Chapter 2

Greedy Algorithms and Dynamic Programming

Knapsack Problems and Investment Decisions

1. (a) The Greedy Algorithm selects in turn items A, D, C with weight 26 and total value 66.

   (b) It is possible to do better. For example, items A, B, D, F have weight 27 and give the larger value 67.

2. (a) The Greedy Algorithm selects in turn items A, B, E with weight 25 and total value 46.

   (b) It is possible to do better. For example, items A, C, D have weight 25 and give the larger value 47.

3. (a) The Greedy Algorithm selects in turn investment options E and C, using \$50,000 and giving a total return of \$8300.

   (b) The table of all selections using \$50,000 is shown below:

A(\$10,000)	B(\$10,000)	C(\$20,000)	D(\$20,000)	E(\$30,000)	Total Return
✓		✓	✓		\$8400
	✓	✓	✓		\$7600
		✓		✓	\$8300
			✓	✓	\$8200
✓	✓			✓	\$8500

   (c) Selecting investments A, B, E gives the maximum return \$8500.

4. (a) The Greedy Algorithm selects in turn projects E and D, requiring an investment of \$35,000 and producing a total benefit of \$55,000.

   (b) The table of all selections using \$35,000 is shown next:

$A(\$5000)$	$B(\$10{,}000)$	$C(\$10{,}000)$	$D(\$15{,}000)$	$E(\$20{,}000)$	Total Benefit
✓	✓			✓	\$58,000
✓		✓		✓	\$60,000
	✓	✓	✓		\$57,000
			✓	✓	\$55,000

The optimal selection is A, C, E which gives the maximum benefit of \$60,000.

5. (a) The Greedy Algorithm selects selects in turn improvements A, B, E, with a cost of \$24,500 and a total benefit of \$34,000.

   (b) Notice that two improvements yield a benefit of at most \$29,000 and that any four improvements exceed the \$25,000 budget. So, we only need to consider options using three improvements, listed in the table below:

$A(\$12{,}000)$	$B(\$9000)$	$C(\$7600)$	$D(\$6800)$	$E(\$3500)$	Total Benefit
✓	✓			✓	\$34,000
✓		✓		✓	\$32,500
✓			✓	✓	\$31,000
	✓	✓	✓		\$34,500
	✓	✓		✓	\$29,500
	✓		✓	✓	\$28,000
		✓	✓	✓	\$26,500

   The optimal choice is to select improvements B, C, D yielding a total benefit of \$34,500.

6. (a) The horizontal edge from $(A, 5)$ to $(B, 5)$ means we do not choose investment A; the diagonal edge from $(B, 5)$ to $(C, 3)$ means we select investment B; the diagonal edge from $(C, 3)$ to $(D, 0)$ means we select investment C. This entirely uses up the given \$50,000 budget, so investments D and E are not selected. The value of this path is $\$0 + \$3400 + \$5400 + \$0 + \$0 = \$8800$.

   (b) The selection of A, C, D corresponds to the path $(A, 5) \to (B, 4) \to (C, 4) \to (D, 1) \to (E, 0) \to (F, 0)$. This path has value $\$2300 + \$0 + \$5400 + \$900 + \$0 = \$8600$.

7. Moving horizontally from $(C, 4)$ means we do not choose investment C and we have \$40,000 left to invest in (possibly) D and E. But D and E together only use up $\$10{,}000 + \$20{,}000 = \$30{,}000$. So there is no way to fully invest the \$50,000.

Greedy Algorithms and Change-Making Problems

8. Using only two coins, it is not possible to make change for 22 cents: with two coins, we can only make change for 32, 25, 21, 18, 17, 14, 10, 6, and 2 cents.

So at least three coins are needed. Since the greedy solution $22 = 16 + 5 + 1$ uses just three coins, it is optimal.

9. If we do not use the 16-cent coin, then the largest amount of change that can be made with three coins is $9 + 9 + 9 = 27$. Thus, we need to use at least one 16-cent coin. However, this leaves $42 - 16 = 26$ cents. It is not possible to make change for 26 cents using only two coins as we can only make change for 32, 25, 21, 18, 17, 14, 10, 6, and 2 cents. This means that we must use at least three coins to make change for 26 cents, and hence at least four coins to make change for 42 cents. Since the Greedy Algorithm uses four coins, it must be optimal.

10. (a) The greedy solution selects one 25-cent coin, one 5-cent coin, and one 1-cent coin: $31 = 25 + 5 + 1$.

    (b) No, there is no solution using only two coins.

11. (a) The greedy solution selects one 25-cent coin, one 5-cent coin, and two 1-cent coins: $32 = 25 + 5 + 1 + 1$.

    (b) Yes, the solution $32 = 12 + 10 + 10$ uses just three coins: one 12-cent coin and two 10-cent coins.

12. (a) The greedy solution selects one 25-cent coin, one 5-cent coin, and four 1-cent coins: $34 = 25 + 5 + 1 + 1 + 1 + 1$.

    (b) Yes, the solution $34 = 12 + 12 + 10$ uses just three coins: two 12-cent coins and one 10-cent coin.

13. (a) The greedy solution selects two 25-cent coins, one 12-cent coin, and one 5-cent coin: $67 = 25 + 25 + 12 + 5$.

    (b) No, there is no solution using only three coins.

14. (a) The greedy solution selects two 12-cent coins, one 4-cent coin, and three 1-cent coins: $31 = 12 + 12 + 4 + 1 + 1 + 1$.

    (b) Yes, the solution $31 = 10 + 10 + 10 + 1$ uses just four coins: three 10-cent coins and one 1-cent coin.

15. (a) The greedy solution selects two 17-cent coins, one 7-cent coin, and one 1-cent coin: $42 = 17 + 17 + 7 + 1$

    (b) The greedy solution $42 = 17 + 17 + 7 + 1$ is optimal, and also the solution $42 = 17 + 12 + 12 + 1$ is optimal.

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16. Given the table below, we iteratively calculate the entries for $n = 8, \dots, 12$.

1	2	3	4	5	6	7	8	9	10	11	12
W	W	W	L	W	W	W					

($n = 8$) Taking 1, 2, 3 flags leaves us in states 7 (W), 6 (W), 5 (W) all winning; so 8 is a losing state.

($n = 9$) Taking 1, 2, 3 flags leaves us in states 8 (L), 7 (W), 6 (W); since 8 is a losing state, 9 is a winning state (by selecting 1 flag).

($n = 10$) Taking 1, 2, 3 flags leaves us in states 9 (W), 8 (L), 7 (W); since 8 is a losing state, 10 is a winning state (by selecting 2 flags).

($n = 11$) Taking 1, 2, 3 flags leaves us in states 10 (W), 9 (W), 8 (L); since 8 is a losing state, 11 is a winning state (by selecting 3 flags).

($n = 12$) Taking 1, 2, 3 flags leaves us in states 11 (W), 10 (W), 9 (W) all winning; so 12 is a losing state.

17. (a) The completed table is given below:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
W	L	W	W	W	W	L	W	L	W	W	W	W	L	W	L	W	W	W	W	L

(b) Since state 21 is a losing state it is better to go second, as any move of your opponent will leave you in a winning state (20, 18, or 17).

18. (a) The completed table is given below:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
W	W	L	W	W	L	W	W	L	W	W	L	W	W	L	W	W	L	W	W	L

(b) Since state 21 is a losing state it is better to go second, as any move of your opponent will leave you in a winning state (20, 19, or 17).

19. (a) The completed table is given below:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
W	L	W	W	L	W	L	W	W	L	W	L	W	W	L	W	L	W	W	L	W

(b) Since state 21 is a winning state it is better to go first, as you can place the opponent in the losing state 20 by selecting one flag.

20. (a) The completed table is given below:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
W	L	W	W	W	W	W	L	W	L	W	W	W	W	W	L	W	L	W	W	W

(b) Since state 21 is a winning state it is better to go first, as you can place the opponent in the losing state 16 by selecting five flags.

21. (a) The completed table is given below:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
W	W	L	W	W	W	L	W	W	L	W	W	W	L	W	W	L	W	W	W	L

(b) Since state 21 is a losing state it is better to go second, as any move of your opponent will leave you in a winning state (20, 19, or 15).

22. (a) The completed table is given below:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
W	L	W	L	W	W	W	W	W	W	L	W	L	W	L	W	W	W	W	W	W

(b) Since state 21 is a winning state it is better to go first, as you can place the opponent in the losing state 15 by selecting six flags.

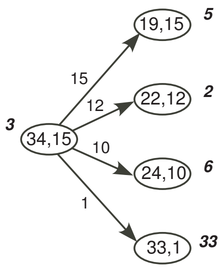
Dynamic Programming and Change-Making Problems

23. (a) It is not possible to make change for 29 cents with just two coins, using denominations 12, 10, 5, or 1. With these denominations the largest amount of change that can be made with two coins is $12+12=24$ cents.

    (b) It is not possible to make change for 31 cents with just three coins, using denominations 10, 5 or 1. With these denominations the largest amount of change that can be made with three coins is $10+10+10=30$ cents.

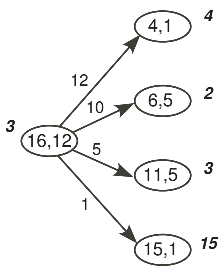
24. (a) The greedy solution selects two 15-cent coins and four 1-cent coins, for a total of six coins.

    (b) The initial vertex $(34, 15)$ is assigned the optimal value 3, obtained from subproblem $(22, 12)$ by using a 12-cent coin. Since subproblem $(22, 12)$ is optimally solved as $22 = 12 + 10$, the original problem only needs three coins: $34 = 12 + 12 + 10$.

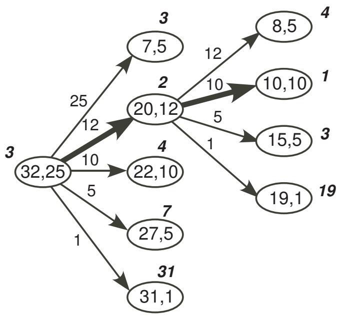


25. (a) The greedy solution selects one 12-cent coin and four 1-cent coins, for a total of five coins.

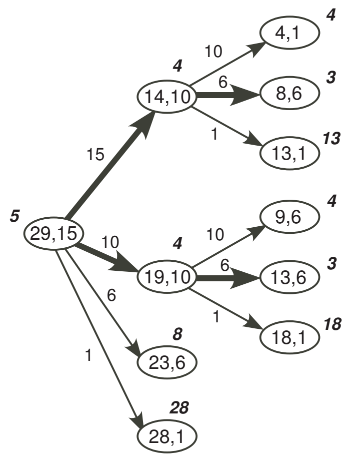
    (b) The initial vertex $(16, 12)$ is assigned the optimal value 3, obtained from subproblem $(6, 5)$ by using a 10-cent coin. Since subproblem $(6, 5)$ is optimally solved as $6 = 5 + 1$, the original problem only needs three coins: $16 = 10 + 5 + 1$.



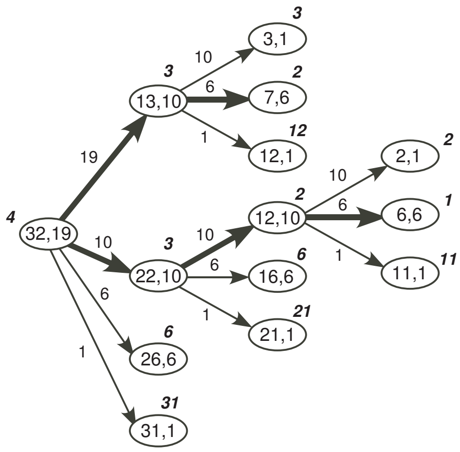
26. Note that a third stage is needed for subproblem $(20, 12)$, where the greedy solution is not optimal. The initial vertex $(32, 25)$ is assigned the optimal value 3, so only three coins are needed. The indicated path corresponds to the optimal solution $32 = 12 + 10 + 10$.



27. The initial vertex $(29, 15)$ is assigned the optimal value 5, so only five coins are needed. There are bolded edges extending to $(14, 10)$ and to $(19, 10)$, both having optimal value 4. The indicated paths correspond to the two optimal solutions $29 = 15 + 6 + 6 + 1 + 1$ and $29 = 10 + 6 + 6 + 6 + 1$.



28. The initial vertex $(32, 19)$ is assigned the optimal value 4, so only four coins are needed. There are bolded edges extending to $(13, 10)$ and to $(22, 10)$, both having optimal value 3. Notice that $(13, 10)$, $(22, 10)$, and $(12, 10)$ need to be expanded to another stage since the greedy solutions for these three vertices are not optimal. The indicated paths correspond to the two optimal solutions $32 = 19 + 6 + 6 + 1$ and $32 = 10 + 10 + 6 + 6$.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21,1
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26,6
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