Chapter 2: Errors in computer simulations

2.1.

514.023 – 514.02 = 0.011 → 1 significant digit

-0.04518 - -0.045113 = 6.7e-5 → 4 significant digits

23.4604 – 23.4213 = 0.0391 → 1 significant digit

2.2.

129 = 10000001

0.1 = 0.000110011…

0.2 = 0.00110011…

0.8125 – 0.1101

1111111111 = 1023

10101.101 = 21.625

0.101010101… = 0.6601

1.3

Estimate output error with Taylor series

$$f(x+\delta x) = f(x) + f'(x)\delta x$$

Or written differently:

$$\delta f = f(x+\delta x) - f(x) = f'(x)\delta x$$

We know:

$$f(x) = 1 - x^2$$

Thus:

$$f'(x) = -2x$$

The condition number is given as the ratio of error in the output and input:

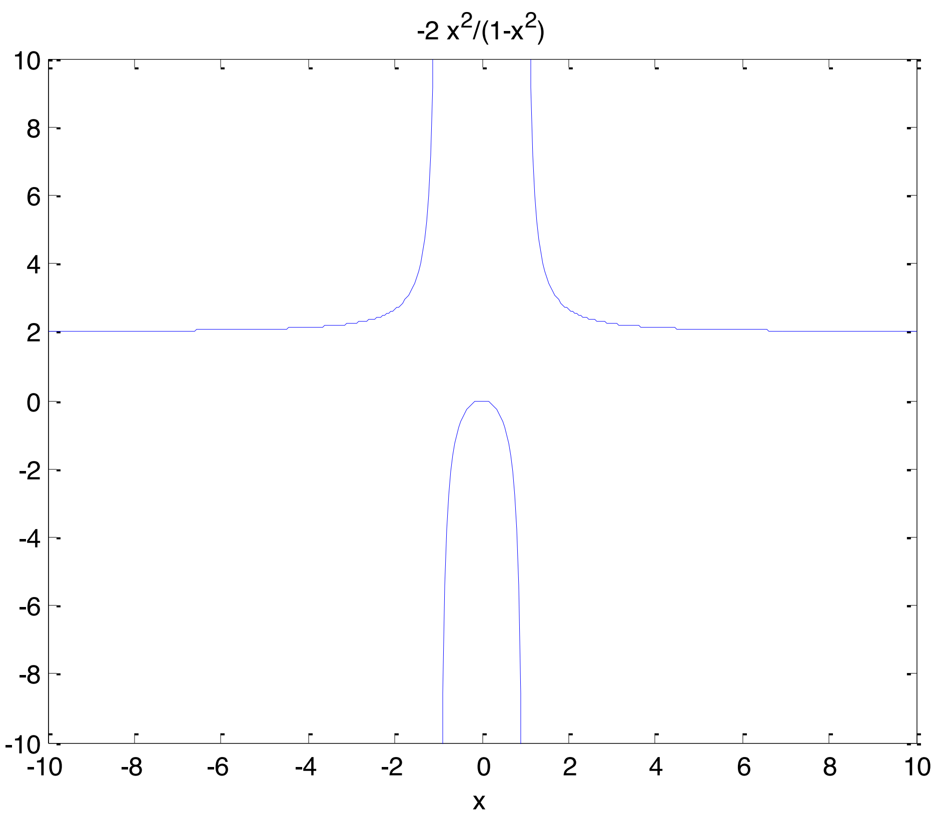
$$C = \max\left|\frac{\frac{\delta f}{f}}{\frac{\delta x}{x}}\right| = max\left|\frac{x\delta f}{f\delta x}\right| = max\left|\frac{xf'(x)}{f}\right| = max\left|\frac{-2x^2}{1-x^2}\right|$$

From this we can find that C<10 for:

$$-\sqrt{10/8} \leq x \leq \sqrt{10/8}$$

In MATLAB we can plot the condition number as function of x:

```
>> ezplot('-2*x^2/(1-x^2)',[-10 10 -10 10])
```



Chapter 3: Linear equations

3.1.a

$$C = \begin{bmatrix} 14 & -4 & -3 \\ -7 & 11 & 3 \\ 0 & -3 & 3 \end{bmatrix} \Rightarrow C^T = \begin{bmatrix} 14 & -7 & 0 \\ -4 & 11 & -3 \\ -3 & 3 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & -1 & 6 \end{bmatrix} \Rightarrow \det(A) = 21$$

$$A^{-1} = \frac{C^T}{det(A)} = \begin{bmatrix} 14/21 & -7/21 & 0 \\ -4/21 & 11/21 & -3/21 \\ -3/21 & 3/21 & 3/21 \end{bmatrix}$$

$$b = [4\ 3\ 1]^T$$

$$Ax = b \Rightarrow x = A^{-1}b$$

$$x = \begin{bmatrix} 14/21 & -7/21 & 0 \\ -4/21 & 11/21 & -3/21 \\ -3/21 & 3/21 & 3/21 \end{bmatrix} [4\ 3\ 1]^T = \begin{bmatrix} 4*14/21 & 3*-7/21 & 1*0 \\ 4*-4/21 & 3*11/21 & 1*-3/21 \\ 4*-3/21 & 3*3/21 & 1*3/21 \end{bmatrix} = [1.67\ 0.67\ 0]^T$$

3.1.b.

$\det(A) = 0$, this matrix is singular

Infinite number of solutions

Rank(A) <0 and rank(A) = rank(Aa)

n =3, rank(A) = 2, for b = [0 1 1], rank(Aa) = 2

No solution

Rank(A) < n

Rank(A)<rank(Aa)

N=3, rank(A) = 2, for b =[1 1 1], rank(Aa) = 3

3.2.

$$\det(M - I\lambda) = 0$$