Chapter 2

P2.1

A simple lossy medium with no source is assumed. Hence

$\nabla \cdot \tilde{\mathbf{E}} = 0 \qquad (1)$

$\nabla \cdot \tilde{\mathbf{H}} = 0 \qquad (2)$

From (1) since $\mathbf{E} = \mathbf{E}(z)$ only

$$\frac{\partial \tilde{E}_z}{\partial z} = 0 \qquad (3)$$

$\tilde{E}_z =$ Constant in space

This constant has to be zero since Maxwell's equation are not satisfied by a spatially constant oscillating field, another way of saying this is

$$\nabla^2 \tilde{E}_z + k^2 \tilde{E}_z = 0 \qquad (4)$$

Is not specified by $\tilde{E}_z$ = constant, unless $k^2 = 0$. If $k = \omega\sqrt{\mu\varepsilon} = 0$ we have static fields and not time harmonic fields. The transverse part of the electric field $\tilde{\mathbf{E}}_t$ satisfies

$$\frac{d^2 \tilde{\mathbf{E}}_t}{dz^2} + k^2 \tilde{\mathbf{E}}_t = 0 \qquad (5)$$

whose solution is

$$\tilde{\mathbf{E}}_t = \tilde{\mathbf{E}}_{to} \begin{Bmatrix} e^{+jkz} \\ e^{-jkz} \end{Bmatrix} \qquad (6)$$

Let us examine one of these solutions and find the corresponding magnetic field from Maxwell's equations.

Substituting

$$\tilde{\mathbf{E}}_t = \tilde{\mathbf{E}}_{t0}^{+} \quad e^{-jkz}$$

in the equation

$$-j\omega\mu\mathbf{H} = \nabla \times \mathbf{E}$$

we can show (2.2) where η is given by (2.5)

P2.2

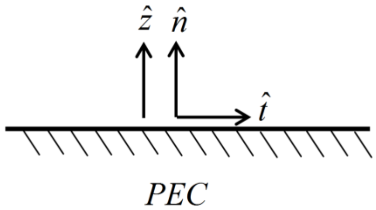
The well known boundary conditions at PEC are

$E_t = 0 \quad , \quad H_n = 0$

If $\hat{n} = \hat{z}$ from (2.23)

$\mathbf{E} = \hat{n}E_n(z) = \hat{z}E_z(z)$

$$\nabla \bullet \mathbf{E} = \frac{\partial E_n}{\partial n} = \frac{\partial E_z}{\partial z} = 0$$



<FigureP2.2_1>

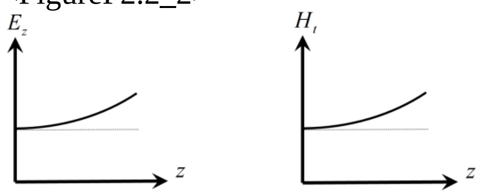
Similarly from (2.25)

$\mathbf{H} = H_t(z)\hat{t}$ and since $\nabla \bullet \mathbf{H} = 0$

$$\frac{\partial H_t}{\partial z} = \frac{\partial H_t}{\partial n} = 0$$

The variation of E_n and H_t near the boundary with the z-coordinate are sketched below

<FigureP2.2_2>



P2.3

(a) In a good conductor the displacement current $j\omega\varepsilon E$ may be neglected in comparison with σ E. That is the implication of saying $T >> 1$. This is equivalent to approximating k^2 by ,from (2.4),

$k^2 \approx -j\omega\mu\sigma \quad (1)$

Thus the fields, including $\tilde{\mathbf{H}}$ satisfy

$\nabla^2\tilde{\mathbf{H}} - j\omega\mu\sigma\tilde{\mathbf{H}} = 0 \qquad (2)$

Since $\quad \tilde{\mathbf{H}} = -\hat{y}\tilde{\mathbf{H}}(z) \quad (3)$

$$\frac{d^2\tilde{\mathbf{H}}}{dz^2} - j\omega\mu\sigma\tilde{\mathbf{H}} = 0 \qquad (4)$$

$$\frac{d^2\tilde{\mathbf{H}}}{dz^2} - \tau^2\tilde{\mathbf{H}} = 0 \qquad (5)$$

$$\tau = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma} \quad e^{j\pi/4} = \sqrt{\frac{\omega\mu\sigma}{2}}(1+j1) = \frac{1}{\delta}(1+j1)$$

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}, \text{ Skin depth}$$

(b) $\tilde{\mathbf{H}} = \begin{Bmatrix} \cosh\tau z \\ \sinh\tau z \end{Bmatrix}$ (6)

Since τ is complex, we need not consider separately trigonometric functions. We choose the even function, with respect to *d*, $\cosh\tau z$ because of the boundary condition

$$\tilde{\mathbf{H}}\Big|_{z=\pm d} = H_1 \quad (7)$$

Thus we choose

$$\tilde{\mathbf{H}} = A \cosh\tau z \qquad (8)$$

A may be determined by imposing (7) and (8)

$$H_1 = A \quad \cosh\tau d \qquad (9)$$

Now (8) becomes

$$\tilde{H}(z) = H_1 \frac{\cosh\tau z}{\cosh\tau d} \quad (10)$$

(b) After we neglect the displacement current we have the Maxwell's equation

$$\tilde{\mathbf{J}} = \nabla \times \tilde{\mathbf{H}} \qquad (11)$$

Substituting for $\tilde{\mathbf{H}}$ in (11) we get

$$\tilde{\mathbf{J}} = \hat{x}\tilde{J}, \qquad (12)$$

Where $\tilde{\mathbf{J}} = \tau H_1 \dfrac{\sinh\tau z}{\cosh\tau d}$ (13)

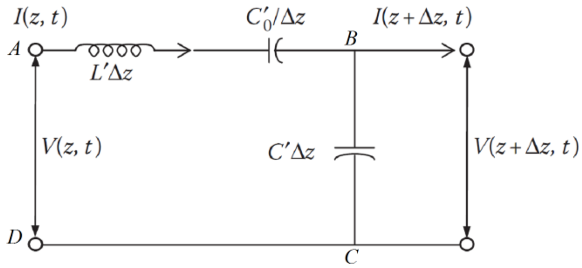
(c) From Poynting theorem, we note that the time averaged power consumed by the material is given by

$$P = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=-d}^{z=d} \frac{\left|\tilde{J}\right|^2}{2\sigma}\, dx\, dy\, dz \quad , \text{ (Ohmic loss)}$$

Evaluating

$$P = \frac{|H_1|^2}{\sigma\delta} \quad \frac{\sinh\frac{2d}{\delta} - \sin\frac{2d}{\delta}}{\cosh\frac{2d}{\delta} + cos\frac{2d}{\delta}}$$

P2.4



<FigureP2.4_1>

From KCL at B

$$I(z,t) - I(z+\Delta z,t) = C'\Delta z \frac{\partial}{\partial t}(z,t)$$

Divide by Δz and take limit $\Delta z \to 0$

$$-\frac{\partial I(z,t)}{\partial z} = C'\frac{\partial}{\partial t}(z,t) \qquad (1)$$

From KVL, ABCDA loop

$$V(z,t) - V(z+\Delta z,t) = L'\Delta z \frac{\partial}{\partial t} I(z,t) + \frac{\Delta z}{C_0'}\int I(z,t)dt$$

Again divide by Δz and take limit $\Delta z \to 0$

$$-\frac{\partial}{\partial t}V(z,t) = L'\frac{\partial I}{\partial t}(z,t) + \frac{1}{C_0'}\int I(z,t)dt$$

Differentiate partially with respect to t

$$-\frac{\partial^2 V(z,t)}{\partial z \partial t} = L'\frac{\partial I}{\partial t^2} + \frac{1}{C_0'}I(z,t) \qquad (2)$$

From (1)

$$-\frac{\partial^2 I(z,t)}{\partial z^2} = C'\frac{\partial^2 V(z,t)}{\partial z \partial t} \qquad (3)$$

From (3) and (2)

$$-\frac{\partial^2 I(z,t)}{\partial z^2} = C'\left[-L'\frac{\partial^2 I(z,t)}{{}^2\partial t} - \frac{1}{C_0'}\,I(z,t)\right]$$

$$\frac{\partial^2 I(z,t)}{\partial z^2} - L'C'\,\frac{\partial^2 I(z,t)}{\partial t^2} - \frac{C'}{C_0'}\,I(z,t) = 0 \qquad (4)$$

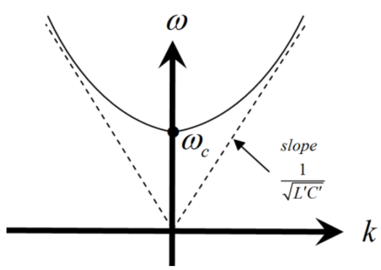
(b) For harmonic variation in space and time $\frac{\partial}{\partial t} \rightarrow j\omega$, $\frac{\partial}{\partial z} = -jk$. Thus (4) becomes the following algebraic equation

$$(-jk)^2 - L'C'(j\omega)^2 - \frac{C'}{C_0'} = 0 \qquad (5)$$

which on simplifying gives

$$\omega^2 = \frac{k^2}{L'C'} + \frac{1}{L'C_0'} \qquad (6)$$

On a $\omega - k$ diagram this is a parabola and easily sketched by noting when $k = 0,\ \omega = \pm\sqrt{\frac{1}{L'C_0'}}$



<FigureP2.4_2>

From this diagram it is obvious that for $\omega < \omega_c$, wave does not propagate (evanescent). To find the slope of the asymptote for large values of ω and k we neglect the last term on the right side of (6) and get $\omega^2_{\text{Asymptote}} = \frac{k^2_{\text{Asymptote}}}{\mathrm{L'C'}}$

Particular case

If $C_0' = \infty$, we have the ideal transmission line and the ω, k diagram is the dotted line. Note that for the particular case $\omega_c = 0$

P2.5

$$E_\rho(\rho) = \frac{\rho_L}{2\pi\varepsilon\rho}, given\, E_\rho(1) = 100$$

$$\therefore \frac{\rho_L}{2\pi\varepsilon} = 100$$

$$E_\rho\Big|_2 = \frac{100}{2} = 50 \quad \Rightarrow \; C = 50$$

$$H_\phi(\rho) = \frac{I}{2\pi\rho} \quad , \quad H_\phi(1) = 2 \quad , \quad \frac{I}{2\pi} = 2$$

$$H_\phi(2) = \frac{2}{2} = 1 \quad \Rightarrow \qquad D = 1$$

(b) For the A.C case, assume received power can be obtained from the power density Assuming RMS values are given:

$$\overset{Powerdensity}{1 < \rho < 2}, \quad \mathrm{E} \times \mathrm{H} = \frac{100}{\rho}\left(\frac{2}{\rho}\right) \hat{a}_\rho \times \hat{a}_\phi = \frac{200}{\rho^2}\hat{z}$$

$$\text{power } = \iint \mathbf{E} \times \mathbf{H} \cdot \mathbf{ds} = \int_{\rho=1}^{2} \int_{\phi=0}^{2\pi} \frac{200}{\rho^2} d\rho\rho d\phi$$

Power received by the load = $400\pi \ln 2 = 871$ watts

Losses: Inner conductor $\mathbf{E} \times \mathbf{H} = 0.05\hat{z} \times 2\hat{\phi} = 0.1\,(-\hat{\rho})$

Power entering the inner conductor = $(0.1)2\pi(1)30 = 6\pi$ watts

Losses: outer conductor

$$|\text{Power density}| = |\mathbf{E} \times \mathbf{H}| = 0.02\; \hat{z} \times 1\hat{\phi} = |0.02\;(-\hat{\rho})|$$

Power entering the outer conductor $= (0.02)(2\pi)(2)(30) = 2.4\pi$ walts

Total Power Loss in the conductor $8.4\pi = 26.4$ walts $=$ Power lost in the line

Source power = Load power + losses in the line =871+26.4= 897.43 watts.

(e) Note that the outer conductor is returning the current, the voltage drop in this conductor is from the receiver end to the source end. So the z component of the electric field at $\rho = 2$ has to be negative.

$$\mathbf{E}\Big|_{\rho=2} = \hat{\rho}50 - \hat{z}0.02\; v/m$$

This may also be seen from the requirement that in the outer conductor $E \times H\big|_{\rho=2} = ()\hat{\rho}$. The power enters the outer conductor from the inner surface at $\rho = 2$ and flows radially outward and gets dissipated.

P2.6

$$V(z,t) - V(z+\Delta z, t) = L'(z)\Delta z \frac{\partial I(z,t)}{\partial t}$$

$$\lim_{\Delta z \to 0} \frac{V(z,t) - V(z+\Delta z,t)}{\Delta z} = L'(z)\frac{\partial I(z,t)}{\partial t}$$

$$-\frac{\partial V(z,t)}{\partial z} = L'(z)\frac{\partial I(z,t)}{\partial t} \qquad (1)$$

$$I(z,t) - I(z+\Delta z,t) = C'(z+\Delta z)\Delta z \frac{\partial V(z\Delta z,t)}{\partial t}$$

$$\lim_{\Delta z \to 0} \frac{I(z,t) - I(z+\Delta z,t)}{\Delta z} = \lim_{\Delta z \to 0} C'(z)\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$-\frac{\partial I(z,t)}{\partial z} = C'(z)\frac{\partial V(z,t)}{\partial t} \qquad (2)$$

From (1)

$$-\frac{\partial V^2(z,t)}{\partial z^2} = L'(z)\frac{\partial I^2(z,t)}{\partial z \partial t} + \frac{\partial I(z,t)}{\partial t}\frac{dL'(z)}{dz} \qquad (3)$$

From (2)

$$-\frac{\partial I^2(z,t)}{\partial z \partial t} = -C'((z)\frac{\partial V^2(z,t)}{\partial t^2} \qquad (4)$$

From (1)

$$-\frac{\partial I}{\partial t} = \frac{1}{L'(z)}\frac{\partial V}{\partial z} \qquad (5)$$

Substituting (4) and (5) in (3)

$$-\frac{\partial V^2(z,t)}{\partial z^2} = L'(z)\left[-C'(z)\frac{\partial V^2}{\partial t^2}\right] + \left[-\frac{1}{L'(z)}\frac{\partial V}{\partial z}\right]\frac{\partial L'(z)}{dz} = 0$$

(A) $-\dfrac{\partial V^2(z,t)}{\partial z^2} = -L'(z)C'(z)\dfrac{\partial V^2}{\partial t^2} - \dfrac{1}{L'(z)}\dfrac{\partial L'(z)}{dz}\dfrac{\partial V}{\partial z} = 0$

(B) $L'(z)C'(z) = L_0 e^{qz} C_0 e^{qz} = L_0 C_0$

$$\frac{1}{L'(z)}\frac{\partial L'(z)}{dz} = \frac{1}{L_0 e^{qz}} q L_0 e^{qz} = q$$

Therefore (1) becomes

$$\frac{\partial V^2}{\partial z^2} - L_0 C_0 \frac{\partial V^2}{\partial t^2} - q\frac{\partial V}{\partial z} = 0$$

For harmonic variation

$$V = V_0 e^{j\omega t - kz}$$

$$\left[(-jk)^2 - L_0 C_0 (j\omega)^2 - (-jkq)\right]V = 0$$

$$-k^2 + L_0 C_0 \omega^2 + jkq = 0$$

$$\omega^2 = +\frac{1}{L_0 C_0}\left(k^2 - jkq\right)$$

P2.7

$$\gamma = jk = \alpha + j\beta = \sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{G'}{C'} + j\omega\right)}\sqrt{L'C'} = \left(\frac{R'}{L'} + j\omega\right)\sqrt{L'C'} = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}$$

$$\alpha = R'\sqrt{\frac{C'}{L'}}, \quad \beta = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\left(\frac{R' + j\omega L'}{G' + j\omega C'}\right)} = \sqrt{\frac{L'}{C'}} = \sqrt{\left(\frac{\frac{R'}{L'} + j\omega}{\frac{C'}{C'} + j\omega}\right)} = \sqrt{\frac{L'}{C'}}$$

P2.8

$$\hat{H}(x) = 2[\hat{y} + j\hat{z}]e^{jkx} \quad (1)$$

$$(a)\ \eta_o \hat{H} \times \hat{k} = \tilde{E};\ \hat{k} = -\hat{x}$$

$$\tilde{E}(x) = \eta_o{}^2[\hat{y} + j\hat{z}]e^{jkx} x(-\hat{x})$$

$$= 2\eta_o[\hat{z} + j(-\hat{y})]e^{jkx}$$

$$= 2\eta_o[\hat{z} - j\hat{y})]e^{jkx}$$

$$E(x,t) = \eta_o 2[\hat{z}\,\mathrm{Re}(e^{jkx}e^{j\omega t}) - \hat{y}\,\mathrm{Re}(e^{jkx}e^{j\pi/2}e^{j\omega t})]$$

$$= \eta_o 2[\hat{z}\cos(\omega t + kx) - \hat{y}\cos(\omega t + kx + \pi/2)]$$

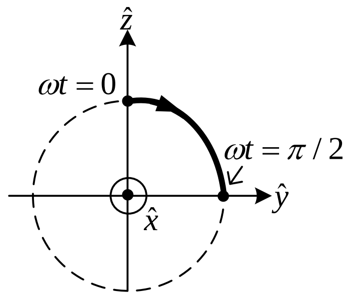
We can simplify further, noting $\cos(\pi/2 + \theta) = -\sin\theta$

$$= 2\eta_o[\hat{z}\cos(\omega t + kx) + \hat{y}\sin(\omega t + kx)]$$

$$(b)\ E(0,t) = 2\eta_o[\hat{y}\sin\omega t + \hat{z}\cos\omega t]$$

Let us set up the transverse plane as shown and set up in the table and the circle

ωt	$E(0,t)$
0	$\eta_o 2\hat{z}$
$\frac{\pi}{2}$	$\eta_o 2\hat{y}$



Electric Field
Figure 1

<FigureP2.8_1>

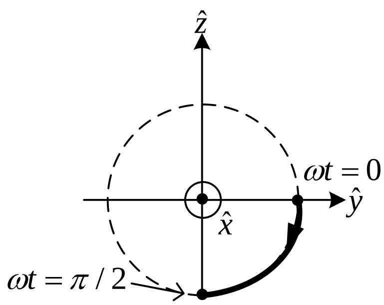
We note that as t increases the circle is traced clockwise in Figure 1(Electric Field). The direction of propagation is $\hat{k} = -\hat{x}$, into the paper. You need a right hand to describe the rotation, the thumb of the right hand into the paper, the other finger curve clockwise: R-wave

$(c)\ \hat{H}(x) = 2[\hat{y} + j\hat{z}]e^{jkx}$

$$\hat{H}(x,t) = 2[\hat{y}\cos(\omega t + kx) + \hat{z}\cos(\omega t + kx + \pi/2)]$$
$$= 2[\hat{y}\cos(\omega t + kx) - \hat{z}\sin(\omega t + kx)]$$

$$\hat{H}(0,t) = 2[\hat{y}\cos\omega t - \hat{z}\sin\omega t]$$

ωt	$H(0,t)$
0	$2\hat{y}$
$\frac{\pi}{2}$	$-2\hat{z}$



Magnetic Field
Figure 2

<FigureP2.8_2>

Once again we see from the clockwise rotation, that it is R-wave

$$(d) < S_R(x) >= \frac{1}{2}\mathrm{Re}[\tilde{E} \times \tilde{H}^*]$$

$$= \frac{1}{2}\mathrm{Re}\{[\eta_o 2(\hat{z} - j\hat{y})e^{jkx}] \times [2(\hat{y} - j\hat{z})e^{-jkx}]$$

$$= \frac{1}{2}\mathrm{Re}\{\eta_o 4(\hat{z} - j\hat{y}) \times (\hat{y} - j\hat{z})\}$$

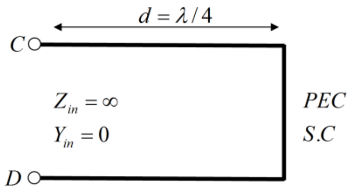
$$= \frac{1}{2}\mathrm{Re}\{\eta_o 4(\hat{z} \times \hat{y} - j\hat{z} \times \hat{z} - j\hat{y} \times \hat{y} - \hat{y} \times \hat{z}\}$$

$$< S_R >= \frac{1}{2}\mathrm{Re}\{\eta_o 4(-2\hat{x})\} = -4\eta_o \hat{x} \text{ as expected}$$

Instantaneous Power Density

$$S(x,t) = E(x,t) \times H(x,t)$$
$$= 2\eta_o[\hat{z}\cos(\omega t + kx) + \hat{y}\sin(\omega t + kx)] \times = 2[\hat{y}\cos(\omega t + kx) - \hat{z}\sin(\omega t + kx)]$$
$$= 4\eta_o[\hat{z} \times \hat{y}\cos^2(\omega t + kx) + (\hat{y} - \hat{z})\sin^2(\omega t + kx)]$$
$$= -4\eta_o\hat{x}[\cos^2(\omega t + kx) + \sin^2(\omega t + kx)]$$
$$S(x,t) = -4\eta_o\hat{x} \; W/m^2$$

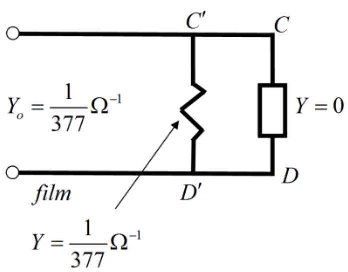
P2.9



<FigureP2.9_1>

At 9 GHz $\quad Y_{C'D'} = \dfrac{1}{377} + 0 = \dfrac{1}{377}\,\Omega^{-1}$

Hence there is matching; reflection coefficient is zero at 9 GHz



<FigureP2.9_2>

For 9 GHz, $\quad \lambda = \dfrac{3\times10^8}{9\times10^9} = \dfrac{1}{3}\times10^{-1}m$

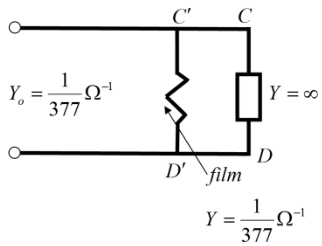
Hence $d = \dfrac{1}{12}\times10^{-1}$. At other frequencies $d \neq \lambda/4$ since λ

changes. For example $f = 18$ *GHz*, $\lambda = \dfrac{3\times10^8}{18\times10^9} = \dfrac{1}{6}\times10^{-1}$, Hence $\dfrac{d}{\lambda} = \dfrac{1}{2}$

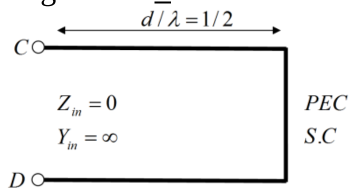
Hence

$$Y_{C'D'} = \infty \text{ Hence } = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{1}{Y_L} - \frac{1}{Y_0}}{\frac{1}{Y_L} + \frac{1}{Y_0}} = \frac{Y_0 - Y_L}{Y_0 - Y_L} = \frac{\frac{1}{377} - \infty}{\frac{1}{377} + \infty}$$

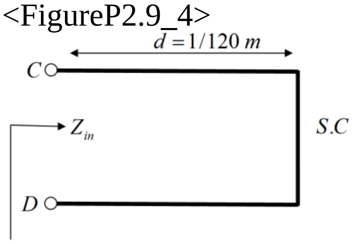
Power reflection coefficient at 18 GHz $\quad |\Gamma|^2 = 1$



<FigureP2.9_3>



<FigureP2.9_4>



<FigureP2.9_5>

For other frequencies

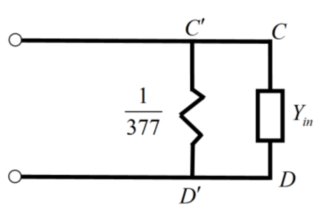
$$Z_{in} = jZ_0 \tan \beta d = jZ_0 \tan \frac{2\pi}{\lambda} d$$

$$\beta d = \frac{2\pi}{\lambda} d = \frac{2\pi}{c} fd = \frac{2\pi}{3\times 10^{-1}}$$

$$Y_{in} = \frac{1}{Z_{in}}$$

$$Y_{C'D'} = Y_{in} + \frac{1}{377}$$

$$\Gamma = \frac{Y_0 - Y_{C'D'}}{Y_0 + Y_{C'D'}} = \frac{\frac{1}{377} - Y_{in} - \frac{1}{377}}{\frac{1}{377} + Y_{in} + \frac{1}{377}}$$

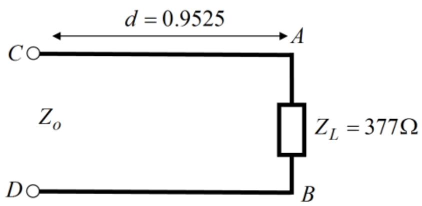


<FigureP2.9_6>

Power reflection coefficient $|\Gamma|^2 =$

$f\ (GHz)$	$\lvert\Gamma\rvert^2$
6	0.076
9	0
15	0.429
18	1

P2.10



<FigureP2.10_1>
Analytical Solution

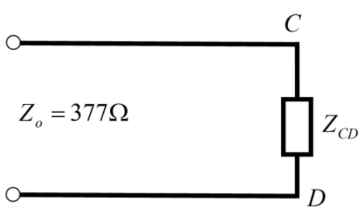
$$\frac{3''}{8} = 0.9525\ cm$$

$$\lambda_0 = 3cm \quad f = \frac{c}{\lambda_0} = \frac{3\times10^8}{3\times10^{-2}} = 10^{10} = 10GHz$$

λ in the dielectric

$$\lambda = \frac{v}{f} = \frac{1}{\sqrt{\mu\varepsilon}}\cdot\frac{1}{f} = \frac{\lambda_0}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{2.8}} = \frac{3}{\sqrt{2.8}} = 1.79cm$$

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon_o}}\cdot\frac{1}{\sqrt{\varepsilon_r}} = \frac{170\pi}{\sqrt{2.8}}$$



<FigureP2.10_2>
Find Z_{CD} from (2.67),

where $\beta d = \dfrac{2\pi}{\lambda} d = \dfrac{2\pi\times0.9525}{\lambda}$.

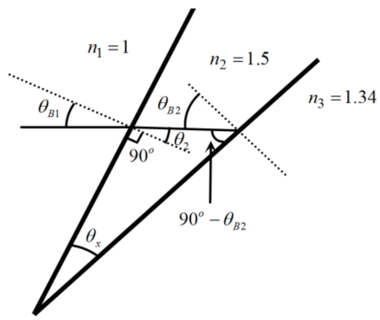
Then $\Gamma = \dfrac{Z_{CD} - Z_o}{Z_{CD} + Z_0}$.

Power Reflection Coefficient $= |\Gamma|^2$

If you calculate on this basis

λ_o (cm)	$|\Gamma|^2$ %
20	6.26
10	17
3	1

P2.11



<FigureP2.11_1>
Make each face at appropriate Brewster angle

$$\theta_{B1} = \tan^{-1}\frac{n_1}{n_2} = \tan^{-1}1.5 = 56.3^0$$

From Snell's law

$$\text{Sin } \theta = \frac{n_1}{n_2}\text{Sin } \theta_{B1} = \frac{\text{Sin } 56.3}{1.5}$$

$$\theta_2 = 33.7^0$$

$$\theta_{B2} = \tan^{-1}\frac{n_3}{n_2} = \tan^{-1}\frac{1.34}{1.5} = 41.8^0$$

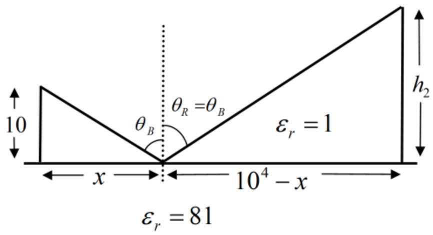
$$\theta_{B2} - \theta_{B1} = 56.3 - 41.8^0 = 14.5^0$$

Angle between faces

$$\theta_x = 180 - (90 + \theta_2) - (90 - \theta_{B2}) = 180 - 180 - \theta_2 + \theta_{B2} = 41.8^0 - 33.7^0 = 8.1^0$$

P2.12

If the reflected wave has no parallel polarized component the angle of incidence has to be the Brewster Angle



<FigureP2.12_1>

$$\theta_i = \theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \tan^{-1} 9 = 83.66^o$$

$$\tan\theta_B = 9 = \frac{x}{10} \quad \Rightarrow x = 90$$

Also $\quad \dfrac{h_2}{10^4 - x} = \dfrac{h_2}{10^4 - 90} = \cot\theta_B = \dfrac{1}{\tan\theta_B} = \dfrac{1}{9}$

$$h_2 = \frac{10^4 - 90}{9} = 1101.9 \text{ meters}$$

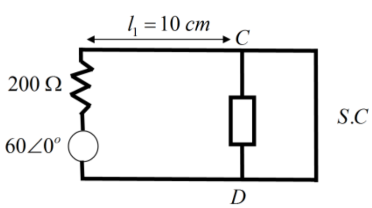
P2.13

(a) $f = 60 \quad \lambda = \dfrac{3\times 10^8}{60} = 5\times 10^6 \text{ meters}$

$$\therefore \beta l_2 \approx \frac{2\pi}{\lambda} l_2 = \frac{2\pi \times 5\times 10^{-2}}{5\times 10^6} \approx 6\times 10^{-8}$$

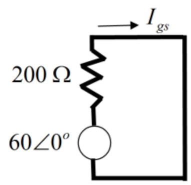
$$\tan\beta l_2 \approx 0 \quad \therefore Z_{in} = Z_L = \text{short}$$

Thus the circuit



<FigureP2.13_1>

Once again when $l_1 = 10\text{ cm}, \quad \tan\beta l_1 \approx 0$ and the circuit reduces to

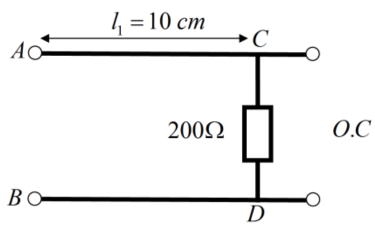


<FigureP2.13_2>

$$I_{gs} = \frac{60\angle 0^o}{200} 0.3\angle 0^o$$

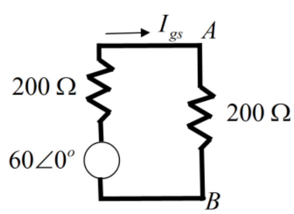
(b) $f = 1.5GHz \quad \lambda = \dfrac{3\times 10^8}{1.5\times 10^9} = 2\times 10^{-1} = 0.2\,\text{cm}$

$l_2 = 5\,\text{cm} \quad \text{l}_2 = \lambda / 4$ Therefore the circuits



<FigureP2.13_3>

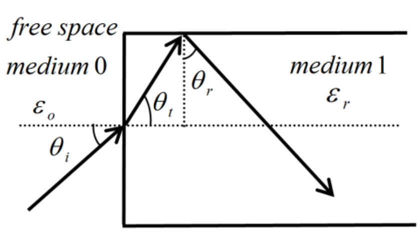
$l_1 = \lambda / 2 \quad \therefore$ The circuit reduces to



<FigureP2.13_4>

$$I_{gs} = \frac{60\angle 0^o}{400} = 0.15\angle 0^o$$

P2.14



<FigureP2.14_1>

$$\sin\theta_{\text{r}} = \sin\left(\frac{\pi}{2} - \theta_t\right) = \cos\theta_{\text{t}}$$

$$\sin\theta_{\mathrm{r}} \geq \sin\theta_{\mathrm{c}} = \sqrt{\frac{\varepsilon_0}{\varepsilon_r\varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_r}}$$

i.e. $\cos\theta_{\mathrm{t}} \geq \frac{1}{\sqrt{\varepsilon_r}}$

From Snell's $k_0 \sin\theta_{\mathrm{i}} = k_1 \sin\theta_{\mathrm{t}}$

$$\sin\theta_{\mathrm{t}} = \frac{k_0}{k_1}\sin\theta_{\mathrm{i}} = \frac{1}{\sqrt{\varepsilon_r}}\sin\theta_{\mathrm{i}}$$

$$\cos\theta_{\mathrm{t}} = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\frac{1}{\varepsilon_r}\sin^2\theta_i} \geq \frac{1}{\sqrt{\varepsilon_r}}$$

$$1-\frac{1}{\varepsilon_r}\sin^2\theta_i \geq \frac{1}{\varepsilon_r}$$

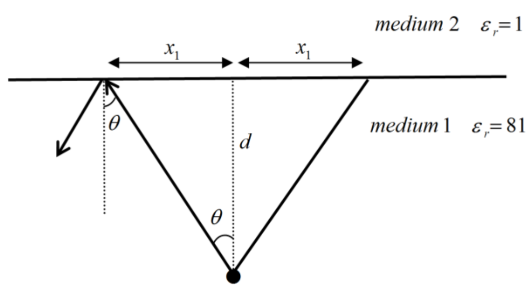
$$\varepsilon_r - \sin^2\theta_i \geq 1$$

$$\varepsilon_r \geq 1+\sin^2\theta_i$$

To accommodate all angles (including $\theta_i = 90^0$)

$$\varepsilon_r \geq 2$$

P2.15



<FigureP2.15_1>

When $\theta = \theta_c$ there is totally reflection

So maximum value for θ, so that there is a transmitted wave is given by

$$\tan\theta_{\mathrm{c}} = \tan\theta = \frac{x_1}{d} =$$

However $\sin\theta_{\mathrm{c}} = \frac{n_2}{n_1} = \frac{1}{9}$

$$\tan\theta_c = \frac{1}{\sqrt{80}} = \frac{x_1}{d}$$

$$x_1 = \frac{d}{\sqrt{80}} = 0.112d$$

P2.16

$$\mathbf{E}_s^t = \hat{y}T_s E_S^I e^{-jk_2 x\sin\theta_t}\ e^{-jk_2 z\sin\theta_t}\ \big|\theta_i > \theta_c$$

$$\sin\theta_t = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta_i\ \big|\theta_i > \theta_c$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t}\ \ \big|\theta_i > \theta_c$$

$$= \sqrt{1-\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_i}\ \big|\theta_i > \theta_c$$

$$e^{-jk_2 z\cos\theta_t} = e^{-k_2 z\sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}\sin^2\theta_i - 1}}\quad \big|\theta_i > \theta_c$$

$$\therefore \mathbf{E}_s^t = \hat{y}T_s E_s^I e^{-\alpha_e z}\ e^{-j\beta_e x} \qquad (1)$$

Where $\alpha_e = k_2\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}\ ,\theta_i > \theta_c \qquad (2)$

$\beta_e = k_2\dfrac{n_1}{n_2}\sin\theta_1$ (Note θ_i same as θ_1)

$$v_{pe} = \frac{\omega}{\beta_e} = \frac{\omega}{k_2\frac{n_1}{n_2}\sin\theta_1} = \frac{v_{p2}}{\frac{n_1}{n_2}\sin\theta_1} < v_{p2} \qquad (3)$$

The magnetic field $\mathbf{H}_s^t = (-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)\dfrac{\mathbf{E}_s^t}{\eta_2} \qquad (4)$

In medium 2

$$\langle\mathbf{S}\rangle = \frac{1}{2}(\mathbf{E}\times\mathbf{H}^*) = \langle\mathbf{S}_2\rangle = \frac{1}{2}(\mathbf{E}_s^t\times\mathbf{H}_s^{t*})$$

$$= \frac{1}{2\eta_2} \hat{y} T_s E_s^t \, e^{-\alpha_e z} \quad e^{-j\beta_e x} \times \left\{ (-\hat{x} \cos\theta_{\mathrm{t}} + \hat{z} \sin\theta_{\mathrm{t}}) T_s^* E_s^{t*} \, e^{-\alpha_e z} \quad e^{+j\beta_e x} \right\}$$

$$= \frac{1}{2} \hat{z} \left[j\sqrt{\frac{n_1^2}{n_2^2} \sin^2\theta_{\mathrm{i}} \text{ -1}} + \hat{x} \left(\frac{n_1}{n_2} \sin^2\theta_{\mathrm{i}} \right) \right] \frac{|T_s|^2 \left|E_s^I\right|^2}{\eta_2} e^{-2\alpha_e z} \quad , \theta_1 > \theta_c$$

The imaginary part is given by

$$\left\langle \mathbf{S}_I \right\rangle = +\frac{1}{2} \hat{z} \sqrt{\left(\frac{n_1}{n_2} \sin\theta_1 \right)^2 - 1} \, \frac{|T_s|^2 \left|E_s^I\right|^2}{\eta_2} e^{-2\alpha_e z} \quad , \theta_1 > \theta_c$$

In the above $\quad n_1 = \sqrt{\mu_{r1}\varepsilon_{r1}}$

$$n_2 = \sqrt{\mu_{r2}\varepsilon_{r2}}$$

$$\eta_2 = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}$$

For non magnetic dielectric $\mu_{r1} = \mu_{r2} = 1$

P2.17

$$\Gamma_s = \frac{\dfrac{\eta_2}{\cos\theta_2} - \dfrac{\eta_1}{\cos\theta_1}}{\dfrac{\eta_2}{\cos\theta_2} + \dfrac{\eta_1}{\cos\theta_1}}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{\eta_o}{n_2} \quad , \quad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \frac{\eta_o}{n_1}$$

$$\Gamma_s = \frac{\dfrac{1}{n_2\cos\theta_2} - \dfrac{1}{n_1\cos\theta_1}}{\dfrac{1}{n_2\cos\theta_2} + \dfrac{1}{n_1\cos\theta_1}} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

$$= \frac{1.5\cos\theta_1 - \cos\theta_2}{1.5\cos\theta_1 + \cos\theta_2}$$

Critical angle $\sin\theta_c = \dfrac{n_2}{n_1} = \dfrac{1}{1.5} \quad \Rightarrow \quad \theta_c = 41.81^o$

At $\theta_1 = \theta_c$, $\sin\theta_2 = 1$ and $\cos\theta_2 = 0$

For incident angle $\theta_1 > \theta_c$, $\sin\theta_2 > 1$ that θ_2 is a complex angle in such a case the angle θ_2 loses its physical interpretation as the angle of refraction for the transmitted wave.

We may write

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_1} = -j\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}$$

Note we have chosen $\sqrt{-1} = -j$ so that

$$e^{-jk_2 z\cos\theta_2} = e^{(-jk_2 z)\left(-j\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}\right)}$$

$$= e^{-k_2 z\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}}$$

And the wave attenuates in the $+z$ direction
For the problem in hand, compute based on

$$\cos\theta_2 = -j\left|\cos\theta_2\right| = -j\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}$$

$$\left|\cos\theta_2\right| = \sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}$$

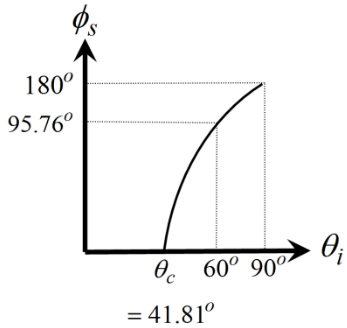
$$\Gamma_s = \frac{n_1\cos\theta_1 + jn_2\left|\cos\theta_2\right|}{n_1\cos\theta_1 - jn_2\left|\cos\theta_2\right|} = \left|\Gamma_s\right| e^{j\phi_s}$$

$$\left|\Gamma_s\right| = 1$$

$$\phi_s = \angle\Gamma_s = 2\tan^{-1}\left(\frac{n_2\left|\cos\theta_2\right|}{n_1\cos\theta_1}\right)$$

Calculation of two typical values

θ_1	$\left\lvert\cos\theta_2\right\rvert = \sqrt{1.5^2\sin^2\theta_1 - 1}$	$\phi_s = 2\tan^{-1}\left(\frac{\left\lvert\cos\theta_2\right\rvert}{1.5\cos\theta_1}\right)$
41.81^o	0	0
60^o	0.83	95.76^o



<FigureP2.17_1>

Similar analysis can be made for *p-wave.*

P2.18

(a) From (2.122)

$$\tilde{H}_\Phi = -k_0 \frac{A_0}{\mu} H_0^{'(2)}(k_0\upsilon)$$

$$= k_0 \frac{A_0}{\mu} H_1^{(2)}(k_0\rho)$$

From Ampere's law

$$\oint_c H_\Phi dl = I_0 \quad (1)$$

Even if c is a circular contour with $\rho \to 0$. Approximate $H_1^{(2)}(k_0\rho)$ in the limit $k_0\rho \to 0$, using (2.127)

$$\underset{k_0\rho\to 0}{H_1^{(2)}(k_0\rho)} = j\frac{2}{\pi}\frac{1}{k_0\rho} \quad (2)$$

Also

$dl = \rho d\Phi$ (3).

Using (2) and (3) in (1) and we get

Limit $k_o\rho \to 0$

$$\oint_c k_0 \frac{A_0}{\mu}\frac{j2}{\pi}\frac{1}{k_0\rho}\rho d\phi = I_0$$

$$k_0 \frac{A_0}{\mu}\frac{j2}{\pi}\frac{1}{k_0}\int_0^{2\pi} d\phi = I_0$$

$$A_0 = -j\frac{\mu I_0}{4I_0}$$

(b) From (2.128), (2.129) and their large argument approximation for $k_o\rho \to \infty$, we have

$$\underset{\lim k_0\rho\to\infty}{\tilde{E}_z} = -\frac{k_0\eta_0}{4}\tilde{I}_0\sqrt{\frac{2j}{\pi k_0\rho}}e^{-jk_0\rho}$$

$$\underset{\lim k_0\rho\to\infty}{\tilde{H}_\Phi} = -\frac{k_0}{4}\tilde{I}_0\sqrt{\frac{2j}{\pi k_0\rho}}e^{-jk_0\rho}e^{-j\frac{3\pi}{4}}$$

Note

$$\underset{k_0\rho\to\infty}{H_1^{(2)}(k_0\rho)} = \sqrt{\frac{2}{\pi k_0\rho}}e^{-jk_0\rho}e^{j\frac{\pi}{4}}e^{j\frac{\pi}{2}}$$

$$\underset{k_0\rho\to\infty}{\frac{\tilde{E}_z}{\tilde{H}_\phi}} = \frac{\eta_0\sqrt{j}}{je^{j\frac{3\pi}{4}}} = -\eta_0$$

There the real part of the Poynting vector is

$$< S_R >= \frac{1}{2}\mathrm{Re}\left[\tilde{\tilde{E}} \times \tilde{\tilde{H}}^*\right]$$

$$= \frac{1}{2}\frac{|E_z|^2}{\eta_0}$$

$$= \frac{1}{2}\frac{k_0^2\eta_0^2}{16\eta_0}\left|\tilde{I}_0\right|^2 \frac{2}{\pi k_0\rho}$$

$$= \frac{\left|\tilde{I}_0\right|^2 k_0\eta_0}{16\pi\rho}$$

<P> per meter length
$$= \frac{\left|\tilde{I}_0\right|^2 k_0\eta_0}{16\pi}\int_{z=0}^{1}\int_{\varphi=0}^{2\pi}\frac{1}{\rho}\rho d\varphi dz$$
$$= \frac{\left|\tilde{I}_0\right|^2}{8} k_0\eta_0$$

P2.18b

From (2.128), (2.129) and the large argument approximations for $k_0\rho \to \infty$ we have

$$\underset{k_0\rho\to\infty}{E_z} = -\frac{k_0\eta_0}{4}\tilde{I}_0\sqrt{\frac{2j}{\pi k_0\rho}}\, e^{-jk_0\rho}$$

$$\underset{k_0\rho\to\infty}{H_\phi} = -\frac{jk_0}{4}\tilde{I}_0\sqrt{\frac{2}{\pi k_0\rho}}\, e^{-jk_0\rho}\, e^{+j\frac{3\pi}{4}}$$

Note $\underset{k_0\rho\to\infty}{H_1^{(2)}}(k_0\rho) = \sqrt{\frac{2}{\pi k_0\rho}}\, e^{-jk_0\rho}\, e^{+j\frac{\pi}{4}}\, e^{+j\pi/2}$

$$\frac{\tilde{E}_z}{\tilde{H}_\rho} = \frac{\eta_0\sqrt{j}}{je^{j3\pi/4}} = -\eta_0$$

Therefore the real part of the Poynting vector is

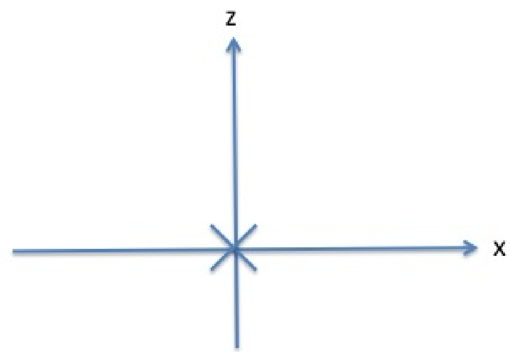
$$\langle S_R\rangle = +\frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right] = \frac{1}{2}\frac{|E_z|^2}{\eta_0} = \frac{1}{2}\frac{k_0^2\eta_0^2}{16\eta_0}\left|\tilde{I}_0\right|^2\frac{2}{\pi k_0\rho} = \frac{\left|\tilde{I}_0\right|^2}{16}\frac{k_0\eta_0}{\pi\rho}$$

$$\langle P\rangle \text{ per meter length } = \frac{\left|\tilde{I}_0\right|^2}{16}\frac{k_0\eta_0}{\pi}\int_{z=0}^{1}\int_{\phi=0}^{2\pi}\rho d\phi dz = \frac{\left|\tilde{I}_0\right|^2}{16}\frac{k_0\eta_0}{\pi}2\pi = \frac{\left|\tilde{I}_0\right|^2}{8}k_0\eta_0$$

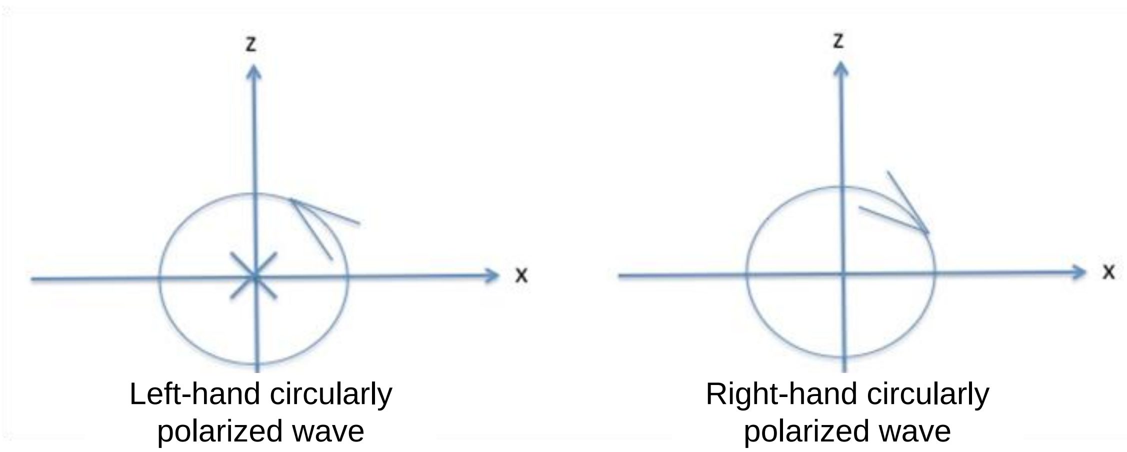
P2.19

$$\overline{E}(y) = \left[\left(75e^{j\frac{\pi}{4}}\hat{a}_x + 75e^{-j\frac{\pi}{4}}\hat{a}_z\right) + \left(25e^{j\frac{\pi}{4}}\hat{a}_x - 25e^{-j\frac{\pi}{4}}\hat{a}_z\right)\right] e^{-jk_0 y}$$

Term 1 Term 2

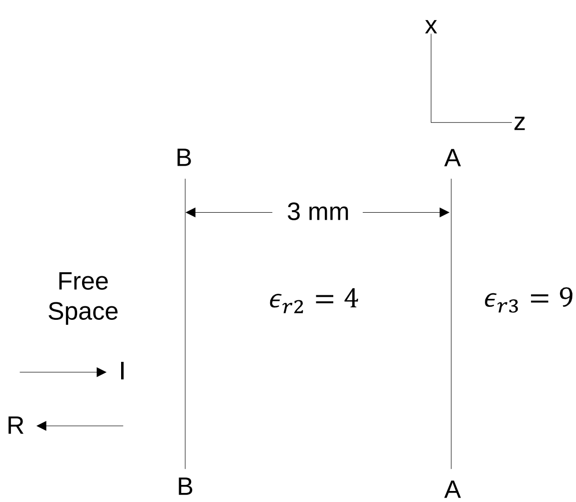


Term 1 Term 2



<FigureP2.19_1>

P2.20



<FigureP2.20_1>
Assume normal incidence:

$\overline{E}^I = 2e^{j[2\pi 10^{10}t - k_0 z]}\hat{a}_x \quad mV/meter$

Determine $\overline{E}^R$

We will do it by Smith Chart

$f = 10GHz, \lambda_0 = \frac{3\times10^8}{10\times10^9} = 3\ \ cm$

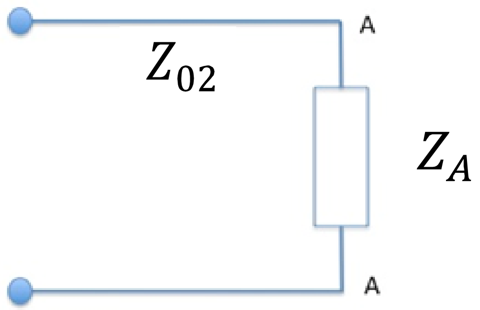
$$\lambda_2 = \frac{V}{f} = \frac{3\times10^8}{2(10\ GHz)} = 1.5\ \ cm$$

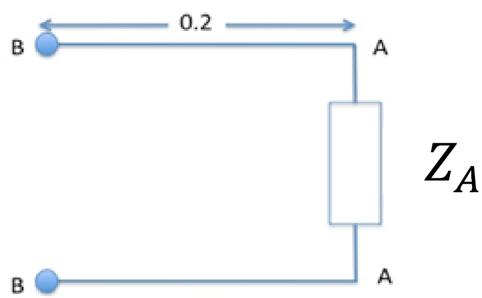
$$\frac{d_2}{\lambda_2} = \frac{3}{15} = \frac{1}{5} = 0.2$$

$Z_A = \sqrt{\frac{\mu_3}{\epsilon_3}} = \frac{120\pi}{3},\ Z_{02} = \frac{120\pi}{2},\ Z_B = 1.31 + j0.32,\ Z_B = \frac{120\pi}{2}[1.31 + j0.32]$

$R = 0.23\angle 149°$

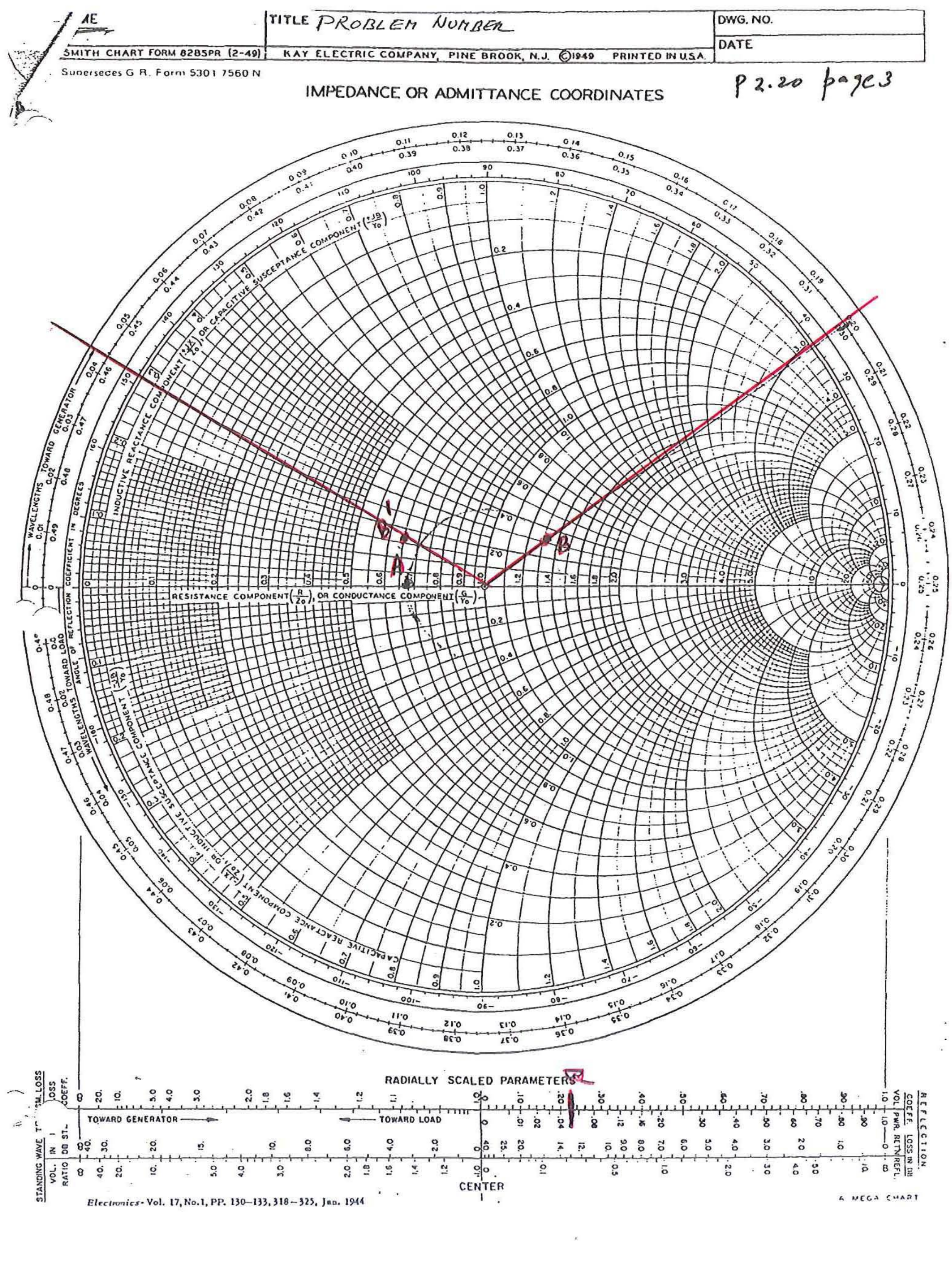
$\overline{E}^R = R\overline{E}^I = (0.23)(2)e^{j\left[2\pi\times10^6 t + k_0 z + \frac{149}{180}\pi\right]}\hat{a}_x \ \ mV/meter$





$$Z_A = \frac{120\pi}{3}\frac{2}{120\pi} = 0.667$$

<FigureP2.20_2>



<FigureP2.20_3>

P2.21

Like homework P. 0.15

$$\frac{x}{10} = \tan\theta_B = \frac{10000 - x}{990}$$

$990x = 10(10000) - 10x$

$1000x = (10)(10000)$

$x = 100$

$$\tan\theta_B = \frac{100}{10} = 10 = \frac{\sqrt{\epsilon_{R2}}}{\sqrt{\epsilon_{R1}}} = \frac{\sqrt{\epsilon_R}}{1}$$

$\epsilon_R = 100$

P2.22

Let $\sqrt{\epsilon_R} = n,\ \sin\theta_3 = \frac{1}{n},\ \theta_2 = \frac{\pi}{2} - \theta_3,\ \sin\theta_2 = \sin\left(\frac{\pi}{2} - \theta_3\right) = \cos\theta_3 = \sqrt{1 - \frac{1}{n^2}}$

Snell's Law

$$\frac{1}{2} = \sin 30° = n\sin\theta_2 = n\sqrt{1 - \frac{1}{n^2}} = \sqrt{n^2 - 1}$$

$\frac{1}{4} = n^2 - 1;\ n^2 = 1 + \frac{1}{4} = \epsilon_R = 1.25$

P2.23

(a)

$E_0 = 5,\ \cos\theta_i = 3/5, \sin\theta_i = 4/5,\ \theta_1 = \theta_i = 53.13°$

(b) Brewster Angle

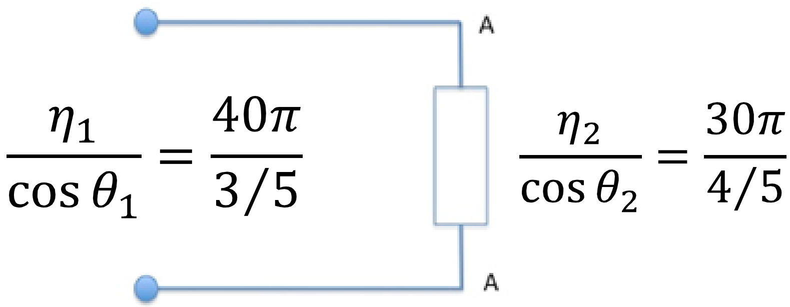
$\tan\theta_B = \frac{n_2}{n_1} = \frac{\sqrt{A}}{3} = \tan 53.13° = \frac{4}{3},\ \sqrt{A} = 4,\ A = 16$

To find Γ_S, use Snell's Law to fine $\theta_2 = \theta_t,\ k_1\sin\theta_1 = k_2\sin\theta_2$,

$\sin\theta_2 = \frac{k_1}{k_2}\sin\theta_1 = \frac{n_1}{n_2}\sin\theta_1 = \frac{3}{4}\frac{4}{5} = \frac{3}{5},\ \theta_2 = 36.87°$

$\cos\theta_2 = \frac{4}{5},\ \eta_1 = \frac{120\pi}{3},\ \eta_2 = \frac{120\pi}{4}$

$$\Gamma_S = \frac{\frac{150\pi}{4} - \frac{200\pi}{3}}{\frac{150\pi}{4} + \frac{200\pi}{3}} = -\frac{7}{25}$$



<FigureP2.23_1>

P2.24

(A)
$\tilde{\tilde{\bar{E}}}_p$ has x and z components and the x component is $E_0 = \cos\theta$ and the z component is $-E_0 \sin\theta$. Comparing with $\tilde{\tilde{\bar{E}}}_p = (3\hat{x} - 4\hat{y})e^{-j(k_x x + k_z z)}$, $E_0 = 5$, $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, $\theta = 53.13°$. If the medium is air, $\eta = \eta_0\sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$, $\tilde{\tilde{\bar{H}}}_p = \hat{y}\frac{5}{120\pi}e^{-j(k_x x + k_z z)}$, $A/meter$

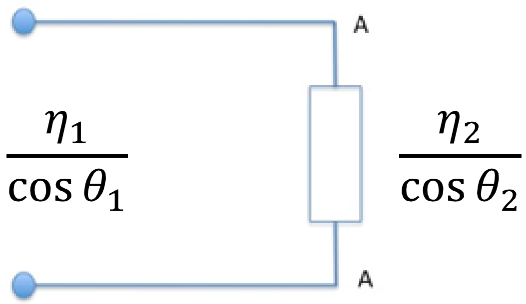
(B)

$$\bar{E}^i = (6\hat{x} - 8\hat{z})\cos(\omega t - k_x x - k_z z) + 10\hat{y}\sin(\omega t - k_x x - k_z z)\,, \qquad V/meter$$

(1) $\theta_1 = \theta = 53.13°$
(2) $\theta_R = \theta_1 = 53.13°$
(3) $0.5 = k_x = k\sin\theta_1 = k(4/5)$, $k = \frac{5}{4}(0.5) = \frac{5}{8}$
$\lambda = \frac{2\pi}{k} = \frac{2\pi}{5/8} = \frac{16\pi}{5} 10.05\, meters$, $f = \frac{v_p}{\lambda} = \frac{3\times10^8/3}{16\pi/5} = 10^8\frac{5}{16\pi} = 0.995\times10^7$, $meters/s$,

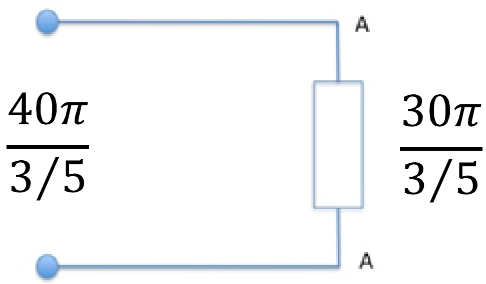
$$\frac{\omega}{c} = k_0 = \frac{2\pi f}{c} = 2\pi\frac{3\times10^8}{3}\frac{5}{10\pi}\frac{1}{3\times10^8} = \frac{5}{24}$$

(4) $\tilde{\tilde{\bar{E}}}_p^I = 10\left(\frac{6}{10}\hat{x} - \frac{8}{10}\hat{z}\right)e^{-j(\sin\theta_1 x + \cos\theta_1 z)}$, $\tilde{\tilde{\bar{E}}}_S^I = 10\hat{y}\cos\big(90° - (\omega t - k_x x - k_z z)\big) =$ $10\hat{y}\cos(\omega t - k_x x - k_z z - 90°)$, $\tilde{\tilde{\bar{E}}}_S^I = -j10\hat{y}e^{-j(k_x x + k_z z)}$
We have two orthogonal components ($p$component and s component) which are equal in magnitude (10), but have a phase difference of $\frac{\pi}{2}$. This corresponds to the case of R-wave equation (0.124). Hence, the polarization is Right Circular Polarization.
(5) Critical angle, $\theta_C = \sin^{-1}\frac{n_2}{n_1}$, $\frac{n_2}{n_1} = \sin\theta_C = \sin 53.13° = \frac{4}{3}$, $n_2 = (n_1)(4/5) = \frac{(3)(4)}{5} = \frac{12}{5}$, $\epsilon_{R2} = \left(\frac{12}{5}\right)^2 = \frac{144}{25} = 5.76$, $\epsilon_2 = 5.76\epsilon_0$
(6) If the p wave is totally transmitted, then the reflected wave has only s component which is linearly polarized. Thus we look for Brewster angle.
$\frac{4}{3} = \tan\theta_B = \frac{n_2}{n_1} = \frac{n_2}{3}$, $n_2 = 4$; $\epsilon_{R2} = 16$, $\epsilon_2 = 16\epsilon_0$
(7) Since $\epsilon_2 = 16\epsilon_0$, as such in (6) p-wave is totally transmitted. s-wave is reflected. Therefore, we calculate the reflection coefficient for s-wave.



<FigureP2.24_1>

$\cos\theta_1 = 3/5$, $\sin\theta_1 = 4/5$. From Snell's Law $k_1 \sin\theta_1 = k_2 \sin\theta_2$, $\sin\theta_2 = \frac{k_1}{k_2}\sin\theta_1 = \frac{n_1}{n_2}\sin\theta_1 = \frac{3}{4}\frac{4}{5} = \frac{3}{5}$, $\theta_2 = 36.87°$, $\cos\theta_2 = \frac{4}{5}$, $\eta_1 = \frac{120\pi}{3}$; $\eta_2 = \frac{120\pi}{4}$



<FigureP2.24_2>

$$\Gamma_S = \frac{\frac{150\pi}{4} - \frac{200\pi}{3}}{\frac{150\pi}{4} + \frac{200\pi}{3}} = \frac{-350}{1250} - \frac{7}{25}$$

$$\tilde{\overline{E}}_S^R = -\frac{7}{25}(10)\sin(\omega t - k_x x - k_z z)$$

<u>Transmitted Wave</u>

Angle of transmission $\theta_2 = 36.87°$, $\cos\theta_2 = 4/5$, $\sin\theta_2 = 3/5$, $k_2 = 4k_0$, $\overline{E}_p^T = (8\hat{x} - 6\hat{z})\cos\left(\omega t - 4k_0\frac{4}{5}z - 4k_0\frac{3}{5}x\right) = (8\hat{x} - 6\hat{z})\cos\left(\omega t - k_0\frac{16}{5}z - k_0\frac{12}{5}x\right)$

Note: $\frac{12}{5}k_0 = \frac{12}{5}\frac{5}{24} = 0.5 = k_{x1} = k_{x2}$. To find $\overline{E}_S^T$ Transmission coefficient

$$T_S = 1 + \Gamma_S = 1 - \frac{7}{25} = \frac{18}{25}$$

$$\overline{E}_S^T = \hat{y}(10)\left(\frac{18}{25}\right)\sin(\omega t - k_{x2}x - k_{z2}x) = \hat{y}\frac{36}{5}\sin\left(\omega t - k_0\frac{16}{5}z - k_0\frac{12}{5}x\right)$$

$$\overline{E}_p^T = (6\hat{x} - 8\hat{z})\cos\left(\omega t - k_0\frac{16}{5}z - k_0\frac{12}{5}x\right) = \hat{y}\frac{36}{5}\sin(\omega t - k_{x2}x - k_{z2}x)$$

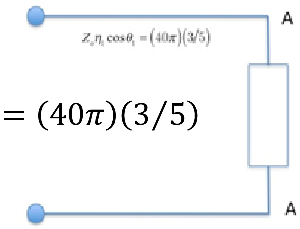
Note: the transmitted wave is elliptically polarized, $k_{x2} = \frac{12}{5}k_0$, $k_{z2} = \frac{16}{5}k_0$.

(8) $\epsilon_2 = 25\epsilon_0$, $\theta_1 = 53.13°$, $\cos\theta_1 = \frac{3}{5}$, $\sin\theta_1 = \frac{4}{5}$. From Snell's Law,

$$k_1\sin\theta_1 = k_2\sin\theta_2 = 3\sin\theta_1 = 5\sin\theta_2$$

$\sin\theta_2 = \frac{3}{5}\sin\theta_1 = \frac{3}{5}\frac{4}{5} = \frac{12}{25}$, $\theta_2 = 28.69°$, $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \frac{21.93}{25} = 0.877$, p-wave analysis:

$Z_0\eta_1\cos\theta_1 = (40\pi)(3/5)$



$Z_L = \dfrac{120\pi}{5}\cos\theta_2 = (24\pi)(0.877)$

<FigureP2.24_3>

$$\Gamma_p = \frac{(24\pi)(0.877)-(40\pi)(0.6)}{(24\pi)(0.877)+(40\pi)(0.6)} = -0.065$$

$$\Gamma_s = \frac{\frac{24\pi}{0.877}-\frac{40\pi}{0.6}}{\frac{24\pi}{0.877}+\frac{40\pi}{0.6}} = -0.418$$

$\overline{E}^R = -0.065(6\hat{x}-8\hat{z})\cos(\omega t - k_{x1}x + k_{z1}z) - \hat{y}0.414(10)\sin(\omega t - k_{x1}x + k_{z1}z)$, $V/meter$,

Note: that $k_x = k_{x1} = k_{x2} = 0.5$, in all cases: $k_{z2} = k_2\cos\theta_2$, $k_{z1} = k_1\cos\theta_1$, $k_2 = \omega\sqrt{\mu_2\epsilon_2}$, $k_1 = \omega\sqrt{\mu_1\epsilon_1}$

P2.25

$$\nabla\times\tilde{E} = -j\omega\mu_o\tilde{H}$$

$$\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 0 & 0\\ \hat{E}_x & \hat{E}_y & 0\end{vmatrix} = -j\omega\mu_o\tilde{H}$$

$$\hat{x}(-\frac{\partial E_y}{\partial E_z}) - \hat{y}(0-\frac{\partial\tilde{E}_x}{\partial E_z}) = -j\omega\mu_o\tilde{H}$$

$$= \hat{x}(-\frac{\partial}{\partial E_z}(-jE_o\cos kz)) + \hat{y}\frac{\partial}{\partial E_z}(E_o\sin kz)$$

$$= \hat{x}[+j(k)]E_o\cos kz + \hat{y}E_o k\cos kz$$

$$(j\hat{x}+\hat{y})E_o\cos kz = -j\omega\mu_o\tilde{H}$$

$$\tilde{H} = -\frac{k}{\omega\mu_o}[\hat{x}+\frac{\hat{y}}{j}]E_o\cos kz$$

$$\tilde{H} = -\frac{\omega\sqrt{\mu_o\varepsilon_o}}{\omega\mu_o}[\hat{x}-j\hat{y}]E_o\cos kz$$

$$(a)\ \tilde{H} = -\frac{1}{\eta_o}[\hat{x}-j\hat{y}]E_o\cos kz$$

$$(b)\ E = \mathrm{Re}[\hat{x} - j\hat{y}]E_o \sin kz e^{j\omega t}$$

$$= \hat{x}E_o \sin kz \cos \omega t + \hat{y}E_o \sin kz \cos(\omega t - \frac{\pi}{2})$$

$$E = E_o[\hat{x}\cos \omega t + \hat{y}\sin \omega t]\sin kz$$

$$H = -\frac{1}{\eta_o}\mathrm{Re}\{[\hat{x} - j\hat{y}]E_o \cos kz e^{j\omega t}\}$$

$$= -\frac{E_o}{\eta_o}\{\hat{x}\cos \omega t + \hat{y}\sin \omega t\}\cos kz$$

$$S = E \times H = -\frac{E_o{}^2}{\eta_o}[\{(\hat{x}\cos \omega t + \hat{y}\sin \omega t)\sin kz\} \times \{(\hat{x}\cos \omega t + \hat{y}\sin \omega t)\cos kz\}]$$

$$= -\frac{E_o{}^2}{\eta_o}[(\hat{x} \times \hat{y})\cos \omega t \sin kz \sin \omega t \cos kz + (\hat{y} \times \hat{x})\sin \omega t \sin kz \cos \omega t \cos kz]$$

$$= -\frac{E_o{}^2}{\eta_o}\hat{z}[\frac{\sin 2\omega t}{2}\frac{\cos 2kz}{2} - \frac{\sin 2\omega t}{2}\frac{\cos 2kz}{2}]$$

$$= 0$$

(c) Time-Average Power Density $= 0 = \frac{1}{T}\int_0^T S dt$

$$(d)\ \tilde{E} = \hat{x}E_o \sin kz$$

$$E = \hat{x}E_o \sin kz \cos \omega t$$

$$\nabla \times \tilde{E} = -j\omega\mu_o \tilde{H}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \partial/\partial z \\ \tilde{E}_x & 0 & 0 \end{vmatrix} = -j\omega\mu_o \tilde{H}$$

$$\tilde{H} = -\frac{1}{-j\omega\mu_o}[-\hat{y}(-\frac{\partial \tilde{E}_x}{\partial z})]$$

$$= \hat{y}\frac{j}{\omega\mu_o}\frac{\partial \tilde{E}_x}{\partial z} = \frac{jE_o}{\omega\mu_o}k \cos kz\ \hat{y}$$

$$= \hat{y}\frac{j}{\eta_o}E_o \cos kz$$

$$H = \hat{y}\frac{E_o}{\eta_o}\cos kz \cos(\omega t + \frac{\pi}{2})$$

$$= -\hat{y}\frac{E_o}{\eta_o}\cos kz \sin \omega t$$

$$S = E \times H = \hat{x}E_o \sin kz \cos \omega t \times (-\hat{y}) \frac{E_o}{\eta_o} \cos kz \sin \omega t$$

$$= -\hat{z} \frac{E_o{}^2}{\eta_o} \sin kz \cos kz \sin \omega t \cos \omega t$$

$$S = -\hat{z} \frac{E_o{}^2}{4\eta_o} \sin 2kz \sin 2\omega t$$

$$S_{av} = -\hat{z} \frac{E_o{}^2}{4\eta_o} \frac{\sin 2kz}{T} \int_0^T \sin 2\omega t dx$$

$$= 0$$

Since $\mathrm{E}_{tan} = 0,\ z = 0\ or\ d$

$\sin kd = 0 \quad kd = m\pi$

$$k = \frac{m\pi}{d} = \omega\sqrt{\mu\varepsilon}$$

$m = 1, 2, \ldots \infty$

$$\omega = \frac{m\pi}{\sqrt{\mu\varepsilon}d}$$

P2.26

$$\tilde{\mathbf{K}}(x, y) = \hat{x}K_o e^{-j\frac{\sqrt{3}}{2}k_o x}$$

The boundary condition

$\left|H_{\tan 1} - H_{\tan 2}\right| = \tilde{K}\Big|_{z=0}$, <u>for all x</u>

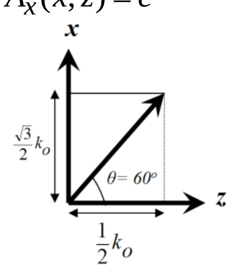
Requires that the x variation of all field quantities must have the exponential factor $e^{-j\frac{\sqrt{3}}{2}k_o x}$.

From the equation for the vector potential $\tilde{\mathbf{A}}$

$\nabla^2 \tilde{\mathbf{A}} + k_o^2 \tilde{\mathbf{A}} = 0$, $z \neq 0$ (1)

And $\tilde{\tilde{A}} = \hat{x}\tilde{A}_x(x, z)$ (2)

$\tilde{A}_x(x, z) = e^{-j\frac{\sqrt{3}}{2}k_o x}\ \tilde{A}_x(z)$ (3)



<FigureP2.26_1>

We get the differential equation

$$\frac{\partial^2 \tilde{A}_x(x,z)}{\partial z^2} + k_o^2 \tilde{A}_x(z) + \left(-j\frac{\sqrt{3}}{2}\right)^2 k_o^2 \tilde{A}_x(z) = 0$$

Let $k_z^2 = k_o^2\left(1-\frac{3}{4}\right)$; $k_z = \frac{1}{2}k_o$ (4)

$$k_x = \frac{\sqrt{3}}{2}k_o \quad (5)$$

$k_o \cos(\theta) = \frac{1}{2}k_o$, $\underline{\theta = 60^o}$

Equation for $A_x(z)$ takes a simple form

$$\frac{\partial^2 \tilde{A}_x(z)}{\partial z^2} + k_z^2 \tilde{A}_x(z) = 0$$

Let us look at the positive going wave solution and negative wave solution.

$$\tilde{A}_x^+(z) = A_o e^{-jk_z z} \quad (6a)$$

$$\tilde{A}_x^-(z) = A_o e^{+jk_z z} \quad (6b)$$

Thus for $z > 0$

$$\tilde{\mathbf{A}}^+(z) = \hat{x}A_o e^{-j(k_x x + k_z z)} \quad (7)$$

From $\tilde{\mathbf{B}} = \overline{\nabla} \times \tilde{\mathbf{A}}^+$

$$\tilde{\mathbf{H}}^+ = -j\frac{k_z}{\mu_o}\tilde{A}_x^+ \hat{y} \quad (8)$$

From $\overline{\nabla} \times \tilde{\mathbf{H}}^+ = +j\omega\varepsilon\tilde{\mathbf{E}}^+$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -jk_x & 0 & -jk_z \\ 0 & \tilde{H}^+ & 0 \end{vmatrix} = j\omega\varepsilon\tilde{\mathbf{E}}^+$$

We find

$$\tilde{\mathbf{E}}^+ = \frac{1}{j\omega\varepsilon}\left[\hat{x}jk_z - \hat{z}jk_x\right]\tilde{H}^+ \quad (9)$$

$$\tilde{\mathbf{E}}^+ = \frac{1}{j\omega\varepsilon}\left[\hat{x}jk_z - \hat{z}jk_x\right]\tilde{H}_y^+$$

$$\tilde{\mathbf{E}}^+ = \frac{1}{\omega\varepsilon}\left[\hat{x}k_z - \hat{z}k_x\right]\tilde{H}_y^+ \quad (10)$$

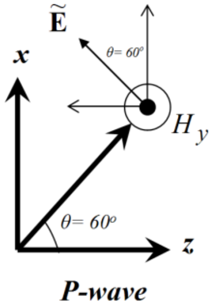
$$= \frac{1}{\omega\varepsilon}\left[\hat{x}\frac{k_o}{2} - \hat{z}\frac{\sqrt{3}}{2}k_o\right]\tilde{H}_y^+$$

$$= \frac{k_o}{\omega\varepsilon}\left[\hat{x}\frac{1}{2} - \hat{z}\frac{\sqrt{3}}{2}\right]\tilde{H}_y^+$$

$$= \frac{\omega\sqrt{\mu_o\varepsilon_o}}{\omega\varepsilon}\left[\hat{x}\frac{1}{2} - \hat{z}\frac{\sqrt{3}}{2}\right]\tilde{H}_y^+$$

$$= \eta_o\left[\hat{x}\sin\left(30^o\right) - \hat{z}\cos\left(30^o\right)\right]\tilde{H}_y^+$$

$$\tilde{\mathbf{E}}^+ = \eta_o\left[\hat{x}\cos\left(60^o\right) - \hat{z}\sin\left(60^o\right)\right]\tilde{H}_y^+ \qquad (11)$$



<FigureP2.26_2>

This solution is a *p-wave* (positive going).

The second solution , valid for z <0:

From $\tilde{\mathbf{B}}^- = \overline{\nabla} \times \tilde{\mathbf{A}}^-$ replace jk_z by $-jk_z$ in the positive going wave solution.

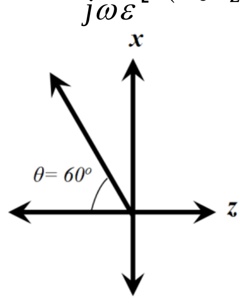
$$\tilde{\mathbf{H}}^- = j\frac{k_z}{\mu_o}\tilde{A}_x^-\hat{y} \qquad (12)$$

Where $\tilde{A}_x^-(z) = A_o^- e^{jk_z z}$ (13)

$$\tilde{\mathbf{H}}^- = j\frac{k_z}{\mu_o}\tilde{A}_x^-\hat{y} \qquad (14)$$

From $\overline{\nabla} \times \tilde{\mathbf{H}}^- = +j\omega\varepsilon\tilde{\mathbf{E}}^-$

$$\tilde{\mathbf{E}}^- = \frac{1}{j\omega\varepsilon}\left[\hat{x}\left(-jk_z\right) - \hat{z}jk_x\right]\tilde{H}_y^- \qquad (15)$$



<FigureP2.26_3>

Following through, like for positive going wave solution

$$\tilde{\mathbf{E}}^- = \eta_o\left[-\hat{x}\cos\left(60^o\right) - \hat{z}\sin\left(60^o\right)\right]\tilde{H}_y^- \qquad (16)$$

Now let us relate A_o to k_o by using Ampere's law as we did in section 2.15

$$A_o = -\frac{1}{2}\frac{jk_o\eta_o}{\omega}$$

Now we can write all the fields:

z>0

$$\tilde{\mathbf{H}}^{+}(z) = -j\frac{k_z}{\mu_o}\tilde{A}_x^{+}\hat{y} = \frac{-jk_z}{\mu_o}A_o e^{-jk_z z}\hat{y}$$

$$\tilde{\mathbf{H}}^{+}(x,z) = -j\frac{k_z}{\mu_o}\left(-\frac{1}{2}\frac{jk_o\eta_o}{\omega}\right)e^{-jk_z z}e^{-j\frac{\sqrt{3}}{2}k_o x}\hat{y}$$

$$= -j\frac{k_z c}{2}\frac{K_o}{\omega}\,e^{-jk_o\left(\frac{1}{2}z+\frac{\sqrt{3}}{2}x\right)}\hat{y}$$

$$= -j\frac{k_o}{2\times 2}\frac{K_o}{k_o}\,e^{-jk_o\left(\frac{1}{2}z+\frac{\sqrt{3}}{2}x\right)}\hat{y}$$

$$\tilde{\mathbf{H}}^{+}(x,z) = -\frac{K_o}{4}\,e^{-jk_o\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}z\right)}\hat{y}\ \ A/m$$

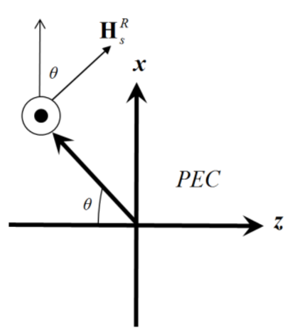
$$\tilde{\mathbf{E}}^{+} = \eta_o\left[\hat{x}\cos\left(60^o\right)-\hat{z}\sin\left(60^o\right)\right]\tilde{H}_y^{+}$$

$$\tilde{\mathbf{E}}^{+} = \eta_o\left[\hat{x}\cos\left(60^o\right)-\hat{z}\sin\left(60^o\right)\right]\left(-\frac{K_o}{4}\,e^{-jk_o\left(\sin\left(60^o\right)x+\cos\left(60^o\right)z\right)}\right)$$

$$\tilde{\mathbf{H}}^{-}(x,z) = \frac{K_o}{4}\,e^{-jk_o\left(\sin\left(60^o\right)x-\cos\left(60^o\right)z\right)}\hat{y}$$

$$\tilde{\mathbf{E}}^{-} = \frac{\eta_o K_o}{4}\left[-\hat{x}\cos\left(60^o\right)-\hat{z}\sin\left(60^o\right)\right]\left(e^{-jk_o\left(\sin\left(60^o\right)x-\cos\left(60^o\right)z\right)}\right)$$

P2.27



<FigureP2.27_1>

(a) $k_1 = \omega\sqrt{\mu_1\varepsilon_1} = \frac{\omega}{c}\sqrt{\varepsilon_{r1}} = \frac{2\pi\times 10^{10}}{3\times 10^{8}}\sqrt{16} = \frac{8\pi\times 100}{3} = \frac{800\pi}{3}\ rad/s$

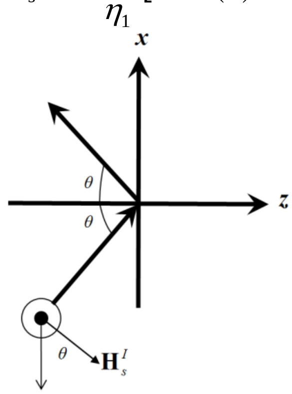
(b) $\tilde{\mathbf{H}}_s^{I} = \frac{E_o}{\eta_1}\left[-\hat{x}\cos(\theta)+\hat{z}\sin(\theta)\right]e^{-jk_1(x\sin(\theta)+z\cos(\theta))}$ *(A/m)*

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \frac{\eta_o}{4} = \frac{120\pi}{4} = 30\pi \ \Omega$$

Transmission line analogy $Z_L = 0$

(ii) $\Gamma_s = -1$

$$\tilde{\mathbf{H}}_s^R = -\frac{E_o}{\eta_1}\left[\hat{x}\cos(\theta) + \hat{z}\sin(\theta)\right]e^{-jk_1(x\sin(\theta) - z\cos(\theta))} \ (A/m)$$



<FigureP2.27_2>

(iii) power reflection coefficient $\left|\Gamma_s\right|^2 = 1$

(iv) $\tilde{\mathbf{H}} = \tilde{\mathbf{H}}_s^I + \tilde{\mathbf{H}}_s^R\Big|_{z=0}$

$$= \frac{E_o}{\eta_1}\left[-\hat{x}\cos(\theta) + \hat{z}\sin(\theta) - \hat{x}\cos(\theta) - \hat{z}\sin(\theta)\right]e^{-jk_1 x\sin(\theta)}$$

$$= -2\frac{E_o}{\eta_1}\hat{x}\cos(\theta)e^{-jk_1 x\sin(\theta)}$$

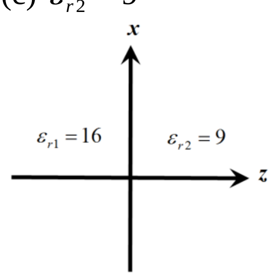
$$\mathbf{H}(x, y, 0, t) = -2\frac{E_o}{\eta_1}\hat{x}\cos(\theta)\cos(\omega t - k_1 x\sin(\theta)) \ (A/m)$$

(v) from B.C (0.64) $\hat{n}_{12} \times \mathbf{H}_2 = \mathbf{K}$, $\hat{n}_{12} = -\hat{z}$

$$\mathbf{K}(x, y, 0, t) = -2\frac{E_o}{\eta_1}(-\hat{z} \times \hat{x})\cos(\theta)\cos(\omega t - k_1 x\sin(\theta))$$

$$\mathbf{K}(x, y, 0, t) = \hat{y}\frac{2E_o}{\eta_1}\cos(\theta)\cos(\omega t - k_1 x\sin(\theta)) \ (A/m)$$

(c) $\varepsilon_{r2} = 9$

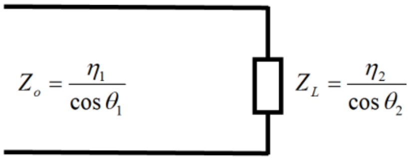


<FigureP2.27_3>

(i) from (0.145)

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{3}{4}\right) = 48.6^o$$

(ii) $\theta_i = \theta_c + 5^o = 53.6^o$



<FigureP2.27_4>

Use *s-wave* transmission line analogy

From Snell's Law

$$k_1 \sin\theta_1 = k_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{k_1}{k_2}\sin\theta_1 = \frac{n_1}{n_2}\sin 53.6^o = \frac{4}{3}\sin 53.6^o = 1.074$$

$$\cos\theta_2 = \sqrt{1^2 - 1.074^2} = \pm j0.3917$$

The sign choice for $\cos\theta_2$ is done on the following basis

$$\mathbf{E}_s^T = \mathbf{E}_s^I T_s e^{j(\omega t - k_2 \sin\theta_2 x - k_2 \sin\theta_2 z)}$$

$$\mathbf{E}_s^T = \mathbf{E}_s^I T_s e^{j(\omega t - k_2 \sin\theta_2 x)} e^{-\alpha_e z}$$

Where $\alpha_e = jk_2 \sin\theta_2 = jk_2 \underbrace{(\pm j0.3917)}_{\cos\theta_2}$

For α_e to be positive we have to choose $\cos\theta_2 = -j0.3917$

Thus $\Gamma_s = \dfrac{\dfrac{\eta_2}{\cos\theta_2} - \dfrac{\eta_1}{\cos\theta_1}}{\dfrac{\eta_2}{\cos\theta_2} + \dfrac{\eta_1}{\cos\theta_1}}$; $\eta_1 = \dfrac{120\pi}{4}$, $\eta_2 = \dfrac{120\pi}{3}$, $\cos\theta_1 = 0.5934$

$$\Gamma_s = \frac{\dfrac{1}{3(-j0.3917)} - \dfrac{1}{4(0.5934)}}{\dfrac{1}{3(-j0.3917)} + \dfrac{1}{4(0.5934)}} = \frac{-2.3736 + j1.1751}{2.3736 + j1.1751} = \frac{1\angle(180^o - 26.33^o)}{1\angle 26.33^o} = 1\angle 127.34^o$$

$$\left.\frac{\tilde{H}_x^R}{\tilde{H}_x^I}\right|_{z=0} = -\Gamma_s = -1\angle 127.34^o = \frac{\left.\tilde{H}_x^R\right|_{z=0}}{-\dfrac{E_o}{\eta_1}\cos\theta_1 e^{-jk_1 \sin\theta_1}}$$

$$\left.\tilde{H}_x^R\right|_{z=0} = \frac{E_o}{\eta_1}\cos\theta_1 e^{-jk_1 \sin\theta_1 x + 127.34^o}$$

$$\tilde{H}_x^R = \frac{E_o}{\eta_1}\cos\theta_1 e^{-jk_1 \sin\theta_1 x + jk_1 \cos\theta_1 z + 127.34^o}$$

$$H_x^R = \frac{E_o}{\eta_1}\cos\theta_1 \cos(\omega t - k_1 \sin\theta_1 x + k_1 \cos\theta_1 z + 127.34^o)$$

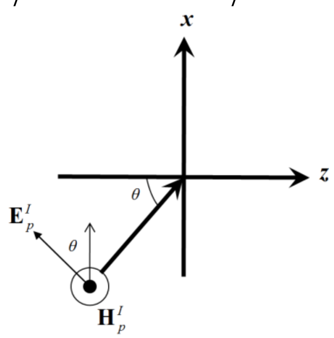
$$A = \frac{E_o \cos\theta_1}{\eta_1} = \frac{0.0198 E_o}{\pi} \ \ (A/m)$$

$$\omega = 2\pi \times 10^{10} \ \ rad/s$$

$$b = -k_1 \sin\theta_1 = \frac{-8000\pi}{3} \sin 53.6^o = -2146.4\pi \ \ rad/m$$

$$c = +k_1 \cos\theta_1 = 1582.4\pi \ \ rad/m$$

$$2\phi = 127.34^o \quad \Rightarrow \phi = 63.67^o$$



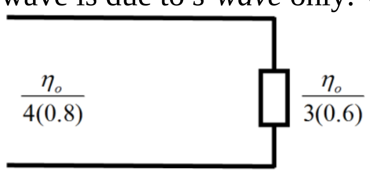
<FigureP2.27_5>

(d) $\tilde{\mathbf{E}}_s^I = E_o[\hat{x}\cos(\theta) - \hat{z}\sin(\theta)]e^{-jk_1(x\sin(\theta)+z\cos(\theta))}$

(i) $\tilde{\mathbf{E}}^I = E_o[\hat{x}2\cos(\theta) + \hat{y} - \hat{z}\sin(\theta)]e^{-jk_1(x\sin(\theta)+z\cos(\theta))}$

(ii) $\theta_B = \tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^o$

(iii) At Brewster Angle the power is totally transmitted. Hence the electric field of the reflected wave is due to *s-wave* only. We next calculate the reflection coefficient for *s-wave*.



<FigureP2.27_6>

$$\theta_1 = \theta_B = 36.87^o \ , \ \cos\theta_1 = 0.8 \ , \ \eta_1 = \frac{\eta_o}{4} \ , \ \eta_2 = \frac{\eta_o}{3}$$

From Snell's Law $k_1 \sin\theta_1 = k_2 \sin\theta_2$

$$4\sin 36.87 = 3\sin\theta_2 \ ; \ \sin\theta_2 = \frac{4}{3}(0.6) = 0.8 \Rightarrow \theta_2 = 53.13^o \text{ and } \cos\theta_2 = 0.6.$$

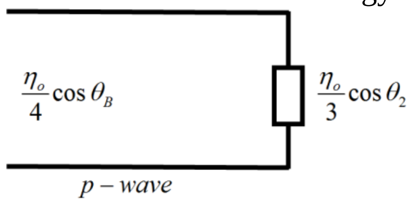
$$\Gamma_s = \frac{\frac{1}{1.8} - \frac{1}{3.2}}{\frac{1}{1.8} + \frac{1}{3.2}} = 0.28 \ ; \ T_s = 1 + \Gamma_s = 1.28$$

$$\tilde{\mathbf{E}}_s^R = \Gamma_s \tilde{\mathbf{E}}_s^I = \hat{y}0.28 E_o \cos(\omega t - 0.6k_1 x + 0.8k_1 z)$$

The transmitted electric field will have both *s* and *p* components. The *s* component is easily obtained fro previous work.

$$\tilde{\mathbf{E}}_s^T = T_s \tilde{\mathbf{E}}_s^I = \hat{y}1.28 E_o e^{-jk_1(0.8x+0.6z)}$$

The reflection and transmission coefficient for *p-wave* are $\Gamma_p = 0$, $T_p = 1$ those are confirmed from transmission line analogy for p-wave and (2.92) & (2.93)



<FigureP2.27_7>

$$\Gamma_{p1} = \frac{\frac{0.8}{4} - \frac{0.6}{3}}{\frac{0.8}{4} + \frac{0.6}{3}} = 0 \text{ , } \cos\theta_B = 0.8 \text{, } \cos\theta_2 = 0.6$$

$$T_{p1} = \frac{\frac{2\eta_o}{3}(0.6)}{\frac{\eta_o(0.8)}{4} + \frac{\eta_o(0.6)}{3}} = 1 \text{ as expected}$$

Note that

$$\Gamma_{p1} = \frac{E_{xp}^R}{E_{xp}^I} \text{ (2.92) ; } T_{p1} = \frac{E_{xp}^T}{E_{xp}^I} \text{ (2.93)}$$

If we wish to use $T_p = \frac{E_p^T}{E_p^I} = \frac{\cos\theta_1}{\cos\theta_2} T_{p1}$ (2.96)

$$T_p = \frac{\cos\theta_B}{\cos\theta_2} T_{p1} = \frac{4}{3}$$

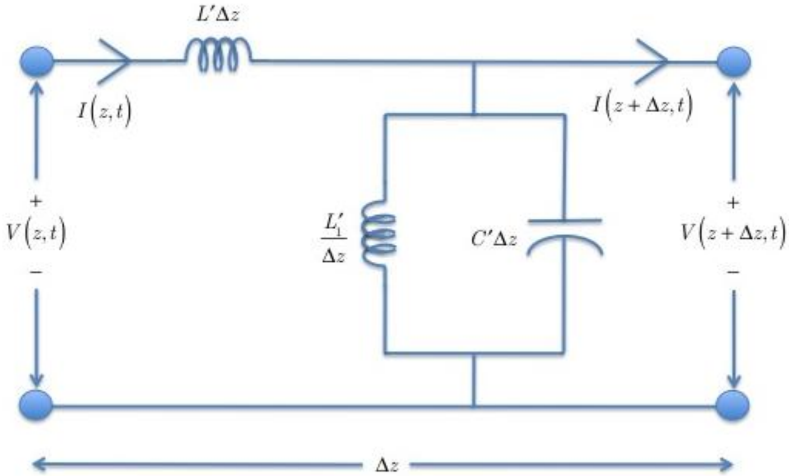
Thus $\tilde{\mathbf{E}}_p^T = \frac{4}{3} 2E_o(\hat{x}\cos\theta_2 - \hat{z}\sin\theta_2)e^{-jk_2(x\sin\theta_2 + z\cos\theta_2)}$

$k_2 = 3k_o$, $\sin\theta_2 = 0.8$, $\cos\theta_2 = 0.6$

$$\mathbf{E}^T = \mathbf{E}_p^T + \mathbf{E}_s^T = E_o(\hat{x}1.6 + \hat{y}1.28 - \hat{z}2.133)\cos(\omega t - 3k_o(0.8x + 0.6z)) \ V/m$$

P2.28

From KVL $V(z,t) - V(z+\Delta z, t) = L'\Delta Z \dfrac{\partial I(z,t)}{\partial t}$



<FigureP2.28_1>

Dividing by Δz and take limit $\Delta z \to 0$

$$-\frac{\partial V(z,t)}{\partial z} = L'\frac{\partial I(z,t)}{\partial t} \qquad (1)$$

From KCL

$$I(z,t) - I(z+\Delta z,t) = C'\Delta Z \frac{\partial V(z+\Delta z,t)}{\partial t} + \frac{\Delta z}{L_1'}\int V(z+\Delta z,t)\,dt$$

Dividing by Δz and take limit $\Delta z \to 0$

$$-\frac{\partial I(z,t)}{\partial z} = C'\frac{\partial V(z,t)}{\partial t} + \frac{1}{L_1'}\int V(z,t)\,dt \qquad (2)$$

Differentiate partially with respect to (*wrt*) *t*

$$-\frac{\partial^2 I(z,t)}{\partial z \partial t} = C'\frac{\partial^2 V(z,t)}{\partial t^2} + \frac{1}{L_1'}V(z,t) \qquad (3)$$

Differentiate (1) *wrt z:*

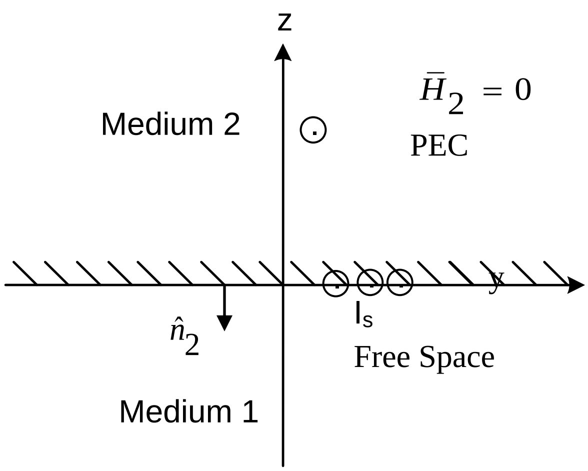
$$-\frac{\partial^2 V(z,t)}{\partial z^2} = L'\frac{\partial^2 I(z,t)}{\partial z \partial t} \qquad (4)$$

From (3) and (4)

$$\frac{1}{L'}\frac{\partial^2 V(z,t)}{\partial z^2} = C'\frac{\partial^2 V(z,t)}{\partial t^2} + \frac{1}{L_1'}V(z,t) \qquad (5)$$

$$\frac{\partial^2 V(z,t)}{\partial z^2} - L'C'\frac{\partial^2 V(z,t)}{\partial t^2} - \frac{L'}{L_1'}V(z,t) = 0 \qquad (6)$$

P2.29



<FigureP2.29_1>

(a)

$$\hat{n}_2 \times \left[\bar{H}_1 - \bar{H}_2\right] = \bar{J}_s$$

$$\bar{H}_2 = 0$$

$$-\bar{z} \times \left[\bar{H}_1\right] = \bar{J}_s$$

$$\bar{J}_s = -\bar{z} \times \bar{y} \cos(3 \times 10^8 t - 4x)$$

$$= \bar{x} \cos(3 \times 10^8 t - 4x)$$

$$\bar{J}_s(4,2,0)\Big|_{T=5ns} = -\bar{z} \times \bar{y} \cos(3 \times 10^8 \times 5 \times 10^{-9} - 4 \times 4)$$

$$= \bar{x} \cos(14.5)$$

$$= \bar{x} 0.355 A/m$$

(b) Find D first

$$\nabla \times \bar{H}_1 = \frac{\partial \bar{D}_1}{\partial t} \qquad \text{in free space}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & H_{y1} & 0 \end{vmatrix} = \hat{z} \frac{\partial H_{y1}}{\partial x}$$

$$= \hat{z} \frac{\partial}{\partial x} \left(\cos(3 \times 10^8 t - 4x)\right)$$

$$= \hat{z} \frac{\partial}{\partial x} (-4)(-1) \left(\cos(3 \times 10^8 t - 4x)\right)$$

$$= \frac{\partial \bar{D}_1}{\partial t}$$

$$\overline{D}_1 = \hat{z}\int 4\left(\cos(3\times10^8 t - 4x)\right)dt$$
$$= \hat{z}\,\frac{(-4)}{3\times10^8}\cos(3\times10^8 t - 4x)$$
$$= -\hat{z}\,\frac{4}{3\times10^8}\cos(3\times10^8 t - 4x)$$
$$\hat{n}\bullet\left[\overline{D}_1 - \overline{D}_2\right] = \rho_s$$
$$-\hat{z}\bullet\overline{D}_1 = \rho_s$$
$$\overline{D}_2 = 0$$
$$\rho_s(4,2,0)\big|_{t=5x10^{-9}} = \frac{4}{3x10^8}\cos(14.5)$$
$$= -0.473\times10^{-8}$$
$$= -4.73nc/m^2$$

P2.30

$$\overline{E}(0,0,0,0) = 2\hat{a}_x - 10\hat{a}_y + 3\hat{a}_z \qquad v/m$$

The origin (0,0,0) lies on a perfectly conducting surface. The electric field is entirely normal to the conducted surface at the origin.

$$\left|\overline{E}_n\right| = \left|\overline{E}\right| = \left(2^2 + 10^2 + 3^2\right)^{1/2}$$
$$\left|\rho_s\right| = \left|\overline{D}_n\right|$$
$$= \varepsilon_0(10)\left|\overline{E}_n\right|$$
$$= 941\times10^{-12}$$
$$= 941\,pc/m^2$$

P2.31

(a) From KVL $V(z,t) - V(z+\Delta z,t) = L'\Delta z\,\frac{\partial I(z,t)}{\partial t}$

Dividing by Δz and take limit $\Delta z \to 0$

$$-\frac{\partial V(z,t)}{\partial z} = L'\,\frac{\partial I(z,t)}{\partial t} \qquad (1)$$

From KCL

$$I(z,t) - I(z+\Delta z,t) = G'\Delta z V(z+\Delta z,t)$$

Again dividing by Δz and take limit $\Delta z \to 0$

$$-\frac{\partial I(z,t)}{\partial z} = G'V(z,t) \quad (2)$$

(b) from (2) $-\dfrac{\partial^2 I(z,t)}{\partial z^2} = G'\dfrac{\partial V(z,t)}{\partial z} = G'(-L')\dfrac{\partial I(z,t)}{\partial t}$

$$\frac{\partial^2 I(z,t)}{\partial z^2} - G'L'\frac{\partial I(z,t)}{\partial t} = 0 \quad (3)$$

(c) since $\dfrac{\partial}{\partial t} \to j\omega$, $\dfrac{\partial}{\partial z} \to -jk$

$$(-jk)^2 - G'L'j\omega = 0$$

$$k^2 = -G'L'j\omega$$

$$k = \mp\sqrt{-G'L'j\omega} = \sqrt{G'L'\omega}\angle -45^o$$

(d) If $k = \beta - j\alpha$

$$\beta = \sqrt{\frac{G'L'\omega}{2}} = \sqrt{\pi f G'L'}$$

$$\alpha = \sqrt{\pi f G'L'}$$

Hence

$$G' = \sigma \quad , \quad L' = \mu$$

P2.32

$$B = 15\times 10^8$$

Incident field amplitude = 60

Transmission coefficient $T = \dfrac{2\eta_2}{\eta_1 + \eta_2}$

$$\eta_2 = \eta_o\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} = \eta_o\sqrt{\frac{20}{5}} = 2\eta_o$$

$$\eta_1 = \eta_o$$

$$T = \frac{2\times 2\eta_o}{\eta_o + 2\eta_o} = \frac{4}{3}$$

$$A = 60\times T = 60\left(\frac{4}{3}\right) = 80$$

Note this number agrees with the B.C $\mathbf{E}_{\tan 1} = \mathbf{E}_{\tan 2}\big|_{z=0}$

$C = \beta_2$

$$\upsilon_{p2} = \frac{1}{\sqrt{\mu_2 \varepsilon_2}} = \frac{c}{\sqrt{5 \times 20}} = \frac{c}{10} = \frac{\omega}{\beta_2}$$

$$\beta_2 = \frac{10\omega}{c} = \frac{10 \times 15 \times 10^8}{3 \times 10^8} = 50$$

$A = 80 \quad , \quad B = 15 \times 10^8 \quad , \quad C = 50$

(b) $\mathbf{H}_2 = \hat{y} \dfrac{80}{\eta_2} \cos\left(15 \times 10^8 t - 50z\right)$

$\eta_2 = 2\eta_o = 240\pi$

$\mathbf{H}_2 = \hat{y} 0.106 \cos\left(15 \times 10^8 t - 50z\right) A/m$

P2.33

(a) Critical angel $\theta_c = \sin^{-1} \dfrac{n_2}{n_1} = 41.81^o$

$\theta_1 = 46.81^o$

$\alpha_e = k_2 \sqrt{2.25 \sin^2 46.81 - 1} = 0.443 k_2$

Depth of penetration $= \dfrac{1}{\alpha_e} = \dfrac{1}{0.443 k_2} = \dfrac{2.26}{k_2}$ *meters*

$$\beta_e = k_2 \frac{n_1}{n_2} \sin\theta_1$$

$= k_2 1.5 \sin 46.81$

$= 1.0936 k_2$

$$v_{pe} = \frac{\omega}{\beta_c}$$

$$= \frac{\omega}{k_2 1.0936}$$

$$= \frac{\omega}{\omega \sqrt{\mu_0 \varepsilon_0} 1.0936}$$

$= 0.914c$

$= 0.914 \times 3 \times 10^8 \quad m/s$

(b) $\theta_{BP} = \tan^{-1} \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \tan^{-1} \dfrac{1}{1.5} = 33.7^o$

Appendix 2C

P2C.1

This problem can be solved by using smith chart just like th 3esolution given in section 2c.4. The answer comes out to be

Reflection coefficient $\Gamma_s = 0.453\angle 194^0$

Another way to solve this problem is through ABCD parameters of section 7.3.1. note that the solution of ABCD parameters of a layer for obliques s-wave incidence is given in the solution for P7.10. Denoting kcosθ of this solution by q

Q=kcosθ

A=D=cos(qd),

B=jZ_1sin(qd)

$$C = \frac{i}{Z}\sin(qd)$$

Where Z=η/cosθ

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{n} = \frac{120\pi}{n}$$

Where n is the refractive index. Note that k_x =ksinθ is the same for all layers, i.e., nsinθ is the same (Snell's law), i.e.,

Sin30°=(1)sinθ_I =n_1 sinθ =$n_2$sinθ_2 = $n_3$sinθ_3

Thus

$$q_1 = k_0 n_1 \left[1 - \frac{1}{n^2}\sin^2\theta\right]^{1/2} = 296.19$$

Note

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{2\pi}{0.03}$$

$$Z_1 = 266.57 \quad \text{convenient to write} \quad Z = \frac{\omega\mu_0}{q_1}$$

$A_1 = D_1$ = -0.9839

B_1 = j 47.64

C_1 = j 0.00067

For layer 2

q_2 = 405.5788

Z_2 = 194.6733

$A_2 = D_2$ = 0.980165

B_2 = -j 38.5808

C_2 = -j 0.001018

Overall ABCD parameters for the layers

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -0.9839 & j47.64 \\ j0.00067 & -0.9839 \end{bmatrix} \begin{bmatrix} 0.980165 & -38.5808 \\ -j0.001018 & 0.980165 \end{bmatrix} = \begin{bmatrix} = -0.9159 & j84.6547 \\ 0.001658 & -0.9385 \end{bmatrix}$$

Input medium q_i = 181.3799, Z_i = 435.906

From (7.47) and (7.48)

$$R_s = \Gamma = \Gamma_s = 0.46\angle 194^0$$
$$T_s = T = 0.53\angle 199.6^0$$