 = \frac{Q}{w_1} = \frac{9.0}{4.5} = 2.0\,\frac{\text{m}^2}{\text{s}}$$

$$q_2 = \frac{9.0}{6.0} = 1.5\,\frac{\text{m}^2}{\text{s}}$$

b. The downstream specific energy is:

$$E_2 = y_2 + \frac{q_2^2}{2gy_2^2} = 0.3 + \frac{(1.5)^2}{2(9.81)(0.3)^2} = 1.57\text{ m}$$

c. $$F_{r,2} = \frac{q_2}{y_2\sqrt{gy_2}} = \frac{1.5}{0.3\cdot\sqrt{(9.81)\cdot(0.3)}} = 2.91$$

d. $E_1 = E_2 = 1.57$ m

e. Looking for the value of y_1 that satisfies,

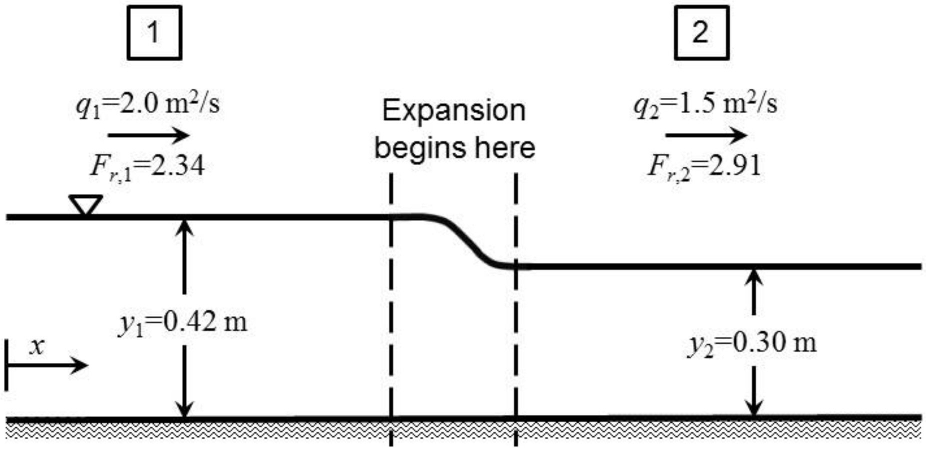
$$E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = y_1 + \frac{(2.0)^2}{2(9.81)(y_1)^2} = 1.57\text{ m}$$

There are two solutions: y_1 = 0.42 m or 1.48 m. Because the downstream flow is supercritical and because the expansion will not make subcritical flow accessible, we choose the root that corresponds to the same flow regime, supercritical in this case. Thus y_1 = 0.42 m.

f. $$F_{r,1} = \frac{q_1}{y_1\sqrt{gy_1}} = \frac{2.0}{0.42\cdot\sqrt{(9.81)\cdot(0.42)}} = 2.34$$

g. At location 1, the water surface is at 0.42 m. At location 2, the water level is at y_2 = 0.30 m. Thus the change in water level from location 1 to location 2 is 0.30 – 0.42 = 0.12 m (the flow has become shallower by 0.12 m).

h. The sketch for this system is shown below:



2-10. Water flows in a horizontal, rectangular channel initially 10.0 feet wide and 2.0 feet deep. The initial velocity is 6.0 ft/s. This flow encounters downstream a simultaneous downward step and constriction. No energy losses are associated with these changes in channel configuration.

a. What is the specific discharge, q:
   i. Upstream (for w = 10.0 feet)?
   ii. Downstream (for w = 6.0 feet)?
b. What specific energy, E, is associated with the flow as initially specified (at the upstream location)?
c. What is the minimum energy needed to pass the full discharge at the downstream location?
d. What is the height (Δz) of the smallest downward step necessary for the upstream flow conditions to remain as specified?

Solution:

a. $q_1 = v_1 \cdot y_1 = (6.0)\cdot(2.0) = 12\,\frac{\text{ft}^2}{\text{s}}$

From knowing q_1, we know the total discharge, $Q = q_1 \cdot w_1 = (12)\cdot(10.0) = 120\,\frac{\text{ft}^3}{\text{s}}$.

By continuity, we then know that,

$$q_2 = \frac{Q}{w_2} = \frac{120}{6} = 20\,\frac{\text{ft}^2}{\text{s}}$$

b. $E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = 2.0 + \frac{(12)^2}{2\cdot(32.2)\cdot(2.0)^2} = 2.56\text{ ft}$

c. Minimum energy downstream is critical energy for a specific discharge of 20 ft^2/s. So,

$$E_{2,\min} = E_c = 1.5\cdot\left(\frac{q_2^2}{g}\right)^{1/3} = 1.5\cdot\left(\frac{(20)^2}{32.2}\right)^{1/3} = 3.47\text{ ft}$$

d. The minimum needed downward step is the energy difference between answers to parts b and c above. Thus,

$$\Delta z_{\min} = E_{2,\min} - E_1 = 3.47 - 2.56 = 0.91\text{ ft}$$

2-11. A trapezoidal channel has a bottom width of 20. ft, side slopes of 2H:1V, and carries a flow of 750 ft^3/s.

a. Find the flow depth at the head of a steep slope.

b. If there is a short but smooth transition to a rectangular section 20. feet wide just before the head of the steep slope, find the depth at the upstream and downstream ends of the transition, assuming that the specific energy remains unchanged through the transition.

Solution:

a. As stated, we can assume critical conditions apply. This is simple application of Figure 2-24 which deals with critical depth. From the provided information the horizontal axis is determined,

$$Z = \frac{Qm^{3/2}}{b^2\sqrt{gb}} = \frac{(750)(2)^{3/2}}{(20)^2\sqrt{(32.2)(20)}} = 0.209$$

Entering the horizontal axis at this value and moving vertically to the line for the trapezoidal cross section, and then moving horizontally to the vertical axis, we pull off a value of,

$$\frac{my_c}{b} = 0.31$$

Therefore,

$$y_c = \frac{(0.31)(20)}{2} = 3.1\,\text{ft}$$

Alternatively, one could note that the Froude number is 1 at critical depth,

$$F_r = \frac{Q/A}{\sqrt{g\left(\frac{A}{B}\right)}} = 1$$

With the trapezoidal geometry known, A and B, are simple functions of depth, y. using the Excel goal seek function, a value of 3.15 ft is determined. Clearly the provided figure performs adequately but slightly greater precision is available if using goal seek.

b. If there is a short but smooth transition to a rectangular cross-section at the head of the slope, then the rectangular critical depth equation applies. At the head of the slope,

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(750/20)^2}{32.2}\right]^{1/3} = 3.52\,\text{ft}$$

The specific energy is 1.5 times this value, or 1.5(3.52)=5.28 ft. The depth at the upstream end of the transition is the depth in a trapezoidal section that satisfies the general specific energy equation,

$$E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2} = y + \frac{(750)^2}{2g\left(by + my^2\right)^2} = 5.28 \text{ ft}$$

The above equation recognizes and shows that area, *A,* is a simple function of depth, so we are seeking the subcritical root to the above equation. By iteration (or goal seek) this depth is y=4.86 ft. This is the trapezoidal depth at the upstream end of the channel before the transition to a rectangular cross-section.

2-12. A horizontal, frictionless channel of circular cross-section and diameter equal to 5.0 feet flows at a depth of 3.0 feet and a velocity of 2.0 ft/s.

a. Find the discharge in the channel.
b. Determine the specific energy of this flow.
c. Determine the Froude number of this flow.
d. What is the maximum upward step height (Δz_{max}) that this flow can negotiate without a choke?

Solution:

a. Using the circular sections table, $(y/d)=(3/5)=0.6$. Searching the table for this value, we find that the associated value of $(A/d^2)=0.4920$. Therefore, the cross-sectional area is,

$$A=\left(\frac{A}{d^2}\right)\cdot D^2=(0.4920)\cdot(5^2)=12.3\,\text{ft}^2$$

The discharge is then simply the product of the area, A, and velocity, v,

$$Q=A\cdot v=(12.3)\cdot(2)=24.6\,\frac{\text{ft}^3}{\text{s}}$$

b. The specific energy is,

$$E=y+\frac{v^2}{2g}=3+\frac{(2)^2}{2\cdot(32.2)}=3.06\,\text{ft}$$

c. The Froude number requires knowing the top-width, B, of the wetted flowing channel. From the circular sections table, for $(y/d)=0.6$, the wetted top-width is $(T/d)=0.9798$,

$$B=\left(\frac{T}{d}\right)\cdot D=(0.9798)\cdot(5)=4.899\,\text{ft}$$

The Froude number is thus,

$$F_r=\frac{v}{\sqrt{g\left(\frac{A}{B}\right)}}=\frac{2}{\sqrt{32.2\cdot\left(\frac{12.3}{4.899}\right)}}=0.22$$

d. The maximum upward step is equal to E - E_c where E was found in part b. and E_c can be determined by using Figure 2-25. The horizontal axis of this figure gives,

$$Z=\frac{Q}{D^2\sqrt{gD}}=\frac{(24.6)}{(5)^2\sqrt{(32.2)(5)}}=0.0776$$

Entering the horizontal axis at this value and moving vertically to the line for the circular cross section, and then moving horizontally to the vertical axis, we pull off a value of,

$$\frac{E_c}{D}=0.36$$

Therefore,

$$E_c=(0.36)\cdot(5)=1.8\,\text{ft}$$

The highest step, is thus E - E_c = 3.06 -1.8 = 1.26 (or 1.3) ft.

2-13. A trapezoidal channel with a base width of 20. feet and side slopes of 2H:1V carries a flow of 2000. ft^3/s at a depth of 8.0 feet. There is a smooth transition to a rectangular section 20. feet wide accompanied by a gradual lowering of the channel bed by 2.0 feet.

a. Find the depth of water within the rectangular section, and the change in the water surface level.

b. What is the minimum amount by which the bed must be lowered for the upstream flow to be possible as specified?

Solution:

a. Assume no losses in energy from upstream to downstream. The energy downstream is thus equal to the energy upstream plus 2.0 feet of energy due to the lowering of the channel bed in the rectangular section. Have the information we need to calculate the energy in the trapezoidal section once we know the cross-sectional area in the upstream section. Firs solve for this area,

$$A = b \cdot y + m \cdot y^2 = (20)\cdot(8) + (2)\cdot(8^2) = 288\,\text{ft}^2$$

$$E = y + \frac{Q^2}{2gA^2} = 8 + \frac{(2000)^2}{(2)\cdot(32.2)\cdot(288)^2} = 8.75\,\text{ft}$$

Although not requested, let's calculate the Froude number so the flow regime is known,

$$F_r = \frac{(Q/A)}{\sqrt{g\left(\frac{A}{B}\right)}} = \frac{(2000/288)}{\sqrt{32.2\cdot\left(\frac{288}{20+(2)\cdot(2)\cdot(8)}\right)}} = 0.52$$

So the upstream flow is subcritical. We therefore expect subcritical flow downstream as well. The energy in the downstream rectangular section is 2 feet greater than the upstream energy, or 2+8.75=10.75 ft. We solve the rectangular specific energy equation to determine the subcritical root,

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(100)^2}{2\cdot(32.2)y^2} = 10.75\,\text{ft}$$

The subcritical root to this equation is y=8.70 ft. Letting the upstream (trapezoidal section) be location 1 and the downstream (rectangular section) be location, 2, the absolute change in water level is calculated as,

$$\Delta y = y_1 + \Delta z - y_2 = 8.0 + 2.0 - 8.70 = 1.3\,\text{ft}$$

(notice that although the channel bottom declined by 2 feet, the water surface did not decline by as much because the flow became more subcritical (from the energy "gift" of the downstream step) and was also pinched into a narrower cross-section (rectangular vs. trapezoidal).

b. The minimum energy "gift" for the step would correspond to the flow being at exactly critical conditions in the downstream (rectangular) section. This energy is,

$$E_c = 1.5\cdot\left(\frac{q^2}{g}\right)^{1/3} = 1.5\cdot\left[\frac{(100)^2}{32.2}\right]^{1/3} = 10.16\,\text{ft}$$

Given that the upstream flow has 8.75 ft of energy and the downstream flow must have at least 10.16 feet of energy, the downwards step must be, at a minimum,

$$\Delta z_{\min} = E_{c,2} - E_1 = 10.16 - 8.75 = 1.41 \text{ ft}$$

Any downward step smaller than this will not provide sufficient energy to pass the upstream discharge at the flow conditions specified, and thus the downstream transition to a rectangular cross-section will serve as a choke to the flow.