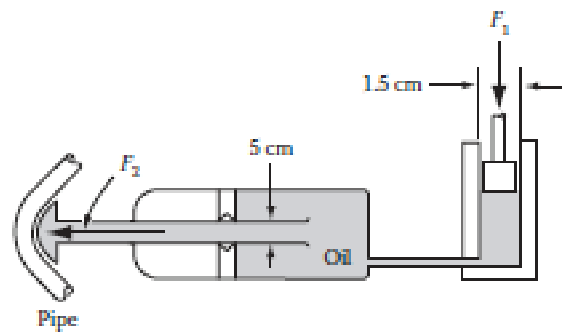
CHAPTER 2

2.1 Repeat the derivation in Section 2.1 for a three-dimensional prism to prove that pressure at a point is independent of direction.

Proof.

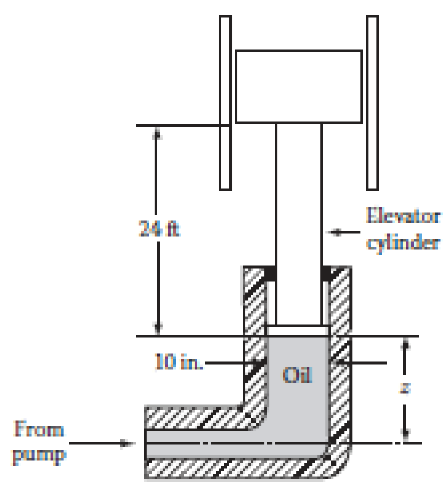
2.2 A hydraulic jack is used to bend pipe as shown in Figure P2.2. What force F_2 is exerted on the pipe if F_1 is 450 N, assuming pressure is constant throughout the system.



pressure inside is constant; $\frac{F_1}{A_1} = \frac{F_2}{A_2}$; $F_2 = F_1 \frac{A_2}{A_1} = 450 \left(\frac{\pi(5)^2/4}{\pi(1.5)^2/4} \right)$

$F_2 = 450 \frac{25}{2.25}$; $\boxed{F_2 = 5\ 000\ \text{N}}$

2.3 A hydraulic elevator is lifted by a 12 in. diameter cylinder under which oil is pumped (Figure P2.3). Determine the output pressure of the pump as a function of z if the weight of the elevator and occupants is 1800 lbf. Take the specific gravity of the oil to be 0.86. The elevator cylinder does not contain oil.



weight acts over cylinder so

$$\frac{1800}{\pi(12/12)^2/4} + \rho_o g z = p; \qquad 2292 + 0.86(1.94)(32.2)z \text{ or}$$

$\boxed{p = 2292 + 53.7z \text{ linear}}$

@ z = 0 p = 2934 psf

@ z = 24 ft p = 4223 psf

2.4 A suited diver can dive to a depth at which the pressure is about 1.8 MPa. Calculate the depth in seawater where this pressure exists.

Table A-5 for sea water, $\rho = 1\,030 \text{ kg/m}^3$; $d = \frac{p}{\rho g} = \frac{1\,800\,000}{1\,030(9.81)}$

or $\boxed{d = 178 \text{ m}}$

2.5 A free diver (unsuited) can dive to a depth at which the pressure is about 70 psi. What is the corresponding depth in fresh water with $\rho = 1.94$ slug/ft^3?

$p = (\rho g)d$; $d = \frac{p}{\rho g}$; $\rho = 1.94)$ slug/ft^3 so $d = \frac{70(144)}{1.94(32.2)}$

or $\boxed{d = 161 \text{ ft}}$

2.6 The deepest descent in seawater of a diving bell is 1370 ft. What is the pressure at this depth?

Table A-5 for sea water, $\rho = 1.03(1.94)$ slug/ft^3

$p = \rho g z$; bell, $z = 1370$ ft; $p = 1.03(1.94)(32.2)(1370)$

$\boxed{p = 88{,}150 \text{ psfg} = 612 \text{ psig}}$

2.7 The deepest descent in seawater of a human-carrying vessel is 35,820 ft. What is the pressure at this depth?

vessel $z = 35820$ ft; $p = 1.03(1.94)(32.2)(35820)$

$\boxed{p = 2.305 \times 10^6 \text{ psfg} = 1.6 \times 10^4 \text{ psig}}$

2.8 A cylindrical tank is used to store methyl alcohol after its production and before it is pumped to its user. The tank is 35 ft in diameter and is filled to a depth of 10 ft. Calculate the pressure exerted at the bottom of the walls.

Table A-5, $\rho = 0.789(1.94)$ slug/ft^3 ; $p = \rho g d = 0.789(1.94)(32.2)(10)$

$\boxed{p = 492 \text{ psf}}$

2.9 Linseed oil can be mixed with powders to produce paint. The oil is shipped via rail in tank cars that are 6 ft in diameter and 40 ft long. Calculate the pressure exerted at the bottom of a full tank.

$\rho = 0.93(1.94)$ slug/ft^3 , from Table A-5; $p = \rho g z = 0.93(1.94)(32.2)(6)$

$\boxed{p = 349 \text{ lbf/ft}^2 = 2.4 \text{ psi}}$

2.10 The pressure at the bottom of an open top tank of ethyl alcohol is 17 psia. What is the depth of the liquid?

Appendix Table A-5 for ethyl alcohol $\rho = 0.787(1.94)$ slug/ft^3

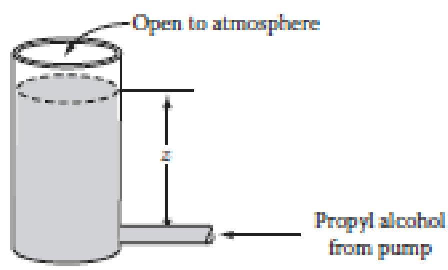
Pressure at tank bottom = $p_B = p_{atm} + \rho g h$; $p_{atm} = 14.7$ psia, so

$p_B = 14.7(144) + 0.787(1.94)(32.2)h = 17(144)$

$$h = \frac{(17 - 14.7)(144)}{0.787(1.94)(32.2)};$$

$\boxed{h = 6.74 \text{ ft}}$

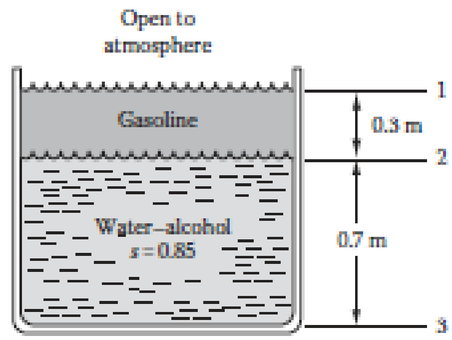
2.11 The output pressure of a pump is 3.5 psi. The pump is used to deliver propyl alcohol to a tank, as shown in Figure P2.11. To what depth can the tank be filled at this pump pressure, provided that inflow stops when pump pressure equals hydrostatic pressure in the tank? Ignore frictional losses.



$$p = 3.5 \text{ psi}; \ p = \rho g z; \ z = \frac{p}{\rho g}; \ \rho = 0.802(1.94 \text{ slug/ft}^3; \text{ so } z = \frac{3.5(144)}{0.802(1.94)(32.2)}$$

$\boxed{z = 10.1 \text{ ft}}$

2.12 Methyl alcohol and gasoline (assume octane) are mixed together in an open fuel tank. The methyl alcohol soon absorbs water from the atmosphere, and the water–alcohol mix separates from the gasoline, as shown in Figure P2.12. Find the pressure at the bottom of the tank wall.

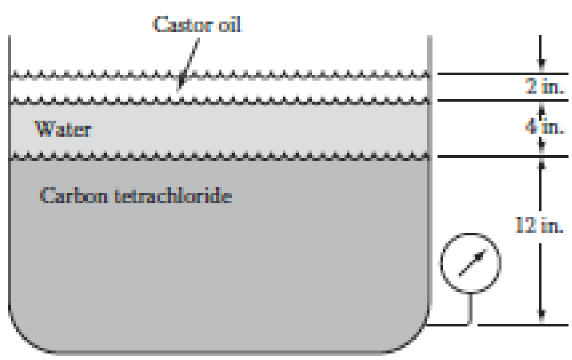


Octane $\rho_o = 701\ kg/m^3$, water alcohol $\rho_m = 850\ kg/m^3$
$p_3 = \rho_o g(0.3) + \rho_m g(0.7) = 701(9.81)(0.3) + 850(9.81)(0.7)$
$\boxed{p_3 = 7\,900\ N/m^2 \text{ gage pressure}}$ or $p = 101\,300 + 7\,900$
$\boxed{p_3 = 109\,200\ N/m^2 \text{ absolute}}$

2.14 A common drinking glass is 7 cm in diameter and filled to a depth of 10 cm with water. Calculate the pressure difference between the top and the bottom of the glass sides.

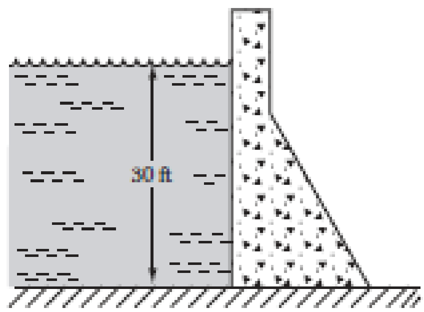
Hydrostatic equation applies with $\rho = 1\,000\ kg/m^3$
$\Delta p = \rho gz = (1\,000\ kg/m^3)(9.81\ m/s^2)(0.10\ m)$
or $\boxed{\Delta p = 981\ N/m^2}$
Result is independent of diameter

2.15 Figure P2.15 shows a tank containing three liquids. What is the expected reading on the pressure gauge?



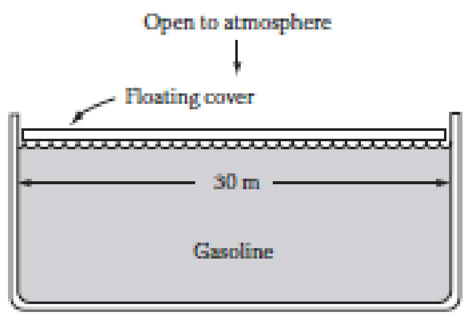
Castor oil $\rho_o = 0.96(1.94)\ slug/ft^3$ from Table A-6; for water, $\rho_{H2O} = 1.94$
Carbon tet, $\rho_c = 1.59(1.94)$; use gage pressure so that $p_{atm} = 0$. For the tank,
$\rho_o g(2/12) + \rho_{H2O} g(4/12) + \rho_c g(12/12) + p_{atm} = p$; substituting,
$p = 0.96(1.94)(32.2)(2/12) + 1.94(32.2)(4/12) + 1.59(1.94)(32.2)(1)$
$\boxed{p = 130 \text{ psfg} = 0.904 \text{ psig}}$

2.16 A concrete dam is constructed as shown in Figure P2.16. When the water level on the left is 30 ft, determine the pressure at the bottom of the dam.



$p = \rho g z = 1.94(32.2)(30) = \boxed{1870 \text{ psfg} = 13.0 \text{ psig}}$

2.17 A floating cover is used in oil- and gasoline-storage tanks to keep moisture out (Figure P2.17). If the cover weighs 1 800 N and the depth of gasoline (assume octane) is 3 m, determine the pressure at the bottom of the tank wall.



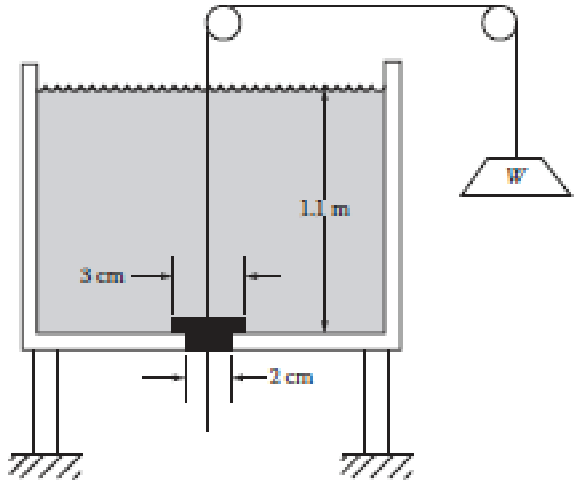
Bottom has weight of liquid + weight of cover

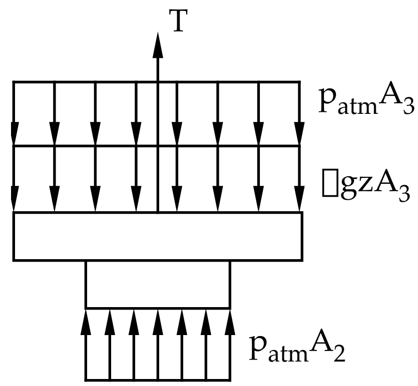
$p = \rho g z + \frac{mg}{A} = 0.701(1\,000)(9.81)(3) + \frac{1\,800}{\pi(30)^2/4}$ where $\rho = 0.701(1\,000)$ for octane is from Table A-5. Thus, $p = 20\,630 + 2.55$ Pa or

$\boxed{p = 21 \text{ kPa}}$

2.18 Figure P2.18 shows a tank containing acetone. The tank bottom has a plug with a cable attached that leads to a weight W. It is desired to have the plug close when the acetone depth is 1.1 m. Calculate the weight required to do this. Neglect the thickness of the plug.

Acetone $\rho = 0.787(1\,000)$ kg/m^3 from Table A-5. p_{atm} must be taken into account. FBD of plug





$\Sigma F = 0$; $T + p_{atm}A_2 = p_{atm}A_3 + \rho g z A_3$; $\quad T = \rho g z A_3 + p_{atm}(A_3 - A_2)$

$T = 0.787(1\,000)(9.81)(1.1)(\pi(0.03)^2/4) + 101\,300(\pi(0.03)^2/4 - \pi(0.02)^2/4)$

$T = 6.0 + 39.8$

$\boxed{T = 45.8 \text{ N}}$ (If atmospheric pressure is neglected, then $T = 6.64$ N)

2.19 Calculate the atmospheric pressure at an elevation of 50,000 ft. Express the answer in kPa, psia, and psig.

Eq Ex 2.3 $\quad z - 11\,000 = \dfrac{286.8(216)}{9.81} \ln \dfrac{22.5}{p}$; now, 50,000 ft = 15 240 m, so

$\dfrac{4240(9.81)}{286.8(216)} = \ln \dfrac{22.5}{p}$; $0.6714 = \ln \dfrac{22.5}{p}$ or $\dfrac{22.5}{p} = 1.957$ and

$\boxed{p = 11.5 \text{ kPa} = 11\,500 \text{ N/m}^2 = 240 \text{ psfa} = 1.67 \text{ psia} = -13.0 \text{ psig}}$

2.20 At what atmospheric elevation is the pressure equal to 15 kPa if the temperature is –57°C? Assume a lapse rate of 6.5°C=km from sea level, where the temperature is taken to be 288 K.

Eq 2.3; $\quad z - 11\,000 = \dfrac{286.8(216)}{9.81} \ln \dfrac{22.5}{15} = 11\,000 + 2\,560$

$\boxed{z = 13\,560 \text{ m}}$

2.21 a) $dp = \rho g dz$; $dp = \dfrac{p}{RT} g dz$

b) $\dfrac{dp}{p} = g \dfrac{dz}{RT}$; $\displaystyle\int_{22.5 \text{ kPa}}^{p} \frac{dp}{p} = \frac{g}{RT} \int_{z}^{11\,000 \text{ m}} dz$;

$\ln \dfrac{p}{22.5} = \dfrac{g}{RT}(11\,000 - z)$; $g = 9.81$; $R = 287$; $T = 216.65$

c) By substitution, $\ln \frac{p}{22.5} = (1.58 \times 10^{-4})(11\ 000 - z)$
d) Sketch given in following problem.

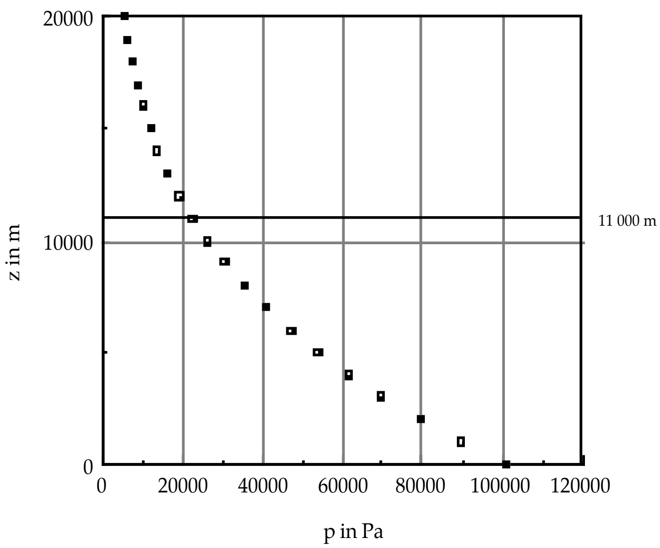
2.22 a) $dp = \rho g dz = \frac{p}{RT} g dz = \frac{gp}{R} \frac{dz}{(T_o - \alpha z)}$

b) $\frac{dp}{p} = \frac{g}{R} \frac{dz}{(T_o - \alpha z)}$; $\int_{101.3 kPa}^{p} \frac{dp}{p} = \int_{0}^{11\ 000\ m} \frac{g dz}{R(T_o - \alpha z)}$

$\ln \frac{p}{101.3} = \frac{g}{R\alpha} \ln \left(\frac{(T_o - \alpha z)}{T_o} \right)$; $\alpha = 6.5°C/km$; $T_o = 15°C = 288$ K; $g = 9.81\ m/s^2$

c) Substituting, $\frac{p}{101.3} = (1 - 0.000\ 022\ 6z)^{5.26}$

d) & e)



2.23 At what elevation is atmospheric pressure equal to 7 kN/m²? Use the equation in Problem 2.21 or in Problem 2.22 as appropriate.

Assume following equation applies and check after solving:

$\ln \frac{22.5}{p} = (1.58 \times 10^{-4})(z - 11\ 000)$; p = 7 kPa, so

$\ln \frac{22.5}{7} = (1.58 \times 10^{-4})(z - 11\ 000)$; $1.17 = (1.58 \times 10^{-4})(z - 11\ 000)$

$z = \frac{1.17}{1.58 \times 10^{-4}} + 11\ 000$; $\boxed{z = 18\ 405\ m}$

Equation used applies for 20 000 ≥ z ≥ 11 000, so result is OK

2.24 At what elevation is atmospheric pressure equal to 4 psia? Use the equation in Problem 2.21 or in Problem 2.22 as appropriate.

$p = 4$ psia $= 2.76 \times 10^4\ N/m^2$; 27.6 kPa > 22.5 kPa, so troposphere equation applies: $p = 101.3(1 - 0.000\,022\,6z)^{5.26}$

Substitute to get $\left(\frac{27.6}{101.3}\right)^{1/5.26} = 1 - 0.000\,022\,6z$;

$\boxed{z = 9\,690 \text{ m} = 31{,}800 \text{ ft}}$

2.25 The elevation above sea level at Denver, Colorado, is 5283 ft, and the local atmospheric pressure there is 83.4 kPa. If the temperature there is 75°F, calculate the local air density and the barometric pressure in centimeters of mercury.

Elevation = 5283 ft; p = 83.4 kPa = 83 400 N/m^2 = 1740 lbf/ft^2 ; T = 75°F

$\rho = \frac{p}{RT}$; R = 1710 ft·lbf/slug·°R from Table A-6

$\rho = \frac{1740}{1710(75 + 460)} = \boxed{0.0019 \text{ slug/ft}^3}$

$p = \rho g h$; $\rho_{Hg} = 13.6(1.94)$ slug/ft^3 so $h = \frac{1740}{32.2(13.6)(1.94)}$; solving,

$\boxed{h = 2.05 \text{ ft} = 0.62 \text{ m Hg} = 62 \text{ cm Hg}}$

2.26 Various Answers

2.27 Determine the height of mercury in a barometer if atmospheric pressure is 12 psia.

$\rho = 13.6(1.94)$ slug/ft^3 from Table A-5; $p = \rho g z$; $z = \frac{p}{\rho g}$

$z = \frac{12(144)}{13.6(1.94)(32.2)}$

$\boxed{z = 2.03 \text{ ft} = 24.4 \text{ in. of Hg}}$

2.28 At an elevation of 12 km, atmospheric pressure is 19 kN/m^2. Determine the pressure at an elevation of 20 km if the temperature at both locations is 57°C.

Select given properties as reference; $z_1 = 11\,000$ m where $p_1 = 22.5$ kPa. With R = 286.8 J/(kg·K) from Appendix Table A-6, T = - 57°C = 216 K, and $z_2 = 20\,000$ m, we get after substituting into general equation:

$z_2 - z_1 = \frac{RT}{-g} \ln \frac{p_2}{p_1}$; $(20\,000 - 11\,000) = \frac{286.8(216)}{-9.81} \ln \frac{p_2}{22.5}$

Solving, $-1.425 = \ln \frac{p_2}{22.5}$

or $\boxed{p_2 = 5.41 \text{ kPa}}$

2.29 Figure P2.29 shows a tank containing linseed oil and water. Attached to the tank 3 ft below the water–linseed oil interface is a mercury manometer. For the dimensions shown, determine the depth of the linseed oil.

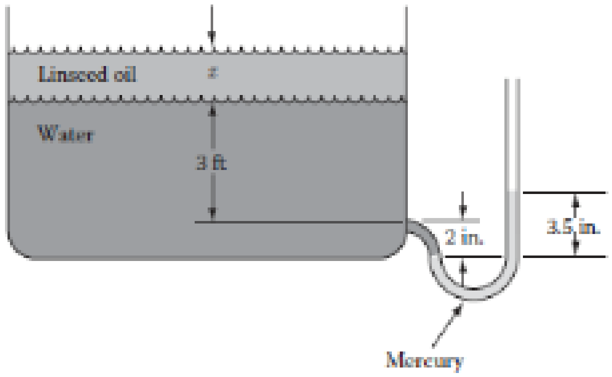


FIGURE P2.29

Linseed oil $\rho_L = 0.93(1.94)$ slug/ft^3 $\rho_{Hg} = 13.6(1.94)$ $\rho_{H2O} = 1.94$
From linseed surface to lowest water level in manometer,
$p_{atm} + \rho_L g z + \rho_{H2O} g(3) + \rho_{H2O} g(2/12) = p$
In right leg of manometer we have:
$p_{atm} + \rho_{Hg} g(3.5/12) = p$; Equating and canceling p_{atm} gives
$\rho_L g z + \rho_{H2O}(3.17)g = \rho_{Hg} g(0.292)$; Canceling g & substituting,
$(0.93)(1.94)z + 1.94(3.17) = 13.6(1.94)(0.292)$
$\boxed{z = 0.86 \text{ ft}}$

2.30 A mercury manometer is used to measure pressure near the bottom of a tank containing acetone as shown in Figure P2.30. What is the depth *d* of the acetone if if *Δh* is measured to be 4 in. of mercury, and *x* is 2 in?

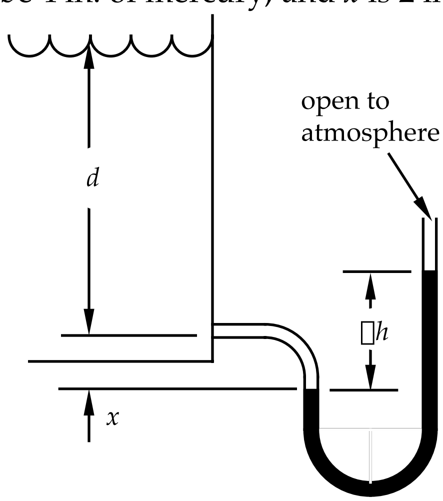


FIGURE P2.30

For the fluids, $\rho_a = 0.787(1.94)$ slug/ft^3; $\rho_{Hg} = 13.6(1.94)$ slug/ft^3

For the manometer, we write

$$p_{atm} + \rho_a g(x + d) = p_{atm} + \rho_{Hg} g \Delta h$$

Canceling p_{atm}, and substituting,

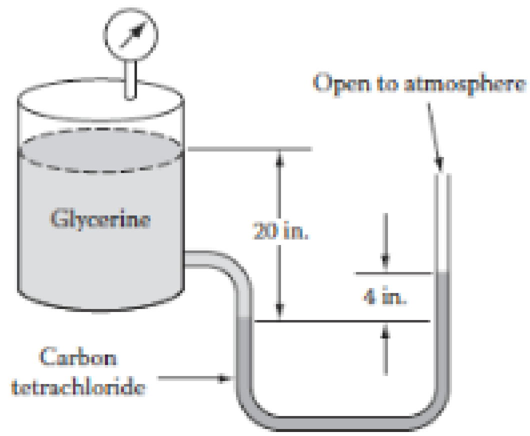
$$0.787(1.94)(32.2)(2/12 + d) = 13.6(1.94)(32.2)(4/12)$$

$$2/12 + d = \frac{13.6(4/12)}{0.787} = 5.76 \text{ ft}$$

Solving,

$$\boxed{d = 5.76 - 2/12 = 5.6 \text{ ft}}$$

2.31 Determine the pressure above the glycerine in Figure P2.31.



$p + \rho_g g z_1 = p_{atm} + \rho_c g z_2$; $\rho_g = 1.263(1.94)$ slug/ft^3 ; $\rho_c = 1.590(1.94)$

substituting,

$p = 14.7(144) + 1.590(1.94)(32.2)(4/12) - 1.263(1.94)(32.2)(20/12)$

$\boxed{p = 2018 \text{ psf} = 14.0 \text{ psi (less than } p_{atm})}$

2.32 Determine the pressure of the water at the point where the manometer attaches. All dimensions are in centimeters.

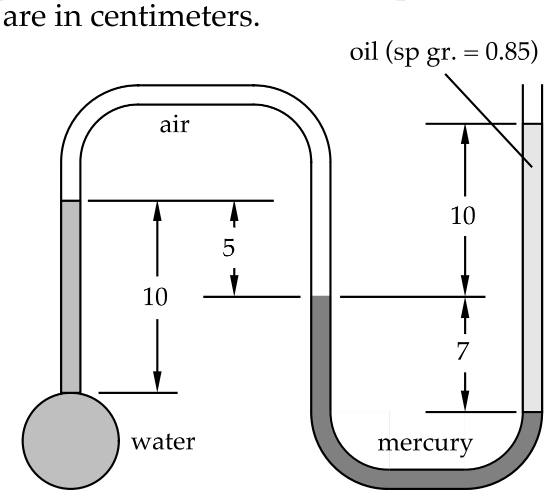
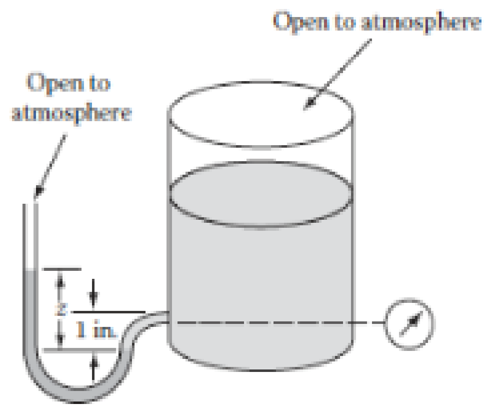


FIGURE P2.32

$\rho_{hg} = 13.6(1\,000)$ kg/m^3 $\rho_o = 0.85(1\,000)$ kg/m^3
$p_w - \rho_w g(0.10) + \rho_{air} g(0.05) + \rho_{Hg} g(0.07) - \rho_o g(0.17) = p_{atm}$
Rearranging and substituting, with ρ_{air} term negligible,
$p_w = 101\,300 + 1\,000(9.81)(0.10) - 13\,600(9.81)(0.07) + 850(9.81)(0.17)$

$\boxed{p_w = 94\,359 \text{ N/m}^2}$

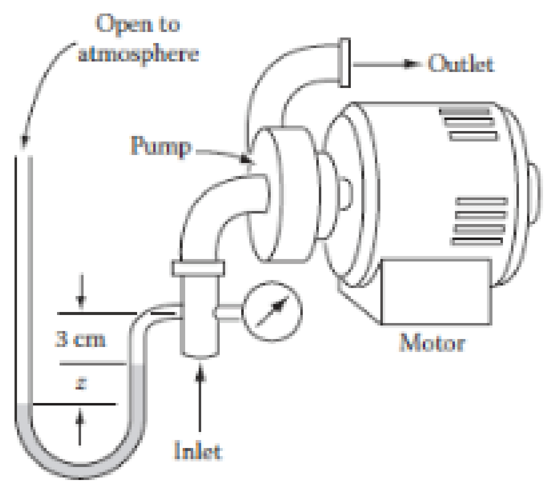
2.33 Figure P2.33 shows a manometer and a gauge attached to the bottom of a tank of kerosene. The gauge reading is 3 psig. Determine the height z of mercury in the manometer.



left leg, $p_{atm} + \rho_{Hg} g z = p_A$; right leg, $144(3 \text{ psig} + 14.7) + \rho_k g(1/12) = p_A$
set $p_{atm} = 0$ and work with gage pressure
$\rho_{Hg} = 13.6(1.94)$; $\rho_k = 0.823(1.94)$
Left & right leg equations equal to each other
with atmospheric pressure equal to zero; substitute to get

$13.6(1.94)(32.2)z = 3(144) + 0.823(1.94)(1/12)(32.2)$
or $\boxed{z = 0.513 \text{ ft} = 6.16 \text{ in}}$

2.34 A pump is a device that puts energy into a liquid in the form of pressure. The inlet side of a pump usually operates at less than atmospheric pressure. As shown in Figure P2.34, a manometer and a vacuum gauge are connected to the inlet side of a pump, and the vacuum gauge reads the equivalent of 34 kPa (absolute).



a. Express the gauge reading in psig.
b. Calculate the deflection when the manometer liquid is mercury.

a) $p = 34\text{ kPa} = 34 - 101.3 = \boxed{-67.3\text{ kPa}}$

b) $p + \rho_{H2O}g(1/12) + \rho_{Hg}gz = p_{atm}$

$$p = 101\,300 - 1\,000(9.81)(0.03) - 13.6(1\,000)(9.81)z = 34\,000$$

so $$z = \frac{34\,000 - 101\,300 + 1\,000(9.81)(0.03)}{-13.6(1\,000)(9.81)}$$

$$\boxed{z = 0.50\text{ m}}$$

2.35 A manometer is used to measure pressure drop in a venturi meter as shown in Figure P2.35. Derive an equation for the pressure drop in terms of Δh. Make all measurements from the meter centerline, and ρ_2 is not negligible.

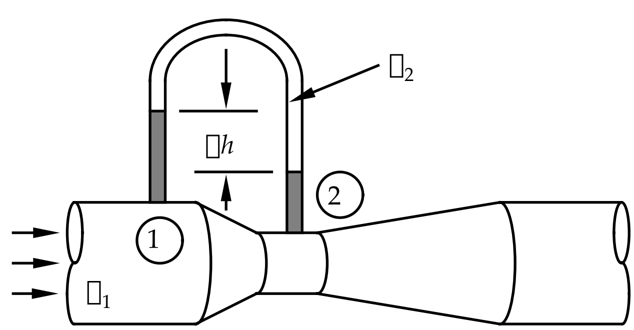
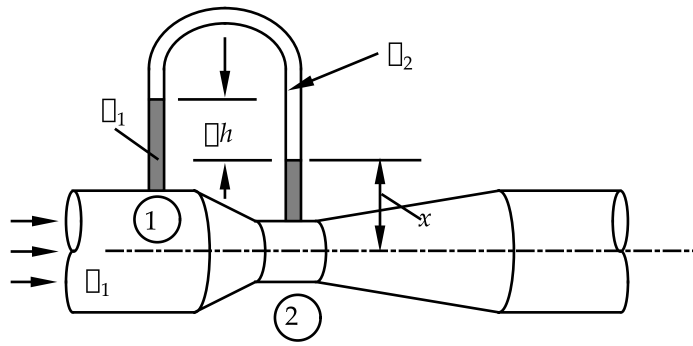


FIGURE P2.35

A more detailed drawing is shown, with the distance x included in the sketch.



For the manometer, we have:

$$p_1 - \frac{\rho_1 g}{g_c} x - \frac{\rho_1 g_c}{g} \Delta h = p_2 - \frac{\rho_1 g}{g_c} x - \frac{\rho_2 g}{g_c} \Delta h$$

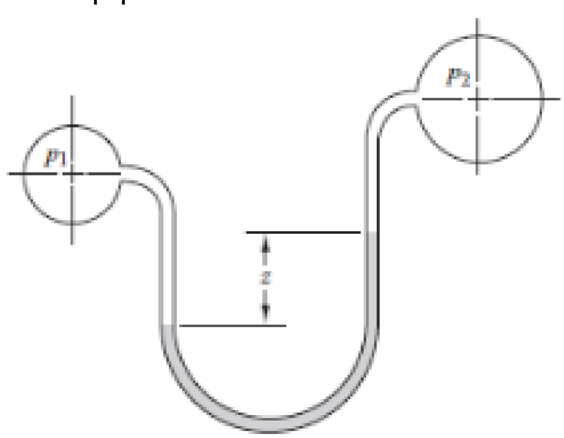
which becomes

$$p_1 - p_2 = \frac{\rho_1 g}{g_c} \Delta h - \frac{\rho_2 g}{g_c} \Delta h = \frac{\rho_1 g}{g_c} \Delta h \left(1 - \frac{\rho_2}{\rho_1}\right)$$

or

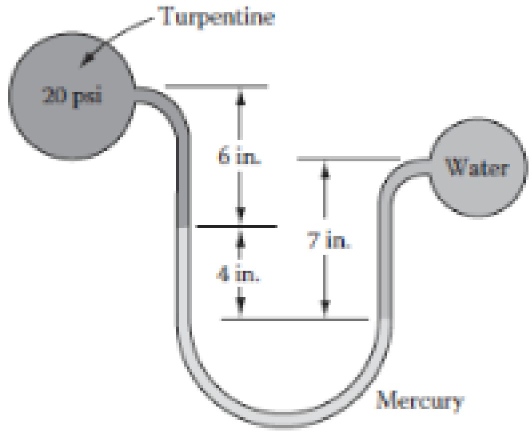
$$\boxed{\Delta h = \frac{(p_1 - p_2) g_c}{\rho_1 g (1 - \rho_2 / \rho_1)}}$$

2.36 Two pipelines are connectedwith a manometer, as shown in Figure P2.33. Determine the pressure p_2 if the manometer deflection is 30 cm of water and p_1 is 55 kPa. The fluid in each pipe is air.



$p_1 = p_2 + \rho g z$; $\quad p_2 = 55\,000 - 1\,000(9.81)(0.30)$ $\quad \boxed{p_2 = 52\,057 \text{ Pa}}$

2.37 If turpentine pressure is 20 psi, find the water pressure for the system shown in Figure P2.37.



$\rho_t = 0.870(1.94)$ slug/ft^3, $\rho_{H2O} = 1.94$ slug/ft^3, $\rho_{Hg} = 13.6(1.94)$ slug/ft^3

$p_t + \rho_t g(6/12) + \rho_{Hg} g(4/12) = p_{H2O} + \rho_{H2O} g(7/12)$

$20(144) + 0.870(1.94)(32.2)(6/12) + 13.6(1.94)(32.2)(4/12) - 1.94(32.2)(7/12) = p_{H2O}$

$\boxed{p_{H2O} = 3154 \text{ psf} = 21.9 \text{ psi}}$

2.38 Determine the pressure difference between the linseed and castor oils of Figure P2.38. All dimensions are in inches.

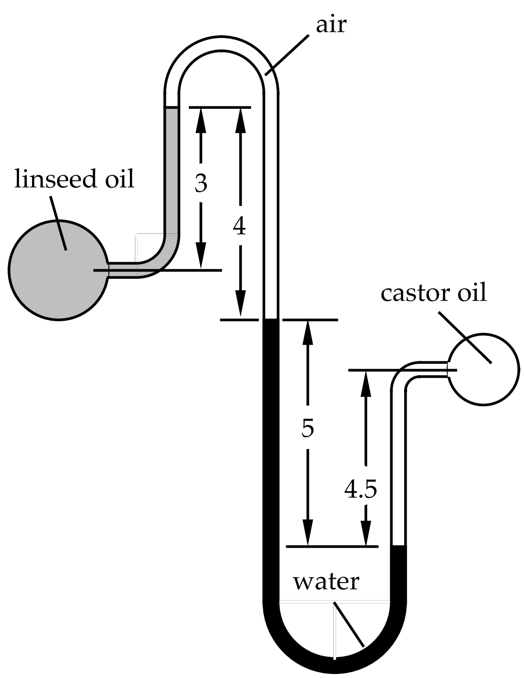


FIGURE P2.38

We can write the solution equation directly:

$$p_{LO} - \rho_{LO}g\ (3/12) + \rho_{air}g\ (4/12) + \rho_{H2O}g\ (5/12) - \rho_{CO}g(4.5/12) = p_{CO}$$

The air term is negligible. With $\rho_{LO} = 0.93(1.94)$ and $\rho_{CO} = 0.96(1.94)$, we have

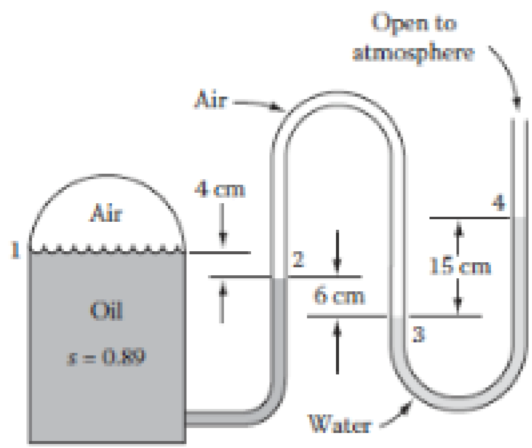
$$p_{LO} - p_{CO} = \rho_{LO}g\ (3/12) - \rho_{H2O}g\ (5/12) + \rho_{CO}g\ (4.5/12)$$

$$p_{LO} - p_{CO} = 0.93(1.94)(32.2)(3/12) - 1.94(32.2)(5/12) + 0.96(1.94)(32.2)(4.5/12)$$

$$p_{LO} - p_{CO} = 14.52 - 26.03 + 22.49$$

$$\boxed{p_{LO} - p_{CO} = 11.0\ \text{lbs/ft}^2}$$

2.39 Calculate the air pressure in the tank shown in Figure P2.39. Take atmospheric pressure to be 101.3 kPa.



$p_1 + \rho_o g(0.04) = p_2$; $p_2 + \rho_{air}g(0.06) = p_3$; $p_3 = p_4 + \rho_{H2O}g(0.15)$

Combining & noting that ρ_{air} negligible,

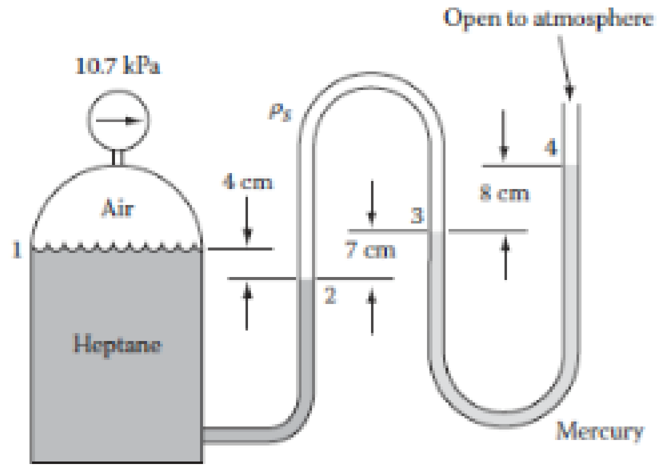
$p_1 + \rho_o g(0.04) = p_4 + \rho_{H2O}g(0.15)$;

$p_1 = 101\ 300 + 1\ 000(9.81)(0.15) - 0.89(1\ 000)(9.81)(0.04)$

$p_1 = 101\ 300 + 1\ 472 - 349$

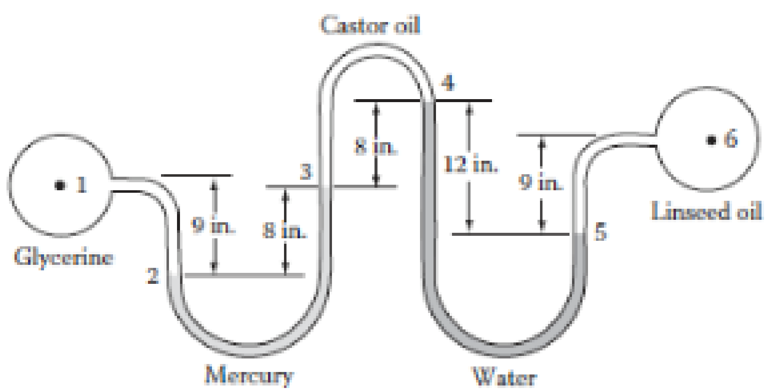
$\boxed{p_1 = 102\ 423\ \text{Pa} = 102\ \text{kPa}}$

2.40 The pressure of the air in Figure P2.40 is 10.7 kPa, and the manometer fluid is of unknown specific gravity s. Determine s for the deflections shown.



$p_1 + \rho_H g(0.04) = p_2$; $p_2 - \rho_s g(0.07) = p_3$; $p_4 + \rho_{Hg} g(0.08) = p_3$
$p_1 - \rho_H g(0.04) = p_4 + \rho_{Hg} g(0.08) + \rho_s g(0.07)$; $\rho_H = 0.681(1\ 000)$
$\rho_{Hg} = 13.6(1\ 000)$; so
$p_1 - p_4 + 681(9.81)(0.04) - 13\ 600(9.81)(0.08) = \rho_s(9.81)(0.07)$
but $p_1 - p_4 = 10.7$ kPa; substituting and simplifying,
$10\ 700 + 267 - 10\ 673 = \rho_s(0.686)$; $\boxed{\rho_s = 428.6 \text{ kg/m}^3}$;
$\boxed{s = 0.429}$

2.41 For the sketch of Figure P2.41 (all dimensions in inches), determine the pressure of the linseed oil if the glycerine pressure is 100 psig.

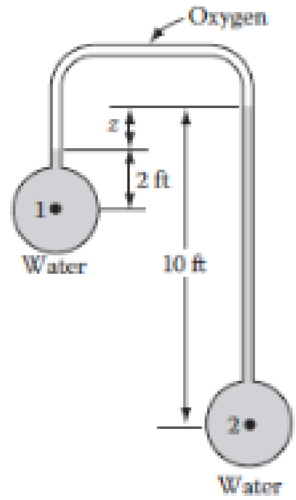


$\rho_g = 1.263(1.94)$; $\rho_c = 0.960(1.94)$; $\rho_L = 0.93(1.94)$; $\rho_{Hg} = 13.6(1.94)$; $\rho_{H2O} = 1.94$
$p_1 + \rho_g g(9/12) - \rho_{Hg}\, g(8/12) - \rho_c g(8/12) + \rho_{H2O} g(12/12) - \rho_L g(9/12) = p_6$

$$p_6 = 100(144) + \frac{32.2}{12}(1.94)[(1.263)(9) - 13.6(8) - 0.96(8) + 1(12) - 0.93(9)]$$

$p_6 = 14400 - 528$
$\boxed{p_6 = 13872 \text{ psfg} = 96.3 \text{ psig}}$

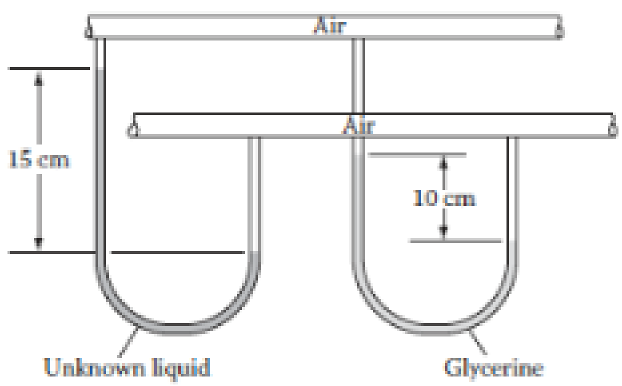
2.42 Find z in Figure P2.42 if $p_2 - p_1 = 2.5$ psig.



ρ_o negligible; $p_1 + \rho_{H2O}g(2 + z) - \rho_{H2O}g(10) = p_2$
$p_1 - p_2 = \rho_{H2O}(g(10 - 2 - z)$; $p_1 - p_2 = -3$ psig; so
$-2.5(144) = 1.94(32.2)(10 - 2 - z)$; $\dfrac{2.5(144)}{1.94(32.2)} = 8 - z$; $z = 8 - 5.76$;

$\boxed{z = 2.23 \text{ ft} = 26.8 \text{ in.}}$

2.43 Two pipes containing air are connected with two manometers, as shown in Figure P2.43. One manometer contains glycerine, and the difference in liquid levels is 10 cm. The other manometer contains an unknown liquid, and it shows a difference in liquid levels of 15 cm. What is the density of the unknown liquid?



glycerine $\rho = 1.263(1\,000)$ kg/m^3 Table A-5; $\Delta p = \rho g h\,|_{gly} = \rho g h\,|_{unk}$
$\rho_{unk} = \rho_{gly}\dfrac{h_{gly}}{h_{unk}} = 1.263(1\,000)\dfrac{10}{15}$ $\boxed{\rho_{unk} = 842 \text{ kg/m}^3}$

2.44 For the system of Figure P2.44, determine the pressure of the air in the tank.

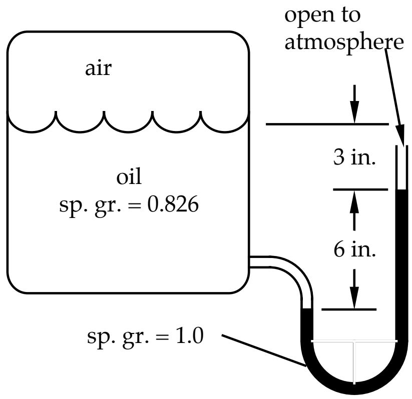


FIGURE P2.44

$$p_{air} + \rho_{oil}g(9/12) = p_{atm} + \rho g(6/12)$$

Rearranging,

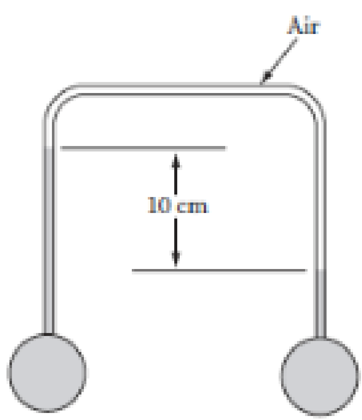
$$p_{air} = p_{atm} + \rho g(6/12) - \rho_{oil}g(9/12)$$

$$p_{air} = 14.7(144) + 1.94(32.2)(6/12) - 0.826(1.94)(32.2)(9/12)$$

$$p_{air} = 2117 + 31.2 - 38.7$$

or $\boxed{p_{air} = 2110 \text{ lbf/ft}^2 = 14.65 \text{ psi}}$

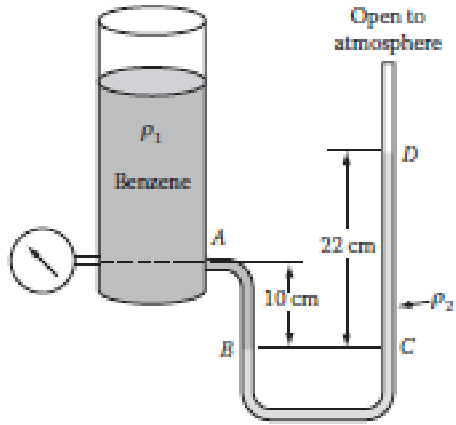
2.45 An inverted U-tube manometer is connected to two sealed vessels, as shown in Figure P2.45. Heptane is in both vessels, and the manometer shows a difference in heptane levels of 10 cm. Calculate the pressure difference between the two vessels.



$\Delta p = \rho g h$; $\rho = 0.681(1\,000)$ kg/m^3 for heptane Table A-5

$\Delta p = 0.681(1\,000)(9.81)(10/100)$ or $\boxed{\Delta p = 668 \text{ N/m}^2}$

2.46 A manometer is used to measure pressure in a tank as a check against a simultaneous measurement with a gauge. The tank liquid is benzene; the manometer fluid is mercury. For the configuration shown in Figure P2.43, determine the gauge pressure at A.



Apply hydrostatic equation to manometer to obtain
$p_A + \rho_1 g(0.10) = p_{atm} + \rho_2 g(0.22)$; from Appendix Table A.5,
$\rho_1 = 876\ kg/m^3$ for benzene and $\rho_2 = 13.6(1\,000\ kg/m^3)$ for mercury
Rearrange & substitute to get
$p_A - p_{atm} = 9.81(13\,600)(0.22) - 876(0.10)$ or
$\boxed{p_A - p_{atm} = 29.3\text{ kPa gage pressure}}$

2.47 Write the equation for z_c and for z_r in terms of depth h for a rectangular gate that is completely in contact with liquid.

$$z_c = \frac{h}{2};\ I_{xxc} = \frac{bh^3}{12};\ A = bh;\ z_r = z_c + \frac{I_{xxc}}{z_c A};\ \text{so } z_r = \frac{h}{2} + \frac{bh^3}{12}\frac{1}{(h/2)bh}$$

$$z_r = \frac{h}{2} + \frac{h}{6} = \boxed{\frac{2h}{3} = z_r}\ \text{rectangular gate}$$

2.48 Write the equation for z_c and for z_r in terms of depth h for a circular gate that is completely in contact with liquid.

$$z_c = \frac{D}{2};\ I_{xxc} = \frac{\pi D^4}{64};\ A = \frac{\pi D^2}{4}(;\ z_r = z_c + \frac{I_{xxc}}{z_c A};\ \text{so } z_r = \frac{D}{2} + \frac{\pi D^4}{64}\frac{1}{\frac{D}{2}\frac{\pi D^2}{4}}$$

$$z_r = \frac{D}{2} + \frac{D}{8} = \boxed{\frac{5D}{8} = z_r}\ \text{circular gate}$$

2.49 Write the equation for z_c and for z_r in terms of depth h for a triangular gate that is completely in contact with liquid.

$$z_c = \frac{2h}{3}\ ; I_{xxc} = \frac{bh^3}{36}\ ; A = \frac{bh}{2};\ z_r = z_c + \frac{I_{xxc}}{z_c A}\ ; \text{so } z_r = \frac{2h}{3} + \frac{bh^3}{36}\frac{1}{\frac{2h}{3}\frac{bh}{2}}$$

$$z_r = \frac{2h}{3} + \frac{h}{12} = \boxed{\frac{3h}{4} = z_r} \text{ triangular gate}$$

2.50 Write the equation for z_c and for z_r in terms of depth h for a semicircular gate that is completely in contact with liquid.

$$z_c = \frac{D}{2} - \frac{2D}{3\pi};\ I_{xxc} = \frac{D^4(9\pi^2 - 64)}{1152\pi}\ ; A = \frac{\pi D^2}{8};\ z_r = z_c + \frac{I_{xxc}}{z_c A};\ \text{so}$$

$$z_r = \frac{D}{2} - \frac{2D}{3\pi} + \frac{D^4(9\pi^2 - 64)}{1152\pi}\frac{1}{\left(\frac{D}{2} - \frac{2D}{3\pi}\right)\frac{\pi D^2}{8}};$$

$$z_r = \frac{3\pi D - 4D}{6\pi} + \frac{D^4(9\pi^2 - 64)}{1152\pi}\frac{6\pi(8)}{(3\pi D - 4D)\pi D^2};$$

$$z_r = \frac{3\pi D - 4D}{6\pi} + \frac{D(9\pi^2 - 64)}{36(3\pi - 4)} = \frac{D}{2} - \frac{2D}{3\pi} + \frac{D(9\pi^2 - 64)}{36(3\pi - 4)}$$

$$z_r = D\left(\frac{1}{2} - \frac{2}{3\pi} + \frac{(9\pi^2 - 64)}{(3\pi - 4)}\right)$$

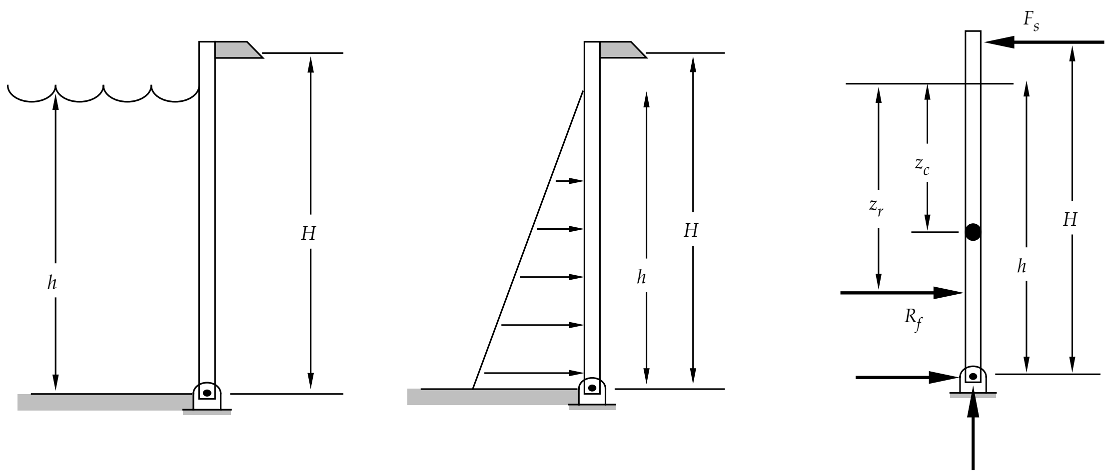
$$\boxed{z_r = 4.864D} \text{ semicircular gate}$$

2.51 Write the equation for z_c and for z_r in terms of depth h for an elliptical gate that is completely in contact with liquid.

$$z_c = b\ ; I_{xxc} = \frac{\pi ab^3}{4}\ ; A = \pi ab;\ z_r = z_c + \frac{I_{xxc}}{z_c A}\ ; \text{so } z_r = b + \frac{\pi ab^3}{4}\frac{1}{b\ \pi ab}$$

$$z_r = b + \frac{b}{4} = \boxed{\frac{5b}{4} = z_r} \text{ elliptical gate}$$

2.52 Figure P2.52 shows a rectangular gate hinged at the bottom, and filled to a depth of $h = 0.3$ m with turpentine. The gate width into the page is 1 m. At the top of the gate is a stop, and the gate height H is 0.4 m. What is the magnitude of the force F_s at the stop?



The figure shows the pressure prism acting on the gate, and figure 3 is a free body diagram showing relevant forces acting on the gate. The force due to the liquid is

$$Rf = \rho g z_c A$$

where $z_c = h/2 = 0.3/2 = 0.15$ m measured from the surface of the liquid.
$A = 0.3(1) = 0.3$ m^2
$\rho = 0.87(1\,000)$ kg/m^3

$$R_f = 870(9.81)(0.15)(0.3) = 384 \text{ N}$$

The line of action of this force acts at $z_r = z_c + \dfrac{I_{xxc}}{z_c A}$; The second moment of the portion of the gate in contact with the liquid is

$$I_{xxc} = \frac{wh^3}{12} = \frac{1(0.30)^3}{12} = 0.002\,25 \text{ m}^4$$

The location of the force then is

$$z_r = 0.15 + \frac{0.002\,25}{(0.15)(0.3)} = 0.2 \text{ m}$$

This distance is measured from the free surface of the liquid. From the hinge, the line of action of the force R_f is $h - z_r = 0.3 - 0.2 = 0.1$ m. Summing moments about the hinge (with $H = 0.4$ m),

$$R_f(0.1) = F_s(0.4) \qquad F_s = 384 \text{ N}(0.1/0.4)$$

$$\underline{F_s = 96 \text{ N}}$$

2.53 Figure P2.53 is a sketch of an apparatus used in an experiment. The apparatus consists of one-fourth of a torus of rectangular cross section and made of clear plastic. The torus is attached to a lever arm, which is free to rotate (within limits) about a pivot point. The torus has inside and outside radii, R_i and R_o respectively, and it is constructed such that the center of these radii is at the pivot point of the lever arm. The torus is submerged in a liquid, and there will exist an unbalanced force that is exerted on the plane of dimensions *h x w*. In order to bring the torus and lever arm back to their balanced position, a weight *W* must be added to the weight hanger. The force and its line of action can be found by summing moments about the pivot.

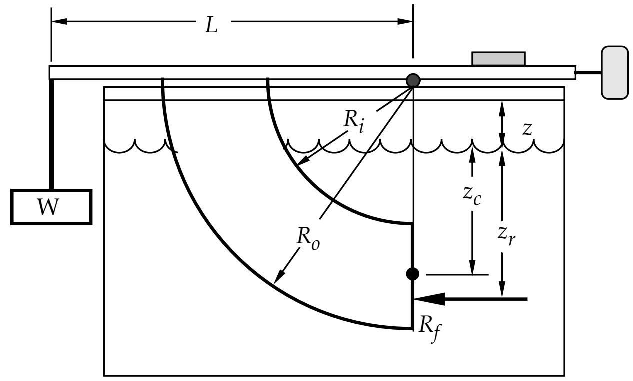
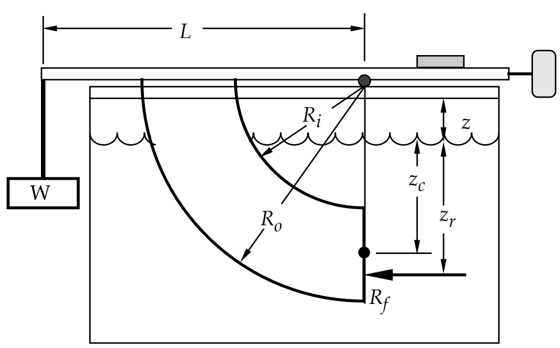


FIGURE P2.53

Following are data obtained on this apparatus, which shows the actual weight required to balance the lever arm while the torus is submerged. Substitute appropriately, and determine if the preceding equations accurately predict the weight *W*.

$L = 33.6$ cm $\quad h = 10.2$ cm $\quad w = 7.6$ cm
tank bottom to bottom of torus = 8.9 cm = d_2
pivot to tank bottom $d_1 = 31.8$ cm $\quad W = 5.4$ N $\quad z = 6.05$ cm



The submerged plane area is $A = hw = 0.007\,742$ m². The distance from the free surface to the centroid of the plane is

$$z_c = d_1 - z - d_2 - h/2 = 31.8 - 6.05 - 8.9 - 10.2/2 = 11.75 \text{ cm}$$

$$R_f = \rho g z_c A = 1\,000(9.81)(0.117\,5)(0.007\,742) = 8.92 \text{ N}$$

$$I_{xxc} = \frac{wh^3}{12} = \frac{0.076(0.102)^3}{12} = 6.66 \times 10^{-6} \text{ m}^4$$

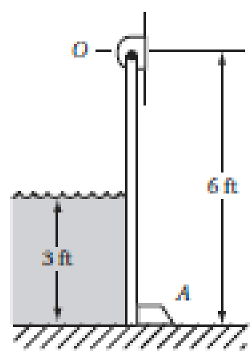
$$z_r = z_c + \frac{I_{xxc}}{z_c A} = 0.117\,5 + \frac{6.66 \times 10^{-6}}{0.117\,5(0.007\,742)} = 0.124\,8 \text{ m}$$

Summing moments about the hinge gives

$$W = R_f(z + z_r)/L = 8.9(0.060\,5 + 0.124\,8)/0.336$$

$\underline{W = 4.9 \text{ N}}$ $\qquad$ $\%\text{error} = (5.4 - 4.9)/5.4 = 0.089 \approx 9\%$

2.54 A hinged rectangular door 2 ft wide is free to rotate about point O but is held securely by a block at A. On the left side of the door is seawater filled to a depth of 3 ft. Find the restraining force at A. (See Figure P2.54.)



$R_f = \rho g z_c A$; $z_c = 1.5$ ft; $A = 3(2) = 6$ ft^2 ; $\rho = 1.03(1.94)$

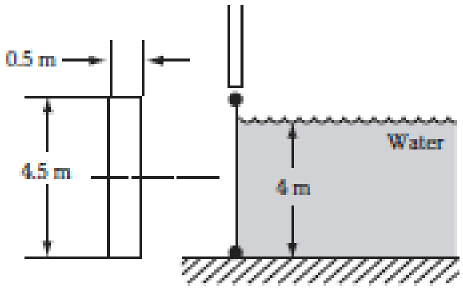
$R_f = 1.03(1.94)(32.2)(1.5)(6) = \boxed{579 \text{ lbf}}$

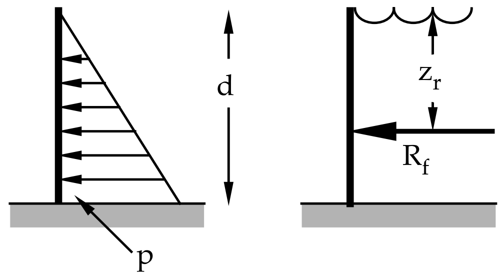
$z_r = z_c + \dfrac{I_{xxc}}{z_c A}$; $I_{xxc} = \dfrac{bh^3}{12} = \dfrac{2(3)^3}{12} = 4.5$;

$z_r = 1.5 + \dfrac{4.5}{1.5(6)} = 2$ ft; Σ M about hinge; $F_A(6) = 579(3+2)$;

$\boxed{F_A = 482 \text{ lbf}}$

2.55 Figure P2.55 shows a rectangular retaining door holding water. Sketch the pressure prism and determine the resultant force on the door and its location.





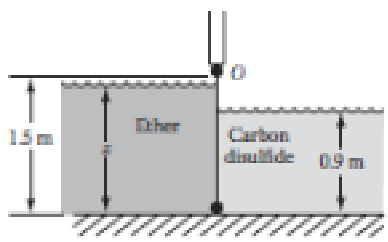
$p = \rho g d = 1\,000(9.81)(4) = 39.24$ kPa at bottom

$R_f = \rho g z_c A$; $z_c = 2$ m; $A = 4(0.5) = 2$ m^2 ; $R_f = 1\,000(9.81)(2)(2) = \boxed{39.24 \text{ kN}}$

$$z_r = z_c + \frac{I_{xxc}}{z_c A}\,;\; I_{xxc} = \frac{bh^3}{12} = \frac{0.5(4)^3}{12} = 2.67;\; z = 2 + \frac{2.67}{2(2)}$$

$\boxed{z_r = 2.66 \text{ m}}$

2.56 A rectangular gate is used to separate carbon disulfide from liquid ether as shown in Figure P2.56. The gate is 0.4 m wide. Determine the height of ether that is required to balance the force exerted by the carbon disulfide such that the moment about O is zero.



Properties from Table A-5; ether $\rho_e = 0.715(1\,000) = 715$ kg/m^3 ;

$\rho_{CS2} = 1\,265$ kg/m^3 ; $R_f = \rho_e g z_c A$; $z = \dfrac{z}{2}$; $A = 0.4z$;

$R_f = 715(9.81)(z/2)(0.4z) = 1\,402z^2$ for ether

$$z_r = z_c + \frac{I_{xxc}}{z_c A}\,;\; I_{xxc} = \frac{bh^3}{12} = \frac{0.4z^3}{12}\,;\text{ so}$$

$$z_r = \frac{z}{2} + \frac{0.4z^3}{12}\,\frac{1}{\frac{z}{2}(0.4z)} = \frac{z}{2} + \frac{z}{6} = \boxed{\frac{2z}{3} = z_r \text{ for ether}}$$

For CS_2, $R_f = 1\,265(9.81)(0.9/2)(0.9\cdot 0.4) = \boxed{2\,010 \text{ N} = R_f \text{ for } CS_2}$

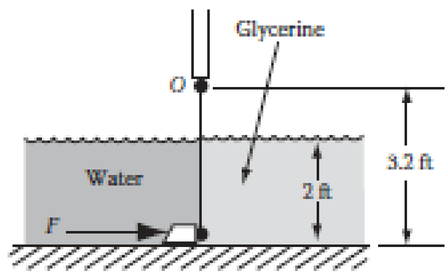
$z = \dfrac{2}{3}0.9$; $\boxed{z_r = 0.6 \text{ m}}$ (as seen above for a rectangular gate)

Summing moments,

$1\,402z^2(1.5 - (z - 2z/3)) = 2\,010(1.5 - (0.9 - 0.6))$

$z^2(1.5 - z/3) = 1.721$; solving by trial and error gives $\boxed{z = 1.26 \text{ m}}$

2.57 A rectangular gate 1.5 ft wide is used as a partition to separate glycerine and water as shown in Figure P2.57. A stop is located on the floor of the water side of the gate. Calculate the force required to hold the door closed.



For water, $R_f = \rho g z_c A = 1.94(32.2)(1)(2\cdot 1.5) = 187$ lbf

$z_r = z_c + \dfrac{I_{xxc}}{z_c A}$; $\quad I_{xxc} = \dfrac{bh^3}{12} = \dfrac{1.5(2)^3}{12} = 1$; $\quad z_r = 1 + \dfrac{1}{1(3)} = 1.333$ both sides

For glycerine, $R_f = \rho g z_c A = 1.263(1.94)(32.2)(1)(3) = 236.7$ lbf

Sum moments, $F(3.2) + 187(1.2 + 1.333) = 236.7(1.2 + 1.333)$

$\boxed{F = 39.3 \text{ lbf}}$

2.58 A rectangular gate is 9 ft high and 8 ft wide (into the page) and separates reservoirs of fresh water and oil. If the oil has a specific gravity of 0.85, determine the depth *h* required to make the reaction at B equal to 0. See Figure P2.58.

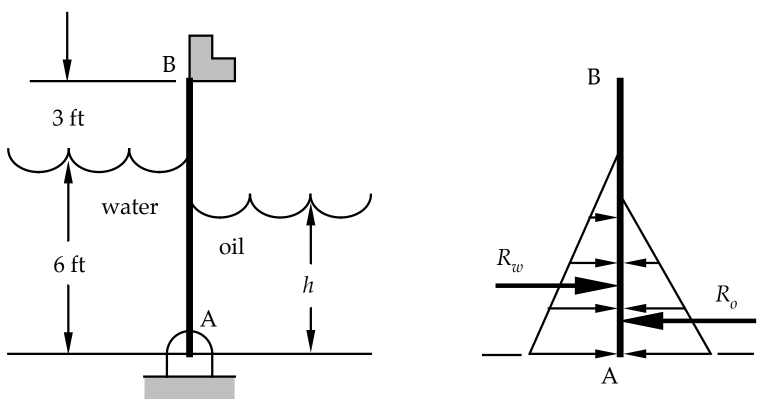


FIGURE P2.58

The pressure prisms and equivalent forces are shown. Eventually we will need to sum moments about the hinge; but before doing so, we have to evaluate the forces. Because the gate is rectangular, the forces will act at distances that are 2/3 of the depth of each respective fluid. From hydrostatics, the water force is

$$R_w = \frac{\rho g}{g_c} z_c A = \frac{62.4(32.2)}{32.2}\left(\frac{6}{2}\right)(6)(8)$$

The oil force is

$$R_o = \frac{\rho g}{g_c} z_c A = \frac{62.4(0.85)(32.2)}{32.2}\left(\frac{h}{2}\right)(h)(8)$$

Taking the ratio of these forces gives

$$\frac{R_o}{R_w} = \frac{0.85h^2}{36}$$

A second equation that relates these forces is obtained by summing moments about the hinge:

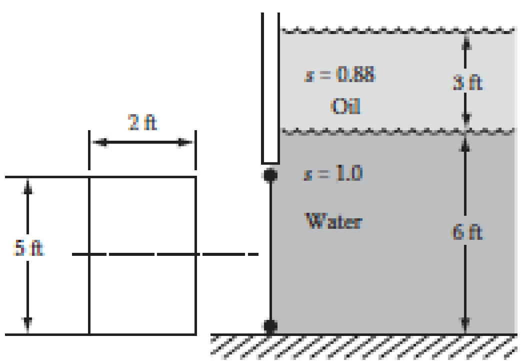
$$\Sigma M_{hinge} = 0 = 2R_w - R_o\frac{h}{3} \qquad \text{which gives} \qquad \frac{R_o}{R_w} = \frac{6}{h}$$

Combining with the previous equation yields $6 = \dfrac{0.85h^3}{36}$ or

$\boxed{h = 6.34 \text{ ft}}$ deeper than the water.

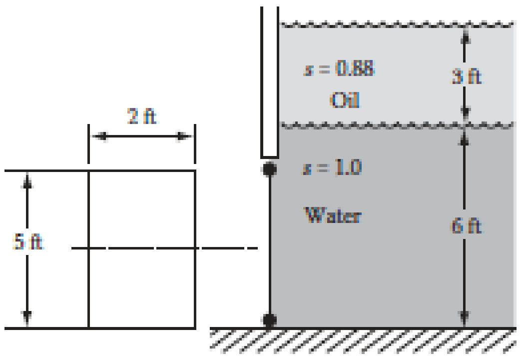
2.59 Figure P2.59 shows a rectangular gate holding water. Floating atop the water is a layer of oil. For the dimension shown,

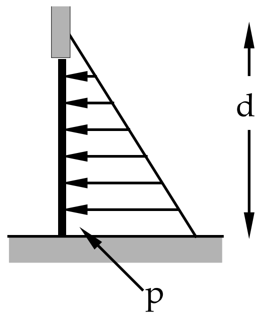
a. Sketch the pressure prism for water only in contact with the door
b. Sketch an additional pressure distribution due to the oil
c. Determine the magnitudes of the resultant forces

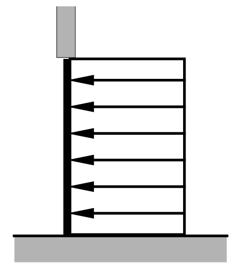


a) $p = \rho g d = 1.94(32.2)(6) = 375$ psf
for the water in contact with the gate, measure from the interface:
$R_f = \rho g z_c A$; $z_c = 1 + 5/2 = 3.5$ ft; $A = 2(5) = 10$ ft^2 ;
$R_f = 1.94(32.2)(3.5)(10) = 2186$ lbf

$$z_r = z_c + \frac{I_{xxc}}{z_c A}; \; I_{xxc} = \frac{bh^3}{12} = \frac{2(5)^3}{12} = 20.8; \; z_r = 4.09 \text{ ft}$$





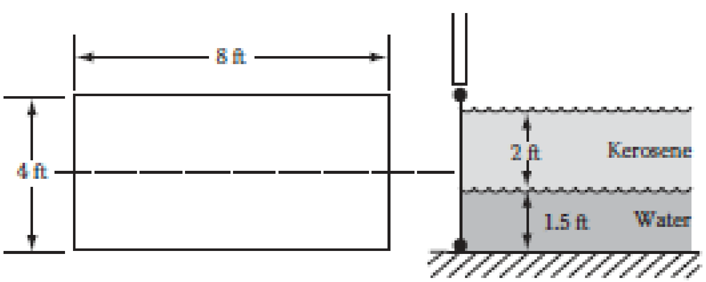


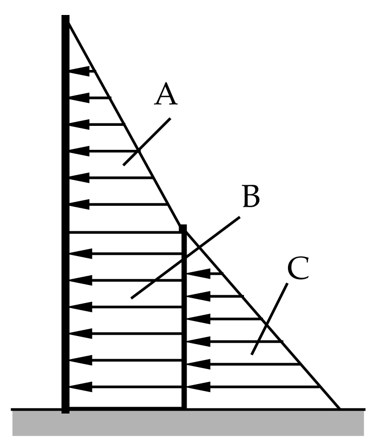
for the oil on top, a constant overpressure is exerted:
$p = \rho g d = 0.88(1.94)(32.2)(3) = 187.4$ psf uniform due to oil
$R_{fo} = \rho g(3)A = 187.4(10) = 1874$ psf; $z = 1 + 5/2 = 3.5$ ft from surface
of water (midpoint of door)

Now sum moments about point where water surface intersects wall:
$$\underset{\text{(water)}}{2186(4.09)} + \underset{\text{(oil)}}{1874(3.5)} = \underset{\text{(total force)}\cdot\text{(average distance)}}{(2186 + 1874)z_{avg}}$$
Solving, $\boxed{z_{avg} = 3.81 \text{ ft from water surface}}$ or
$\boxed{z_{avg} = 2.81 \text{ ft from top of door}}$

2.60 A rectangular gate is holding water that has a layer of kerosene over it. With both liquids in contact with the gate as shown in Figure P2.60:
a. Sketch the pressure prisms
b. Determine the magnitude of the resultant forces

Draw pressure profiles; separate profiles for each component





Profile A is due to kerosene in contact with the gate; for A,
$R_f = \rho g z_c A$; $z_c = 1$ ft; $A = 2(8) = 16$ ft^2
$R_f = 0.823(1.94)(32.2)(1)(16) = \boxed{822 \text{ lbf} = R_f \text{ for A}}$
Profile B is due to kerosene over water; $R_f = pA = 0.823(1.94)(32.2)(24)$

$\boxed{R_f = 1230 \text{ lbf for B}}$

Profile C is due to water in contact with the gate; $R_f = \rho g z_c$; $z = \frac{1.5}{2}$;

$A = 1.5(8)$; so $R_f = 1.94(32.2)(1.5/2)(1.5)(8) = \boxed{562 \text{ lbf} = R_f \text{ for C}}$

$\boxed{\text{Total force on gate} = 2620 \text{ lbf}}$

2.61 A rectangular gate 1.6 m wide is used as a partition as shown in Figure P2.61. One side has linseed oil filled to a depth of 10 m. The other has two fluids—castor oil over water. Determine the depth of castor oil required to keep the door stationary (no moment about point O).

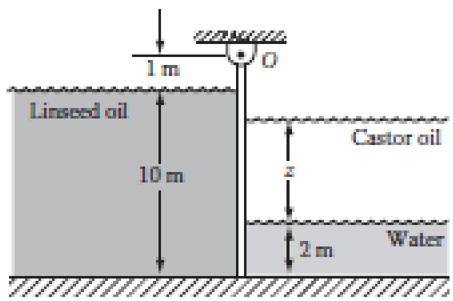


Table A-5 for fluid properties; linseed oil,

$R_f = \rho g z_c A = 0.93(1\,000)(9.81)(5)(10 \cdot 1.6) = 729.8$ kN

$$z_r = z_c + \frac{I_{xxc}}{z_c A} = 5 + \frac{1.6(10)^3}{12}\frac{1}{5(10 \cdot 1.6)} = 6.67 \text{ m}$$

Castor oil $R_f = \rho g z_c A = 0.96(1\,000)(9.81)(z/2)(1.6z) = 7.534z^2$ kN

$$z_r = z_c + \frac{I_{xxc}}{z_c A} = \frac{z}{2} + \frac{1.6z^3}{12}\frac{1}{(z/2)(1.6z)} = \frac{z}{2} + \frac{z}{3} = \frac{2z}{3}$$

Castor oil on water, $R_f = \rho g z_c A = 960(9.81)z(1.6z) = 15.068z^2$ kN

$z_r = z + 1$;

Water, $R_f = \rho g z_c A = 1\,000(9.81)(1)(2 \cdot 1.6) = 31.392$ kN

$$z_r = z_c + \frac{I_{xxc}}{z_c A} = 1 + \frac{1.6(2)^3}{12}\frac{1}{1(2 \cdot 1.6)} = 4/3 \text{ m}$$

Sum moments about O:

$729.9(6.67 + 1) = (7.534z^2)(11 - 2 - z/3) + 15.07z^2(11 - 1) + 31.4(11 - 2/3)$

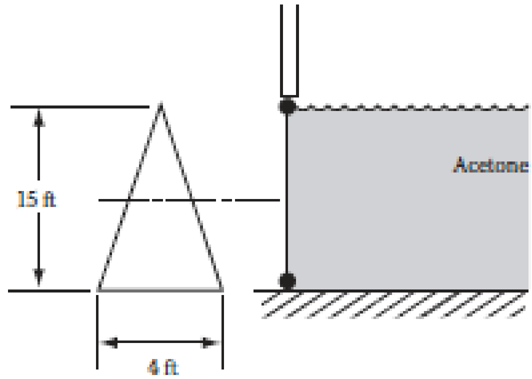
$5598.1 = 67.81z^2 - 2.511z^3 + 150.68z^2 + 324.4$

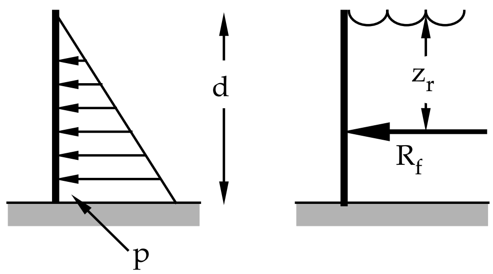
$2.511z^3 - 218.5z^2 + 5273.7 = 0$ or $\boxed{z^3 - 87.01z^2 + 2100 = 0}$ Trial & error

Assume	LHS
z = 5	49.95
5.5	-365.5
5.1	-30.28
5.06	1.98 close enough

$\boxed{z = 5.06 \text{ m}}$

2.62 Figure P2.62 shows triangular retaining door holding acetone. Sketch the pressure prism and determine the resultant force on the door and its location.



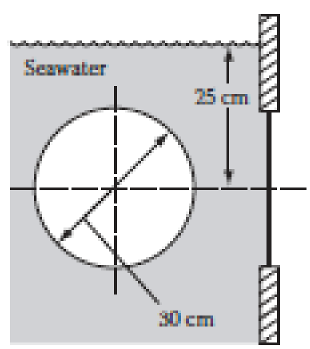


$\rho = 0.787(1.94)$; $p = \rho g d = 0.787(1.94)(32.2)(15) = 737$ psf bottom
$R_f = \rho g z_c A$; $z_c = 2h/3 = 10$ ft; $A = bh/2 = 30$ ft^2 ;
$R_f = 0.787(1.94)(32.2)(10)(30) = \boxed{14700 \text{ lbf}}$

$$z_r = z_c + \frac{I_{xxc}}{z_c A}\,;\; I_{xxc} = \frac{bh^3}{36} = \frac{4(15)^3}{36} = 375$$

$$z_r = 10 + \frac{375}{10(30)} = \boxed{11.25 \text{ ft}}$$

2.63 A porthole in the wall of a loaded ship is just below the surface of the water as shown in Figure P2.63. Sketch the pressure prism for the window, determine the magnitude of the resultant force, and find its location.



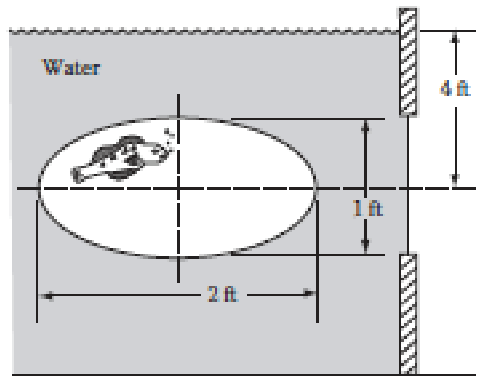
$\rho = 1\,030$ kg/m^3 for sea water; $p = \rho g z$; $p_1 = \rho g(0.25 - 0.15)$
$p_1 = 1\,030(9.81)(0.1) = 101.0$ kPa; $p_2 = 1\,030(9.81)(0.25 + 0.15) = 404.2$ kPa
$R_f = \rho g z_c A$; $z_c = 25$ cm $= 0.25$ m; $A = \pi D^2/4 = \pi(0.3)^2/4 = 0.0707$ m^2 ;
$R_f = 1\,030(9.81)(0.25)(0.0707)$;

$\boxed{R_f = 178.6\text{ N}}$ $\quad I_{xxc} = \dfrac{\pi R^4}{4} = \pi(0.15)^4/4 = 3.976 \times 10^{-4}$

$$z_r = z_c + \frac{I_{xxc}}{z_c A} = 0.25 + \frac{3.976 \times 10^{-4}}{0.25(0.0707)}$$

$\boxed{z_r = 0.2725\text{ m}}$

2.64 An aquarium is designed such that a viewing window is elliptical, as shown in Figure P2.64. Sketch the pressure prism for the window, determine the magnitude of the resultant force, and find its location.



$\rho = 1.94\text{ slug/ft}^3$; $p = \rho g z$; $p_1 = 1.94(32.2)(4 - 1) = 187.4\text{ psf}$

$p_2 = 1.94(32.2)(4 + 1) = 312.3\text{ psf}$; $R_f = \rho g z_c A$; $z_c = 4\text{ ft}$; $A = \pi ab$

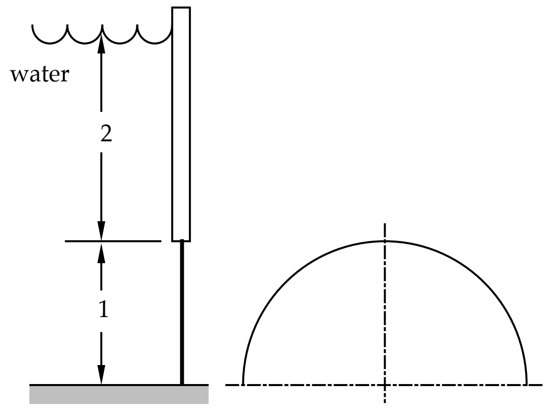
$A = \pi(2/2)(1/2) = 1.571\text{ ft}^2$; $R_f = 1.94(32.2)(4)(1.571) =$ $\boxed{392.5\text{ lbf} = R_f}$

$$I_{xxc} = \frac{\pi ab^3}{4} = \pi(2/2)(1/2)^3/4 = 0.09817$$

$$z_r = z_c + \frac{I_{xxc}}{z_c A} = 4 + \frac{0.09817}{4(1.571)};$$

$\boxed{z_r = 4.02\text{ ft}}$

2.65 A semicircular gate located in a wall is in contact with 3 ft of water, as shown in Figure P2.65. Sketch the pressure prism for the window, determine the magnitude of the resultant force, and find its location.



The centroid of the gate is located at a distance of $4r/3\pi$ from the bottom. From the free surface,

$z_c = 3 - 4(1)/3\pi = 2.58$ m

The force is $R_f = \rho g z_c A$

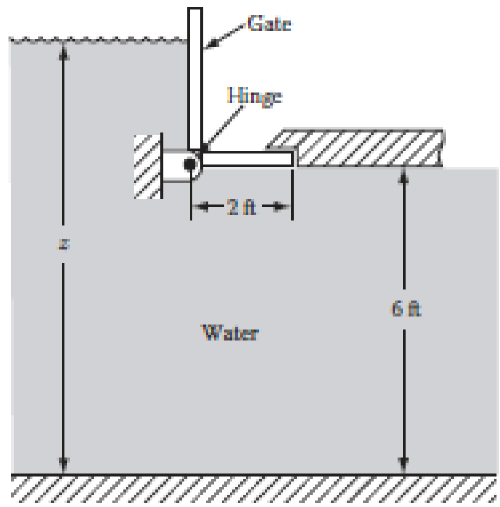
For this gate, $A = \pi r^2/2 = \pi(1)^2/2 = 1.57$ m^2

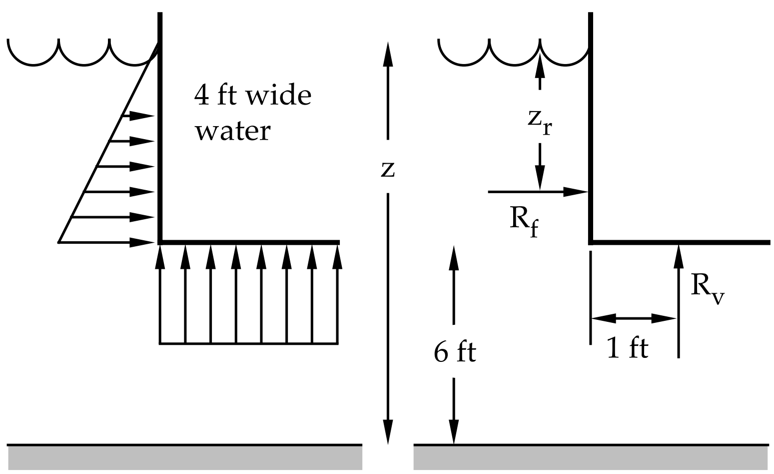
The second moment is $I_{xxc} = \pi r^4/8 = \pi(1)^4/8 = 0.393$ m^4

The location of the force is $z_r = z_c + \dfrac{I_{xxc}}{z_c A} = 2.58 + \dfrac{0.393}{2.58(1.57)}$

$\boxed{z_r = 2.67 \text{ m from the free surface}}$

2.66 Figure P2.66 shows a 4-ft-wide (into the page) gate that has an L-shaped cross section and is hinged at its corner. The gate is used to ensure that the water level does not get too high. Once the level rises over a certain point, the fluid forces acting on the gate tend to open it and release some liquid. Determine the height z above which the gate tends to open.





Gate is 4 ft wide and the fluid is water; FBD:

$R_f = \rho g z_c A \sin\theta$; $z_c = \dfrac{z - 6}{2}$; $A = (z - 6)4$; $\theta = 90°$

$R_f = 1.94(32.2)((z\text{-}6)/4)(z\text{-}6)(4) = \boxed{125(z - 6)^2}$

To find z_r, evaluate $I_{xxc} = \dfrac{bh^3}{12} = \dfrac{4(z\text{-}6)^3}{12}$; $z_r = z_c + \dfrac{I_{xxc}}{z_c A}$

$$z_r = \frac{z - 6}{2} + \frac{4(z\text{-}6)^3}{12((z - 6)/2)(z - 6)4} = \frac{z - 6}{2} + \frac{z - 6}{6}$$

$\boxed{z_r = \dfrac{2(z - 6)}{3}}$ To find R_v , find pressure at bottom of vertical portion,

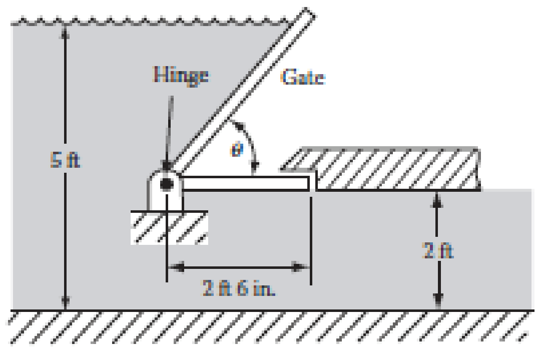
$p = \rho_{H2O} g(z - 6) = 1.94(32.2)(z - 6) = 62.5(z - 6)$
$R_v = pA$ where $A = 2(4)\ ft^2$
$R_v = 62.5(2)(4)(z - 6) = 500(z - 6)$; R_v acts at centroid of horizontal portion of gate, 1 ft from hinge. Sum moments about hinge: $\Sigma M = 0$ for gate about to open: $R_f(z - 6 - z_r) - 500(z - 6) = 0$; $R_f(z - 6 - \dfrac{2(z - 6)}{3}) = 500(z - 6)$
$R_f\ ((z - 6)/3) = 500(z - 6)$ or $R_f = 1500$; Substitute in R_f equation:

$R_f = 125(z - 6)^2 = 1500$; $(z - 6)^2 = 12$; $(z - 6) = 3.46$;

$\boxed{z = 9.46 \text{ ft gate about to open}}$ when $z > 9.46$ gate will open

---

2.67 Figure P2.67 shows a hinged gate used as a retainer for castor oil. The liquid depth to the horizontal portion of the gate is 2 ft, and the gate itself is to be designed so that the oil depth does not exceed 5 ft. When the depth is greater than 5 ft, the fluid forces act to open the gate, and some oil escapes through it. The gate is 1.2 ft wide (into the page). Determine the angle θ required for the gate to open when necessary.



Castor Oil $\rho = 0.960(1.94)$ slug/ft^3 Table A-5; 1.2 ft wide gate

$R_f = \rho g z_c A \sin\theta$; $z = \dfrac{3 \text{ ft}}{2 \sin\theta}$; $A = \dfrac{3(1.2)}{\sin\theta}$;

$R_f = 0.96(1.94)(32.2)(1.5/\sin\theta)(3.6/\sin\theta)(\sin\theta)$; so $R_f = \dfrac{324}{\sin\theta}$

To find z_r, first evaluate $I_{xxc} = \dfrac{bh^3}{12} = \dfrac{1.2(3/\sin\theta)^3}{12} = \dfrac{2.7}{\sin^3\theta}$

$z_r = \dfrac{3}{2\sin\theta} + \dfrac{0.5}{\sin\theta} = \dfrac{2}{\sin\theta}$; the force R_v is pressure at 3 ft depth x area where area is (2 ft 6 in.)(1.2 ft) = 3 ft^2; $R_v = \rho g(3)(3) = 0.96(1.94)(32.2)(9)$

$R_v = 539$ lbf acting at 1.25 ft from hinge. Next sum moments about hinge; $\Sigma M = 0$ when gate is about to open:

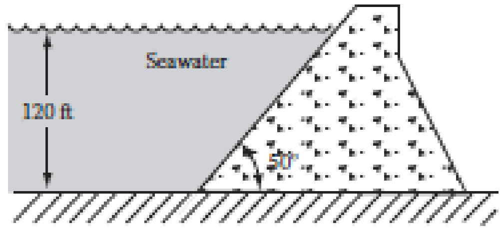
$R_f\left(\dfrac{3}{\sin\theta} - z_r\right) - R_v(1.25 \text{ ft}) = 0$; substituting,

$\dfrac{324}{\sin\theta}\left(\dfrac{3 - 2}{\sin\theta}\right) = 539(1.25)$; $\sin^2\theta = \dfrac{324}{539(1.25)} = 0.48$;

$\sin\theta = \pm 0.693$ and we reject the negative value as having no physical meaning. So

$\boxed{\theta = 43.9°}$ (Note: the interested reader may verify that this angle is independent of the width into the page.)

2.68 A dam is constructed as in Figure P2.68. Determine the resultant force and its location acting on the inclined surface. Perform the calculations assuming a unit width into the page.



z coordinate measured along slanted side $R_f = \rho g z_c A$; $\rho = 1.03(1.94)$ slug/ft^3 from Table A-5; area $A = \dfrac{1(120)}{\sin\theta}$; $z_c = \dfrac{60}{\sin\theta}$ so

$R_f = 1.03(1.94)(32.2)\dfrac{60}{\sin 50}\dfrac{120}{\sin 50}\sin 50$; $\boxed{R_f = 6.05 \times 10^5 \text{ lbf}}$

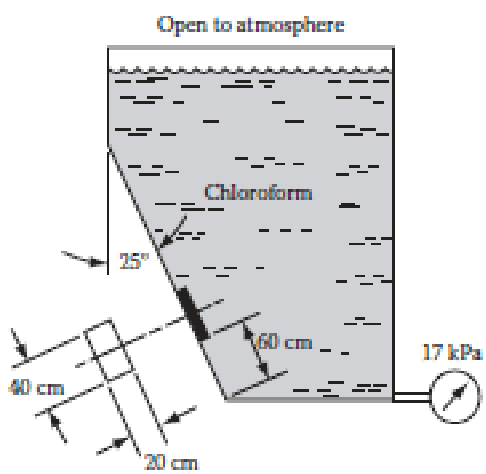
$z_r = z_c + \dfrac{I_{xxc}}{z_c A}$; $I_{xxc} = \dfrac{bh^3}{12} = \dfrac{1(120/\sin 50)^3}{12} = 3.2 \times 10^5 \text{ ft}^4$

$z_r = \dfrac{60}{\sin 50} + \dfrac{3.2 \times 10^5}{(60/\sin 50)(120/\sin 50)}$

$\boxed{z_r = 104.4 \text{ ft along surface of dam}}$ or $z_r = 104.4(\sin 50)$

$\boxed{z_r = 80 \text{ ft straight down}}$

2.69 A tank having one inclined wall contains chloroform as shown in Figure P2.69. The inclined wall has a rectangular plug 20 cm wide by 40 cm high. Determine the force exerted on the plug if a pressure gauge at the tank bottom reads 17 kPa (gauge).



$p = \rho g z$; $p = 17$ kPa; $\rho = 1.47(1\,000)$ (Table A-5); $z = \dfrac{p}{\rho g} = \dfrac{17\,000}{1\,470(9.81)}$

or $z = 1.18$ m vertical depth of liquid.

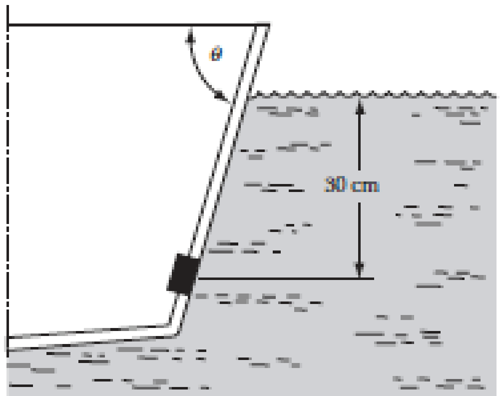
$R_f = \rho g z A \sin\theta$; z_c measured from free surface along incline

$z_c = \dfrac{1.18}{\sin 65} - 0.06 - 0.2 = 0.5$ m; $A = 0.4(0.2) = 0.08$ m^2

$R_f = 1\,470(9.81)(0.5)(0.08) \sin 65°$

$\boxed{R_f = 523 \text{ N}}$

2.70 A small, narrow, flat-bottomed fishing boat is loaded down with two crew members and a catch of fish. The rear of the boat is a plane inclined at an angle u of 758 with the horizontal. Thirty centimeters below the water surface is a hole in the boat plugged with a cork that is 8 cm in diameter. Determine the force and its location on the cork due to the saltwater if the liquid density is 1025 kg=m3. (See Figure P2.70.)



$R_f = \rho g z_c A \sin\theta$; $\quad z_c = \dfrac{0.30}{\sin\theta} = 0.311$ m; $\quad A = \pi(0.08)^2/4 = 0.005$ m^2

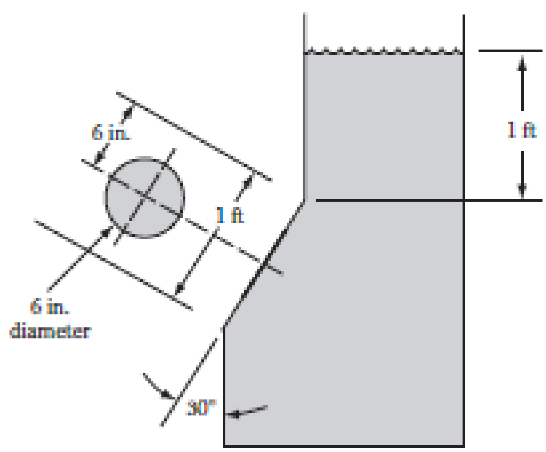
$R_f = (1\,025)(9.81)\left(\dfrac{0.30}{\sin\theta}\right)(0.005)(\sin\theta)$; or $\boxed{R_f = 15.1 \text{ N}}$

$$I_{xxc} = \frac{\pi D^4}{64} = \frac{\pi(0.08)^4}{64} = 2.01 \times 10^{-6} \text{ m}^4$$

$$z_r = z_c + \frac{I_{xxc}}{z_c A} = 0.311 + \frac{2.01 \times 10^{-6}}{(0.311)(0.005)} \text{ or}$$

$\boxed{z_r = 31.2 \text{ cm}}$

2.71 Figure P2.71 shows a tank that contains hexane. A circular gate in the slanted portion of one wall is 6 in. in diameter. Determine the resultant force acting on the gate and its location.



$\rho = 0.657(1.94 \text{ slug/ft}^3)$ from Appendix Table A-5

$R_f = \rho g z_c A \sin\theta$; θ is with horizontal in this equation; therefore $\theta = 60°$

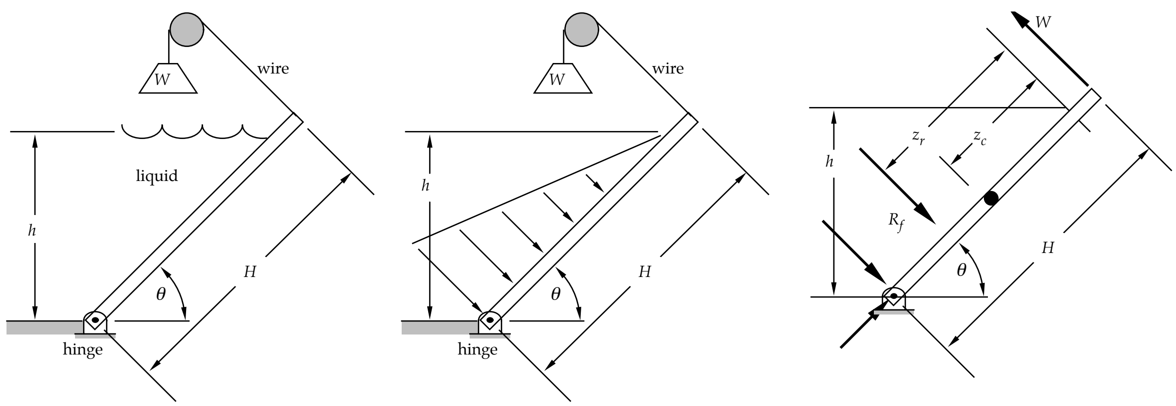
z_c along slant of surface; $z_c = 1/\sin 60° + 6/12 = 1.65$ ft

$$A = \frac{\pi D^2}{4} = \frac{\pi(6/12)^2}{4} = 0.19\ \text{ft}^2; \qquad R_f = 0.657(1.94)(32.2)(1.65)(0.19)\sin 60°$$

$$\boxed{R_f = 11.1\ \text{lbf}} \qquad I_{xxc} = \frac{\pi D^4}{64} = \frac{\pi(6/12)^4}{64} = 0.003067$$

$$z_r = z_c + \frac{I_{xxc}}{z_c A} = 1.65 + \frac{0.003067}{1.65(0.19)}; \quad \text{Solving,} \qquad \boxed{z_r = 1.66\ \text{ft}}$$

2.72 Figure P2.72 shows a rectangular gate hinged at the bottom, and filled to a depth of $h = 1$ m with castor oil. The gate width into the page is 0.5 m. At the top of the gate is a wire that goes over a pulley and is attached to a weight. What is the magnitude of the weight W required to hold the gate in position? The angle θ is 45°, and the distance $H = 1.8$ m.



The pressure prism acting on the gate, and the free body diagram are shown. The force due to the liquid is

$$R_f = \rho g z_c A$$

Along the slant, we calculate the lengthe of the plate in contact with liquid as

$$\frac{h}{\sin 45°} = \frac{1}{0.707} = 1.414\ \text{m}$$

where $z_c = 1.414/2 = 0.707$ m measured from the surface of the liquid along the gate.

$A = 1.414(0.5) = 0.707\ \text{m}^2$
$\rho = 0.960(1\ 000)\ \text{kg/m}^3$

Substituting,

$$R_f = 960(9.81)(0.707)(0.707) = 4\ \ 707\ \text{N}$$

The line of action of this force acts at $z_r = z_c + \dfrac{I_{xxc}}{z_c A}$

The second moment of the portion of the gate in contact with the liquid is

$$I_{xxc} = \frac{w(1.414)^3}{12} = \frac{0.5(1.414)^3}{12} = 0.118\ \text{m}^4$$

The location of the force then is

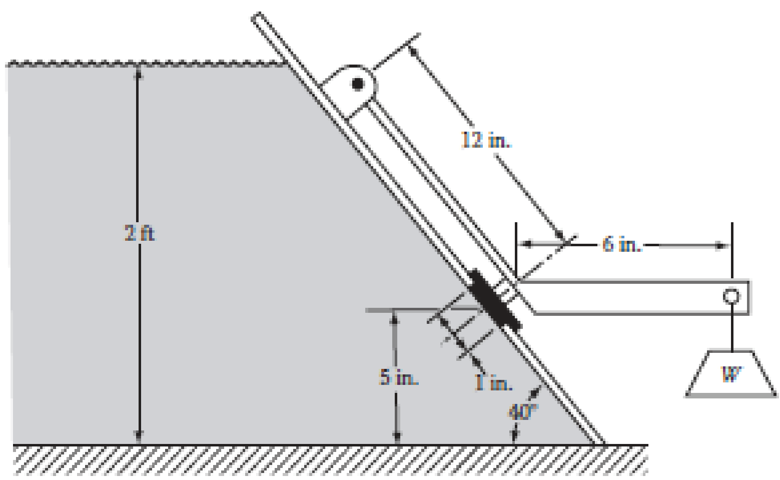
$$z_r = 0.707 + \frac{0.118}{(0.707)(0.707)} = 0.943 \text{ m}$$

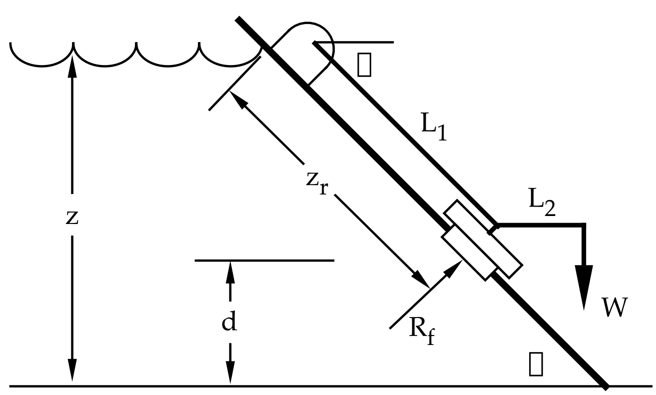
This distance is measured from the free surface of the liquid. The free surface of the liquid as measured along the gate is at a distance $H - h/\sin\theta = 1.8 - 1.414 = 0.386$ m from the wire. From the hinge, the line of action of R_f is at $H - 0.386 - z_r = 1.8 - 0.386 - 0.943 = 0.471$ m. Summing moments about the hinge,

$$R_f(0.471) = W(1.8) \qquad W = 4\,707(0.471/1.8)$$

$$\boxed{W = 1\,232 \text{ N}}$$

2.73 Figure P2.73 shows an inclined wall of a tank containing water. The tank wall contains a plug held in place by a weight. The liquid depth can be maintained constant in the tank, and the depth can be controlled by the amount of weight W that is used. For the conditions shown, determine the weight required to keep the plug in place.





Liquid ρ, area for R_f is $A = \pi D^2/4$; $R_f = \rho g z_c A \sin\theta$; $z_c = \dfrac{z - z/n}{\sin\theta}$

Given: $z = 2$ ft; $d = 5/12$ ft; $\theta = 40°$; $L_2 = 6/12$ ft; $L_1 = 12/12$ ft
$\rho = 1.94$ slug/ft^3; $A = \pi D^2/4 = \pi(1/12)^2/4 = 0.00545$ ft^2
$z_c = (2 - 5/12)/\sin 40° = 2.46$ ft
$R_f = \rho g z_c A \sin\theta = 1.94(32.2)(2.46)(0.00545) \sin 40°$
$R_f = 0.539$ lbf

$$z_r = z_c + \frac{I_{xxc}}{z_c A}; \; I_{xxc} = \frac{\pi R^4}{4} = \frac{\pi(D/2)^4}{4} = \frac{\pi D^4}{64}$$

$$I_{xxc} = \frac{\pi(1/12)^4}{64} = 2.37 \times 10^{-6}$$

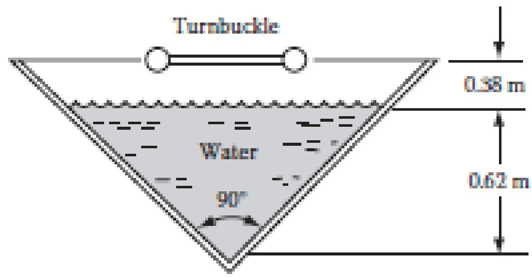
$$z_r = 2.46 + \frac{2.37 \times 10^{-6}}{2.46(0.00545)} = 2.46 \text{ ft}$$

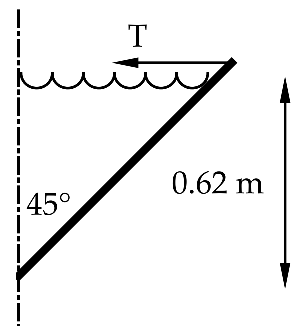
W acts at $L = L_1\cos\theta + L_2$ from hinge; $L = (12 \cos 40° + 6)/12 = 1.27$ ft
sum moments about hinge = 0

$$R_f(12/12) - WL = 0; \; W = \frac{R_f(1)}{L} = \frac{0.539}{1.27} \text{ or}$$

$\boxed{W = 0.424 \text{ lbf}}$

2.74 A trough formed by two sides of wood is used to convey water. Every 0.8 m, a turnbuckle and wire are attached to support the sides. Calculate the tension in the wire using the data in Figure P2.74.





The z coordinate is measured along slant. Consider only half of trough:

$$R_f = \rho g z_c A \sin\theta; \; \rho = 1\,000 \text{ kg/m}^3; \; z_c = \frac{0.31}{\sin 45}$$

and $A = 0.8(0.62)/\sin 45$

$R_f = 1\,000(9.81)(0.31/\sin 45)(0.8)(0.62) = 2\,130$ N

$$z_r = z_c + \frac{I_{xxc}}{z_c A}; \; I_{xxc} = \frac{bh^3}{12} = \frac{0.8}{12}\frac{0.62}{(\sin 45)^3} = 0.044\,9 \text{ m}^4$$

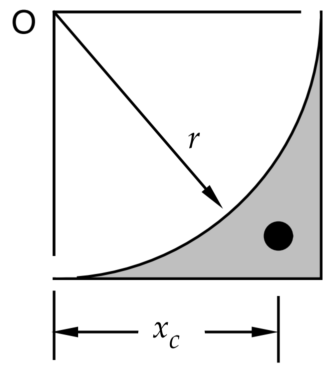
$$z_r = \frac{0.31}{\sin 45} + \frac{0.044\,9}{\frac{0.62}{\sin 45}\frac{0.8(0.62)}{\sin 45}} = 0.511 \text{ m along surface}$$

Sum moments about bottom:

$$2\,130\left(\frac{0.62}{\sin 45} - 0.511\right) = T(1);$$ $T = 780$ N; but this is due to a resultant force acting on only one side of the trough. For a two sided trough, the total tension is equal to 2(780); or,

$\boxed{T = 1\,560 \text{ N}}$

2.75 Shown in Figure P2.75 is an ex-quarter circle area, and it is desired to derive an expression for the distance to the centroid. The region consists of a square area with an inscribed quarter circle removed.



From calculus, the centroid of the region may be found with:

$$x_c = \frac{1}{A}\int_0^r \frac{1}{4}\,[f(x)]^2\,dx$$

where

$$f(x) = \left(r^2 - x^2\right)^{1/2}$$

Substitute this function into the integral, perform the integration, and show that

$$\underline{x_c = \frac{2r}{3(4 - \pi)}}$$

Solution:

$$x_c = \frac{1}{A}\int_0^r \frac{1}{4}\,[f(x)]^2\,dx \qquad\qquad A = r^2 - \frac{\pi r^2}{4}$$

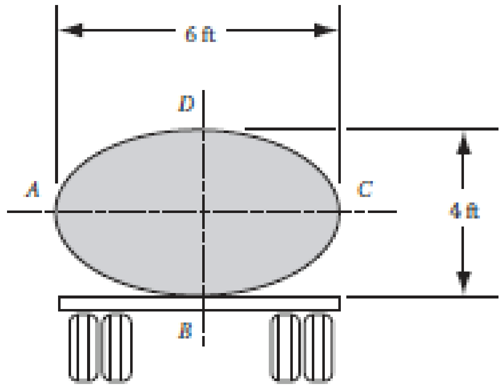
$$x_c = \frac{4}{4r^2 - \pi r^2}\int_0^r \frac{1}{4}\,[\left(r^2 - x^2\right)^{1/2}]^2\,dx = \frac{1}{4r^2 - \pi r^2}\int_0^r \left(r^2 - x^2\right)dx$$

$$x_c = \frac{1}{4r^2 - \pi r^2}\left(r^2 x - \frac{x^3}{3}\right)\Bigg|_0^r = \frac{1}{4r^2 - \pi r^2}\,\frac{2r^3}{3}$$

or

$$\underline{x_c = \frac{2r}{3(4 - \pi)}}$$

2.76 The reservoir of a tank truck is elliptical in cross section—4 ft high, 6 ft wide, and 7 ft long. Calculate the horizontal and vertical forces exerted on one of the lower quadrants of the tank when half-filled with gasoline (Figure P2.76). Take gasoline properties to be the same as those for octane.



$R_h = \rho g z_c A_v$; $\rho = 0.701(1.94)$ for octane; $z_c = 1$ ft; $A_v = 2(7)$;
$R_h = 0.701(1.94)(32.2)(1)(14)$;

$\boxed{R_h = 613 \text{ lbf}}$ $\qquad R_v = \rho g \cancel{V}$; $\cancel{V} = \dfrac{\pi abL}{4}$

$$R_v = 0.701(1.94)(32.2)\frac{\pi(3)(2)(7)}{4} = \boxed{1444 \text{ lbf} = R_v}$$

$$R = \sqrt{R_h^2 + R_v^2} = \boxed{1568 \text{ lbf} = R}$$

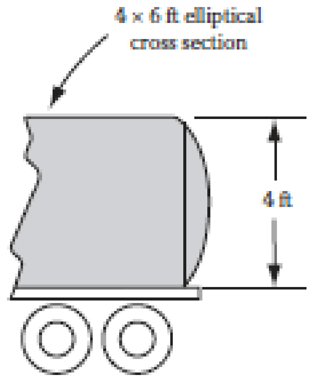
2.77 Find the horizontal and vertical forces exerted on the portion labeled ABC of the elliptical tank truck of Figure P2.76 when it is half-filled with gasoline. Take gasoline properties to be the same as those for octane. The length of the tank is 7 ft.

$R_h = 0$ due to symmetry; $R_v = \rho g \cancel{V}$; $\cancel{V} = \dfrac{\pi abL}{2}$;

$$R_v = 0.701(1.94)(32.2)\frac{\pi(3)(2)(7)}{2}$$

$\boxed{R_v = 2888 \text{ lbf}}$

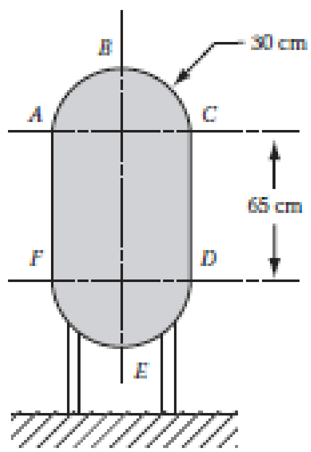
2.78 The tank truck of Figure P2.76 has ends as sketched as in Figure P2.78. Determine the horizontal force acting on the end when the tank is filled with gasoline (assume the same properties as octane).



$R_h = \rho g z_c A_v$; $\rho = 0.701(1.94)$; $z_c = 2$ ft; $A_v = \pi ab$; $R_h = 0.701(1.94)(32.2)(2)\pi(3)(2)$

$\boxed{R_h = 1650 \text{ lbf}}$

2.79 In many regions of the United States, home heating is effected by burning propane. A side view of a typical propane storage tank is sketched in Figure P2.79. Determine the forces exerted on quadrant DE for the case when the tank is filled to the top (point B). Tank length into the page is 2 m.



$\rho = 0.495(1\,000)$ kg/m^3 for propane from Table A-5; $R_h = \rho g z_c A_v$;

$z_c = 0.3 + 0.65 + 0.3/2 = 1.10$ m; $A_v = 0.3(2) = 0.6$ m^2

$R_h = 495(9.81)(1.1)(0.6) = \boxed{3.205 \text{ kN} = R_h}$

R_v is taken as being due to volume over DE only

$$R_v = \rho g \cancel{V};\ \cancel{V} = \left(\frac{\pi(0.3)^2}{4} + 0.3(0.65) + \frac{\pi(0.3)^2}{4}\right)(2) = 0.6727 \text{ m}^3$$

$R_v = 495(9.81)(0.6727) = \boxed{3.267 \text{ kN} = R_v}$

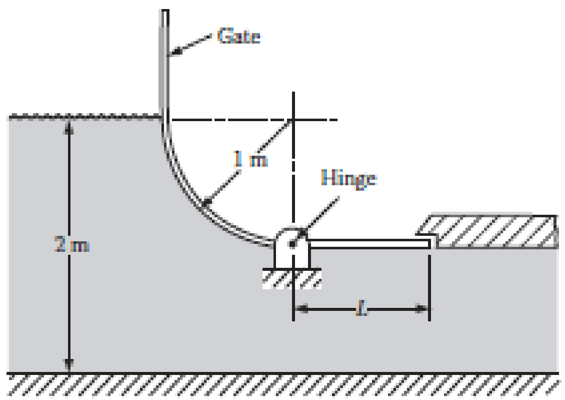
2.80 In many regions of the United States, home heating is effected by burning propane. A side view of a typical propane storage tank is sketched in Figure P2.79. Determine the forces exerted on the bottom portion labeled FED for the case when the tank is filled to the top (point B). Tank length into the page is 2 m.

$\boxed{R_h = 0}$ due to symmetry; $R_v = \rho g \cancel{V}$; $\rho = 0.495(1\,000)$

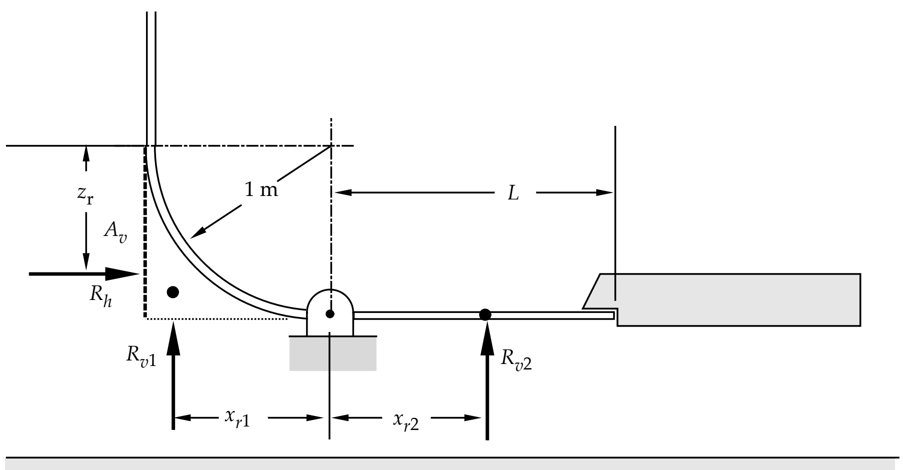
$$\cancel{V} = \left(\frac{\pi(0.3)^2}{2} + 0.6(0.65) + \frac{\pi(0.3)^2}{2}\right)(2) = 1.35 \text{ m}^3$$

$R_v = 495(9.81)(1.35) = \boxed{6\,530 \text{ N} = R_v}$

2.81 Figure P2.81 shows a seawater-retaining gate with a curved portion. When the water level gets to a certain height, the fluid forces acting on the gate open it, and the seawater passes through. The gate is 0.7 m wide (into the page) and is to be designed so that the water depth does not exceed 2 m. Determine the length *L* of the straight portion of the gate required to do this.



seawater $\rho = 1.03(1\,000)$ kg/m^3; gate width = 1 m; force diagram below.



Water depth = 2 m; $R_h = \rho g z_c A \sin\theta$; $z_c = 0.5$ m; $A = 0.7 \times 1 = 0.7$ m^2;

$\theta = 90°$; $R_h = 1.03(1\,000)(9.81)(0.5)(0.7) = 3\,537$ N; $\quad z_r = z_c + \dfrac{I_{xxc}}{z_c A}$;

$$I_{xxc} = \frac{bh^3}{12} = \frac{0.7(1)^3}{12} = 0.058\,3 \text{ m}^4;\ z_r = 0.5 + \frac{0.058\,3}{0.5(0.7)} = 0.667 \text{ m}$$

R_{v1} = weight of volume of displaced liquid so

$R_{v1} = \rho g \cancel{V} = 1.03(1\,000)(9.81)(\pi(1)^2/4)(0.7) = 5\,555$ N from Section 2.3, $x_{r1} = \dfrac{2R}{3(4-\pi)} = \dfrac{2(1)}{3(4-\pi)} = 0.777$ m

R_{v2} = pressure at 1 m depth x area; A = (L)(0.7); so

$R_{v2} = \rho g(1)(L)(0.7) = 1.03(1\ 000)(9.81)(0.7)L = 7\ 073L$ which acts at L/2

Sum moments about hinge = 0

$\Sigma M = 0$ about to open; CW is (+).

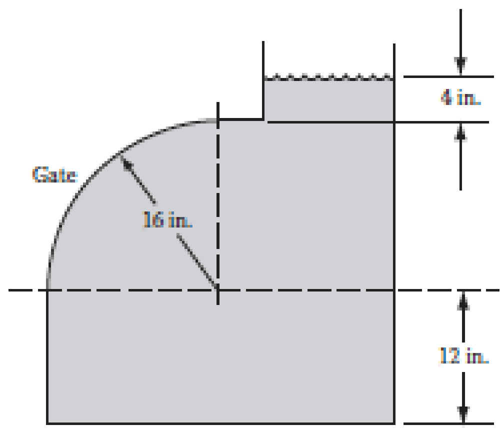
$R_h(1 - z_r) + R_{v1}(x_{r1}) - R_{v2}(L/2) = 0$

$3\ 537(1 - 0.667) + 5\ 555(0.777) = 7\ 073L^2/2$

$1\ 178 \quad + \quad 4\ 316 \quad = 3\ 537L^2$; $L^2 = 1.55$; so

$\boxed{L = 1.25 \text{ m}}$

2.82 Figure P2.82 shows the side view of a tank with a curved gate. The gate dimensions are 36 inches by 24 inches (into the page). The tank contains glycerine and is filled to a depth of 32 in. Determine the magnitude of the forces acting on the gate.



$\rho = 1.263(1.94)$ slug/ft^3 from Appendix Table A-5; Two forces R_h & R_v

horizontal force is $R_h = \rho g z_c A_v$; $\quad z_c = (4 + 16/2)/12 = 1$ ft

$A_v = (16)(24)/144 = 2.67$ ft^2 $\quad R_h = 1.263(1.94)(32.2)(1)(2.67)$

$\boxed{R_h = 210 \text{ lbf}}$

$I_{xxc} = \dfrac{bh^3}{12} = \dfrac{(24/12)(16/12)^3}{12} = 0.395$ ft^4;

$\boxed{z_r = 1 + \dfrac{0.395}{1(2.67)} = 1.15 \text{ ft}}$ from the free surface

R_v is found with: $\quad R_v = \rho g \cancel{V}$ where $\cancel{V}$ is volume above gate extending to free surface. The volume will be made up of 3 cross sections:

$\cancel{V} = \pi(16/12)^2(24/12)/4 + (4/12)(24/12) + (36/12 - 16/12)(24/12)$

$\cancel{V} = 2.79 + 0.666 + 3.33 = 6.79$ ft^3

$R_v = 1.263(1.94)(32.2)(6.79)$ or

$\boxed{R_v = 536 \text{ lbf}}$

2.83 At what acceleration must the car of Figure 2.25 be moving for the fluid to spill over the rear wall? The tank attached to the car is 6 in. long and 4 in. high. It is filled to a depth of 2 in. with linseed oil.

Refer to Fig 2.21; for fluid to tip over, z_o = height of wall - z_f = 4 - 2,

z_o = 2 in.; x_o given as 3 in. From the geometry, $\frac{z_o}{x_o} = \frac{a}{g}$; so

$a = 32.2(2/3)$; or $\boxed{a = 21.46 \text{ ft/s}^2}$

2.84 The car of Figure 2.25 is traveling at a constant velocity of 15 cm=s but goes around a curve of radius 56 cm. Viewing the car from the front, what is the equation of the surface? The tank attached to the car is 20 cm long and 10 cm high. It is filled to a depth of 5 cm with linseed oil.

Equation of surface is $z = \frac{r^2\omega^2}{2g} + z_p$; and the free surface height is

$z_p + \frac{R^2\omega^2}{4g} = 5$ cm; so $z_p = 0.05 - (0.56)^2(0.15/0.56)^2(\frac{1}{2(9.81)})$

or $z_p = 0.049$ m; $\quad z = \frac{r^2}{2g}\frac{V^2}{R^2} + z_p = r^2 \; \frac{0.15^2}{0.56^2} \; \frac{1}{2(9.81)}$

$\boxed{z = 0.003\,66\, r^2 + 0.049 \text{ where r varies from wall to wall}}$

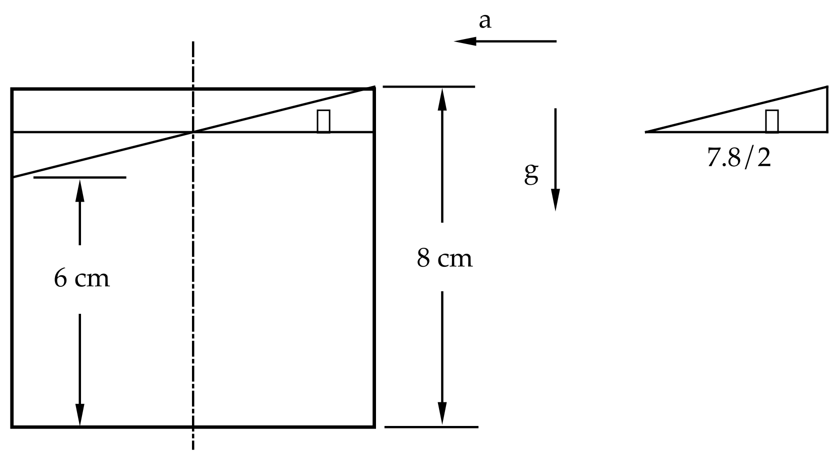
2.85 The car of Figure 2.25 is decelerating uniformly at 12 g. Determine the free surface of the liquid. The car is 6 in. long and 4 in. high. It is filled to a depth of 2 in. with linseed oil.

Analysis is reversed with $a = -g/2$; so $z = z_i - ax/g$ becomes

$z = z_i + \frac{g}{g}\frac{x}{2}$ or $\boxed{z = z_i + \frac{x}{2}}$

2.86 A passenger in a car is holding a cup of coffee. The cup has an inside diameter of 7.8 cm and an internal height of 8 cm. The cup is filled with liquid to a height of 7 cm. The car accelerates uniformly from rest until it reaches 40 mph. What is the maximum acceleration rate that can be attained without spilling coffee over the top of the cup if the passenger holds the cup level?

$\tan\theta = \frac{a}{g}$; $\tan\theta = \frac{1}{7.8/2} = 0.256 = \frac{1}{9.81}$ or $\boxed{a = 2.52 \text{ m/s}^2}$



2.87 At what rate of deceleration will the liquid level of Figure 2.26 just reach the top of the tank?

For liquid to spill over the front, equation of surface is

$\tan\theta = \dfrac{z_o}{x_o} = \dfrac{a}{g}$; $z_o = 1$ ft; $x_o = 2$ ft; so $\quad a = \dfrac{1}{2}g = 32.2/2$

$\boxed{a = 16.1 \text{ ft/s}^2}$

2.88 The liquid container of Figure 2.27 is rotating at 1.5 rev=s. Determine the shape of the liquid surface. The tank diameter is 1 ft; tank height is 18 in.; and the liquid depth when the tank is stationary is 6 in.

D = 1 ft; R = 0.5 ft; Equation of surface is

$z = \dfrac{r^2\omega^2}{2g} + z_p$; $\quad$ with $\quad \omega = 1.5 \text{ rev/s} \times 2\pi \text{ rad/rev} = 9.42 \text{ rad/s}$

$z_p + \dfrac{R^2\omega^2}{4g} = 6/12 = 0.5$ ft; $\quad z_p = 0.5 - \dfrac{(9.42)^2(0.5)^2}{4(32.2)} = 0.328$ ft

$z = \dfrac{r^2\,(9.42)^2}{2(32.2)} + 0.328$; $\boxed{z = 1.38\,r^2 + 0.328}$

2.89 The liquid acceleration experiment depicted in Figure 2.25 is to be run with glycerin. The tank measures 6 in. long and 4 in. high. Glycerine is poured in until its height is 2 in. The weight is allowed to fall freely. Owing to wheel and pulley friction, the car accelerates at only 0.7g. Determine the free surface of the glycerine.

$\tan\theta = \dfrac{z_o}{x_o} = \dfrac{a}{g} = 0.7$ given; $x_o = 3$ in.; $\quad z_o = \dfrac{a}{g}x_o$

$z_o = 0.7(3) = 2.1$ in.; $z_i = z_f + z_o = 2 + 2.1 = 4.1$ in.

The free surface is $\quad z = z_i - \dfrac{a}{g}x \quad$ or

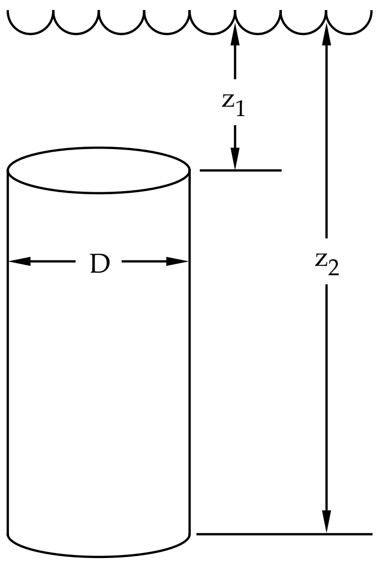
$\boxed{z = 4.1 - 0.7x \quad \text{in.}}$

2.90 Derive an equation for the buoyant force exerted on a submerged cylinder with its axis vertical.

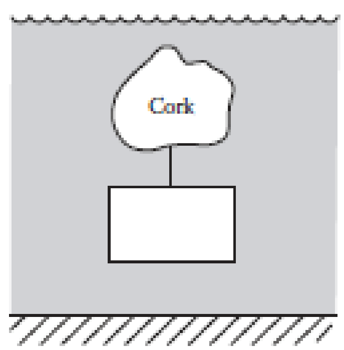
$$\text{upper surface } p_u = \rho g z_1;\ F_u = p_u A_u = \rho g z_1 \frac{\pi D^2}{4}$$

$$\text{lower surface } p_l = \rho g z_2;\ F_l = p_l A_l = \rho g z_2 \frac{\pi D^2}{4}\text{; by definition,}$$

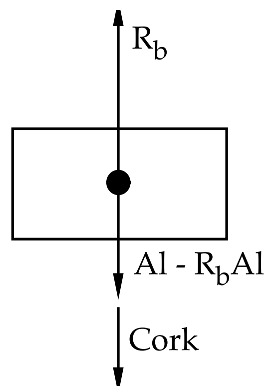
$$\boxed{R_b = p_l - p_u = \rho g \frac{\pi D^2}{4}(z_2 - z_1)} \text{ where } z_2 - z_1 = \text{length of cylinder}$$



2.91 A 1.2-ft^3 block of aluminum is tied to a piece of cork, as shown in Figure P2.91. What volume of cork is required to keep the aluminum from sinking in castor oil?



Free body diagram of cork; $R_b = \rho_a g \cancel{V}_a - \rho g \cancel{V}_a + \rho_c g \cancel{V}_c$



$\rho_a = 2.7(1.94)$ Al [Table A-8]; $\rho_c = 0.24(1.94)$ Cork [Table A-7]
$\cancel{V}_a = 1.2\ \text{ft}^3$ given; $R_b = \rho g \cancel{V}_c$ and substituting into & rearranging the FBD equation, $\rho g \cancel{V}_c = \rho_a g \cancel{V}_a - \rho g \cancel{V}_a + \rho_c g \cancel{V}_c$; or

$\rho \cancel{V}_c - \rho_c \cancel{V}_c = \rho_a \cancel{V}_a - \rho \cancel{V}_a$; or $\quad \cancel{V}_c = \dfrac{\rho_a \cancel{V}_a - \rho \cancel{V}_a}{\rho - \rho_c}$; for the fluid,

$\rho = 0.960(1.94)$ [Table A-5] $\cancel{V}_c = \dfrac{\cancel{V}_a(\rho_a - \rho)}{\rho - \rho_c}$; the 1.94's cancel so

$$\cancel{V}_c = \frac{1.2(2.70 - 0.960)}{0.960 - 0.24} = \boxed{2.9\ \text{ft}^3 = \cancel{V}_c}$$

2.92 What percentage of total volume of an ice cube will be submerged when the ice cube is floating in water?

For ice, $\rho_i = 0.917(1\,000)$ from Table A-7. Weight = Buoyant Force;
$\rho_i g \cancel{V}_i = \rho g \cancel{V}_s$; where $\cancel{V}_i$ = total volume of ice and $\cancel{V}_s$ = submerged volume; so

$$\frac{\cancel{V}_s}{\cancel{V}_i} = \frac{\rho_i}{\rho} = \frac{0.917(1\,000)}{1\,000} = 0.917 \text{ or } \boxed{91.7\% \text{ of total volume is submerged}}$$

2.93 A copper cylinder of diameter 4 cm and length 15 cm weighs only 14.5 N when submerged in liquid. Determine the liquid density.

Cylinder volume $\cancel{V}_c = \pi D^2 L/4 = \pi(0.04)^2(0.15)/4 = 0.000\,188\ \text{m}^3$;
$\rho_c = 8.96(1\,000)$ from Table A-8;
Weight of cylinder $= \rho_c g \cancel{V}_c = 8.96(1\,000)(9.81)(0.000\,188) = 16.52$ N;
Buoyant force $= R_b$ = weight in air - weight submerged
$R_b = 16.6 - 14.5 = 2.02$ N; also, $R_b = \rho g \cancel{V}_c$ so

$$\rho = \frac{R_b}{g\cancel{V}_c} = \frac{2.02}{9.81(0.000\,188)} \text{ or } \boxed{\rho = 1\,095\ \text{kg/m}^3}$$

2.94 A cylinder 8 cm in diameter is filled to a depth of 30 cm with liquid. A cylindrical piece of aluminum is submerged in the liquid, and the level rises 4 cm. The weight of the aluminum when submerged is 2.5 N. Determine the liquid density.

$D = 0.08$ m; $\Delta L = 0.04$ m; $\cancel{V}_a$ = volume of aluminum

$\cancel{V}_a = \dfrac{\pi D^2 L}{4} = \dfrac{\pi(0.08)^2(0.04)}{4} = 0.000\,201\ \text{m}^3$; R_b = wt in air - wt submerged;

so $\rho_a g \cancel{V}_a - 2.5 \text{ N} = \rho g \cancel{V}_a$; $\rho = \dfrac{\rho_a g \cancel{V}_a - 2.5}{g \cancel{V}_a} = \rho_a - \dfrac{2.5}{g \cancel{V}_a}$

$\rho_a = 2.7(1\,000) = 2\,700 \text{ kg/m}^3 =$; $\rho = 2\,700 - \dfrac{2.5}{9.81(0.000\,201)}$

$\boxed{\rho = 1\,432 \text{ kg/m}^3}$

2.95 A submerged steel spherical buoy weighs 220 lbf out of water. A chain anchors the center of the buoy at a depth of 30 in. below the surface of the water (salt water of density 1.99 slug/ft^3). Calculate the tension in the chain.

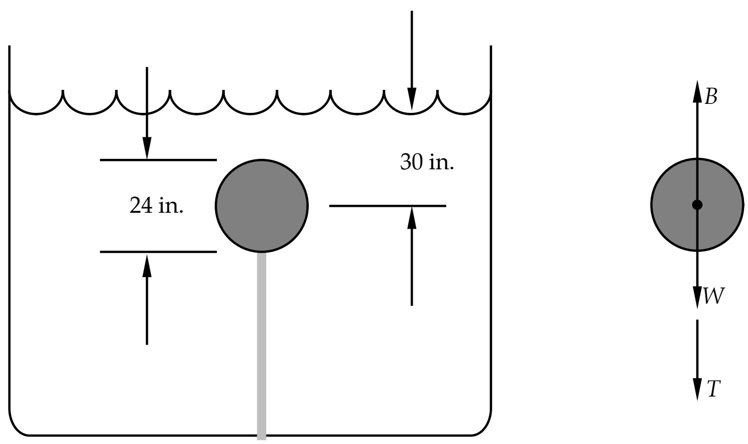


FIGURE P2.95

Perform a force balance on the sphere that includes the tension T, the buoyant force B, and the weight W. The buoyant force is found as

$$B = \rho g, \cancel{V} = 1.99(32.2)\left(\frac{4}{3}\pi R^3\right) = 268 \text{ lbf}$$

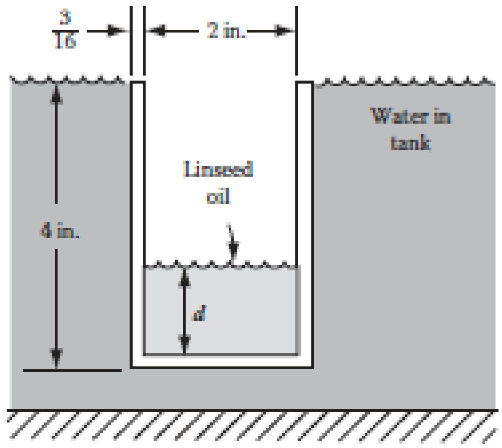
The tension is found by summing forces

$$T = B - W = 268 - 220$$

$$\boxed{T = 48 \text{ lbf}}$$

Depth of submergence has no effect.

2.96 A glass is filled to a depth d with linseed oil and allowed to float in a tank of water. To what depth can the glass be filled with ethyl alcohol such that the tank water will just reach the brim of the glass? (See Figure P2.96).



FBD of system gives: weight glass + weight oil = R_b(glass + oil)

Weight of glass = $\rho_g g \cancel{V}_g$; $\cancel{V}_g = AL|_{walls} + At|_{bottom}$

$A_{wall} = \pi[(1^3/_{16})^2 - (1)^2] = 1.29 \text{ in.}^2 = 0.00895 \text{ ft}^2$;

$\cancel{V}_g = 0.00895(4/12) + \pi(1/12)^2(3/16)(1/12) = 0.00332 \text{ ft}^3$

$\rho_g = 2.6(1.94)$ slug/ft^3 from Table A-7.

Weight of glass = 2.6(1.94)(32.2)(0.00332) = <u>0.54 lbf</u>

Weight of oil = $\rho_o g \cancel{V}_o$; $\rho_o = 0.787(1.94)$; $\cancel{V}_o = \pi(1/12)^2 d$

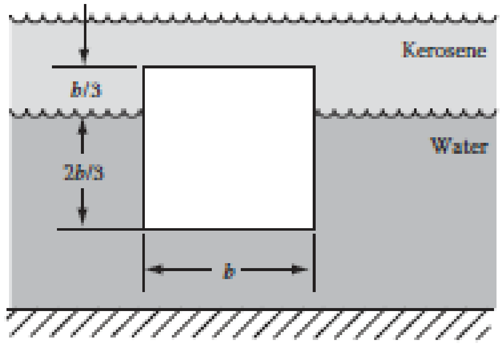
Weight of oil = $0.787(1.94)(32.2)\pi(1/12)^2 d = 1.07d$

$R_b = \rho g \cancel{V}_s = 1.94(32.2)\pi(1^3/_{16})^2(1/12)^2(4/12) =$ <u>0.641 lbf</u>

Substituting, $0.54 + 1.07d = 0.641$; $d = \dfrac{0.641 - 0.54}{1.07}$ or

$\boxed{d = 0.094 \text{ ft} = 1.13 \text{ in.}}$

2.97 A cube of material is 4 in. on a side and floats at the interface between kerosene and water, as shown in Figure P2.97. Find the specific gravity of the material.



Top surface $F_u = p_u A = \rho g z_1 b^2$; ρ = that for kerosene; z_1 = depth to top of cube from free surface;
Lower surface $F_l = p_l A = [\rho g(z_1 + b/3) + \rho_{H2O} g(2b/3)]b^2$
Weight = $\rho_c g \cancel{V}_c$; where $\cancel{V}_c$ = cube volume. Free body diagram,
R_b = Weight; substituting,
$[\rho g(z_1 + b/3) + \rho_{H2O} g(2b/3)]b^2 - \rho g z_1 b^2 = \rho_c g b^3$

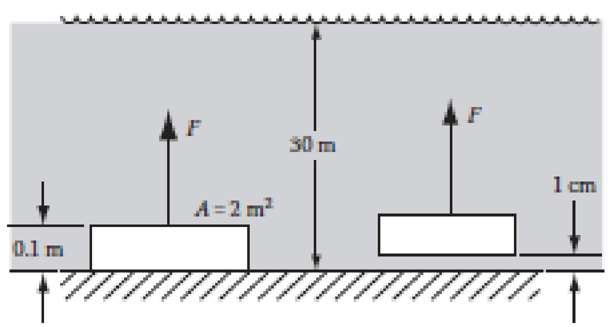
$\rho z_1 + \frac{\rho b}{3} + \rho_{H2O}\frac{2b}{3} - \rho z_1 = \rho_c b$; $\frac{\rho}{3} + \frac{2\rho_{H2O}}{3} = \rho_c$; Divide by ρ_{H2O}

$\frac{1}{3}\frac{\rho}{\rho_{H2O}} + \frac{2}{3} = \frac{\rho_c}{\rho_{H2O}}$; using specific gravity, s, notation;

$\frac{s}{3} + \frac{2}{3} = s_c$; for kerosene, s = 0.823; so $s_c = \frac{0.823}{3} + \frac{2}{3}$ or

$\boxed{s = 0.941}$ regardless of dimensions of cube

2.98 A block of tin of cross-sectional area 2 m^2 is resting on the flat bottomof a tank, as in Figure P2.98. No water can get beneath the block, which is only 0.1 m high. The water depth is 30 m.
a. Determine the force required to lift the block just off the bottom.
b. When the block has been raised just 1 cm and the water can get beneath it, determine the force required to lift the block further.



(a) No force on lower surface; only forces acting are force on upper surface and weight of tin. Thus
$F = p_u A_u + Wt = \rho_{H2O} g(30 - 1)(2) + \rho_t g \cancel{V}_t$
F = 1 000(9.81)(29.9)(2) + 7.31(1 000)(9.81)(2)(0.1) = 586 kn + 14.3 kN

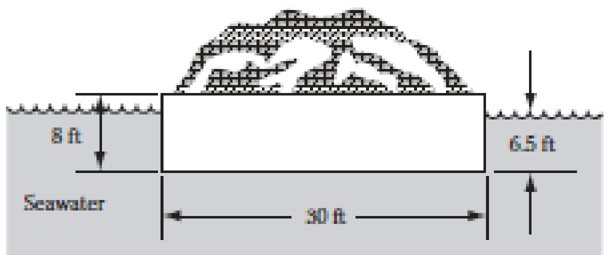
(a) $\boxed{F = 600 \text{ kN}}$

When lifted, water gets beneat and force on lower surface ≠ 0. Thus from FBD,

$F = Wt - R_b$; $R_b = \rho g \cancel{V} = 14.3 \text{ kN} - 1\,000(9.81)(2)(0.1)$ or

(b) $\boxed{F = 12.3 \text{ kN}}$

2.99 A flat barge carrying a load of dirt is sketched in Figure P2.99. The barge is 30 ft wide, 60 ft long, and 8 ft high. The barge is submerged 6.5 ft. Determine the weight of the dirt load if the weight of the barge is 400,000 lbf.



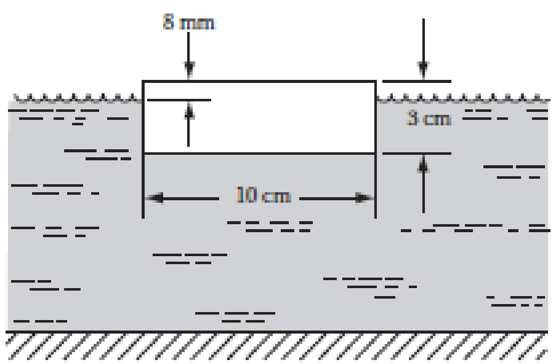
Wt dirt + Wt barge = $R_b = \rho g \cancel{V}_s$;

$R_b = 1.03(1.94)(32.2)(6.5)(30)(60) = 7.53 \times 10^5$ lbf

Wt dirt = $7.53 \times 10^5 - 4 \times 10^5$

$\boxed{\text{Wt dirt} = 3.53 \times 10^5 \text{ lbf}}$

2.100 A bar of soap of dimensions 10 cm long, 5 cm wide, and 3 cm tall is floating in a basin of water with 8 mm extending above the surface. If the water density is 997 kg=m3, determine the density of the soap. (See Figure P2.100.)



A free body diagram of the soap includes gravity and buoyant forces. Summing these

$\rho_o g \cancel{V} = \rho g \cancel{V}_s$;   The soap density then becomes

$$\rho_o = \frac{\rho \cancel{V}_s}{\cancel{V}} = \frac{997(0.1)(0.05)(0.022)}{0.1(0.05)(0.03)}; \quad \text{solving,}$$

$\boxed{\rho_o = 731 \text{ kg/m}^3}$

2.101 Rework Example 2.19 for a balsa log, whose specific gravity is 0.125.

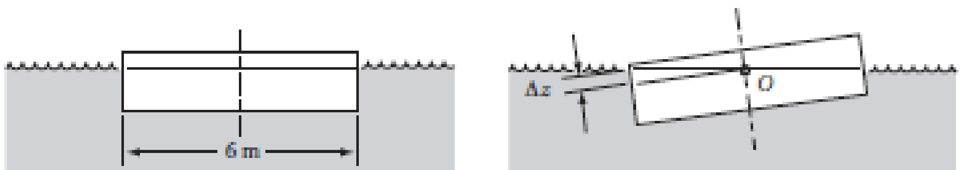
Balsa s = 0.125; B is 0.125L/2 from bottom; we have

$$GB = \frac{L}{2} - 0.125\,\frac{L}{2} = 0.438L;\ \frac{\pi D^4/64}{0.125\pi D^2 L/4} - 0.438L = 0;\ \text{then,}$$

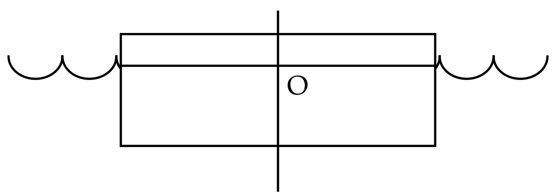
$$\frac{D^2}{16(0.125)} = 0.438L^2;\ L^2 = \frac{(0.3)^2}{16(0.125)(0.438)} = 0.103;\ \text{and,}$$

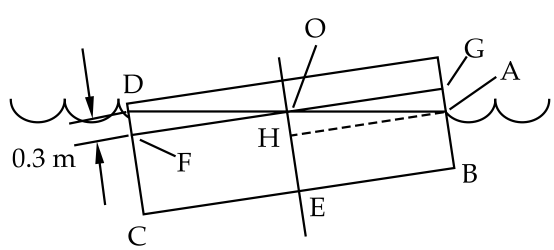
$$\boxed{L = 0.321\ \text{m} = 32.1\ \text{cm}}$$

2.102 Figure P2.102 shows a prismatic body, such as a barge, floating in water. The body is 6 m wide and 18 m long, and it weighs 1.96 MN. Its center of gravity is located 30 cm above the water surface. Determine the metacentric height if Dz is 30 cm.



b = 6 m; Δz = 30 cm = 0.3 m; L = 18 m; mg = 1 960 000 N;
OG = 0.3 m above surface; mg = $\rho g\cancel{V}$; 1 960 000 = 1 000(9.81)(6)(18)h
where h = depth of submergence; solving,
h = 1.85 m;





To find centroid in tipped position,

$$\bar{x}(ABCD) = \bar{x}_1(OEFC) + \bar{x}_2(OFD) + \bar{x}_3(OABE)$$

$$\bar{x}(1.85)(6) = \frac{1.85}{2}(1.85)(3) + (1.85 + \frac{0.3}{3})\,\frac{0.3(3)}{2}$$

$$+ \frac{1.85 - 0.3}{2}(3)(1.85 - 0.3) \qquad \text{(ABEH)}$$

$$+ (\frac{0.3}{3} + (1.85 - 0.3))\,\frac{0.3(3)}{2} \qquad \text{(HOA)}$$

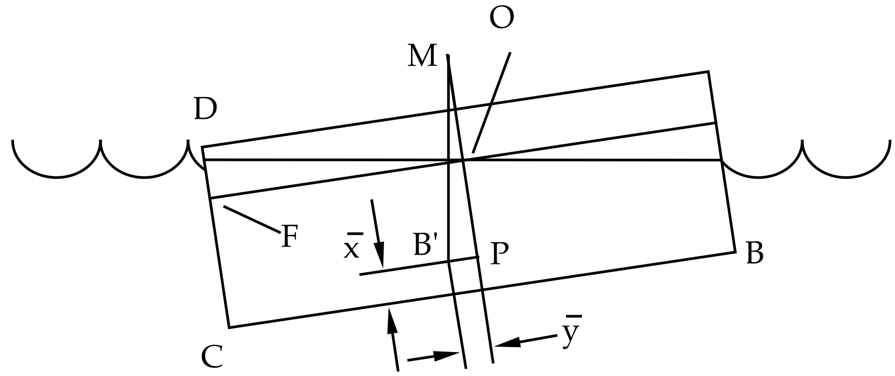
where $\bar{x}$ is measured from CB. Continuing, $11.1\bar{x} = 10.35$;

$\bar{x} = 0.933$ measured in direction of OE from CD

$$\bar{y}\,(1.85)(6) = \frac{3}{2}(1.85)(3) + \frac{2}{3}(3)\frac{0.3(3)}{2} - [+ (1.85 - 0.3)(3)\frac{3}{2} + \frac{1}{3}(3)\frac{0.3(3)}{2}]$$

where $\bar{y}$ is measured from OE; solving,

$\bar{y} = 0.162$ m to left of OE



Similar triangles, $\frac{B'P}{FD} = \frac{MP}{FO}$; $B'P = \bar{y} = 0.162$; FD = 0.3; FO = 3 m; so

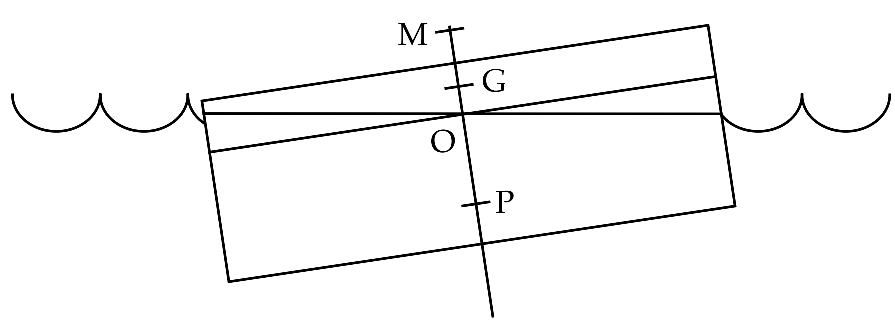
$MP = \frac{3(0.162)}{0.3} = 1.62$ m. So, as measured from BC,

$\boxed{M = \bar{x} + MP = 2.533 \text{ m}}$

2.103 Figure P2.103 shows a prismatic body, such as a barge, floating in water. The body is 6 m wide and 18 m long, and it weighs 1.96 MN. Its center of gravity is located 30 cm above the water surface. Determine the restoring couple if Dz is 30 cm.

From the above, OG = 0.3; MG = MP - GP or

$MG = MO - OG = \underbrace{(2.533 - 1.85)}_{(MO)} - \underbrace{0.3}_{(OG)} = 0.403$; positive = stable.



$$\text{Righting moment} = (mg)MG \sin\theta = 1\,960\,000(0.403)\frac{FD}{OD}$$

$$= 7.9 \times 10^5 \frac{FD}{\sqrt{FD^2 + OF^3}}$$

$$= 7.9 \times 10^5 \frac{0.3}{\sqrt{0.3^2 + 3^2}} \text{; solving,}$$

$\boxed{\text{Righting moment} = 7.86 \times 10^4 \text{ N·m}}$
