Chapter 2

Analyzing Continuous-Time Systems in the Time Domain

2.1.

a.

$$y_1(t) = \text{Sys}\left\{x_1(t)\right\} = \left|x_1(t)\right| + x_1(t)$$
$$y_2(t) = \text{Sys}\left\{x_2(t)\right\} = \left|x_2(t)\right| + x_2(t)$$

Using $x(t) = \alpha_1\, x_1(t) + \alpha_2\, x_2(t)$ as input we obtain

$$\begin{aligned} y(t) =& \text{Sys}\left\{\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right\} \\ =& \left|\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right| + \alpha_1\, x_1(t) + \alpha_2\, x_2(t) \\ \neq& \alpha_1\, y_1(t) + \alpha_2\, y_2(t) \end{aligned}$$

The system is not linear.

$$\text{Sys}\left\{x_1(t-\tau)\right\} = \left|x_1(t-\tau)\right| + x_1(t-\tau) = y_1(t-\tau)$$

The system is time-invariant.

b.

$$y_1(t) = \text{Sys}\left\{x_1(t)\right\} = t\, x_1(t)$$
$$y_2(t) = \text{Sys}\left\{x_2(t)\right\} = t\, x_2(t)$$

Using $x(t) = \alpha_1\, x_1(t) + \alpha_2\, x_2(t)$ as input we obtain

$$\begin{aligned} y(t) =& \text{Sys}\left\{\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right\} \\ =& t\left[\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right] \\ =& \alpha_1\, t\, x_1(t) + \alpha_2\, t\, x_2(t) \\ =& \alpha_1\, y_1(t) + \alpha_2\, y_2(t) \end{aligned}$$

The system is linear.

$$\text{Sys}\left\{x_1(t-\tau)\right\} = t\, x_1(t-\tau) \neq y_1(t-\tau)$$

The system is not time-invariant.

c.

$$y_1(t) = \text{Sys}\left\{x_1(t)\right\} = e^{-t}\, x_1(t)$$

$$y_2(t) = \text{Sys}\left\{x_2(t)\right\} = e^{-t}\, x_2(t)$$

Using $x(t) = \alpha_1\, x_1(t) + \alpha_2\, x_2(t)$ as input we obtain

$$\begin{aligned} y(t) =& \text{Sys}\left\{\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right\} \\ =& e^{-t}\left[\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right] \\ =& \alpha_1\, e^{-t}\, x_1(t) + \alpha_2\, e^{-t}\, x_2(t) \\ =& \alpha_1\, y_1(t) + \alpha_2\, y_2(t) \end{aligned}$$

The system is linear.

$$\text{Sys}\left\{x_1(t-\tau)\right\} = e^{-t}\, x_1(t-\tau) \neq y_1(t-\tau)$$

The system is not time-invariant.

d.

$$y_1(t) = \text{Sys}\left\{x_1(t)\right\} = \int_{-\infty}^{t} x_1(\lambda)\, d\lambda$$

$$y_2(t) = \text{Sys}\left\{x_2(t)\right\} = \int_{-\infty}^{t} x_2(\lambda)\, d\lambda$$

Using $x(t) = \alpha_1\, x_1(t) + \alpha_2\, x_2(t)$ as input we obtain

$$\begin{aligned} y(t) =& \text{Sys}\left\{\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right\} \\ =& \int_{-\infty}^{t} \left[\alpha_1\, x_1(\lambda) + \alpha_2\, x_2(\lambda)\right] d\lambda \\ =& \alpha_1 \int_{-\infty}^{t} x_1(\lambda)\, d\lambda + \alpha_2 \int_{-\infty}^{t} x_2(\lambda)\, d\lambda \\ =& \alpha_1\, y_1(t) + \alpha_2\, y_2(t) \end{aligned}$$

The system is linear.

$$\text{Sys}\left\{x_1(t-\tau)\right\} = \int_{-\infty}^{t} x_1(\lambda-\tau)\, d\lambda$$

Let $\gamma = \lambda - \tau$. It follows that $d\gamma = d\lambda$. Substituting these into the integral and adjusting the limits yields

$$\text{Sys}\left\{x_1(t-\tau)\right\} = \int_{-\infty}^{t-\tau} x_1(\gamma)\, d\gamma = y_1(t-\tau)$$

The system is time-invariant.

e.

$$y_1(t) = \text{Sys}\left\{x_1(t)\right\} = \int_{t-1}^{t} x_1(\lambda)\, d\lambda$$

$$y_2(t) = \text{Sys}\left\{x_2(t)\right\} = \int_{t-1}^{t} x_2(\lambda)\, d\lambda$$

Using $x(t) = \alpha_1\, x_1(t) + \alpha_2\, x_2(t)$ as input we obtain

$$\begin{aligned} y(t) =& \mathrm{Sys}\left\{\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right\} \\ =& \int_{t-1}^{t} \left[\alpha_1\, x_1(\lambda) + \alpha_2\, x_2(\lambda)\right]\, d\lambda \\ =& \alpha_1 \int_{t-1}^{t} x_1(\lambda)\, d\lambda + \alpha_2 \int_{t-1}^{t} x_2(\lambda)\, d\lambda \\ =& \alpha_1\, y_1(t) + \alpha_2\, y_2(t) \end{aligned}$$

The system is linear.

$$\mathrm{Sys}\left\{x_1(t-\tau)\right\} = \int_{t-1}^{t} x_1(\lambda-\tau)\, d\lambda$$

Let $\gamma = \lambda - \tau$. It follows that $d\gamma = d\lambda$. Substituting these into the integral and adjusting the limits yields

$$\mathrm{Sys}\left\{x_1(t-\tau)\right\} = \int_{t-\tau-1}^{t-\tau} x_1(\gamma)\, d\gamma = y_1(t-\tau)$$

The system is time-invariant.

f.

$$y_1(t) = \mathrm{Sys}\left\{x_1(t)\right\} = (t+1) \int_{-\infty}^{t} x_1(\lambda)\, d\lambda$$

$$y_2(t) = \mathrm{Sys}\left\{x_2(t)\right\} = (t+1) \int_{-\infty}^{t} x_2(\lambda)\, d\lambda$$

Using $x(t) = \alpha_1\, x_1(t) + \alpha_2\, x_2(t)$ as input we obtain

$$\begin{aligned} y(t) =& \mathrm{Sys}\left\{\alpha_1\, x_1(t) + \alpha_2\, x_2(t)\right\} \\ =& (t+1) \int_{-\infty}^{t} \left[\alpha_1\, x_1(\lambda) + \alpha_2\, x_2(\lambda)\right]\, d\lambda \\ =& \alpha_1\, (t+1) \int_{-\infty}^{t} x_1(\lambda)\, d\lambda + \alpha_2\, (t+1) \int_{-\infty}^{t} x_2(\lambda)\, d\lambda \\ =& \alpha_1\, y_1(t) + \alpha_2\, y_2(t) \end{aligned}$$

The system is linear.

$$\mathrm{Sys}\left\{x_1(t-\tau)\right\} = (t+1) \int_{-\infty}^{t} x_1(\lambda-\tau)\, d\lambda$$

Let $\gamma = \lambda - \tau$. It follows that $d\gamma = d\lambda$. Substituting these into the integral and adjusting the limits yields

$$\mathrm{Sys}\left\{x_1(t-\tau)\right\} = (t+1) \int_{-\infty}^{t-\tau} x_1(\gamma)\, d\gamma \neq y_1(t-\tau)$$

The system is not time-invariant.

2.2.

a.

$$\begin{aligned} w(t) &= 3\,x(t) \\ y(t) &= w(t-2) = 3\,x(t-2) \end{aligned}$$

b.

$$\begin{aligned} \bar{w}(t) &= x(t-2) \\ \bar{y}(t) &= 3\,\bar{w}(t) = 3\,x(t-2) \end{aligned}$$

Input-output relationship of the system does not change when the order of the two subsystems is changed.

2.3.

a. Using the first configuration:

$$\begin{aligned} w(t) &= 3\,x(t) \\ y(t) &= t\,w(t) = 3t\,x(t) \end{aligned}$$

Using the second configuration:

$$\begin{aligned} \bar{w}(t) &= t\,x(t) \\ \bar{y}(t) &= 3\,\bar{w}(t) = 3t\,x(t) \end{aligned}$$

Input-output relationship of the system does not change when the order of the two subsystems is changed.

b. Using the first configuration:

$$\begin{aligned} w(t) &= 3\,x(t) \\ y(t) &= w(t)+5 = 3\,x(t)+5 \end{aligned}$$

Using the second configuration:

$$\begin{aligned} \bar{w}(t) &= x(t)+5 \\ \bar{y}(t) &= 3\,\bar{w}(t) = 3\left[x(t)+5\right] = 3\,x(t)+15 \end{aligned}$$

Input-output relationship of the system changes when the order of the two subsystems is changed.

2.4.

Writing the KVL around the loop on the left yields

$$\begin{aligned} x(t) &= R\left[i_L(t)+i_C(t)\right]+y(t) \\ &= R\,i_L(t)+R\,i_C(t)+y(t) \end{aligned}$$

Recognizing that

$$i_C(t) = C\frac{dv_c(t)}{dt} = C\frac{dy(t)}{dt}$$

we have

$$x(t) = R\,i_L(t) + RC\frac{dy(t)}{dt} + y(t)$$

Differentiating both sides of this result and recognizing that

$$y(t) = v_L(t) = L\frac{di_L(t)}{dt}$$

we get

$$\begin{aligned}\frac{dx(t)}{dt} =& R\frac{di_L(t)}{dt} + RC\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt}\\ =& \frac{R}{L}\,y(t) + RC\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt}\end{aligned}$$

Thus the differential equation for the circuit is

$$\frac{d^2y(t)}{dt^2} + \frac{1}{RC}\frac{dy(t)}{dt} + \frac{1}{LC}\,y(t) = \frac{1}{RC}\frac{dx(t)}{dt}$$

Initial conditions are found through

$$y(0) = v_C(0) = 2$$

and

$$R\,i_L(0) + RC\left.\frac{dy(t)}{dt}\right|_{t=0} + y(0) = x(0) \quad\Rightarrow\quad \begin{aligned}\left.\frac{dy(t)}{dt}\right|_{t=0} &= -\frac{1}{RC}\,y(0) - \frac{1}{C}\,i_L(0) - \frac{1}{RC}\,x(0)\\ &= -\frac{2}{RC} - \frac{1}{C} - \frac{1}{RC}\,x(0)\end{aligned}$$

2.5. Let the currents of the two capacitors be $i_1(t)$ and $i_2(t)$. Begin by writing the nodal equations for the circuit:

$$\frac{v_1(t) - x(t)}{R_1} + \frac{v_1(t) - v_2(t)}{R_2} + i_1(t) = 0$$

$$\frac{v_2(t) - v_1(t)}{R_2} + i_2(t) = 0$$

Using the relationships

$$v_2(t) = y(t)\,, \quad i_1(t) = C_1\frac{dv_1(t)}{dt}\,,\text{and} \quad i_2(t) = C_2\frac{dv_2(t)}{dt} = C_2\frac{dy(t)}{dt}$$

nodal equations become

$$\frac{v_1(t) - x(t)}{R_1} + \frac{v_1(t) - y(t)}{R_2} + C_1\frac{dv_1(t)}{dt} = 0 \qquad \text{(P.2.5.1)}$$

$$\frac{y(t) - v_1(t)}{R_2} + C_2\frac{dy(t)}{dt} = 0 \qquad \text{(P.2.5.2)}$$

Next, let us solve for $v_1(t)$ from Eqn. (P2.5.2)

$$v_1(t) = y(t) + R_2C_2\,\frac{dy(t)}{dt}$$

and differentiate both sides to obtain

$$\frac{dv_1(t)}{dt} = \frac{dy(t)}{dt} + R_2C_2\,\frac{d^2y(t)}{dt^2}$$

Substituting the last two results into Eqn. ((P.2.5.1) and simplifying the differential equation obtained yields

$$R_1R_2C_1C_2\,\frac{d^2y(t)}{dt^2} + \left[R_1(C_1+C_2) + R_2C_2\right]\frac{dy(t)}{dt} + y(t) = x(t)$$

The initial conditions are

$$y() = v_2(0) = 2\ \text{V} \qquad \text{and} \qquad \left.\frac{dy(t)}{dt}\right|_{t=0} = \frac{1}{R_2C_2}\left[v_1(0) - v_2(0)\right]$$

2.6.

Let $w(t)$ be the number of encounters between prey and predators at time t:

$$w(t) = K\,x(t)\;y(t)$$

The growth rate of prey is

$$\begin{aligned}\frac{dx(t)}{dt} =& A\,x(t) - C\,w(t)\\ =& A\,x(t) - \bar{C}\,x(t)\;y(t)\end{aligned}$$

The growth rate of predators is

$$\begin{aligned}\frac{dy(t)}{dt} =& -B\,y(t) + D\,w(t)\\ =& -B\,y(t) + \bar{D}\,x(t)\;y(t)\end{aligned}$$

The differential equations derived form a nonlinear system.

2.7.

Using Eqn. (2.57) with $t_0 = 0$ yields the solution

$$\begin{aligned}y(t) =& e^{-4t}\,y(0) + \int_0^t e^{-4(t-\tau)}\,r(\tau)\,dt\\ =& e^{-4t}\,y(0) + 4\int_0^t e^{-4(t-\tau)}\,u(\tau)\,dt\\ =& e^{-4t}\,y(0) + 4\,e^{-4t}\int_0^t e^{4\tau}\,dt\\ =& e^{-4t}\,y(0) + 1 - e^{-4t}\end{aligned}$$

a. $y(t) = 1 - e^{-4t}\ , \qquad t \geq 0$

b. $y(t) = 1 + 4\,e^{-4t}\ , \qquad t \geq 0$

c. $y(t) = 1\ , \qquad t \geq 0$

d. $y(t) = 1 - 2\,e^{-4t}\ , \qquad t \geq 0$

e. $y(t) = 1 - 4\,e^{-4t}\ , \qquad t \geq 0$

2.8.

a.

$$\begin{aligned} y(t) &= e^{-4t}\,(-1) + e^{-4t}\int_0^t e^{4\tau}\,u(\tau)\,d\tau \\ &= e^{-4t}\,(-1) + e^{-4t}\int_0^t e^{4\tau}\,d\tau \\ &= \frac{1}{4} - \frac{5}{4}\,e^{-4t}\ , \qquad t \geq 0 \end{aligned}$$

b.

$$y(t) = e^{-2t}\,(2) + e^{-2t}\int_0^t e^{2\tau}\,(2)\,[u(\tau) - u(\tau-5)]\,d\tau$$

If $0 < t < 5$ then

$$\begin{aligned} y(t) &= 2\,e^{-2t} + 2\,e^{-2t}\int_0^t e^{2\tau}\,d\tau \\ &= 1 + e^{-2t} \end{aligned}$$

If $t > 5$, then

$$\begin{aligned} y(t) &= 2\,e^{-2t} + 2\,e^{-2t}\int_0^5 e^{2\tau}\,d\tau \\ &= \left[e^{10} + 1\right]\,e^{-2t} \end{aligned}$$

Therefore, the complete solution is

$$y(t) = \begin{cases} 1 + e^{-2t}\,, & 0 < t < 5 \\ \left[e^{10} + 1\right]\,e^{-2t}\,, & t > 5 \end{cases}$$

c.

$$\begin{aligned} y(t) &= e^{-5t}\,(0.5) + e^{-5t}\int_0^t 3\,e^{5\tau}\,\delta(\tau)\,d\tau \\ &= 0.5\,e^{-5t} + 3\,e^{-5t} = 3.5\,e^{-5t}\ , \qquad t > 0 \end{aligned}$$

d.

$$\begin{aligned} y(t) =& e^{-5t}\,(-4) + e^{-5t}\int_0^t e^{5\tau}\,3\tau\,u(\tau)\,d\tau \\ =& -4\,e^{-5t} + 3\,e^{-5t}\int_0^t \tau\,e^{5\tau}\,d\tau \end{aligned}$$

Using Eqn. (B.16) from Appendix B.2 we get

$$\int_0^t \tau\,e^{5\tau}\,d\tau = \frac{1}{25}\left[5\,t\,e^{5t} - e^{5t} + 1\right]$$

and

$$y(t) = \frac{3}{5}\,t - \frac{3}{25} - \frac{97}{25}\,e^{-5t}\;, \qquad t \ge 0$$

e.

$$\begin{aligned} y(t) =& e^{-t}\,(-1) + e^{-t}\int_0^t e^{\tau}\,2\,e^{-2\tau}\,u(\tau)\,d\tau \\ =& -e^{-t} + 2\,e^{-t}\int_0^t e^{-\tau}\,d\tau \\ =& e^{-t} - 2\,e^{-2t}\;, \quad t \ge 0 \end{aligned}$$

2.9.

a. Characteristic equation is

$$s^2 + 3s + 2 = 0 \qquad \Rightarrow \qquad (s+1)\,(s+2) = 0$$

The solutions of the characteristic equation are $s_1 = -1$ and $s_2 = -2$. The homogeneous solution is in the form

$$y(t) = c_1\,e^{-t} + c_2\,e^{-2t}\;, \qquad t \ge 0$$

In order to satisfy the initial conditions we need

$$y(0) = c_1 + c_2 = 3$$

and

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = -c_1 - 2\,c_2 = 0$$

which can be solved to yield $c_1 = 6$ and $c_2 = -3$. The homogeneous solution is

$$y(t) = 6\,e^{-t} - 3\,e^{-2t}\;, \qquad t \ge 0$$

b. Characteristic equation is

$$s^2 + 4s + 3 = 0 \qquad \Rightarrow \qquad (s+1)\,(s+3) = 0 \qquad \Rightarrow \qquad s_{1,2} = -1\,,\,-3$$

The solutions of the characteristic equation are $s_1 = -1$ and $s_2 = -3$. The homogeneous solution is in the form

$$y(t) = c_1\, e^{-t} + c_2\, e^{-3t}\ , \qquad t \ge 0$$

In order to satisfy the initial conditions we need

$$y(0) = c_1 + c_2 = -2$$

and

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = -c_1 - 3\,c_2 = 1$$

which can be solved to yield $c_1 = -5/2$ and $c_2 = 1/2$. The homogeneous solution is

$$y(t) = -\frac{5}{2}\, e^{-t} + \frac{1}{2}\, e^{-3t}\ , \qquad t \ge 0$$

c. Characteristic equation is

$$s^2 - 1 = 0 \qquad \Rightarrow \qquad (s+1)\,(s+2) = 0$$

The solutions of the characteristic equation are $s_1 = 1$ and $s_2 = -1$. The homogeneous solution is in the form

$$y(t) = c_1\, e^{t} + c_2\, e^{-t}\ , \qquad t \ge 0$$

In order to satisfy the initial conditions we need

$$y(0) = c_1 + c_2 = 1$$

and

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = c_1 - c_2 = -2$$

which can be solved to yield $c_1 = -1/2$ and $c_2 = 3/2$. The homogeneous solution is

$$y(t) = -\frac{1}{2}\, e^{t} + \frac{3}{2}\, e^{-t}\ , \qquad t \ge 0$$

d. Characteristic equation is

$$s^3 + 6s^2 + 6s + 2 = 0 \qquad \Rightarrow \qquad (s+1)\,(s+2)\,(s+3) = 0$$

The solutions of the characteristic equation are $s_1 = -1$, $s_2 = -2$ and $s_3 = -3$. The homogeneous solution is in the form

$$y(t) = c_1\, e^{-t} + c_2\, e^{-2t} + c_3\, e^{-3t}\ , \qquad t \ge 0$$

In order to satisfy the initial conditions we need

$$y(0) = c_1 + c_2 + c_3 = 2$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = -c_1 - 2\,c_2 - 3\,c_3 = -1$$

and

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=0} = c_1 + 4\,c_2 + 9\,c_3 = 1$$

which can be solved to yield $c_1 = 4$, $c_2 = -3$ and $c_3 = 1$. The homogeneous solution is

$$y(t) = 4\,e^{-t} - 3\,e^{-2t} + e^{-3t}\;, \qquad t \ge 0$$

2.10.

a. The characteristic equation is

$$s^2 + 3 = 0 \qquad \Rightarrow \qquad \left(s + j\sqrt{3}\right)\left(s - j\sqrt{3}\right) = 0$$

Therefore the homogeneous solution is in the form

$$y(t) = d_1 \cos\left(\sqrt{3}\,t\right) + d_2 \sin\left(\sqrt{3}\,t\right)\;, \qquad t \ge 0$$

Coefficients d_1 and d_2 are determined through the initial conditions.

$$y(0) = d_1 = 2$$

$$\frac{dy(t)}{dt} = -\sqrt{3}\,d_1 \sin\left(\sqrt{3}\,t\right) + \sqrt{3}\,d_2 \cos\left(\sqrt{3}\,t\right)$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = \sqrt{3}\,d_2 = 0 \qquad \Rightarrow \qquad d_2 = 0$$

Therefore

$$y(t) = 2\cos\left(\sqrt{3}\,t\right)\;, \qquad t \ge 0$$

b. The characteristic equation is

$$s^2 + 2s + 2 = 0 \qquad \Rightarrow \qquad (s+1)^2 + 1 = 0 \qquad \Rightarrow \qquad (s + 1 + j)\,(s + 1 - j) = 0$$

Therefore the homogeneous solution is in the form

$$y(t) = d_1\,e^{-t}\cos(t) + d_2\,e^{-t}\sin(t)\;, \qquad t \ge 0$$

Coefficients d_1 and d_2 are determined through the initial conditions.

$$y(0) = d_1 = -2$$

$$\frac{dy(t)}{dt} = e^{-t}\,(d_2 - d_1)\cos(t) + e^{-t}\,(-d_1 - d_2)\sin(t)$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = d_2 - d_1 = -1 \qquad \Rightarrow \qquad d_2 - 1 + d_1 = -3$$

Therefore

$$y(t) = -2\,e^{-t}\cos(t) - 3\,e^{-t}\sin(t)\;, \qquad t \ge 0$$

c. The characteristic equation is

$$s^2+4s+13=0 \quad \Rightarrow \quad (s+2)^2+9=0 \quad \Rightarrow \quad \left(s+2+j3\right)\left(s+2-j3\right)=0$$

Therefore the homogeneous solution is in the form

$$y(t)=d_1\,e^{-2t}\cos(3t)+d_2\,e^{-2t}\sin(3t)\ , \qquad t\ge 0$$

Coefficients d_1 and d_2 are determined through the initial conditions.

$$y(0)=d_1=5$$

$$\frac{dy(t)}{dt}=e^{-2t}\,(-2\,d_1+3\,d_2)\cos(3t)+e^{-2t}\,(-3\,d_1-3\,d_2)\sin(t)$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0}=-2\,d_1+3\,d_2=0 \quad \Rightarrow \quad d_2=\frac{2}{3}\,d_1=\frac{10}{3}$$

Therefore

$$y(t)=5\,e^{-2t}\cos(3t)+\frac{10}{3}\,e^{-2t}\sin(3t)\ , \qquad t\ge 0$$

d. The characteristic equation is

$$s^3+3\,s^2+4s+2=0 \quad \Rightarrow \quad (s+1)\left(s+1+j\right)\left(s+1-j\right)=0$$

Therefore the homogeneous solution is in the form

$$y(t)=c_1\,e^{-t}+d_2\,e^{-t}\cos(t)+d_3\,e^{-t}\sin(t)\ , \qquad t\ge 0$$

The derivatives are

$$\frac{dy(t)}{dt}=-c_1\,e^{-t}+(d_3-d_2)\,e^{-t}\cos(t)+(-d_3-d_2)\,e^{-t}\sin(t)$$

and

$$\frac{d^2y(t)}{dt^2}=c_1\,e^{-t}-2d_3\,e^{-t}\cos(t)+2d_2\,e^{-t}\sin(t)$$

Imposing the initial conditions yields

$$y(0)=c_1+d_2=1$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0}=-c_1-d_2+d_3=0$$

and

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=0}=c_1-2d_3=-2$$

Coefficient values are $c_1=0$, $d_2=1$ and $d_3=1$. The homogeneous solution is

$$y(t)=e^{-t}\cos(t)+e^{-t}\sin(t)\ , \qquad t\ge 0$$

2.11.

a. The characteristic equation is

$$s^2+2s+1=0 \qquad \Rightarrow \qquad (s+1)^2=0$$

Therefore the homogeneous solution is in the form

$$y(t)=c_1\,e^{-t}+c_2\,t\,e^{-t}\;, \qquad t\ge 0$$

Coefficients c_1 and c_2 are determined through the initial conditions.

$$y(0)=c_1=1$$

$$\frac{dy(t)}{dt}=-c_1\,e^{-t}+c_2\,e^{-t}-c_2\,t\,e^{-t}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0}=-c_1+c_2=0 \qquad \Rightarrow \qquad c_2=1$$

Therefore

$$y(t)=e^{-t}+t\,e^{-t}\;, \qquad t\ge 0$$

b. The characteristic equation is

$$s^3+7s^2+16s+12=0 \qquad \Rightarrow \qquad (s+2)^2\,(s+3)=0$$

Therefore the homogeneous solution is in the form

$$y(t)=c_1\,e^{-2t}+c_2\,t\,e^{-2t}+c_3\,e^{-3t}\;, \qquad t\ge 0$$

Coefficients c_1, c_2 and c_3 are determined through the initial conditions.

$$y(0)=c_1+c_3=1 \tag{P.2.11.1}$$

$$\frac{dy(t)}{dt}=-2\,c_1\,e^{-2t}+c_2\,e^{-2t}-2\,c_2\,t\,e^{-2t}-3\,c_3\,e^{-3t}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0}=-2\,c_1+c_2-3\,c_3=-2 \tag{P.2.11.2}$$

$$\frac{d^2y(t)}{dt^2}=4\,c_1\,e^{-2t}-2\,c_2\,e^{-2t}-2\,c_2\,e^{-2t}+4\,c_2\,t\,e^{-2t}+9\,c_3\,e^{-3t}$$

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=0}=4\,c_1-4\,c_2+9\,c_3=1 \tag{P.2.11.3}$$

Solving Eqns. (P.2.11.1), (P.2.11.2) and (P.2.11.3) for the coefficients leads to

$$c_1=4\;, \qquad c_2=-3\;, \qquad c_3=-3\;,$$

Therefore

$$y(t)=4\,e^{-2t}-3\,t\,e^{-2t}-3\,e^{-3t}\;, \qquad t\ge 0$$

c. The characteristic equation is

$$s^3+6s^2+12s+8=0 \qquad \Rightarrow \qquad (s+2)^3=0$$

Therefore the homogeneous solution is in the form

$$y(t)=c_1\,e^{-2t}+c_2\,t\,e^{-2t}+c_3\,t^2\,e^{-2t}\;, \qquad t\geq 0$$

Coefficients c_1, c_2 and c_3 are determined through the initial conditions.

$$y(0)=c_1=-1 \tag{P.2.11.4}$$

$$\frac{dy(t)}{dt}=-2\,c_1\,e^{-2t}+c_2\,e^{-2t}-2\,c_2\,t\,e^{-2t}+2\,c_3\,t\,e^{-2t}-2\,c_3\,t^2\,e^{-2t}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0}=-2\,c_1+c_2=0 \qquad \Rightarrow \qquad c_2=2\,c_1=-2 \tag{P.2.11.5}$$

$$\frac{d^2y(t)}{dt^2}=4\,c_1\,e^{-2t}-2\,c_2\,e^{-2t}-2\,c_2\,e^{-2t}+4\,c_2\,t\,e^{-2t}+2\,c_3\,e^{-2t}-4\,c_3\,t\,e^{-2t}-4\,c_3\,t\,e^{-2t}+4\,c_3\,t^2\,e^{-2t}$$

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=0}=4\,c_1-4\,c_2+2\,c_3=1 \tag{P.2.11.6}$$

Solving Eqns. (P.2.11.1), (P.2.11.2) and (P.2.11.3) for the coefficients leads to

$$c_1=-1\;, \qquad c_2=-2\;, \qquad c_3=-1.5;\;,$$

Therefore

$$y(t)=-e^{-2t}+-2\,t\,e^{-2t}-1.5\,e^{-3t}\;, \qquad t\geq 0$$

2.12.

The particular solution is in the form

$$y_p=k_1\,t+k_2$$

Since it must satisfy the differential equation, we have

$$k_1+4\left[k_1\,t+k_2\right]=4t$$

which leads to coefficient values $k_1=1$ and $k_2=-1/4$. The characteristic equation is

$$s+4=0$$

Therefore the homogeneous solution is in the form

$$y_h(t)=c_1\,e^{-4t}\;, \qquad t\geq 0$$

and the total solution is in the form

$$y(t)=c_1\,e^{-4t}+t-\frac{1}{4}\;, \qquad t\geq 0$$

The total solution must satisfy the initial conditions.

$$y(0) = c_1 - \frac{1}{4} = 0 \qquad \Rightarrow \qquad c_1 = \frac{1}{4}$$

Therefore

$$\begin{aligned} y(t) =& \frac{1}{4}\,e^{-4t} + t - \frac{1}{4} \\ =& t - \frac{1}{4}\left[1 - e^{-4t}\right] \;, \qquad t \ge 0 \end{aligned}$$

2.13.

a. The particular solution is in the form

$$y_p = k_1$$

Since it must satisfy the differential equation, we have

$$4\,k_1 = 1 \qquad \Rightarrow \qquad k_1 = -\frac{1}{4}$$

The characteristic equation is

$$s + 4 = 0$$

Therefore the homogeneous solution is in the form

$$y_h(t) = c_1\,e^{-4t} \;, \qquad t \ge 0$$

and the total solution is in the form

$$y(t) = c_1\,e^{-4t} + \frac{1}{4} \;, \qquad t \ge 0$$

The total solution must satisfy the initial condition.

$$y(0) = c_1 + \frac{1}{4} = -1 \qquad \Rightarrow \qquad c_1 = -\frac{5}{4}$$

Therefore

$$y(t) = -\frac{5}{4}\,e^{-4t} \;, \qquad t \ge 0$$

b. The particular solution is in the form

$$y_p = k_1 \sin(2t) + k_2 \cos(2t) + k_3 \cos(t) + k_4 \sin(t) \tag{P.2.13.1}$$

The particular solution must satisfy the differential equation.

$$\frac{dy_p(t)}{dt} = 2\,k_1 \cos(2t) - 2\,k_2 \sin(2t) - k_3 \sin(t) + k_4 \cos(t) \tag{P.2.13.2}$$

Using Eqns. (P.2.13.1) and (P.2.13.2) in the differential equation we have

$$\begin{aligned} \frac{dy_p(t)}{dt} + 2\,y_p(t) =& \left[2\,k_1 + 2\,k_2\right] \cos(2t) + \left[2\,k_1 - 2\,k_2\right] \sin(2t) + \left[2\,k_3 + k_4\right] \cos(t) + \left[-k_3 + 2\,k_4\right] \sin(t) \\ =& 2 \sin(2t) + 4 \cos(t) \end{aligned}$$

which leads to the set of equations

$$
\begin{aligned}
2k_1 + 2k_2 &= 0 \\
2\,k_1 - 2\,k_2 &= 2 \\
2\,k_3 + k_4 &= 4 \\
-k_3 + 2\,k_4 &= 0
\end{aligned}
$$

and can be solved to yield

$$k_1 = \frac{1}{2}\ , \qquad k_2 = -\frac{1}{2}\ , \qquad k_3 = \frac{8}{5}\ , \qquad k_4 = \frac{4}{5}$$

The characteristic equation is

$$s + 2 = 0$$

Therefore the homogeneous solution is in the form

$$y_h(t) = c_1\, e^{-2t}\ , \qquad t \ge 0$$

and the total solution is in the form

$$y(t) = c_1\, e^{-2t} + \frac{1}{2}\sin(2t) - \frac{1}{2}\cos(2t) + \frac{8}{5}\cos(t) + \frac{4}{5}\sin(t)\ , \qquad t \ge 0$$

The total solution must satisfy the initial condition.

$$y(0) = c_1 - \frac{1}{2} + \frac{8}{5} = 2 \qquad \Rightarrow \qquad c_1 = \frac{9}{10}$$

Therefore

$$y(t) = \frac{9}{10}\, e^{-2t} + \frac{1}{2}\sin(2t) - \frac{1}{2}\cos(2t) + \frac{8}{5}\cos(t) + \frac{4}{5}\sin(t)\ , \qquad t \ge 0$$

c. The particular solution is in the form

$$y_p = k_1\, t + k_2$$

The particular solution must satisfy the differential equation.

$$k_1 + 5\left[k_1\, t + k_2\right] = 3\,t$$

We obtain the set of equations

$$
\begin{aligned}
5\,k_1 &= 3 \\
k_1 + 5\,k_2 &= 0
\end{aligned}
$$

The coefficients of the particular solution are $k_1 = 3/5$ and $k_2 = -3/25$. The characteristic equation is

$$s + 5 = 0$$

Therefore the homogeneous solution is in the form

$$y_h(t) = c_1\, e^{-5t}\ , \qquad t \ge 0$$

and the total solution is in the form

$$y(t) = c_1\, e^{-5t} + \frac{3}{5}\, t - \frac{3}{25}\ , \qquad t \ge 0$$

The total solution must satisfy the initial condition.

$$y(0) = c_1 - \frac{3}{25} = -4 \qquad \Rightarrow \qquad c_1 = -\frac{97}{25}$$

Therefore

$$y(t) = -\frac{97}{25}\, e^{-5t} + \frac{3}{5}\, t - \frac{3}{25}\ , \qquad t \ge 0$$

d. The particular solution is in the form

$$y_p = k_1\, e^{-2t} \tag{P.2.13.1}$$

The particular solution must satisfy the differential equation.

$$-2\, k_1\, e^{-2t} + k_1\, e^{-2t} - 2\, e^{-2t} \tag{P.2.13.1}$$

which leads to $k_1 = -2$. The characteristic equation is

$$s + 1 = 0$$

Therefore the homogeneous solution is in the form

$$y_h(t) = c_1\, e^{-t}\ , \qquad t \ge 0$$

and the total solution is in the form

$$y(t) = c_1\, e^{-t} - 2\, e^{-2t}\ , \qquad t \ge 0$$

The total solution must satisfy the initial condition.

$$y(0) = c_1 - 2 = -1 \qquad \Rightarrow \qquad c_1 = 1$$

Therefore

$$y(t) = e^{-t} - 2\, e^{-2t}\ , \qquad t \ge 0$$

2.14.

a. The particular solution is in the form

$$y_p = k_1$$

Since it must satisfy the differential equation, we have $k_1 = 1/2$. The characteristic equation is

$$s^2 + 3s + 2 = 0 \qquad \Rightarrow \qquad (s+1)\,(s+2) = 0$$

Therefore the homogeneous solution is in the form

$$y_h(t) = c_1\, e^{-t} + c_2\, e^{-2t}\ , \qquad t \ge 0$$

and the total solution is in the form

$$y(t) = c_1 e^{-t} + c_2 e^{-2t} + \frac{1}{2}\ , \qquad t \geq 0$$

The total solution must satisfy the initial conditions.

$$y(0) = c_1 + c_2 + \frac{1}{2} = 3 \qquad \Rightarrow \qquad c_1 + c_2 = \frac{5}{2}$$

$$\frac{dy(t)}{dt} = -c_1 e^{-t} - 2c_2 e^{-2t}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = -c_1 - 2c_2 = 0$$

The coefficients are found as $c_1 = 5$ and $c_2 = -5/2$. Therefore

$$y(t) = 5e^{-t} - \frac{5}{2}e^{-2t} + \frac{1}{2}\ , \qquad t \geq 0$$

b. The particular solution is in the form

$$y_p = k_1 t + k_2$$

The particular solution must satisfy the differential equation.

$$\frac{dy_p(t)}{dt} = k_1$$

$$4k_1 + 3(k_1 t + k_2) = t + 1 \qquad \Rightarrow \qquad k_1 = \frac{1}{3}\ , \quad k_2 = -\frac{1}{9}$$

The characteristic equation is

$$s^2 + 4s + 3 = 0 \qquad \Rightarrow \qquad (s+1)(s+3) = 0$$

Therefore the homogeneous solution is in the form

$$y_h(t) = c_1 e^{-t} + c_2 e^{-3t}\ , \qquad t \geq 0$$

and the total solution is in the form

$$y(t) = c_1 e^{-t} + c_2 e^{-3t} + \frac{1}{3}t - \frac{1}{9}\ , \qquad t \geq 0$$

The total solution must satisfy the initial conditions.

$$y(0) = c_1 + c_2 - \frac{1}{9} = 2 \qquad \Rightarrow \qquad c_1 + c_2 = \frac{19}{9}$$

$$\frac{dy(t)}{dt} = -c_1 e^{-t} - 3c_2 e^{-3t} + \frac{1}{3}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = -c_1 - 3c_2 + \frac{1}{3} = 0 \qquad \Rightarrow \qquad -c_1 - 3c_2 = -\frac{1}{3}$$

The coefficients are found as $c_1 = 3$ and $c_2 = -8/9$. Therefore

$$y(t) = 3\,e^{-t} - \frac{8}{9}\,e^{-3t} + \frac{1}{3}\,t - \frac{1}{9}\ , \qquad t \ge 0$$

c. The particular solution is in the form

$$y_p = k_1$$

Since it must satisfy the differential equation, we have $k_1 = 1/3$. The characteristic equation is

$$s^2 + 3 = 0 \qquad \Rightarrow \qquad \left(s + j\sqrt{3}\right)\left(s - j\sqrt{3}\right) = 0$$

Therefore the homogeneous solution is in the form

$$y_h(t) = d_1\,\cos\left(\sqrt{3}\,t\right) + d_2\,\sin\left(\sqrt{3}\,t\right)\ , \qquad t \ge 0$$

and the total solution is in the form

$$y(t) = d_1\,\cos\left(\sqrt{3}\,t\right) + d_2\,\sin\left(\sqrt{3}\,t\right) + \frac{1}{3}\ , \qquad t \ge 0$$

The total solution must satisfy the initial conditions.

$$y(0) = d_1 + \frac{1}{3} = 1 \qquad \Rightarrow \qquad d_1 = \frac{2}{3}$$

$$\frac{dy(t)}{dt} = -\sqrt{3}\,d_1\,\sin\left(\sqrt{3}\,t\right) + \sqrt{3}\,d_2\,\cos\left(\sqrt{3}\,t\right)$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = \sqrt{3}\,d_2 = 0 \qquad \Rightarrow \qquad d_2 = 0$$

Therefore

$$y(t) = \frac{2}{3}\,\cos\left(\sqrt{3}\,t\right) + \frac{1}{3}\ , \qquad t \ge 0$$

d. The particular solution is in the form

$$y_p = k_1\,e^{-2t}$$

Since it must satisfy the differential equation, we have

$$4\,k_1\,e^{-2t} - 4\,k_1\,e^{-2t} + k_1\,e^{-2t} = e^{-2t}$$

leading to $k_1 = 1$. The characteristic equation is

$$s^2 + 2s + 1 = 0 \qquad \Rightarrow \qquad (s+1)^2 = 0$$

Therefore the homogeneous solution is in the form

$$y_h(t) = c_1\,e^{-t} + c_2\,t\,e^{-t}\ , \qquad t \ge 0$$

and the total solution is in the form

$$y(t) = c_1\,e^{-t} + c_2\,t\,e^{-t} + e^{-2t}\ , \qquad t \ge 0$$

The total solution must satisfy the initial conditions.

$$y(0) = c_1 + 1 = 1 \qquad \Rightarrow \qquad c_1 = 0$$

$$\frac{dy(t)}{dt} = -c_1\,e^{-t} + c_2\,e^{-t} - c_2\,t\,e^{-t} - 2\,e^{-2t}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = -c_1 + c_2 - 2 = 0 \qquad \Rightarrow \qquad c_2 = 2$$

Therefore

$$y(t) = 2t\,e^{-t} + e^{-2t}\,, \qquad t \ge 0$$

2.15. Using the intermediate variable $w(t)$ we have

$$\frac{d^2w(t)}{dt^2} + 4\,\frac{dw(t)}{dt} + 3\,w(t) = x(t)$$

and the output signal $y(t)$ is computed as

$$y(t) = \frac{dw(t)}{dt} - 2\,w(t)$$

Using the output equation, the initial conditions can be expressed as

$$y(0) = \left.\frac{dw(t)}{dt}\right|_{t=0} - 2\,w(0) = -2 \tag{P.2.15.1}$$

and

$$\left.\frac{dy(0)}{dt}\right|_{t=0} = \left.\frac{d^2w(t)}{dt^2}\right|_{t=0} - 2\,\left.\frac{dw(t)}{dt}\right|_{t=0} = 1 \tag{P.2.15.2}$$

The second derivative in Eqn. (P.2.15.2) can be resolved as

$$\left.\frac{d^2w(t)}{dt^2}\right|_{t=0} = -4\,\left.\frac{dw(t)}{dt}\right|_{t=0} - 3\,w(0) + x(0)$$

which can be used in Eqn. (P.2.15.2) to yield

$$-6\,\left.\frac{dw(t)}{dt}\right|_{t=0} - 3\,w(0) = 1 \tag{P.2.15.3}$$

where we have assumed that $x(0) = 0$. To simplify the notation, let

$$a = \left.\frac{dw(t)}{dt}\right|_{t=0} \qquad \text{and} \qquad b = w(0)$$

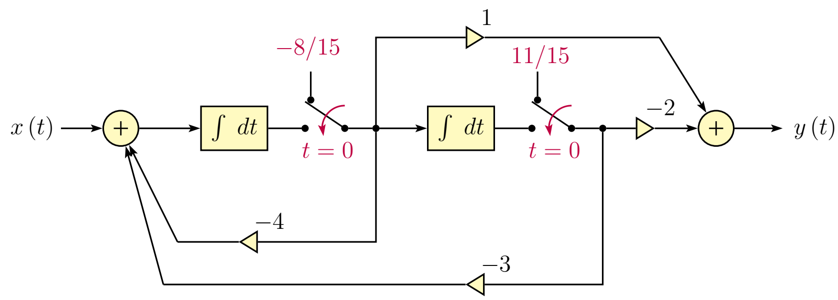
so that the Eqns. (P.2.15.1) and (P.2.15.3) become

$$a - 2b = -2$$

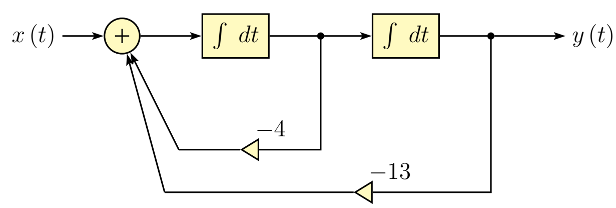
$$-6a - 3b = 1$$

with the solutions

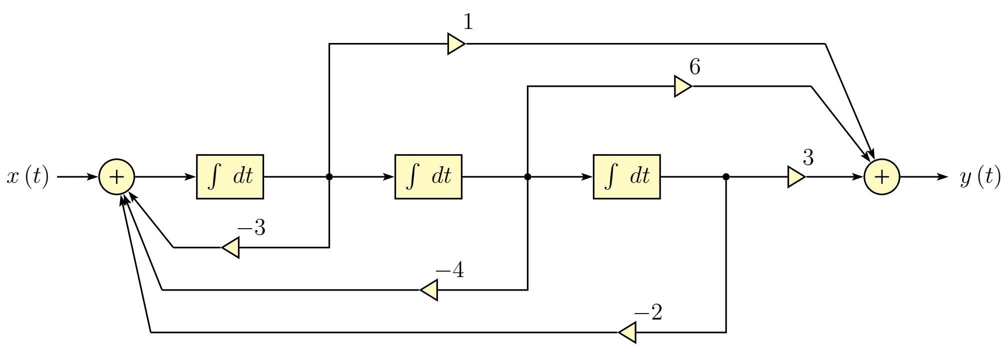
$$a = \left.\frac{dw(t)}{dt}\right|_{t=0} = -8/15 \qquad \text{and} \qquad b = w(0) = 11/15$$


1
−8/15
11/15
−2
x (t)
+
∫ dt
t = 0
∫ dt
t = 0
+
y (t)
−4
−3

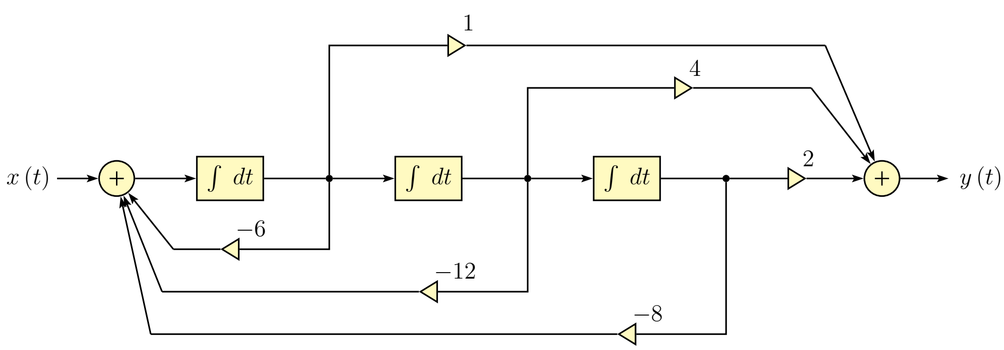
2.16. a.


x (t)
+
∫ dt
∫ dt
y (t)
−4
−13

b.


1
6
3
x (t)
+
∫ dt
∫ dt
∫ dt
+
y (t)
−3
−4
−2

c.


1
4
2
x (t)
+
∫ dt
∫ dt
∫ dt
+
y (t)
−6
−12
−8

2.17.

a.

$$\begin{aligned} w(t) &= h_1(t) * x(t) \\ y(t) &= h_2(t) * w(t) \\ &= h_2(t) * \left[h_1(t) * x(t)\right] \\ &= \left[h_2(t) * h_1(t)\right] * x(t) \end{aligned}$$

Therefore

$$h_{eq}(t) = h_2(t) * h_1(t) = h_1(t) * h_2(t)$$

b.

$$h_{eq}(t) = \int_{-\infty}^{\infty} \Pi(\tau - 0.5)\, \Pi(t - \tau - 0.5)\, d\tau$$

Since

$$\Pi(\tau - 0.5) = \begin{cases} 1\,, & 0 < \tau < 1 \\ 0\,, & \text{otherwise} \end{cases}$$

and

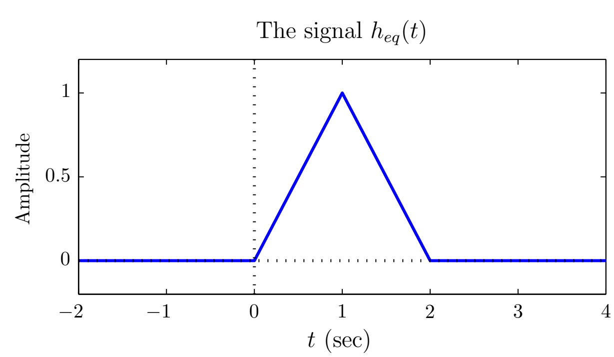
$$\Pi(t - \tau - 0.5) = \begin{cases} 1\,, & t-1 < \tau < t \\ 0\,, & \text{otherwise} \end{cases}$$

the convolution integral can be written as follows:

$$\begin{aligned} t<0: &\quad h_{eq}(t) = 0 \\ 0<t<1: &\quad h_{eq}(t) = \int_0^t (1)\,(1)\, d\tau = t \\ 1<t<2: &\quad h_{eq}(t) = \int_{t-1}^1 (1)\,(1)\, d\tau = 2-t \\ t>2: &\quad h_{eq}(t) = 0 \end{aligned}$$

The equivalent impulse response is

$$h_{eq}(t) = \begin{cases} t\,, & 0<t<1 \\ 2-t\,, & 1<t<2 \\ 0\,, & \text{otherwise} \end{cases}$$



c.

$$w(t) = \int_{-\infty}^{\infty} h_1(\tau)\, u(t-\tau)\, d\tau$$

Since

$$u(t-\tau) = \begin{cases} 1, & \tau < t \\ 0, & \tau > t \end{cases}$$

the convolution integral can be written as

$$w(t) = \int_{-\infty}^{t} h_1(\tau)\, d\tau$$

and can be evaluated as

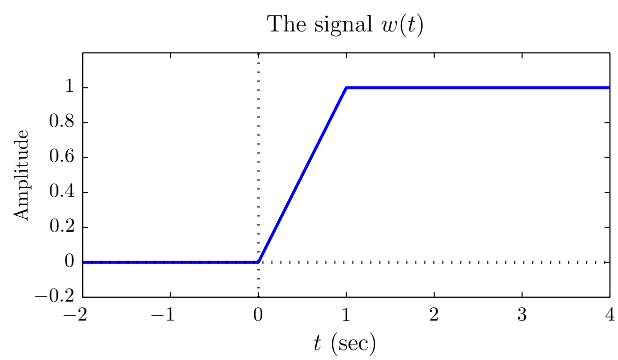
$$t<0: \qquad w(t) = 0$$

$$0<t<1: \qquad w(t) = \int_0^t (1)\, d\tau = t$$

$$t>1: \qquad w(t) = \int_0^1 (1)\, d\tau = 1$$

The signal $w(t)$ is

$$w(t) = \begin{cases} 0, & t<0 \\ t, & 0<t<1 \\ 1, & t>1 \end{cases}$$



Similarly

$$y(t) = \int_{-\infty}^{\infty} h_{eq}(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} h_{eq}(\tau)\, d\tau$$

which can be evaluated as

$$t<0: \qquad y(t) = 0$$

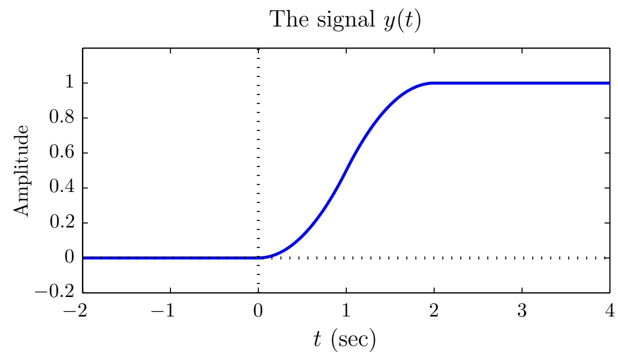
$$0<t<1: \qquad y(t) = \int_0^t \tau\, d\tau = \frac{t^2}{2}$$

$$1<t<2: \qquad y(t) = \int_0^1 \tau\, d\tau + \int_1^t (2-\tau)\, d\tau = -\frac{t^2}{2} + 2t - 1$$

$$t>2: \qquad y(t) = \int_0^1 \tau\, d\tau + \int_1^2 (2-\tau)\, d\tau = 1$$

The response of the system is

$$y(t) = \begin{cases} 0\,, & t<0 \\ \dfrac{t^2}{2}\,, & 0<t<1 \\ -\dfrac{t^2}{2}+2t-1\,, & 1<t<2 \\ 1\,, & t>2 \end{cases}$$



2.18.

a.

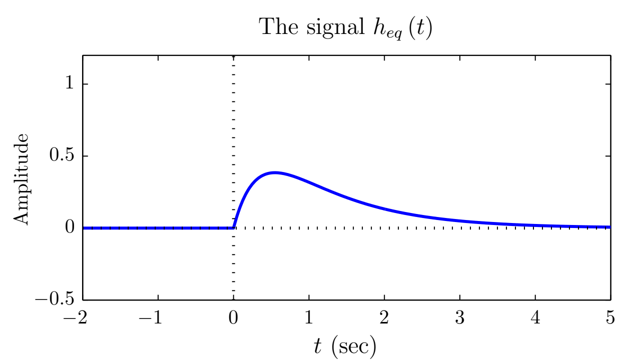
$$\begin{aligned} y_1(t) &= h_1(t) * x(t) \\ y_2(t) &= h_2(t) * x(t) \\ y(t) &= y_1(t) + y_2(t) \\ &= h_1(t) * x(t) + h_2(t) * x(t) \\ &= \left[h_1(t) + h_2(t)\right] * x(t) \end{aligned}$$

Therefore

$$h_{eq}(t) = h_1(t) + h_2(t)$$

b.

$$h_{eq}(t) = \left(e^{-t} - e^{-3t}\right) u(t)$$



c.

$$y_1(t) = \int_{-\infty}^{\infty} h_1(\tau)\, u(t-\tau)\, d\tau$$

Since

$$u(t-\tau) = \begin{cases} 1\,, & \tau<t \\ 0\,, & \tau>t \end{cases}$$

the convolution integral can be written as

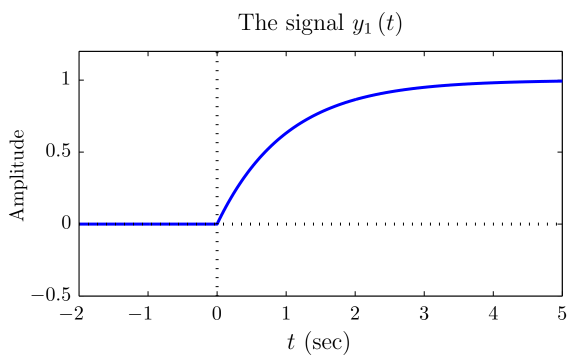
$$y_1(t) = \int_{-\infty}^{t} h_1(\tau)\, d\tau = \int_{-\infty}^{t} e^{-\tau}\, d\tau = 1 - e^{-t}\ , \qquad t \ge 0$$

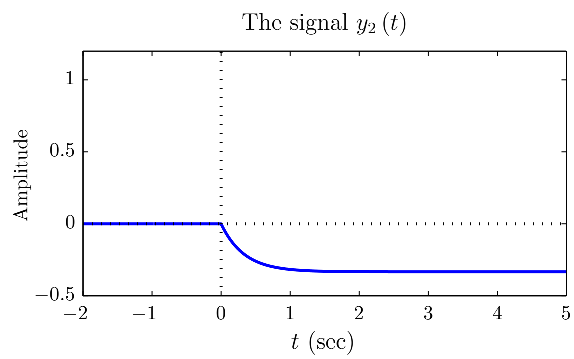
Similarly for $y_2(t)$ we obtain

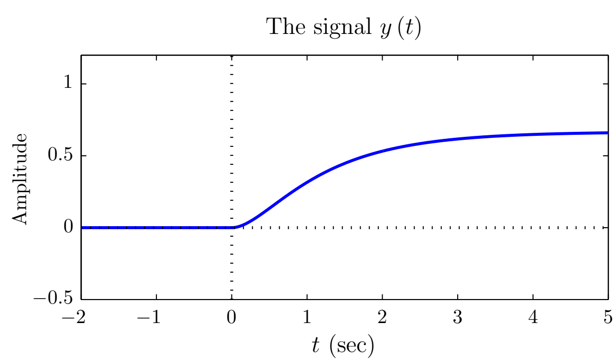
$$y_2(t) = \int_{-\infty}^{t} h_2(\tau)\, d\tau = \int_{-\infty}^{t} -e^{-3\tau}\, d\tau = -\frac{1}{3}\left[1 - e^{-3t}\right]\ , \qquad t \ge 0$$

and the output signal is

$$y(t) = y_1(t) + y_2(t) = \left[\frac{2}{3} - e^{-t} + \frac{1}{3}\, e^{-3t}\right] u(t)$$







2.19.

a.

$$y_1(t) = h_1(t) * x(t)$$

$$w(t) = h_2(t) * x(t)$$

$$y_3(t) = h_3(t) * w(t) = h_2(t) * h_3(t) * x(t)$$

The output signal is

$$y(t) = y_1(t) + y_2(t)$$
$$= \left[h_1(t) + h_2(t) * h_3(t)\right] * x(t)$$

and the equivalent impulse response is

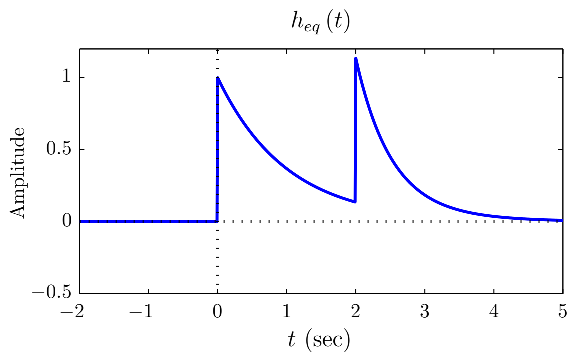
$$h_{eq}(t) = h_1(t) + h_2(t) * h_3(t)$$

b. Carrying out convolution operation we obtain

$$h_2(t) * h_3(t) = h_3(t-2) = e^{-2(t-2)}\, u(t-2)$$

and the equivalent impulse response is

$$h_{eq}(t) = e^{-t}\, u(t) + e^{-2(t-2)}\, u(t-2)$$



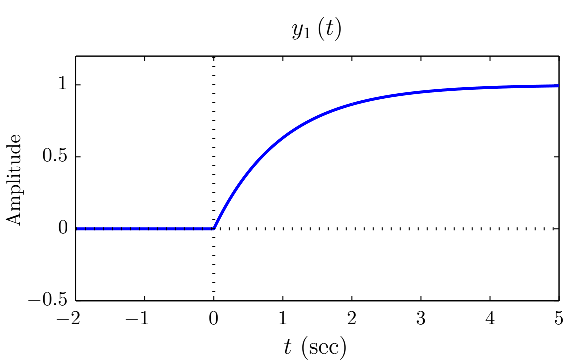
c.

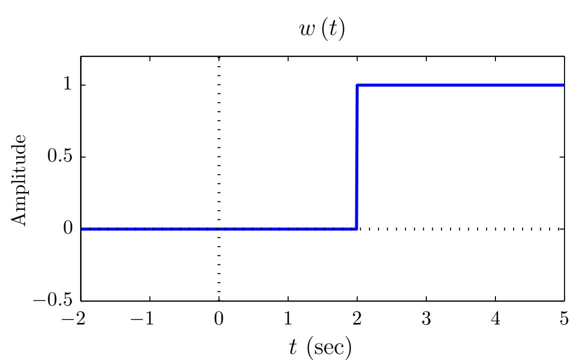
$$y_1(t) = u(t) * h_1(t) = \left[1 - e^{-t}\right] u(t)$$

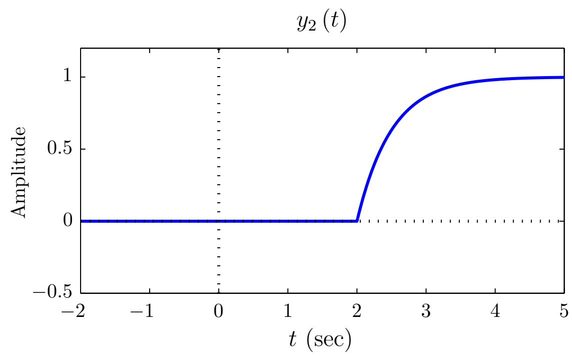
$$w(t) = u(t) * h_2(t) = u(t-2)$$

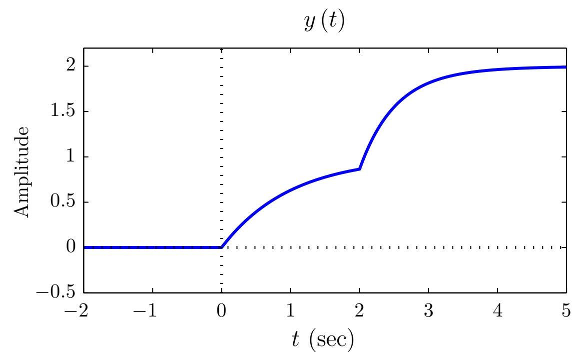
$$y_2(t) = w(t) * h_3(t) = \left[1 - e^{-2(t-2)}\right] u(t-2)$$

$$y(t) = \left[1 - e^{-t}\right] u(t) + \left[1 - e^{-2(t-2)}\right] u(t-2)$$









2.20.

a.

$$y_1(t) = h_1(t) * x(t)$$
$$w(t) = h_2(t) * x(t)$$
$$y_3(t) = h_3(t) * w(t) = h_2(t) * h_3(t) * x(t)$$
$$y_4(t) = h_4(t) * w(t) = h_2(t) * h_4(t) * x(t)$$

The output signal is

$$\begin{aligned} y(t) &= y_1(t) + y_3(t) + y_4(t) \\ &= \left[h_1(t) + h_2(t) * h_3(t) + h_2(t) * h_4(t)\right] * x(t) \end{aligned}$$

and the equivalent impulse response is

$$h_{eq}(t) = h_1(t) + h_2(t) * h_3(t) + h_2(t) * h_4(t)$$

b. Carrying out convolution operations we obtain

$$h_2(t) * h_3(t) = \begin{cases} t\,, & 0 \le t < 1 \\ 2-t\,, & 1 \le t < 2 \\ 0\,, & \text{otherwise} \end{cases} \qquad \Rightarrow \qquad h_2(t) * h_3(t) = \Lambda(t-1)$$

and

$$h_2(t) * h_4(t) = u(t-1) - u(t-2) \qquad \Rightarrow \qquad h_2(t) * h_4(t) = \Pi(t-1.5)$$

The equivalent impulse response is

$$h_{eq}(t) = e^{-t}\,u(t) + \Lambda(t-1) + \Pi(t-1.5)$$

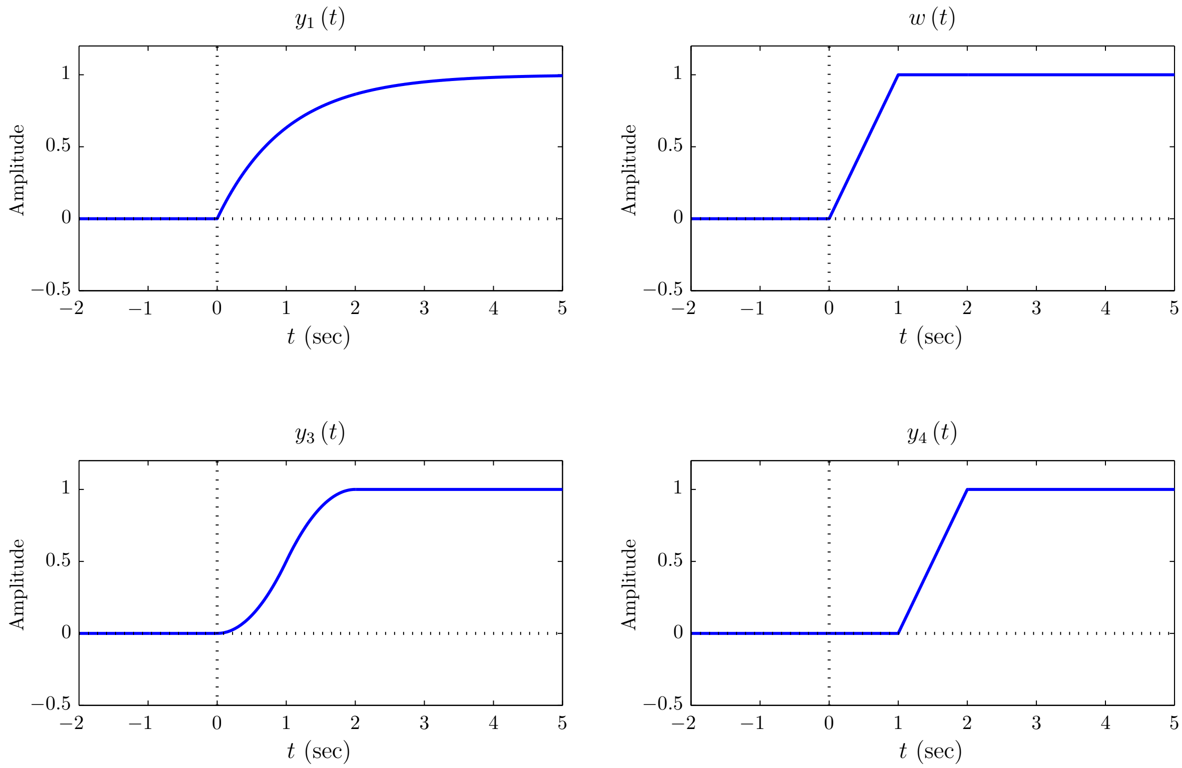
c.

$$y_1(t) = u(t) * h_1(t) = \left(1 - e^{-t}\right)\,u(t)$$

$$w(t) = u(t) * h_2(t) = \begin{cases} t\,, & 0 \le t < 1 \\ 1\,, & t \ge 1 \\ 0\,, & \text{otherwise} \end{cases}$$

$$y_3(t) = w(t) * h_3(t) = \begin{cases} t^2/2\,, & 0 \le t < 1 \\ -t^2/2 + 2t - 1\,, & 1 \le t < 2 \\ 1\,, & t \ge 2 \\ 0\,, & \text{otherwise} \end{cases}$$

$$y_4(t) = h_4(t) * w(t) = w(t-1)$$



2.21.

$$y(t) = \text{Sys}\left\{x(t)\right\} = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Let $w(t) = \dfrac{dx(t)}{dt}$.

$$\text{Sys}\left\{\frac{dx(t)}{dt}\right\} = \text{Sys}\left\{w(t)\right\} = \int_{-\infty}^{\infty} h(\tau)\, w(t-\tau)\, d\tau$$

$$\begin{aligned}
\frac{dy(t)}{dt} &= \frac{d}{dt}\left[\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau\right] \\
&= \int_{-\infty}^{\infty} \frac{d}{dt}\left[h(\tau)\, x(t-\tau)\right] d\tau \\
&= \int_{-\infty}^{\infty} h(\tau)\, \frac{d}{dt}\left[x(t-\tau)\right] d\tau \\
&= \int_{-\infty}^{\infty} h(\tau)\, w(t-\tau)\, d\tau = \text{Sys}\left\{\frac{dx(t)}{dt}\right\}
\end{aligned}$$

2.22.

a. $x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau = x(t)$

b. $x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-t_0-\tau)\, d\tau = x(t-t_0)$

c.

$$x(t) * u(t-2) = \int_{-\infty}^{\infty} x(\tau)\, u(t-2-\tau)\, d\tau$$

Since

$$u(t-2-\tau) = \begin{cases} 1\,, & \tau < t-2 \\ 0\,, & \tau > t-2 \end{cases}$$

the convolution integral can be written as

$$x(t) * u(t-2) = \int_{-\infty}^{t-2} x(\tau)\, d\tau$$

d.

$$x(t) * u(t-t_0) = \int_{-\infty}^{\infty} x(\tau)\, u(t-t_0-\tau)\, d\tau$$

Since

$$u(t-t_0-\tau) = \begin{cases} 1\,, & \tau < t-t_0 \\ 0\,, & \tau > t-t_0 \end{cases}$$

the convolution integral can be written as

$$x(t) * u(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$$

e.

$$x(t) * \Pi\left(\frac{t-t_0}{T}\right) = \int_{-\infty}^{\infty} x(\tau)\, \Pi\left(\frac{t-t_0-\tau}{T}\right) d\tau$$

Since

$$\Pi(t-t_0-\tau) = \begin{cases} 1\,, & t-t_0-T/2 < \tau < t-t_0+T/2 \\ 0\,, & \text{otherwise} \end{cases}$$

the convolution integral can be written as

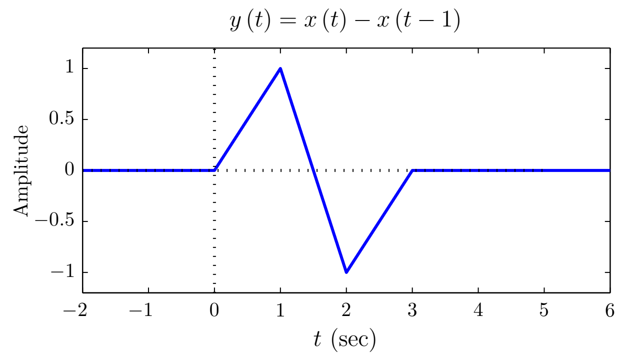
$$x(t) * \Pi\left(\frac{t-t_0}{T}\right) = \int_{t-t_0-T/2}^{t-t_0+T/2} x(\tau)\, d\tau$$

2.23.

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \\
&= \int_{-\infty}^{\infty} \left[\delta(\tau) - \delta(\tau-1)\right] x(t-\tau)\, d\tau \\
&= \int_{-\infty}^{\infty} \delta(\tau)\, x(t-\tau)\, d\tau - \int_{-\infty}^{\infty} \delta(\tau-1)\, x(t-\tau)\, d\tau
\end{aligned}
$$

Using the sifting property of the unit-impulse function, we have

$$
y(t) = x(t) - x(t-1)
$$

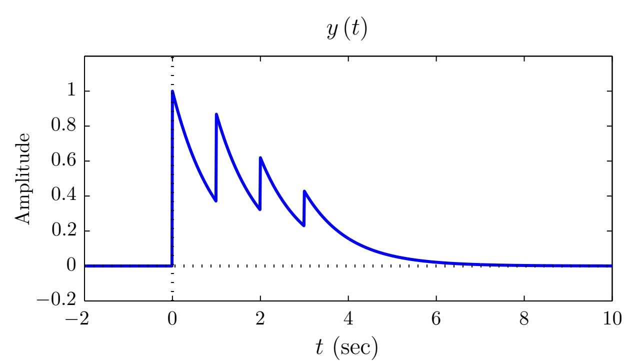


2.24.

$$
\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \\
&= \int_{-\infty}^{\infty} \left[\delta(\tau) + 0.5\,\delta(\tau-1) + 0.3\,\delta(\tau-2) + 0.2\,\delta(\tau-3)\right] x(t-\tau)\, d\tau \\
&= \int_{-\infty}^{\infty} \delta(\tau)\, x(t-\tau)\, d\tau + 0.5 \int_{-\infty}^{\infty} \delta(\tau-1)\, x(t-\tau)\, d\tau \\
&\quad + 0.3 \int_{-\infty}^{\infty} \delta(\tau-2)\, x(t-\tau)\, d\tau + 0.2 \int_{-\infty}^{\infty} \delta(\tau-3)\, x(t-\tau)\, d\tau
\end{aligned}
$$

Using the sifting property of the unit-impulse function, we have

$$
y(t) = x(t) + 0.5\, x(t-1) + 0.3\, x(t-2) + 0.2\, x(t-3)
$$



2.25.

$$\tilde{x}(t) = \left[\sum_{n=-\infty}^{\infty} \delta(t - nT_s)\right] * x(t)$$

$$= \int_{-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} \delta(\tau - nT_s)\right] x(t-\tau)\, d\tau$$

Changing the order of integration and summation yields

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\tau - nT_s)\, x(t-\tau)\, d\tau$$

Using the sifting property of the unit-impulse function on each integral leads to the result

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} x(t - nT_s)\, d\tau$$

which is clearly a periodic extension of the signal $x(t)$.

2.26.

a.

$$y(t) = \int_{-\infty}^{\infty} u(\lambda)\, e^{-2(t-\lambda)}\, u(t-\lambda)\, d\lambda = \int_{0}^{\infty} e^{-2(t-\lambda)}\, u(t-\lambda)\, d\lambda$$

Case 1: $t < 0$

$$y(t) = 0$$

Case 2: $t \ge 0$

$$\int_0^t e^{-2(t-\lambda)}\, d\lambda = \frac{1}{2}\left(1 - e^{-2t}\right)$$

b.

$$y(t) = \int_{-\infty}^{\infty} u(\lambda)\left[e^{-(t-\lambda)} - e^{-2(t-\lambda)}\right] u(t-\lambda)\, d\lambda = \int_{0}^{\infty} \left[e^{-(t-\lambda)} - e^{-2(t-\lambda)}\right] u(t-\lambda)\, d\lambda$$

Case 1: $t < 0$

$$y(t) = 0$$

Case 2: $t \ge 0$

$$y(t) = \int_0^t \left[e^{-(t-\lambda)} - e^{-2(t-\lambda)}\right] d\lambda = 1 - e^{-t} - \frac{1}{2}\left(1 - e^{-2t}\right)$$

c.

$$y(t) = \int_{-\infty}^{\infty} u(\lambda - 2)\, e^{-2(t-\lambda)}\, u(t-\lambda)\, d\lambda = \int_{2}^{\infty} e^{-2(t-\lambda)}\, u(t-\lambda)\, d\lambda$$

Case 1: $t < 2$

$$y(t) = 0$$

Case 2: $t \ge 2$

$$\int_2^t e^{-2(t-\lambda)}\, d\lambda = \frac{1}{2}\left(1 - e^{-2(t-2)}\right)$$

d.

$$y(t) = \int_{-\infty}^{\infty} \left[u(\lambda) - u(\lambda - 2) \right] e^{-2(t-\lambda)} \, u(t-\lambda) \, d\lambda = \int_{0}^{2} e^{-2(t-\lambda)} \, u(t-\lambda) \, d\lambda$$

Case 1: $t < 0$

$$y(t) = 0$$

Case 2: $0 \le t < 2$

$$\int_{0}^{t} e^{-2(t-\lambda)} \, d\lambda = \frac{1}{2} \left(1 - e^{-2t} \right)$$

Case 3: $t \ge 2$

$$\int_{0}^{2} e^{-2(t-\lambda)} \, d\lambda = \frac{1}{2} \left(e^{4} - 1 \right) e^{-2t}$$

e.

$$y(t) = \int_{-\infty}^{\infty} e^{-\lambda} \, u(\lambda) \, e^{-2(t-\lambda)} \, u(t-\lambda) \, d\lambda = \int_{0}^{\infty} e^{-2t+\lambda} \, u(t-\lambda) \, d\lambda$$

Case 1: $t < 0$

$$y(t) = 0$$

Case 2: $t \ge 0$

$$y(t) = \int_{0}^{t} e^{-2t+\lambda} \, d\lambda = e^{-t} - e^{-2t}$$

2.27.

a.

$$y(t) = \int_{-\infty}^{\infty} \Pi\left(\frac{\lambda - 2}{4} \right) u(t-\lambda) \, d\lambda = \int_{0}^{4} u(t-\lambda) \, d\lambda$$

Case 1: $t < 0$

$$y(t) = 0$$

Case 2: $0 \le t < 4$

$$y(t) = \int_{0}^{t} (1) \, d\lambda = t$$

Case 3: $t > 4$

$$y(t) = \int_{0}^{4} (1) \, d\lambda = 4$$

b.

$$y(t) = \int_{-\infty}^{\infty} 3 \, \Pi\left(\frac{\lambda - 2}{4} \right) e^{-(t-\lambda)} \, u(t-\lambda) \, d\lambda = \int_{0}^{4} 3 \, e^{-(t-\lambda)} \, u(t-\lambda) \, d\lambda$$

Case 1: $t < 0$

$$y(t) = 0$$

Case 2: $0 \le t < 4$

$$y(t) = \int_{0}^{t} 3 \, e^{-(t-\lambda)} \, d\lambda = 3 \left(1 - e^{-t} \right)$$

Case 3: $t > 4$

$$y(t) = \int_0^4 3\,e^{-(t-\lambda)}\,d\lambda = 3\,e^{-t}\left(e^4 - 1\right)$$

c.

$$y(t) = \int_{-\infty}^{\infty} \Pi\left(\frac{\lambda-2}{4}\right)\Pi\left(\frac{t-\lambda-2}{4}\right)d\lambda = \int_0^4 \Pi\left(\frac{t-\lambda-2}{4}\right)d\lambda$$

Case 1: $t < 0$

$$y(t) = 0$$

Case 2: $0 \le t < 4$

$$y(t) = \int_0^t (1)\,d\lambda = t$$

Case 3: $4 \le t < 8$

$$y(t) = \int_{t-4}^4 (1)\,d\lambda = 8 - t$$

Case 4: $t > 8$

$$y(t) = 0$$

d.

$$y(t) = \int_{-\infty}^{\infty} \Pi\left(\frac{\lambda-2}{4}\right)\Pi\left(\frac{t-\lambda-3}{6}\right)d\lambda = \int_0^4 \Pi\left(\frac{t-\lambda-3}{6}\right)d\lambda$$

Case 1: $t < 0$

$$y(t) = 0$$

Case 2: $0 \le t < 4$

$$y(t) = \int_0^t (1)\,d\lambda = t$$

Case 3: $4 \le t < 6$

$$y(t) = \int_0^4 (1)\,d\lambda = 4$$

Case 4: $6 \le t < 10$

$$y(t) = \int_{t-6}^4 (1)\,d\lambda = 10 - t$$

Case 5: $t > 10$

$$y(t) = 0$$

2.28.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

Since $x(t) = 0$ outside the interval $t_1 < t < t_2$, the convolution integral can be written as

$$y(t) = x(t) * h(t) = \int_{t_1}^{t_2} x(\tau)\,h(t-\tau)\,d\tau$$

The nonzero range of the signal $h(t)$ is specified to be $t_3 < t < t_4$.

$$h(t) = 0 \quad \text{except for } \; t_3 < t < t_4$$

and

$$h(t-\tau) = 0 \quad \text{except for } \; t_3 < t-\tau < t_4$$

Equivalently

$$h(t-\tau) = 0 \quad \text{except for } \; t-t_4 < \tau < t-t_3$$

For the integrand to be nonzero, we need

$$t - t_3 > t1 \qquad \text{and} \qquad t - t_4 < t_2$$

which can also be expressed as

$$t_1 + t_3 < t < t_2 + t_4$$

Therefore

$$t_5 = t_1 + t_3 \qquad \text{and} \qquad t_6 = t_2 + t_4$$

2.29.

Using the convolution integral, the output signal is

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

First, let us assume that $|h(t)| < \infty$ for all t, and we can select the input signal to be $x(t) = h^*(-t)$ so that

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, h^*(\lambda - t)\, d\lambda$$

At $t = 0$ the output signal is

$$y(0) = \int_{-\infty}^{\infty} h(\lambda)\, h^*(\lambda)\, d\lambda = \int_{-\infty}^{\infty} \left|h(\lambda)\right|^2 d\lambda$$

If the integral

$$\int_{-\infty}^{\infty} \left|h(\lambda)\right| d\lambda$$

does not converge, then neither does the integral

$$y(0) = \int_{-\infty}^{\infty} \left|h(\lambda)\right|^2 d\lambda$$

If the assumption $|h(t)| < \infty$ is not valid, then there is at least one value of t for which $h(t)$ is infinitely large. In that case choosing the input signal to be $x(t) = \delta(t)$ leads to the output signal

$$y(t) = h(t)$$

which is also infinitely large for at least one value of t.

2.30.

a. Let the input signal to the system be $x_1(t)$.

$$y_1(t) = \text{Sys}\left\{x_1(t)\right\} = x_1(t) + \alpha_1\, x_1(t-\tau_1) + \alpha_2\, x_1(t-\tau_2)$$

Similarly, if the input signal is $x_2(t)$

$$y_2(t) = \text{Sys}\left\{x_2(t)\right\} = x_2(t) + \alpha_1\, x_2(t-\tau_1) + \alpha_2\, x_2(t-\tau_2)$$

The response of the system to the input signal $x(t) = \beta_1\, x_1(t) + \beta_2\, x_2(t)$ is

$$\begin{aligned}\text{Sys}\left\{\beta_1\, x_1(t) + \beta_2\, x_2(t)\right\} &= \beta_1\left[x_1(t) + \alpha_1\, x_1(t-\tau_1) + \alpha_2\, x_1(t-\tau_2)\right] + \beta_2\left[x_2(t) + \alpha_1\, x_2(t-\tau_1) + \alpha_2\, x_2(t-\tau_2)\right]\\ &= \beta_1\, y_1(t) + \beta_2\, y_2(t)\end{aligned}$$

The system is linear.

b. The response to $x_1(t-a)$ is

$$\text{Sys}\left\{x_1(t-a)\right\} = x_1(t-a) + \alpha_1\, x_1(t-\tau_1-a) + \alpha_2\, x_1(t-\tau_2-a) = y_1(t-a)$$

The system is time-invariant.

c. The system is causal provided that $\tau_1 > 0$ and $\tau_2 > 0$.

d. The system is stable provided that $\alpha_1,\ \alpha_2 < \infty$.

2.31.

a. Let $x(t) = \delta(t)$.

$$h(t) = \text{Sys}\left\{\delta(t)\right\} = \int_{-\infty}^{t} \delta(\lambda)\, d\lambda = \begin{cases} 1\,, & t > 0 \\ 0\,, & \text{otherwise}\end{cases}$$

Therefore

$$h(t) = u(t)$$

Since $h(t) = 0$ for $t < 0$, the system is causal. However, since $h(t)$ is not absolute summable, the system is not stable.

b. Let $x(t) = \delta(t)$.

$$h(t) = \text{Sys}\left\{\delta(t)\right\} = \int_{t-T}^{t} \delta(\lambda)\, d\lambda = \begin{cases} 1\,, & 0 < t < T \\ 0\,, & \text{otherwise}\end{cases}$$

Therefore

$$h(t) = \Pi\left(\frac{t - T/2}{T}\right)$$

Since $h(t) = 0$ for $t < 0$, the system is causal. Also, since $h(t)$ is absolute summable, the system is stable.

c. Let $x(t) = \delta(t)$.

$$h(t) = \text{Sys}\left\{\delta(t)\right\} = \int_{t-T}^{t+T} \delta(\lambda)\, d\lambda = \begin{cases} 1\,, & -T < t < T \\ 0\,, & \text{otherwise}\end{cases}$$

Therefore

$$h(t) = \Pi\left(\frac{t}{2T}\right)$$

Since $h(t)$ has nonzero values for some $t < 0$, the system is not causal. It is stable, however, since $h(t)$ is absolute summable.

2.32.

a.

```
x = @(t) exp(-t).*cos(2*t).*(t>=0);
t = [-1:0.01:5];
```

b. Compute and graph $w(t)$:

```
w = @(t) 3*x(t);
plot(t,w(t));
axis([-1,5,-1,4]);
title('w(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

Compute and graph $y(t)$:

```
y = @(t) w(t-2);
plot(t,y(t));
axis([-1,5,-1,4]);
title('y(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

c. Compute and graph $\bar{w}(t)$:

```
wbar = @(t) x(t-2);
plot(t,wbar(t));
axis([-1,5,-1,4]);
title('wbar(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

Compute and graph $\bar{y}(t)$:

```
ybar = @(t) 3*wbar(t);
plot(t,ybar(t));
axis([-1,5,-1,4]);
```

```
title('ybar(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

2.33.

Create an anonymous function to compute the signal $x(t)$:

```
x = @(t) exp(-t).*cos(2*t).*(t>=0);
t = [-1:0.01:5];
```

a. Compute and graph $w(t)$:

```
w = @(t) 3*x(t);
plot(t,w(t));
axis([-1,5,-2,4]);
title('w(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

Compute and graph $y(t)$:

```
y = @(t) t.*w(t);
plot(t,y(t));
axis([-1,5,-2,4]);
title('y(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

Compute and graph $\bar{w}(t)$:

```
wbar = @(t) t.*x(t);
plot(t,wbar(t));
axis([-1,5,-2,4]);
title('wbar(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

Compute and graph $\bar{y}(t)$:

```
ybar = @(t) 3*wbar(t);
plot(t,ybar(t));
axis([-1,5,-2,4]);
title('ybar(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

b. Compute and graph $w(t)$:

```
w = @(t) 3*x(t);
plot(t,w(t));
axis([-1,5,-1,20]);
title('w(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

Compute and graph $y(t)$:

```
y = @(t) w(t)+5;
plot(t,y(t));
axis([-1,5,-1,20]);
title('y(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

Compute and graph $\bar{w}(t)$:

```
wbar = @(t) x(t)+5;
plot(t,wbar(t));
axis([-1,5,-1,20]);
title('wbar(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

Compute and graph $\bar{y}(t)$:

```
ybar = @(t) 3*wbar(t);
plot(t,ybar(t));
axis([-1,5,-1,20]);
title('ybar(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

2.34.

a.

```
t = [0:0.001:2];
% Compute the exact solution
y = 0.25-1.25*exp(-4*t);
% Compute the approximate solution using Euler method
Ts = 1/40;
```

```
t2 = [0:Ts:2];
yhat = zeros(size(t2));
yhat(1) = -1;  % Initial value
for k=1:length(yhat)-1,
  g = -4*yhat(k)+1;
  yhat(k+1) = yhat(k)+Ts*g;  % Eqn. (2.185)
end;
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','SouthEast');
```

b.

```
t = [0:0.001:10];
% Compute the exact solution
y = (1+exp(-2*t)).*((t>=0)&(t<5))...
    +(exp(10)+1)*exp(-2*t).*(t>=5);
% Compute the approximate solution using Euler method
Ts = 1/20;
t2 = [0:Ts:10];
yhat = zeros(size(t2));
yhat(1) = 2;  % Initial value
for k=1:length(yhat)-1,
  if ((k-1)*Ts<5),
    g = -2*yhat(k)+2;
  else
    g = -2*yhat(k);
  end;
  yhat(k+1) = yhat(k)+Ts*g;  % Eqn. (2.185)
end;
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','NorthEast');
```

c.

```
t = [0:0.001:2];
% Compute the exact solution
y = 3.5*exp(-5*t);
% Compute the approximate solution using Euler method
Ts = 1/50;
t2 = [0:Ts:2];
```

```
yhat = zeros(size(t2));
yhat(1) = 0.5;  % Initial value
for k=1:length(yhat)-1,
  if (k==1),
    g = -5*yhat(k)+3/Ts;  % Approximate unit impulse with rectangle
                          % that has a width of Ts and area of 1.
  else
    g = -5*yhat(k);
  end;
  yhat(k+1) = yhat(k)+Ts*g;  % Eqn. (2.185)
end;
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','NorthEast');
```

d.

```
t = [0:0.001:2];
% Compute the exact solution
y = 0.6*t-0.12-3.88*exp(-5*t);
% Compute the approximate solution using Euler method
Ts = 1/50;
t2 = [0:Ts:2];
yhat = zeros(size(t2));
yhat(1) = -4;  % Initial value
for k=1:length(yhat)-1,
  x = (k-1)*Ts;
  g = -5*yhat(k)+3*x;
  yhat(k+1) = yhat(k)+Ts*g;  % Eqn. (2.185)
end;
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','SouthEast');
```

e.

```
t = [0:0.001:2];
% Compute the exact solution
y = exp(-t)-2*exp(-2*t);
% Compute the approximate solution using Euler method
Ts = 1/10;
t2 = [0:Ts:2];
```

```
yhat = zeros(size(t2));
yhat(1) = -1;  % Initial value
for k=1:length(yhat)-1,
  x = exp(-2*(k-1)*Ts);
  g = -yhat(k)+2*x;
  yhat(k+1) = yhat(k)+Ts*g;  % Eqn. (2.185)
end;
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','SouthEast');
```

2.35.

a.

```
t = [0:0.001:2];
% Compute the exact solution
y = 0.25-1.25*exp(-4*t);
% Compute the approximate solution using Euler method
Ts = 1/40;
t2 = [0:Ts:2];
ga = @(t,yhat) -4*yhat+1;
[t2,yhat] = ode45(ga,t2,-1);
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','SouthEast');
```

b.

```
t = [0:0.001:10];
% Compute the exact solution
y = (1+exp(-2*t)).*((t>=0)&(t<5))...
    +(exp(10)+1)*exp(-2*t).*(t>=5);
% Compute the approximate solution using Euler method
Ts = 1/20;
t2 = [0:Ts:10];
gb = @(t,yhat) -2*yhat+2*((t>=0)&(t<=5));
[t2,yhat] = ode45(gb,t2,2);
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
```

```
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','NorthEast');
```

c.

```
t = [0:0.001:2];
% Compute the exact solution
y = 3.5*exp(-5*t);
% Compute the approximate solution using Euler method
Ts = 1/50;
t2 = [0:Ts:2];
gc = @(t,yhat) -5*yhat+3/Ts*(t==0);
[t2,yhat] = ode45(gc,t2,3.5);
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','NorthEast');
```

d.

```
t = [0:0.001:2];
% Compute the exact solution
y = 0.6*t-0.12-3.88*exp(-5*t);
% Compute the approximate solution using Euler method
Ts = 1/50;
t2 = [0:Ts:2];
gd = @(t,yhat) -5*yhat+3*t.*(t>=0);
[t2,yhat] = ode45(gd,t2,-4);
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','SouthEast');
```

e.

```
t = [0:0.001:2];
% Compute the exact solution
y = exp(-t)-2*exp(-2*t);
% Compute the approximate solution using Euler method
Ts = 1/10;
```

```
t2 = [0:Ts:2];
ge = @(t,yhat) -yhat+2*exp(-2*t).*(t>=0);
[t2,yhat] = ode45(ge,t2,-1);
% Graph exact and approximate solutions
clf;
plot(t,y,'-',t2,yhat,'r.'); grid;
title('Exact and approximate solutions');
xlabel('Time (sec)');
ylabel('Amplitude');
legend('Exact solution','Approximate solution',...
  'Location','SouthEast');
```

2.36.

a. The unit-ramp function can be expressed with an anonymous function as

```
xr = @(t) t.*(t>=0);
```

b. The unit-ramp response of the circuit is

$$y_r(t) = \left[t - \frac{1}{4} + \frac{1}{4}e^{-4t}\right] u(t)$$

and can be expressed with an anonymous function as

```
yr = @(t) (t-0.25+0.25*exp(-4*t)).*(t>=0);
```

c. The input signal can be expressed using unit-ramp functions as

$$x(t) = r(t-1) - r(t-1) - r(t-2) + r(t-3)$$

which can be produced with MATLAB statements

```
t = [-1:0.001:5];
inp = xr(t)-xr(t-1)-xr(t-2)+xr(t-3);
```

d. The response of the circuit to the signal $x(t)$ is

$$y(t) = y_r(t-1) - y_r(t-1) - y_r(t-2) + y_r(t-3)$$

and can be computed in MATLAB using

```
t = [-1:0.001:5];
out = yr(t)-yr(t-1)-yr(t-2)+yr(t-3);
```

e. The input and the output signals can be graphed with the following statements:

```
plot(t,inp,'b',t,out,'r');
title('x(t) and y(t)');
xlabel('t (sec)');
ylabel('Amplitude');
legend('x(t)','y(t)');
axis([-1,5,-0.2,1.2]);
grid;
```

2.37.

a. The input signal can be expressed as

$$x(t) = r(t-1) - 2\,u(t-1) + r(t-2) - r(t-3)$$

b. The unit-step response of the circuit is

$$y_u(t) = \left[1 - e^{-4t}\right] u(t)$$

The unit-ramp response of the circuit is

$$y_r(t) = \left[t - \frac{1}{4} + \frac{1}{4}e^{-4t}\right] u(t)$$

The output signal $y(t)$ can be computed through the following statements:

```
xu = @(t) 1*(t>=0);
xr = @(t) t.*(t>=0);
yu = @(t) (1-exp(-4*t)).*(t>=0);
yr = @(t) (0.25*exp(-4*t)+t-0.25).*(t>=0);
t = [-1:0.001:5];
inp = xr(t)-xr(t-1)-2*xu(t-1)+xr(t-2)-xr(t-3);
out = yr(t)-yr(t-1)-2*yu(t-1)+yr(t-2)-yr(t-3);
```

c. Use the following statements to graph the input and the output signals:

```
plot(t,inp,'b',t,out,'r');
axis([-1,5,-1.2,1.2]);
title('x(t) and y(t)');
xlabel('t (sec)');
ylabel('Amplitude');
legend('x(t)','y(t)');
grid;
```

2.38.

```
x = @(t) ss_tri(t-1);
t = [-1:0.01:5];
y = x(t)-x(t-1);
plot(t,y);
axis([-1,5,-1.2,1.2]);
title('y(t)=x(t)-x(t-1)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```

2.39.

```
x = @(t) exp(-t).*(t>=0);
t = [-1:0.01:7];
y = x(t)+0.5*x(t-1)+0.3*x(t-2)+0.2*x(t-3);
plot(t,y);
axis([-1,7,-0.2,1.2]);
title('y(t)');
xlabel('t (sec)');
ylabel('Amplitude');
grid;
```